1. P = h f n(f) Analysis for a Rectangular pulse

It is well known that $E = hf$ for a single photon of frequency f where h is Planck's constant and $P = hfn(f)$ where P is the **total power** of the signal of frequency f and $n(f)$ is the number of photons **per second** at that frequency. (link).]

Let us consider the **total power** P_T of a **sinc pulse** given by $g(t) = 2W sinc(2Wt)$ using Planck's relation $E = hf$. Its Fourier Transform is given by $G(f) = rect(\frac{f}{2W})$.

We know that **Power** of a signal at each frequency f is **proportional** to $|G(f)|^2$. For example, power of a sinusoidal signal $A\cos(\omega_0 t)$ is given by $\frac{A^2}{2}$.

If we want to **conserve power** between time domain and frequency domain, as per **Parseval's theorem**, given by $\int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df$, **then we require** $P(f)$ proportional to $|G(f)|^2$ and $n(f)$ proportional to $\frac{|G(f)|^2}{f}$, so that $P(f) = hfn(f)$ would **obey** Parseval's theorem.

1.1. P = h f n(f) Analysis for a Rectangular pulse in frequency domain

If $n(f)$ is proportional to $\frac{|G(f)|^2}{f}$, it means that $n(f) = \frac{|rect(\frac{f}{2W})|^2}{f}$ **becomes** ∞ at $f = 0$! Also, **total number of photons per second** in the pulse given by $n_T = 2\int_0^{\infty} n(f)df = \int_0^{W} \frac{1}{f} df = [\log f]_0^W = \log W - \log 0$ **becomes infinity**. This **cannot** be the case because $g(t) = 2W sinc(2Wt)$ has finite time window $2W$ in the time domain and total number of photons in a time limited pulse **cannot** be infinity.

Hence the **assumption** of the relation $E = hf$ and $P(f) = hfn(f)$ leads to the result that the **total number of photons per second** in a rectangular pulse is **infinity** which is **NOT possible**.

1.2. ***P = h f n(f) Analysis for a Triangular pulse***

Let us consider a Triangular pulse given by $g(t) = (\frac{T-t}{T})u(t) + (\frac{T+t}{T})u(-t)$. Its Fourier Transform is given by $G(f) = (T)^2 * sinc^2(fT)$ which is an **even function** of variable f. If $n(f)$ is proportional to $\frac{|G(f)|^2}{f}$, **total number of photons per second** in the triangular pulse given by $n_T = 2\int_0^\infty n(f)df = K\int_0^\infty \frac{|G(f)|^2}{f}df = [n_T(t)]_{t=0}$ where $n_T(t) = K\int_0^\infty \frac{|G(f)|^2}{f}e^{i\omega t}df = K\int_{-\infty}^\infty U_2(f)\frac{|G(f)|^2}{f}e^{i\omega t}df$ where unit-step function $U_2(f) = 1$ for $f >= 0$ and zero for $f < 0$ and its inverse fourier transform is $u_2(t) = \frac{-1}{i2\pi t} + \frac{\delta(t)}{2}$.

Hence $n_T(t) = K[u_2(t)\otimes b(t)]$ where $b(t)$ is the inverse fourier transform of $B(f) = \frac{A(f)}{f}$ where $A(f) = |G(f)|^2$, $\otimes$ denotes convolution operation $z(t) = x(t)\otimes y(t) = \int_{-\infty}^\infty x(\tau)y(t-\tau)d\tau$.

We can see that the fourier transform of **Heaviside unit step** function $u(t)$ is given by $U(f) = \frac{1}{i2\pi f} + \frac{\delta(f)}{2}$. We consider $h(t) = a(t)\otimes u(t) = \int_{-\infty}^t a(\tau)d\tau$. Fourier transform of $h(t) = a(t)\otimes u(t)$ is given by $H(f) = A(f)U(f) = \frac{A(f)}{i2\pi f} + \frac{A(f)\delta(f)}{2} = \frac{A(f)}{i2\pi f} + \frac{A(0)\delta(f)}{2}$. We want the Inverse Fourier Transform of the function $B(f) = \frac{A(f)}{f}$ which is given by $b(t) = i2\pi[h(t) - \frac{A(0)}{2}] = i2\pi[\int_{-\infty}^t a(\tau)d\tau - \frac{A(0)}{2}]$.

Hence $n_T(t) = K[u_2(t)\otimes b(t)]$ where $u_2(t) = \frac{-1}{i2\pi t} + \frac{\delta(t)}{2}$ and $b(t) = i2\pi[\int_{-\infty}^t a(\tau)d\tau - \frac{A(0)}{2}]$ and $A(f) = |G(f)|^2 = (T)^4 * \frac{\sin^4(\omega\frac{T}{2})}{(\omega\frac{T}{2})^4}$ and $a(t) = g(t)\otimes g(t)$. We see that $u_2(t)$ is **infinity** at $t = 0$.

We see that $n_T = [n_T(t)]_{t=0} = K[u_2(t)\otimes b(t)]_{t=0}$ is **infinity**! Which is **NOT possible**. Thus we have produced a **contradiction** if we assume that $P = hfn(f)$ and $n(f)$ is proportional to $\frac{|G(f)|^2}{f}$.

2. Millikan's Plot shows Photo-current increases with frequency in photo-electric effect, disproving textbooks!

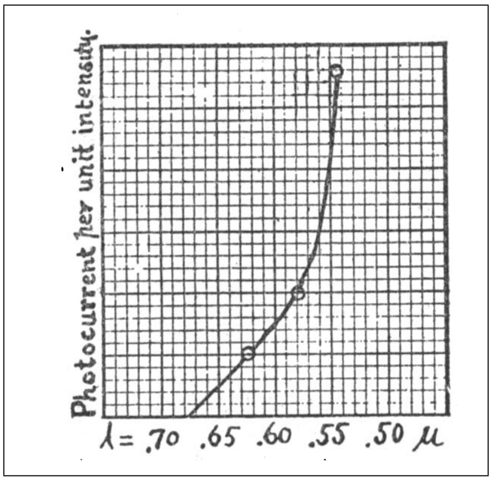

Figure 1:

Millikan's 1915 paper on photo-electric effect has a plot of **Photo-current** per unit intensity Vs **decreasing** wavelength of incident light in microns in Fig 7 in page 5. (link) That plot is reproduced in figure above. We can see that photo-current per unit intensity increases with **decreasing** wavelength, which means photo-current per unit intensity **increases** with **increasing** frequency of incident light, given that frequency $f = \frac{c}{\lambda}$ where c, λ are speed of light in vacuum and light wavelength respectively. This **disproves** statements in textbooks that "**photo-current is independent of the frequency of the incident radiation**".hence **photocurrent** is linearly proportional to incident **light intensity.**(link).

Photocurrent	Wavelength in microns	Frequency in 10^{15} Hz
1	0.625	0.48
2	0.58	0.517
5.6	0.54	0.555

We also know that there is a **one to one corespondence** between incident photons and emitted photo-electrons which make the photocurrent, hence **photocurrent** is linearly proportional to incident **light intensity.**(link).

We can summarize above observations as follows.

1) **Photo-current** per unit intensity **increases** with the frequency of incident light, for a fixed light intensity.

2) **Photocurrent** is **linearly proportional** to Incident light intensity $P(f)$, for a fixed light frequency.

3. Equivalence of Particle and Wave Energy

Let us consider a particle P of rest mass m_0 travelling with a velocity v. According to Einstein's mass-energy equivalence principle,its energy is given by $E = mc^2 = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$ where m is the relativistic mass of the particle. According to De-Broglie, every particle has a wave associated with it of wavelength $\lambda = \frac{h}{mv}$ where $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$.

Given that both the particle and the wave **must travel at the same velocity**, it turns out that wave group velocity equals particle velocity $v_g = v$. We can write $\lambda = \frac{v_p}{f}$ where v_p is the wave phase velocity given by $v_p = \frac{c^2}{v}$. [See De-Broglie relations.]

$$\begin{aligned} \lambda = \frac{h}{mv} = \frac{v_p}{f} &= \frac{c^2}{vf} \\ hf &= mc^2 \\ m &= \frac{m_0}{\sqrt{(1-\frac{v^2}{c^2})}} \end{aligned} \tag{1}$$

This means total relativistic energy of the particle given by $E = mc^2$ should equal **energy of a single photon** hf ? Is the associated wave a **real wave made of photons** or an **unreal probabilistic** metaphysical wave similar to Copenhagen interpretation of quantum mechanics? **Does not make sense**!

[As the particle velocity v **increases** towards velocity c, particle mass "m" approaches **close to infinity**. Above equation $hf = mc^2$ means that **frequency** of associated particle wave f must increase towards **infinity** and $\lambda = \frac{v_p}{f}$ approaches zero! This means **wavelength of the particle wave**, which is given by the distance travelled by the wave in a single wave cycle must **approach zero**!]