Quantum Simulation and Noise Analysis of the Heisenberg Spin Chain

Project Overview

- Goal: simulate quantum systems and study the impact of noise
- **Schrödinger equation**
  - $|\psi(t)\rangle = e^{-iHt}\,|\psi(0)\rangle$
- **Time evolution operator**
  - $U(t) = e^{-iHt}$

Phase 1: Single-Qubit Rabi

Hamiltonian: $H_1 = \dfrac{\Omega}{2}\,\sigma_x$

Probability: $P_1(t) = \sin^2\left(\dfrac{\Omega t}{2}\right)$

Phase 2: Ising + Trotterization

Hamiltonian: $H_2 = J\,\sigma_{z_1}\sigma_{z_2} + \dfrac{\Omega}{2}(\sigma_{x_1} + \sigma_{x_2})$

Coupling term: $H_{ZZ} = J\,\sigma_{z_1}\sigma_{z_2}$

Local term: $H_{\text{local}} = \dfrac{\Omega}{2}(\sigma_{x_1} + \sigma_{x_2})$

1st-order Trotter: $\left(e^{-iH_{\text{local}}\Delta t}\,e^{-iH_{ZZ}\Delta t}\right)^n$

2nd-order Trotter: $\left(e^{-iH_{\text{local}}\Delta t/2}\,e^{-iH_{ZZ}\Delta t}\,e^{-iH_{\text{local}}\Delta t/2}\right)^n$

Phase 3: Heisenberg + VQE

Hamiltonian: $H_3 = J\sum_{\langle i,j\rangle}(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y + \sigma_i^z\sigma_j^z)$

Energy functional: $E(\theta) = \langle\psi(\theta)|H_3|\psi(\theta)\rangle$

Simulation Results & Noise Analysis