

Quantum Simulation and Noise Analysis of the Heisenberg Spin Chain

Project Overview

- ▶ Goal: simulate quantum systems and study the impact of noise
- ▶ **Schrödinger equation**
 - ▶ $|\psi(t)\rangle = e^{-iHt} |\psi(0)\rangle$
- ▶ **Time evolution operator**
 - ▶ $U(t) = e^{-iHt}$

Phase 1: Single-Qubit Rabi

Hamiltonian: $H_1 = \frac{\Omega}{2} \sigma_x$

Probability: $P_1(t) = \sin^2\left(\frac{\Omega t}{2}\right)$

Phase 2: Ising + Trotterization

Hamiltonian: $H_2 = J \sigma_{z_1} \sigma_{z_2} + \frac{\Omega}{2} (\sigma_{x_1} + \sigma_{x_2})$

Coupling term: $H_{ZZ} = J \sigma_{z_1} \sigma_{z_2}$

Local term: $H_{\text{local}} = \frac{\Omega}{2} (\sigma_{x_1} + \sigma_{x_2})$

1st-order Trotter: $\left(e^{-iH_{\text{local}}\Delta t} e^{-iH_{ZZ}\Delta t} \right)^n$

2nd-order Trotter: $\left(e^{-iH_{\text{local}}\Delta t/2} e^{-iH_{ZZ}\Delta t} e^{-iH_{\text{local}}\Delta t/2} \right)^n$

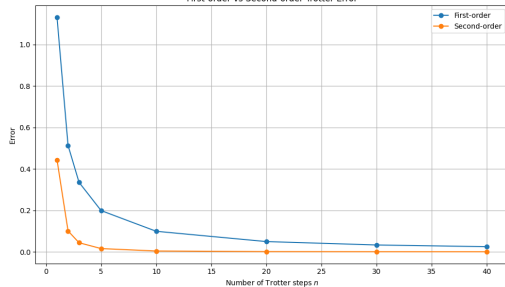
Phase 3: Heisenberg + VQE

Hamiltonian: $H_3 = J \sum_{\langle i,j \rangle} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y + \sigma_i^z \sigma_j^z)$

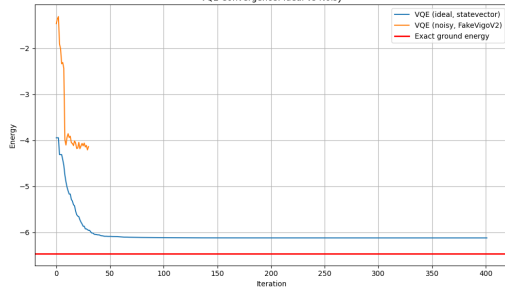
Energy functional: $E(\theta) = \langle \psi(\theta) | H_3 | \psi(\theta) \rangle$

Simulation Results & Noise Analysis

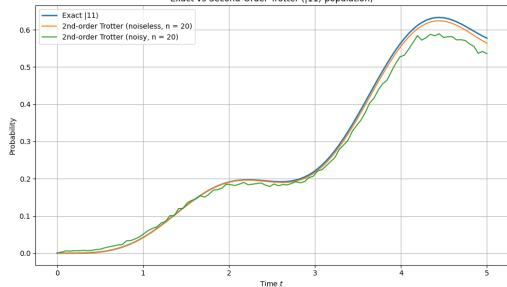
First-order vs Second-order Trotter Error



VQE Convergence: Ideal vs Noisy



Exact vs Second-Order Trotter ($|11\rangle$ population)



Variance of Noisy VQE Final Energies Across Independent Runs

