Вариант 1

#1

Размер кода не ясен, но должен быть больше 20...

#2

Двоичный код (3, 6) с образующим многочленом $g(x) = x^3 + x + 1$

Не является циклическим:

$$\frac{x^{6}-1}{x^{3}+x+1}=x^{3}+x+1+\frac{x^{2}}{x^{3}+x+1}$$
, так как не делится без остатка.

Значит он просто полиномиальный.

Кодирующая матрица (3x6) строится как сдвиг образующего многочлена [0, 0, 1, 0, 1, 1] влево, начиная с нижней строки и до верхней:

1	0	1	1	0	0
0	1	0	1	1	0
0	0	1	0	1	1

Построим множество кодовых слов:

0	0	[0,0,0,0,0,0]
1	$x^3 + x + 1$	[0,0,1,0,1,1]
x	$x^4 + x^2 + x$	[0,1,0,1,1,0]
x + 1	$x^4 + x^3 + x^2 + 1$	[0,1,1,1,0,1]
x^2	$x^5 + x^3 + x^2$	[1,0,1,1,0,0]
$x^2 + 1$	$x^5 + x^2 + x + 1$	[1,0,0,1,1,1]
$x^2 + x$	$x^5 + x^4 + x^3 + x$	[1,1,1,0,1,0]
$x^2 + x + 1$	$x^5 + x^4 + 1$	[1,1,0,0,0,1]

Найдем остатки от деления на g(x):

1	1	[0,0,1]
x	x	[0,1,0]
x^2	x^2	[1,0,0]
x^3	x + 1	[0,1,1]
x ⁴	$x^2 + x$	[1,1,0]

$ x^{5} x^{2} + x + 1 [1,1,1]$		x^5		
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Запишем проверочную матрицу, используя эти остатки (вертикально):

1	1	0	1	0	0
1	1	1	0	1	0
1	0	1	0	0	1

Транспонированная проверочная матрица должна выдавать нулевой вектор при умножении на правильное кодовое слово:

Например на $x^5 + x^4 + 1$.

В результате получается нулевой вектор по модулю 2.

Кодовые слова - векторы размера 6 с элементами из множества {0,1}. Их общее количество составляет 64, но при кодировании используются только 8 из них, которые образуют подгруппу. Построим смежные с ней классы, поэтапно прибавляя вектор из общей группы ко всем векторам данной подгруппы:

Итого имеется 8 смежных классов:

Номер класса	Что прибавляли	Состав класса
	[0, 0, 0, 0, 0, 0]	[0, 0, 0, 0, 0, 0]
	[0, 0, 1, 0, 1, 1]	[0, 0, 1, 0, 1, 1]
	[0, 1, 0, 1, 1, 0]	[0, 1, 0, 1, 1, 0]
1	[0, 1, 1, 1, 0, 1]	[0, 1, 1, 1, 0, 1]
ı	[1, 0, 0, 1, 1, 1]	[1, 0, 1, 1, 0, 0]
	[1, 0, 1, 1, 0, 0]	[1, 0, 0, 1, 1, 1]
	[1, 1, 0, 0, 0, 1]	[1, 1, 1, 0, 1, 0]
	[1, 1, 1, 0, 1, 0]	[1, 1, 0, 0, 0, 1]
2	[0, 0, 0, 0, 0, 1]	[0, 0, 0, 0, 0, 1]

$ \begin{bmatrix} [0,0,1,0,1,0] & [0,0,1,0,1,0] \\ [0,1,0,1,1,1] & [0,1,0,1,1,1] \\ [0,1,1,1,0,0] & [0,1,1,1,0,0] \\ [1,0,0,1,1,0] & [1,0,1,1,0,1] \\ [1,0,1,1,0,1] & [1,0,0,1,1,0] \\ [1,1,1,0,0,0] & [1,1,1,0,0,0] \\ [1,1,1,0,0,0] & [1,1,1,0,0,0] \\ [0,0,0,0,1,0] & [0,0,0,0,1,0] \\ [0,0,1,0,0,1] & [0,0,1,0,0,1] \\ [0,0,1,0,1,0] & [0,0,1,0,0,1] \\ [0,1,0,1,0,1] & [1,0,0,1,0,0] \\ [0,1,1,1,1,1] & [1,0,1,1,1,1] \\ [1,0,0,1,0,1] & [1,0,0,1,0,1] \\ [1,1,0,0,1,0] & [1,1,0,0,1,1] \\ [0,0,0,0,0] & [0,1,0,0,0] \\ [0,1,1,1,1,0] & [1,0,0,1,0] \\ [0,1,1,1,1,0] & [0,0,0,0,1,1] \\ [0,0,0,0,0] & [0,0,1,0,0,0] \\ [0,1,1,1,1,0] & [0,1,0,1,0,1] \\ [0,1,1,1,1,0] & [1,0,0,1,0,0] \\ [0,1,1,1,1,1] & [1,0,0,1,0] \\ [1,0,0,1,0,0] & [1,1,1,0,0,1] \\ [1,0,0,1,0,0] & [1,1,1,0,0,1] \\ [1,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,0,0] & [0,1,0,0,0,1] \\ [1,0,0,0,1] & [1,0,0,0,1] \\ [1,0,0,0,1] & [1,1,0,0,0] \\ [1,0,1,0,0,1] & [1,0,0,0,1] \\ [1,0,0,0,1,1] & [0,0,0,1,1,1,0] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,1,0] \\ [0,1,1,0,0,0] & [0,1,1,0,0,1] \\ [0,1,1,0,0,0] & [1,0,0,0,1,1] \\ [0,1,0,0,0,1,0] & [0,0,0,1,1,0] \\ [1,0,1,0,0,1] & [1,0,0,0,1,1] \\ [1,0,0,0,1,0] & [1,0,0,0,1,1] \\ [1,0,1,0,0,1] & [1,0,0,0,1] \\ [1,0,0,0,1,0] & [1,0,0,0,1,1] \\ [1,0,0,0,1,0] & [1,0,0,0,1,1] \\ [1,0,0,0,1,0] & [1,0,0,0,1,1] \\ [1,0,0,0,1,0] & [1,0,0,0,1,1] \\ [1,0,1,0,0,0] & [1,1,1,1,1,1] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,0] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] & [1,1,1,1,0] \\ [1,1,1,1,1,1,0] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1,0] & [1,1,1,1,0] \\ [1,1,1,1,1,1,0] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [$			
$ \begin{bmatrix} [0,1,1,1,0,0] & [[0,1,1,1,0,0] \\ [1,0,0,1,1,0] & [1,0,1,1,0,1] \\ [1,0,1,1,0,1] & [1,0,0,1,1,0] \\ [1,1,1,0,0,0] & [1,1,1,0,0,0] \\ [1,1,1,0,1,1] & [1,1,0,0,0] \\ [0,0,0,0,1,0] & [0,0,0,0,1,0] \\ [0,0,1,0,0,1] & [0,0,1,0,0,1] \\ [0,1,1,1,1,1] & [0,1,1,1,1,1] \\ [1,0,0,1,0,1] & [1,0,0,1,0,0] \\ [1,1,1,0,0,1] & [1,0,0,1,0,0] \\ [1,1,1,0,0,1] & [1,0,0,1,0,0] \\ [1,1,1,0,0,1] & [1,1,0,0,1,1] \\ [0,0,0,0,0,1,1] & [0,0,0,0,1,1] \\ [0,0,0,0,0,1,1] & [0,0,0,0,1,1] \\ [0,0,1,0,0,0] & [0,0,1,0,0,0] \\ [0,1,1,1,1,0] & [1,0,0,1,0,1] \\ [0,1,1,1,1,0] & [1,0,0,1,0,1] \\ [1,0,0,1,0,0] & [1,0,0,1,0,1] \\ [1,0,0,1,0,0] & [1,0,0,1,0,0] \\ [1,1,1,0,0,1,0] & [1,1,1,0,0,1] \\ [1,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,1,1,0,0,1] & [0,1,1,0,0,1] \\ [1,0,0,0,1,0] & [0,1,1,0,0,1] \\ [1,0,1,0,0,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,0] & [1,1,0,1,0,0] \\ [1,0,0,0,1,0] & [0,0,0,1,0,1] \\ [1,0,0,0,1,0] & [0,0,0,1,0,1] \\ [1,0,0,0,1,0] & [0,0,0,1,0,1] \\ [1,0,0,0,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,0,0] & [0,0,0,1,0,1] \\ [0,0,0,1,0,0] & [0,0,0,1,0,1] \\ [0,0,0,1,0,0] & [0,1,1,0,0,0] \\ [1,0,0,0,1,0] & [1,0,0,0,1,0] \\ [1,0,0,0,1,0] & [1,0,0,0,1,0] \\ [1,0,0,0,1,0] & [1,0,0,0,1,0] \\ [1,0,0,0,1,0] & [1,0,0,0,1,0] \\ [1,0,0,0,1,0] & [1,0,0,0,1,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1$		[0, 0, 1, 0, 1, 0]	[0, 0, 1, 0, 1, 0]
$ \begin{bmatrix} [1,0,0,1,1,0] & [1,0,1,1,0,1] \\ [1,0,1,1,0,1] & [1,0,0,1,1,0] \\ [1,1,0,0,0,0] & [1,1,1,0,1,1] \\ [1,1,1,0,1,0] & [1,1,1,0,1,0] \\ [0,0,0,0,1,0] & [0,0,0,0,1,0] \\ [0,0,0,0,1,0] & [0,0,0,0,1,0] \\ [0,1,0,1,0,0] & [0,1,0,1,0,0] \\ [0,1,1,1,1,1] & [0,1,1,1,1,1] \\ [1,0,0,1,0,1] & [1,0,1,1,1,1] \\ [1,0,0,1,0,1] & [1,0,0,1,0,1] \\ [1,1,1,0,0,1,1] & [1,1,1,0,0,1] \\ [1,1,1,0,0,0] & [1,1,1,0,0,1] \\ [0,0,0,0,0,1] & [0,0,0,0,1,1] \\ [0,0,1,0,0,0] & [0,0,0,0,1,1] \\ [0,0,1,0,0,0] & [0,1,0,1,0,1] \\ [1,0,1,1,1,1] & [1,0,0,1,0,1] \\ [1,0,1,1,1,1] & [1,0,0,1,0] \\ [1,0,1,1,1,1] & [1,0,0,1,0] \\ [1,0,0,1,0,0] & [1,1,1,0,0,1] \\ [1,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,1,1,1] & [0,0,1,1,1,1] \\ [0,1,0,0,1,0] & [0,1,1,0,0,1] \\ [1,0,0,0,1,1] & [1,1,0,0,1] \\ [1,0,0,0,0,1,1] & [1,1,1,1,0,0] \\ [1,1,1,0,1,0,0] & [1,1,1,0,0,1] \\ [0,0,0,1,1,1,1] & [0,0,0,1,1] \\ [1,0,1,0,0,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,0] & [1,1,0,0,1] \\ [0,0,0,1,1,1] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0,0] & [0,1,1,0,0,1] \\ [0,0,0,1,1,1,0,0] & [0,1,1,0,0,1] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,0,0,1,1,1,0,0,0] & [1,1,0,0,0] \\ [1,1,0,0,0,1] & [1,0,0,0,1,0] \\ [1,0,0,0,1,0] & [1,0,1,0,0,1] \\ [1,0,0,0,1,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,0,0,1,0] \\ [1,1,0,1,0,0] & [1,1,1,1,1,1] \\ [1,1,0,1,0,0] & [1,1,1,1,1,1] \\ [1,1,0,1,0,0] & [1,1,1,0,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\$		[0, 1, 0, 1, 1, 1]	[0, 1, 0, 1, 1, 1]
$ \begin{bmatrix} [1,0,1,1,0,1] & [1,0,0,1,1,0] \\ [1,1,0,0,0,0] & [1,1,1,0,1,1] \\ [1,1,1,0,1,1] & [1,1,0,0,0] \\ [0,0,0,0,0,1] & [0,0,0,0,1,0] \\ [0,0,1,0,0,1] & [0,0,1,0,1,0] \\ [0,1,0,1,0,0] & [0,1,0,1,0,0] \\ [0,1,1,1,1,1] & [0,1,1,1,1,1] \\ [1,0,0,1,0,1] & [1,0,1,1,1,0] \\ [1,0,0,1,1,1] & [1,0,0,1,0,1] \\ [1,0,0,1,0,1] & [1,1,0,0,1,1] \\ [1,0,0,1,0,0] & [1,1,0,0,1,1] \\ [0,0,0,0,0,1,1] & [0,0,0,0,1,1] \\ [0,0,0,1,0,0] & [0,0,1,0,1,0] \\ [0,1,1,1,1,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,1,1,1] & [0,0,0,1,0,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [1,0,0,0,1,0] & [0,1,0,0,1] \\ [1,0,0,0,1,0] & [0,1,0,0,1] \\ [1,0,0,0,1,0] & [1,0,0,0,1] \\ [1,0,0,0,1,0] & [0,0,0,1,0,1] \\ [1,0,0,0,1,0] & [0,0,0,1,0,1] \\ [1,0,0,0,1,0] & [0,0,0,1,0,1] \\ [1,0,1,0,0,1] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,1,0,0,0] \\ [1,0,0,0,1,0] & [1,0,0,0,1] \\ [1,0,1,0,0,1] & [0,1,0,0,0] \\ [1,0,0,0,1,0] & [1,0,1,0,0] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,$		[0, 1, 1, 1, 0, 0]	[0, 1, 1, 1, 0, 0]
$ \begin{bmatrix} [1,1,0,0,0,0] & [1,1,1,0,1,1] \\ [1,1,1,0,1,1] & [1,1,0,0,0] \\ [0,0,0,0,1,0] & [0,0,0,0,1,0] \\ [0,0,1,0,0,1] & [0,0,1,0,0,1] \\ [0,1,0,1,0,0] & [0,1,0,1,0,0] \\ [0,1,1,1,1,1] & [0,1,1,1,1,1] \\ [1,0,0,1,0,1] & [1,0,1,1,1,1] \\ [1,0,0,1,0,1] & [1,0,0,1,0,1] \\ [1,0,1,1,1,0] & [1,0,0,1,0,1] \\ [1,1,1,0,0,1,1] & [1,1,1,0,0,1] \\ [0,0,0,0,0,1,1] & [0,0,0,0,1,1] \\ [0,0,1,0,0,0] & [0,0,1,0,0,0] \\ [0,1,1,1,1,0] & [0,1,1,1,1,0] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,1,0,0,1,0] \\ [1,1,1,0,0,1,0] & [1,1,0,0,1,0] \\ [0,0,0,1,1,1,1] & [0,0,1,0,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,1,1,1] & [0,1,0,0,1] \\ [1,0,0,0,1,0] & [0,1,1,0,0,1] \\ [1,0,0,0,1,0] & [0,1,1,0,0,1] \\ [1,0,0,0,1,1] & [1,0,1,0,0,1] \\ [1,0,0,0,1,1] & [0,0,1,1,1,0] \\ [0,0,0,1,1,1,0] & [0,0,1,1,1,0] \\ [0,1,1,0,0,1] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,1,1,1,0] \\ [0,1,1,0,0,1] & [0,1,0,0,0] \\ [1,0,0,0,1,0] & [0,1,1,0,0,0] \\ [1,0,0,0,1,0] & [1,0,0,0,1] \\ [1,0,0,0,1,0] & [1,0,0,0,1] \\ [1,0,0,0,1,0] & [1,0,0,0,1] \\ [1,0,0,0,1,0] & [1,0,0,0,1] \\ [1,0,0,0,1,0] & [1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,$		[1, 0, 0, 1, 1, 0]	[1, 0, 1, 1, 0, 1]
$ \begin{bmatrix} [1,1,1,0,1,1] & [1,1,0,0,0,0] \\ [0,0,0,0,1,0] & [0,0,0,0,1,0] \\ [0,0,1,0,0,1] & [0,0,1,0,0,1] \\ [0,1,0,1,0,0] & [0,1,0,1,0,0] \\ [0,1,1,1,1,1] & [0,1,1,1,1,1] \\ [1,0,0,1,0,1] & [1,0,1,1,1,0] \\ [1,1,1,0,0,1,1] & [1,0,0,1,0,1] \\ [1,1,1,0,0,0] & [1,1,1,0,0,1] \\ [1,1,1,0,0,0] & [1,1,1,0,0,1,1] \\ [0,0,0,0,0,1,1] & [0,0,0,0,1,1] \\ [0,0,1,0,1,0,1] & [0,0,0,0,1,1] \\ [0,0,1,0,1,0,1] & [0,1,0,1,0,1] \\ [0,1,0,1,0,1] & [0,1,1,1,1,0] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,1,1,0,0,1] \\ [1,1,1,0,0,1] & [1,1,0,0,1,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,1,1,1,1] & [0,1,0,0,1,0] \\ [0,0,1,1,1,1] & [0,1,0,0,1,0] \\ [0,0,1,1,0,0,1] & [0,1,0,0,1,0] \\ [0,1,0,0,1,0] & [0,1,0,0,1,0] \\ [1,0,1,0,0,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,0] & [0,0,0,1,0,1] \\ [0,0,1,1,0,0,1] & [0,0,0,1,1,1,0] \\ [0,0,1,1,0,0,1] & [0,0,0,1,1,1,0] \\ [0,0,1,1,0,0,0] & [0,1,0,0,1,1] \\ [0,0,0,1,0,0] & [0,1,0,0,0,1] \\ [1,0,1,0,0,1] & [0,0,0,1,1,1,0] \\ [0,1,0,0,0,1,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,0] & [1,0,1,0,0,1] \\ [1,1,0,1,0,0] & [1,1,1,1,1,1] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,$		[1, 0, 1, 1, 0, 1]	[1, 0, 0, 1, 1, 0]
$ \begin{bmatrix} [0,0,0,0,1,0] & [0,0,0,0,1,0] \\ [0,0,1,0,0,1] & [0,0,1,0,0,1] \\ [0,1,0,1,0,0] & [0,1,0,1,0,0] \\ [0,1,1,1,1,1] & [0,1,1,1,1] \\ [1,0,0,1,0,1] & [1,0,1,1,1,0] \\ [1,0,1,1,1,0] & [1,0,0,1,0,1] \\ [1,1,1,0,0,1] & [1,0,0,1,0,1] \\ [1,1,1,0,0,0] & [1,1,1,0,0,0] \\ [1,1,1,0,0,0] & [1,1,0,0,1,1] \\ [0,0,0,0,0,1,1] & [0,0,0,0,1,1] \\ [0,0,1,0,1,0,1] & [0,1,0,1,0,1] \\ [0,1,1,1,1,0] & [0,1,1,1,1,0] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,1,1,0,0,1] \\ [1,1,1,0,0,1] & [1,1,0,0,1,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,1,1,1] & [0,0,1,0,0,1,0] \\ [0,1,1,0,0,1] & [1,0,0,0,1,0] \\ [0,1,1,0,0,1] & [1,0,0,0,1,1] \\ [1,0,0,0,1,1] & [1,0,0,0,1,1] \\ [1,0,0,0,1,1] & [1,0,0,0,1,0] \\ [0,0,0,1,1,1,0] & [1,1,1,1,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,1,0,0,1,1] & [0,0,0,1,0,1] \\ [0,1,0,0,1,1] & [0,0,0,1,0,1] \\ [0,1,0,0,1,1] & [0,0,0,1,0,1] \\ [0,1,0,0,0,1,0] & [1,0,0,0,1,1] \\ [1,0,1,0,0,0] & [1,0,0,0,1,0] \\ [1,0,1,0,0,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,1] \\ [1,1,1,1,1,1,1]$		[1, 1, 0, 0, 0, 0]	[1, 1, 1, 0, 1, 1]
$ \begin{bmatrix} [0,0,1,0,0,1] & [0,0,1,0,0,1] \\ [0,1,0,1,0,0] & [0,1,0,1,0,0] \\ [0,1,1,1,1,1] & [0,1,1,1,1,1] \\ [1,0,0,1,0,1] & [1,0,1,1,1,0] \\ [1,0,1,1,1,0] & [1,0,0,1,0,1] \\ [1,1,1,0,0,1] & [1,1,1,0,0,0] \\ [1,1,1,0,0,1] & [1,1,1,0,0,1] \\ [0,0,0,0,0,1,1] & [0,0,0,0,1,1] \\ [0,0,1,0,0,0] & [0,0,1,0,0,0] \\ [0,1,0,1,0,0] & [0,1,1,1,1,0] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,1,1,0,0,1] \\ [1,1,0,0,1,0] & [1,1,1,0,0,1] \\ [1,1,1,0,0,1] & [1,1,0,0,1,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,1,1,1] & [0,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,0,1,0,0] \\ [0,1,1,0,0,1] & [1,0,0,1,0,0] \\ [1,0,1,0,0,0] & [1,0,0,1,1] \\ [1,0,1,0,0,0] & [1,0,0,1,0] \\ [1,1,1,1,1,0] & [1,1,1,1,0] \\ [1,1,1,1,1,0] & [1,1,0,1,0,1] \\ [0,0,0,1,1] & [0,0,0,1,0] \\ [0,0,0,1,0,1] & [0,0,0,1,0,1] \\ [0,0,0,1,1] & [0,0,0,1,0] \\ [1,0,0,0,1,0] & [0,1,0,0,1,1] \\ [0,0,0,1,1] & [0,1,0,0,1,1] \\ [0,0,1,1,0,0,0] & [1,0,0,0,1,0] \\ [1,0,0,0,1,0] & [1,0,0,0,1,0] \\ [1,0,1,0,0,1] & [1,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,1,0,0,1] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] &$		[1, 1, 1, 0, 1, 1]	[1, 1, 0, 0, 0, 0]
3 [0, 1, 0, 1, 0, 0] [0, 1, 0, 1, 0, 0] [0, 1, 1, 1, 1, 1] [1, 0, 0, 1, 0, 1] [1, 0, 1, 1, 1, 0] [1, 0, 1, 1, 1, 0] [1, 0, 0, 1, 0, 1] [1, 1, 1, 0, 0, 0] [1, 1, 0, 0, 1, 0] [1, 1, 1, 0, 0, 0] [1, 1, 0, 0, 0, 0] [1, 1, 0, 0, 0, 0] [1, 1, 0, 0, 0, 0] [0, 0, 1, 0, 0, 0] [0, 0, 1, 0, 0, 0] [0, 1, 1, 1, 1, 0] [1, 0, 0, 1, 0, 0, 0] [1, 0, 1, 1, 1, 1] [1, 0, 0, 1, 0, 0] [1, 1, 1, 0, 0, 1] [1, 1, 1, 0, 0, 1] [1, 1, 1, 0, 0, 1, 0] [1, 1, 1, 0, 0, 1, 0] [1, 1, 1, 0, 0, 1, 0] [1, 1, 1, 0, 0, 1, 0] [1, 1, 1, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0, 0] [1, 0, 0, 1, 1, 1, 1, 1] [0, 1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0, 0] [1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 0, 0, 0, 1, 0] [0, 0, 0, 1, 0, 1] [0, 0, 0, 1, 0, 1] [0, 0, 0, 1, 0, 1] [0, 0, 0, 1, 0, 1] [0, 1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 0, 1, 0] [1, 0, 0, 0, 0, 1, 0] [1, 0, 0, 0, 0, 1, 0] [1, 0, 0, 0, 0, 1, 0] [1, 1, 1, 1, 1, 1, 1] [1, 1, 0, 1, 0, 0] [1, 1, 1, 1, 1, 1, 1] [1, 1, 0, 1, 0, 0] [1, 1, 1, 1, 1, 1, 1] [1, 1, 0, 1, 0, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 1, 1] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 1, 1] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1		[0, 0, 0, 0, 1, 0]	[0, 0, 0, 0, 1, 0]
3 [0, 1, 1, 1, 1, 1] [0, 1, 1, 1, 1, 1] [1, 0, 0, 1, 0, 1] [1, 0, 1, 1, 1, 0] [1, 0, 1, 1, 1, 0] [1, 0, 0, 1, 0, 1] [1, 1, 1, 0, 0, 0] [1, 1, 1, 0, 0, 0] [1, 1, 1, 0, 0, 0] [1, 1, 1, 0, 0, 0] [1, 1, 0, 0, 1, 1] [0, 0, 0, 0, 0, 1, 1] [0, 0, 0, 0, 0, 1, 1] [0, 0, 1, 0, 0, 0] [0, 1, 0, 1, 0, 0] [0, 1, 0, 1, 0, 0] [0, 1, 1, 1, 1, 0] [1, 0, 0, 1, 0, 0] [1, 0, 1, 1, 1, 1] [1, 0, 0, 1, 0, 0] [1, 1, 1, 0, 0, 1] [1, 1, 1, 0, 0, 1] [1, 1, 1, 0, 0, 1] [1, 1, 1, 0, 0, 1] [1, 1, 0, 0, 1, 0] [0, 0, 0, 1, 0, 0] [0, 0, 0, 1, 0, 0] [0, 1, 1, 1, 1, 1] [0, 1, 0, 0, 0, 1, 1] [1, 0, 1, 0, 0, 0, 1] [1, 1, 1, 1, 0, 0, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 0, 0, 0] [1, 0, 0, 0, 1, 1] [0, 0, 0, 1, 1, 1] [0, 0, 0, 1, 0, 1] [0, 0, 0, 1, 0, 1] [0, 0, 0, 1, 0, 1] [0, 0, 0, 1, 0, 1] [0, 0, 0, 1, 0, 1] [0, 0, 0, 1, 0, 1] [0, 0, 0, 1, 0, 1] [0, 0, 0, 1, 0, 1] [0, 1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0] [1, 0, 0, 0, 1, 0] [1, 1, 1, 1, 1, 1, 1] [1, 1, 0, 1, 0, 0] [1, 1, 1, 1, 1, 1, 1] [1, 1, 1, 1, 1, 1] [1, 1, 0, 1, 0, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1, 0] [1, 1, 1, 1, 1		[0, 0, 1, 0, 0, 1]	[0, 0, 1, 0, 0, 1]
$ \begin{bmatrix} 1,0,0,1,0,1 \\ [1,0,1,1,1,0] \\ [1,0,0,1,0,1] \\ [1,1,0,0,1,0,1] \\ [1,1,1,0,0,0] \\ [1,1,1,0,0,0] \\ [1,1,1,0,0,1,1] \\ [0,0,0,0,0,1,1] \\ [0,0,0,0,0,1,1] \\ [0,0,0,1,0,0,0] \\ [0,1,0,1,0,1] \\ [0,1,0,1,0,1] \\ [0,1,0,1,1,1,0] \\ [0,1,0,1,1,1,1] \\ [1,0,0,1,0,0] \\ [1,1,0,0,1,0,0] \\ [1,1,1,0,0,1] \\ [1,1,1,0,0,1] \\ [1,1,1,0,0,1] \\ [1,1,1,0,0,1] \\ [1,1,1,0,0,1] \\ [1,1,1,0,0,1] \\ [1,1,1,0,0,1] \\ [1,1,1,0,0,1] \\ [1,1,1,0,0,1] \\ [1,0,0,0,1,0,0] \\ [0,0,0,1,0,0] \\ [0,0,0,1,0,0] \\ [0,1,0,0,1,0] \\ [0,1,0,0,0,1] \\ [1,0,1,0,0,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,0,0,0,1,0] \\ [1,0,0,0,1,0] \\ [1,0,0,0,1,0] \\ [1,0,0,0,1,0] \\ [1,0,0,0,1,0] \\ [1,0,0,0,1,0] \\ [1,0,0,0,1,0] \\ [1,0,0,0,1,0] \\ [1,0,0,0,1,0] \\ [1,0,0,0,1,0] \\ [1,0,0,0,1,0] \\ [1,0,0,0,1,0] \\ [1,0,0,0,1,0] \\ [1,0,0,0,1,0] \\ [1,0,0,0,1,0] \\ [1,0,1,0,0,1] \\ [1,0,1,0,0,0] \\ [1,1,1,1,1,1] \\ [1,1,1,1,1,1] \\ [1,1,1,1,1,1] \\ [1,1,1,1,1,1] \\ [1,1,1,1,1,1] \\ [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] \\ $		[0, 1, 0, 1, 0, 0]	[0, 1, 0, 1, 0, 0]
$ \begin{bmatrix} [1,0,0,1,0,1] & [1,0,1,1,1,0] \\ [1,0,1,1,1,0] & [1,0,0,1,0,1] \\ [1,1,1,0,0,1] & [1,1,1,0,0,0] \\ [1,1,1,0,0,0] & [1,1,0,0,1,1] \\ [0,0,0,0,0,1,1] & [0,0,0,0,1,1] \\ [0,0,1,0,0,0] & [0,0,1,0,0,0] \\ [0,1,0,1,0,1] & [0,1,0,1,0,1] \\ [0,1,1,1,1,0] & [0,1,1,1,1,0] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,1] & [1,0,0,1,0,0] \\ [1,1,1,0,0,1] & [1,1,0,0,1] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,0,0] & [0,1,0,0,1,0] \\ [0,1,1,0,0,1] & [0,1,1,0,0,1] \\ [0,1,0,0,0,1] & [0,1,1,0,0,1] \\ [1,0,0,0,1,1] & [1,0,1,0,0,0] \\ [1,0,1,0,0,0] & [1,0,0,0,1,0] \\ [1,1,1,1,1,0] & [1,1,1,1,0] \\ [0,0,0,1,0,1] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,1,1,0,0,0] & [0,1,1,0,0,0] \\ [1,0,1,0,0,1] & [0,1,0,0,1,1] \\ [0,1,1,0,0,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,1,0,0,1] \\ [1,1,1,1,1,1] & [1,1,1,1,1,1] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,1] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,1] \\ [1,1,1,1,1,1,1] & [1,1,1,$	2	[0, 1, 1, 1, 1, 1]	[0, 1, 1, 1, 1, 1]
$ \begin{bmatrix} [1,1,0,0,1,1] & [1,1,1,0,0,0] \\ [1,1,1,0,0,0] & [1,1,0,0,1,1] \\ [0,0,0,0,0,1,1] & [0,0,0,0,1,1] \\ [0,0,1,0,0,0] & [0,0,1,0,0,0] \\ [0,1,0,1,0,1] & [0,1,0,1,0,1] \\ [0,1,1,1,1,0] & [0,1,1,1,1,0] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,1,0,0,1,0] & [1,1,1,0,0,1] \\ [1,1,1,0,0,1] & [1,1,0,0,1,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,1,1,1] & [0,0,0,1,1,1,1] \\ [0,1,0,0,1,0] & [0,1,0,0,1,0] \\ [0,1,1,0,0,1] & [0,1,1,0,0,1] \\ [1,0,0,0,1,1] & [1,0,1,0,0,0] \\ [1,0,1,0,0,0] & [1,0,0,0,1,1] \\ [1,1,1,1,1,0] & [1,1,1,1,1,0] \\ [1,1,1,1,1,0] & [1,1,0,1,0,1] \\ [0,0,0,1,0,1] & [0,0,0,1,0,1] \\ [0,0,0,1,0,1] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,0,0,0,1,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,1,0,0,1] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,1] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0,0] \\ [1,1,1,1,1,1,1,1] & [1,1,1,1,1,0,0] \\ [1,1,1,1,1,1,1,1] & [1,1,1,1,1,0,0] \\ [1,1,1,1,1,1,1,1] & [1,1,1,1,1,0,0] \\ [1,1,1,1,1,1,1,1] & [1,1,1,1,1,0,0] \\ [1,1,1,1,1,1,1,1] & [1,1,1,1,1,0,0] \\ [1,1,1,1,1,1,1,1,1] & [1,1,1,1,1,1,1] \\ [1,1,1,1,1,1,1,1,1] & [1,1,1,1,1,1,1] \\ [1,1,1,1,1,1,1,1,1,1] & [1,1,1,1,1,1,1] \\ [1,1,1,1,1,1,1,1,1] & [1,1,1,1,1,1,1] \\ [1,1,1,1,1,1,1,1,1,1] & [1,$	J	[1, 0, 0, 1, 0, 1]	[1, 0, 1, 1, 1, 0]
$ \begin{bmatrix} [1,1,1,0,0,0] & [1,1,0,0,1,1] \\ [0,0,0,0,1,1] & [0,0,0,0,1,1] \\ [0,0,1,0,0,0] & [0,0,1,0,0,0] \\ [0,1,0,1,0,1] & [0,1,0,1,0,1] \\ [0,1,1,1,1,0] & [0,1,1,1,1,0] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0] & [1,0,0,1,0,0] \\ [1,1,1,0,0,1,0] & [1,1,1,0,0,1] \\ [1,1,1,0,0,1] & [1,1,0,0,1,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,1,1,1] & [0,0,1,1,1,1] \\ [0,1,0,0,1,0] & [0,1,0,0,1,0] \\ [0,1,1,0,0,1] & [1,0,1,0,0,1] \\ [1,0,0,0,1,1] & [1,0,1,0,0,0] \\ [1,0,0,0,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,0] & [1,1,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,1,1,0] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,0,0,0,1,0] & [1,0,1,0,0,1] \\ [1,0,0,0,1,0] & [1,0,0,0,1,0] \\ [1,1,0,1,0,0] & [1,1,1,1,1,1] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0,0] \\ [1,1,1,1,1,1,1,1] & [1,1,1,1,1,0,0] \\ [1,1,1,1,1,1,1,1] & [1,1,1,1,1,0,0] \\ [1,1,1,1,1,1,1,1] & [1,1,1,1,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,0,0] \\ [1,1,1,1,1,1,1,1] & [1,1,1,1,1,0,0] \\ [1,1,1,1,1,1,1,1] & [1,1,1,1,1,0,0] \\ [1,1,1,1,1,1,1,1] & [1,1,1,1,1,1,1] \\ [1,1,1,1,1,1,1,1] & [1,1,1,1,1,1,1] \\ [1,1,1,1,1,1,1,1] & [1,1,1,1,1,1,1] \\ [1,1,1,1,1,1,1,1,1] & [1,1,1,1,1,1,1,1] \\ [1,1,1,1,1,1,1,1,1] & [1,1,1,1,1,1,1] \\ [1,1,1,1,1,1,1,1,1,1] & [1,1,1,1,1,1,1] \\ [1,1,1,1,1,1,1,$		[1, 0, 1, 1, 1, 0]	[1, 0, 0, 1, 0, 1]
$ \begin{bmatrix} [0,0,0,0,1,1] & [0,0,0,0,1,1] \\ [0,0,1,0,0,0] & [0,0,1,0,0,0] \\ [0,1,0,1,0,1] & [0,1,0,1,0,1] \\ [0,1,1,1,1,0] & [0,1,1,1,1,0] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0] & [1,0,0,1,0,0] \\ [1,1,1,0,0,1,0] & [1,1,1,0,0,1] \\ [1,1,1,0,0,1] & [1,1,0,0,1,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,1,1,1] & [0,0,0,1,0,0] \\ [0,1,1,0,0,1] & [0,1,0,0,1,0] \\ [0,1,1,0,0,1] & [0,1,0,0,0,1] \\ [1,0,0,0,0,1,1] & [1,0,0,0,1,1] \\ [1,1,0,1,0,0] & [1,1,0,1,0,1] \\ [1,1,1,1,1,0] & [1,1,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,1,1,0] \\ [0,1,0,0,1,1] & [0,0,0,1,1,1,0] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,1,0,0,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,0,0,1,0] \\ [1,1,0,1,0,0] & [1,1,1,1,1,1] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ \end{bmatrix} $		[1, 1, 0, 0, 1, 1]	[1, 1, 1, 0, 0, 0]
$ \begin{bmatrix} [0,0,1,0,0,0] & [0,0,1,0,0,0] \\ [0,1,0,1,0,1] & [0,1,0,1,0,1] \\ [0,1,1,1,1,0] & [0,1,1,1,1,0] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,1,1,0,0,1] \\ [1,1,1,0,0,1] & [1,1,0,0,1,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,1,1,1,1] & [0,0,1,1,1,1] \\ [0,1,0,0,1,0] & [0,1,0,0,1,0] \\ [0,1,1,0,0,1] & [0,1,0,0,0] \\ [1,0,1,0,0,0] & [1,0,0,0,1,1] \\ [1,1,0,1,0,0] & [1,1,1,1,1,0] \\ [1,1,1,1,1,0] & [1,1,1,1,1,0] \\ [1,1,1,1,1,0] & [0,0,0,1,1,1,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,1,1,0] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,0,0,0,1] & [0,1,0,0,1,1] \\ [1,0,1,0,0,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,0] & [1,0,1,0,0,1] \\ [1,1,0,1,0,0] & [1,1,1,1,1,1] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ \end{bmatrix} $		[1, 1, 1, 0, 0, 0]	[1, 1, 0, 0, 1, 1]
$ \begin{bmatrix} [0,1,0,1,0,1] & [0,1,0,1,0,1] \\ [0,1,1,1,1,0] & [0,1,1,1,1,0] \\ [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,0,1,0,0] & [1,0,0,1,0,0] \\ [1,1,0,0,1,0] & [1,1,1,0,0,1] \\ [1,1,1,0,0,1] & [1,1,0,0,1,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,1,1,1,1] & [0,0,1,1,1,1] \\ [0,1,0,0,0,1] & [0,1,0,0,0] \\ [1,0,1,0,0,0] & [1,0,0,0,1] \\ [1,0,1,0,0,0] & [1,0,0,0,1,1] \\ [1,1,1,1,1,0] & [1,1,1,1,1,0] \\ [1,1,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,0,0] & [0,0,0,1,0,1] \\ [0,0,0,1,1] & [0,0,0,1,1,1,0] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,0,0,0,1] & [0,1,0,0,0] \\ [1,0,1,0,0,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,0] & [1,0,1,0,0,1] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ \end{bmatrix} $		[0, 0, 0, 0, 1, 1]	[0, 0, 0, 0, 1, 1]
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		[0, 0, 1, 0, 0, 0]	[0, 0, 1, 0, 0, 0]
5 [1,0,0,1,0,0] [1,0,1,1,1,1] [1,0,0,1,0,0] [1,1,1,0,0,1] [1,1,1,0,0,1] [1,1,1,0,0,1] [1,1,1,0,0,1] [0,0,0,1,0,0] [0,0,1,1,1,1] [0,1,0,0,1,0] [1,0,1,0,0,1] [1,0,1,0,0,0] [1,1,1,1,1,0] [1,1,1,1,1,0] [1,1,1,1,1,0] [0,0,1,1,1,0] [0,0,0,1,0,1] [1,1,1,1,1,0] [1,1,1,1,1,0] [1,1,0,0,0,1] [0,0,0,1,0,1] [0,0,0,1,0,1] [0,0,0,1,0,1] [0,0,0,1,0,1] [0,0,0,1,0,1] [0,1,0,0,1,1] [0,1,0,0,0,1] [1,0,1,0,0,0] [1,0,1,0,0,1] [1,0,1,0,0,0] [1,0,1,0,0,1] [1,1,1,1,1,1] [1,1,1,1,1,1] [1,1,1,1,1,1] [1,1,1,1,1,1]		[0, 1, 0, 1, 0, 1]	[0, 1, 0, 1, 0, 1]
$ \begin{bmatrix} [1,0,0,1,0,0] & [1,0,1,1,1,1] \\ [1,0,1,1,1,1] & [1,0,0,1,0,0] \\ [1,1,1,0,0,1,0] & [1,1,1,0,0,1] \\ [1,1,1,0,0,1] & [1,1,0,0,1,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,1,1,1,1] & [0,0,1,1,1,1] \\ [0,1,0,0,1,0] & [0,1,0,0,1] \\ [1,0,1,0,0,0] & [1,0,0,0,1] \\ [1,0,1,0,0,0] & [1,0,0,0,1,1] \\ [1,1,0,1,0,1] & [1,1,1,1,0] \\ [1,1,1,1,1,0] & [1,1,0,1,0,1] \\ [0,0,0,1,0,1] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,1,0,0,0] & [0,1,1,0,0,0] \\ [1,0,0,0,1,0] & [1,0,0,0,1,0] \\ [1,0,0,0,1,0] & [1,0,0,0,1,0] \\ [1,1,0,1,0,0,1] & [1,1,1,1,1,1] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \\ \end{bmatrix} $	4	[0, 1, 1, 1, 1, 0]	[0, 1, 1, 1, 1, 0]
$\begin{bmatrix} [1,1,0,0,1,0] & [1,1,1,0,0,1] \\ [1,1,1,0,0,1] & [1,1,0,0,1,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,1,1,1,1] & [0,0,1,1,1,1] \\ [0,1,0,0,1,0] & [0,1,0,0,1,0] \\ [1,0,1,0,0,1] & [0,1,1,0,0,1] \\ [1,0,1,0,0,0] & [1,0,0,0,1,1] \\ [1,1,0,1,0,0] & [1,1,1,1,1,0] \\ [1,1,1,1,1,0] & [1,1,1,1,1,0] \\ [0,0,0,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,0,1,1,1] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,0,0,0,1,0] & [0,1,0,0,1,1] \\ [1,0,1,0,0,0] & [1,0,0,0,1,0] \\ [1,0,1,0,0,0] & [1,0,0,0,1,0] \\ [1,1,1,1,1,1,1] & [1,1,1,1,1,1] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \end{bmatrix}$	4	[1, 0, 0, 1, 0, 0]	[1, 0, 1, 1, 1, 1]
$\begin{bmatrix} [1,1,1,0,0,1] & [1,1,0,0,1,0] \\ [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,1,1,1,1] & [0,0,1,1,1,1] \\ [0,1,0,0,1,0] & [0,1,0,0,1,0] \\ [0,1,1,0,0,1] & [0,1,1,0,0,1] \\ [1,0,0,0,1,1] & [1,0,1,0,0,0] \\ [1,0,1,0,0,0] & [1,0,0,0,1,1] \\ [1,1,1,1,1,0] & [1,1,1,1,1,0] \\ [1,1,1,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,0,1] & [0,0,0,1,0,1] \\ [0,0,1,1,1,0] & [0,0,1,1,1,0] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,0,0,0,1,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,1,0,0,1] \\ [1,1,1,1,1,1] & [1,1,1,1,1] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \end{bmatrix}$		[1, 0, 1, 1, 1, 1]	[1, 0, 0, 1, 0, 0]
$\begin{bmatrix} [0,0,0,1,0,0] & [0,0,0,1,0,0] \\ [0,0,1,1,1,1] & [0,0,0,1,1,1,1] \\ [0,1,0,0,1,0] & [0,1,0,0,1,0] \\ [0,1,1,0,0,1] & [0,1,1,0,0,1] \\ [1,0,0,0,1,1] & [1,0,1,0,0,0] \\ [1,0,1,0,0,0] & [1,0,0,0,1,1] \\ [1,1,1,1,1,0] & [1,1,1,1,1,0] \\ [1,1,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,0,1] & [0,0,0,1,0,1] \\ [0,0,1,1,1,0] & [0,0,0,1,1,1,0] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,0,0,0,1,0] & [0,1,1,0,0,0] \\ [1,0,0,0,1,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,0,0,1,0] \\ [1,1,1,1,1,1,1] & [1,1,0,1,0,0] \end{bmatrix}$		[1, 1, 0, 0, 1, 0]	[1, 1, 1, 0, 0, 1]
$\begin{bmatrix} [0,0,1,1,1,1] & [0,0,1,1,1,1] \\ [0,1,0,0,1,0] & [0,1,0,0,1,0] \\ [0,1,1,0,0,1] & [0,1,1,0,0,1] \\ [1,0,0,0,1,1] & [1,0,1,0,0,0] \\ [1,0,1,0,0,0] & [1,0,0,0,1,1] \\ [1,1,1,1,1,0] & [1,1,1,1,1,0] \\ [1,1,1,1,1,0] & [0,0,0,1,0,1] \\ [0,0,0,1,0,1] & [0,0,0,1,0,1] \\ [0,0,1,1,1,0] & [0,0,1,1,1,0] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,0,0,0,1,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,0,0,1,0] \\ [1,0,1,0,0,1] & [1,0,0,0,1,0] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \end{bmatrix}$		[1, 1, 1, 0, 0, 1]	[1, 1, 0, 0, 1, 0]
$\begin{bmatrix} [0,1,0,0,1,0] & [0,1,0,0,1,0] \\ [0,1,1,0,0,1] & [0,1,1,0,0,1] \\ [1,0,0,0,1,1] & [1,0,1,0,0,0] \\ [1,0,1,0,0,0] & [1,0,0,0,1,1] \\ [1,1,0,1,0,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,0] & [1,1,0,1,0,1] \\ [0,0,0,1,0,1] & [0,0,0,1,0,1] \\ [0,0,0,1,1,1,0] & [0,0,1,1,1,0] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,1,0,0,0] & [0,1,1,0,0,0] \\ [1,0,0,0,1,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,0,0,1,0] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \end{bmatrix}$		[0, 0, 0, 1, 0, 0]	[0, 0, 0, 1, 0, 0]
$ \begin{bmatrix} [0,1,1,0,0,1] & [0,1,1,0,0,1] \\ [1,0,0,0,1,1] & [1,0,1,0,0,0] \\ [1,0,1,0,0,0] & [1,0,0,0,1,1] \\ [1,1,0,1,0,1] & [1,1,1,1,0] \\ [1,1,1,1,1,0] & [1,1,0,1,0,1] \\ [0,0,0,1,0,1] & [0,0,0,1,0,1] \\ [0,0,1,1,1,0] & [0,0,1,1,1,0] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,1,0,0,0] & [0,1,1,0,0,0] \\ [1,0,0,0,1,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,0,0,1,0] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ \end{bmatrix} $		[0, 0, 1, 1, 1, 1]	[0, 0, 1, 1, 1, 1]
$ \begin{bmatrix} [1,0,0,0,1,1] & [1,0,1,0,0,0] \\ [1,0,1,0,0,0] & [1,0,0,0,1,1] \\ [1,1,0,1,0,1] & [1,1,1,1,0] \\ [1,1,1,1,1,0] & [1,1,0,1,0,1] \\ [0,0,0,1,0,1] & [0,0,0,1,0,1] \\ [0,0,1,1,1,0] & [0,0,1,1,1,0] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,1,0,0,0] & [0,1,1,0,0,0] \\ [1,0,0,0,1,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,0] & [1,1,1,1,1,1] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ \end{bmatrix} $		[0, 1, 0, 0, 1, 0]	[0, 1, 0, 0, 1, 0]
$ \begin{bmatrix} [1,0,0,0,1,1] & [1,0,1,0,0,0] \\ [1,0,1,0,0,0] & [1,0,0,0,1,1] \\ [1,1,0,1,0,1] & [1,1,1,1,1,0] \\ [1,1,1,1,1,0] & [1,1,0,1,0,1] \\ [0,0,0,1,0,1] & [0,0,0,1,0,1] \\ [0,0,1,1,1,0] & [0,0,1,1,1,0] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,1,0,0,0] & [1,0,1,0,0,0] \\ [1,0,0,0,1,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,1] & [1,0,0,0,1,0] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ \end{bmatrix} $	5	[0, 1, 1, 0, 0, 1]	[0, 1, 1, 0, 0, 1]
$ \begin{bmatrix} [1,1,0,1,0,1] & [1,1,1,1,0] \\ [1,1,1,1,1,0] & [1,1,0,1,0,1] \\ [0,0,0,1,0,1] & [0,0,0,1,0,1] \\ [0,0,1,1,1,0] & [0,0,1,1,1,0] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,1,0,0,0] & [0,1,1,0,0,0] \\ [1,0,0,0,1,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,0] & [1,1,1,1,1,1] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ \end{bmatrix} $	3	[1, 0, 0, 0, 1, 1]	[1, 0, 1, 0, 0, 0]
		[1, 0, 1, 0, 0, 0]	[1, 0, 0, 0, 1, 1]
$[0,0,0,1,0,1] \qquad [0,0,0,1,0,1] \\ [0,0,1,1,1,0] \qquad [0,0,0,1,1,1,0] \\ [0,1,0,0,1,1] \qquad [0,1,0,0,1,1] \\ [0,1,1,0,0,0] \qquad [0,1,1,0,0,0] \\ [1,0,0,0,1,0] \qquad [1,0,1,0,0,1] \\ [1,0,1,0,0,0] \qquad [1,1,1,1,1,1] \\ [1,1,1,1,1,1] \qquad [1,1,0,1,0,0]$		[1, 1, 0, 1, 0, 1]	[1, 1, 1, 1, 1, 0]
$ \begin{bmatrix} [0,0,1,1,1,0] & [0,0,1,1,1,0] \\ [0,1,0,0,1,1] & [0,1,0,0,1,1] \\ [0,1,1,0,0,0] & [0,1,1,0,0,0] \\ [1,0,0,0,1,0] & [1,0,1,0,0,1] \\ [1,0,1,0,0,0] & [1,1,1,1,1] \\ [1,1,1,1,1,1] & [1,1,0,1,0,0] \\ \end{bmatrix} $		[1, 1, 1, 1, 1, 0]	[1, 1, 0, 1, 0, 1]
[0, 1, 0, 0, 1, 1] [0, 1, 0, 0, 1, 1] [0, 1, 0, 0, 1, 1] [0, 1, 1, 0, 0, 0] [0, 1, 1, 0, 0, 0] [1, 0, 0, 0, 1, 0] [1, 0, 1, 0, 0, 1] [1, 0, 0, 0, 1, 0] [1, 1, 1, 1, 1, 1] [1, 1, 1, 1, 1, 1] [1, 1, 0, 1, 0, 0]		[0, 0, 0, 1, 0, 1]	[0, 0, 0, 1, 0, 1]
6 [0, 1, 1, 0, 0, 0] [0, 1, 1, 0, 0, 0] [1, 0, 0, 0, 1, 0] [1, 0, 1, 0, 0, 1] [1, 0, 1, 0, 0, 1, 0] [1, 1, 1, 1, 1, 1] [1, 1, 1, 1, 1, 1] [1, 1, 0, 1, 0, 0]		[0, 0, 1, 1, 1, 0]	[0, 0, 1, 1, 1, 0]
[1, 0, 0, 0, 1, 0] [1, 0, 1, 0, 0, 1] [1, 0, 1, 0, 0, 1] [1, 0, 0, 0, 1, 0] [1, 1, 1, 1, 1, 1] [1, 1, 0, 1, 0, 0]		[0, 1, 0, 0, 1, 1]	[0, 1, 0, 0, 1, 1]
[1, 0, 0, 0, 1, 0] [1, 0, 1, 0, 0, 1] [1, 0, 1, 0, 0, 1] [1, 0, 0, 0, 1, 0] [1, 1, 0, 1, 0, 0] [1, 1, 1, 1, 1] [1, 1, 1, 1, 1] [1, 1, 0, 1, 0, 0]	6	[0, 1, 1, 0, 0, 0]	[0, 1, 1, 0, 0, 0]
[1, 1, 0, 1, 0, 0] [1, 1, 1, 1, 1] [1, 1, 1, 1, 1] [1, 1, 0, 1, 0, 0]	U	[1, 0, 0, 0, 1, 0]	[1, 0, 1, 0, 0, 1]
[1, 1, 1, 1, 1] [1, 1, 0, 1, 0, 0]		[1, 0, 1, 0, 0, 1]	[1, 0, 0, 0, 1, 0]
		[1, 1, 0, 1, 0, 0]	[1, 1, 1, 1, 1, 1]
7 [0, 0, 0, 1, 1, 0] [0, 0, 0, 1, 1, 0]		[1, 1, 1, 1, 1, 1]	[1, 1, 0, 1, 0, 0]
	7	[0, 0, 0, 1, 1, 0]	[0, 0, 0, 1, 1, 0]

	[0, 0, 1, 1, 0, 1]	[0, 0, 1, 1, 0, 1]
	[0, 1, 0, 0, 0, 0]	[0, 1, 0, 0, 0, 0]
	[0, 1, 1, 0, 1, 1]	[0, 1, 1, 0, 1, 1]
	[1, 0, 0, 0, 0, 1]	[1, 0, 1, 0, 1, 0]
	[1, 0, 1, 0, 1, 0]	[1, 0, 0, 0, 0, 1]
	[1, 1, 0, 1, 1, 1]	[1, 1, 1, 1, 0, 0]
	[1, 1, 1, 1, 0, 0]	[1, 1, 0, 1, 1, 1]
	[0, 0, 0, 1, 1, 1]	[0, 0, 0, 1, 1, 1]
	[0, 0, 1, 1, 0, 0]	[0, 0, 1, 1, 0, 0]
	[0, 1, 0, 0, 0, 1]	[0, 1, 0, 0, 0, 1]
8	[0, 1, 1, 0, 1, 0]	[0, 1, 1, 0, 1, 0]
0	[1, 0, 0, 0, 0, 0]	[1, 0, 1, 0, 1, 1]
	[1, 0, 1, 0, 1, 1]	[1, 0, 0, 0, 0, 0]
	[1, 1, 0, 1, 1, 0]	[1, 1, 1, 1, 0, 1]
	[1, 1, 1, 1, 0, 1]	[1, 1, 0, 1, 1, 0]

Наименьшее расстояние может быть найдено либо, сравнивая вектора из таблицы напрямую и считая число разных элементов, либо как наименьшее число ненулевых элементов в векторе среди всех векторов, кроме нулевого. В данном случае это 3.

Чтобы код мог обнаружить k ошибок, минимальное расстояние должно составлять k+1, в данном случае код обнаруживает 2 ошибки.

Чтобы код мог исправить k ошибок, минимальное расстояние должно составлять 2k + 1, в данном случае код исправляет 1 ошибку.

В многочлене $x^5 + x^2 + 1$ **имеется ошибка**, во-первых, имеется не нулевой остаток (x) от деления на g(x), во-вторых, это вектора нету среди множества кодовых слов, в-третьих, при умножении на проверочную матрицу появляется ненулевой элемент по модулю 2:

Данную ошибку можно исправить.

Первый способ — это найти ближайшие правильные коды, минимально отличающиеся от полученного вектора, если такой правильный код один, то использовать его, в данном случае получим этот вектор:

$$x^5 + x^2 + x + 1$$

Второй способ заключается в подсчете количества ненулевых элементов в векторе остатка (x). В данном случае он 1 и не превышает число ошибок, проверяемых кодом, поэтому чтобы исправить можно сложить этот вектор с полученным сообщением $(x^5 + x^2 + 1$ и x), тогда получим аналогичный результат.

#3

Группа должна состоять из чисел, взаимно простых с 27, т. е. не кратных 3:

$$Z_{27}^* = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, 25, 26\}$$

Найдем порождающий элемент напрямую, перебирая все числа:

1 - не подходит

1

2 - подходит:

$$2 \rightarrow 4 \rightarrow 8 \rightarrow 16 \rightarrow 5 \rightarrow 10 \rightarrow 20 \rightarrow 13 \rightarrow 26 \rightarrow 25 \rightarrow 23 \rightarrow 19 \rightarrow 11 \rightarrow 22 \rightarrow 17 \rightarrow 7 \rightarrow 14 \rightarrow 1$$

4 - не подходит:

$$4 \rightarrow 16 \rightarrow 10 \rightarrow 13 \rightarrow 25 \rightarrow 19 \rightarrow 22 \rightarrow 7 \rightarrow 1$$

5 **- подходит**:

$$5 \rightarrow 25 \rightarrow 17 \rightarrow 4 \rightarrow 20 \rightarrow 19 \rightarrow 14 \rightarrow 16 \rightarrow 26 \rightarrow 22 \rightarrow 2 \rightarrow 10 \rightarrow 23 \rightarrow 7 \rightarrow 8 \rightarrow 13 \rightarrow 11 \rightarrow 1$$

7 - не подходит

$$7 \rightarrow 22 \rightarrow 19 \rightarrow 25 \rightarrow 13 \rightarrow 10 \rightarrow 16 \rightarrow 4 \rightarrow 1$$

8 - не подходит:

$$8 \rightarrow 10 \rightarrow 26 \rightarrow 19 \rightarrow 17 \rightarrow 1$$

10 - не подходит:

$$10 \rightarrow 19 \rightarrow 1$$

11 - подходит:

$$11 \rightarrow 13 \rightarrow 8 \rightarrow 7 \rightarrow 23 \rightarrow 10 \rightarrow 2 \rightarrow 22 \rightarrow 26 \rightarrow 16 \rightarrow 14 \rightarrow 19 \rightarrow 20 \rightarrow 4 \rightarrow 17 \rightarrow 25 \rightarrow 5 \rightarrow 1$$

13 - не подходит:

$$13 \rightarrow 7 \rightarrow 10 \rightarrow 22 \rightarrow 16 \rightarrow 19 \rightarrow 4 \rightarrow 25 \rightarrow 1$$

14 - подходит:

$$14 \rightarrow 7 \rightarrow 17 \rightarrow 22 \rightarrow 11 \rightarrow 19 \rightarrow 23 \rightarrow 25 \rightarrow 26 \rightarrow 13 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

16 - не подходит:

$$16 \rightarrow 13 \rightarrow 19 \rightarrow 7 \rightarrow 4 \rightarrow 10 \rightarrow 25 \rightarrow 22 \rightarrow 1$$

17 - не подходит:

$$17 \rightarrow 19 \rightarrow 26 \rightarrow 10 \rightarrow 8 \rightarrow 1$$

19 - не подходит:

$$19 \rightarrow 10 \rightarrow 1$$

20 - подходит:

$$20 \rightarrow 22 \rightarrow 8 \rightarrow 25 \rightarrow 14 \rightarrow 10 \rightarrow 11 \rightarrow 4 \rightarrow 26 \rightarrow 7 \rightarrow 5 \rightarrow 19 \rightarrow 2 \rightarrow 13 \rightarrow 17 \rightarrow 16 \rightarrow 23 \rightarrow 1$$

22 - не подходит:

$$22 \rightarrow 25 \rightarrow 10 \rightarrow 4 \rightarrow 7 \rightarrow 19 \rightarrow 13 \rightarrow 16 \rightarrow 1$$

23 - подходит:

$$23 \rightarrow 16 \rightarrow 17 \rightarrow 13 \rightarrow 2 \rightarrow 19 \rightarrow 5 \rightarrow 7 \rightarrow 26 \rightarrow 4 \rightarrow 11 \rightarrow 10 \rightarrow 14 \rightarrow 25 \rightarrow 8 \rightarrow 22 \rightarrow 20 \rightarrow 1$$

25 - не подходит:

$$25 \rightarrow 4 \rightarrow 19 \rightarrow 16 \rightarrow 22 \rightarrow 10 \rightarrow 7 \rightarrow 13 \rightarrow 1$$

26 - не подходит

$$26 \rightarrow 1$$

Итого: {2, 5, 11, 14, 20, 23} — порождающие элементы

#4

(не совпадает формула в 4-й части алгоритма: формула по лекции дает дробную степень, брал формулу из интернета)

Посчитать символ Якоби:

$$\left(\frac{221}{539}\right)$$

По определению:

Символ Лежандра:

$$\left(\frac{a}{p}\right) = 0$$
, $a \mod p = 0$

$$\left(\frac{a}{p}\right) = 1$$
, a квадратичный вычет по модулю p

$$\left(\frac{a}{p}\right) = -1$$
, a квадратичный невычет по модулю p

Разложим 539:

$$539 = 7 \cdot 7 \cdot 11$$

$$\left(\frac{221}{539}\right) = \left(\frac{221}{7}\right) \cdot \left(\frac{221}{7}\right) \cdot \left(\frac{221}{11}\right)$$

$$\left(\frac{221}{7}\right) = \left(\frac{4}{7}\right) = 1$$

$$\left(\frac{221}{11}\right) = \left(\frac{1}{11}\right) = 1$$

Получаем, что $\left(\frac{221}{539}\right) = 1 \cdot 1 \cdot 1 = 1$

По алгоритму: /(221, 539)

- **1.1**) 221 < 539
- 1.2) 221 не делится на 4
- 1.3) 221 не делится на 2

1.4)
$$(-1)^{\frac{221-1}{2}\cdot\frac{539-1}{2}} \cdot J(539, 221)$$

$$(-1)^{110\cdot 269} \cdot J(539, 221) = J(539, 221)$$

- 2.1) $539 > 221 \rightarrow J(97, 221)$
- 3.1) 97 < 221
- 3.2) 97 не делится на 4
- 3.3) 97 не делится на 2

3.4)
$$(-1)^{\frac{97-1}{2},\frac{221-1}{2}}J(221, 97)$$

$$(-1)^{48\cdot110}J(221, 97) = J(221, 97)$$

4.1)
$$221 > 97 \rightarrow J(27, 97)$$

- **5.2**) 27 не делится на 4
- 5.3) 27 не делится на 2

5.4)
$$J(27, 97) = (-1)^{\frac{27-1}{2} \cdot \frac{97-1}{2}} J(97, 27)$$

$$(-1)^{13\cdot48}I(97, 27) = I(97, 27)$$

6.1)
$$97 > 27 \rightarrow J(16,27)$$

- 7.1)16 < 27
- 7.2) 16 делится на $4 \rightarrow J(4, 27)$
- 8.1)4 < 27
- 8.2) 4 делится на $4 \rightarrow J(1, 27)$

$$J\left(\frac{1}{m}\right) = 1$$

Т. к. при рекурсивных вызовах знак не менялся $\rightarrow J(221, 539) = J(1, 27) = 1$

#5

Алгоритм Соловея-Штрассена работает с числами: [2, n-1]

В данной задаче это [2, 20]

Число является составным по алгоритму в двух случаях:

- 1) Если HOД(a, n) > 1, тогда подойдут: $\{3, 6, 7, 9, 12, 14, 15, 18\}$
 - 3: HOД(3,21) = 3
 - 6: HOД(6,21) = 3
 - 7: HOД(7,21) = 7
 - 9: HOД(9,21) = 3
 - 12: HOД(12,21) = 3
 - 14: HOД(14,21) = 7
 - 15: HOД(15,21) = 3
 - 18: HOJ(18,21) = 3
- 2) Если $\left(\frac{a}{n}\right) \neq a^{\frac{n-1}{2}} \pmod{n}$

$$\left(\frac{a}{21}\right) = \left(\frac{a}{3}\right) \cdot \left(\frac{a}{7}\right)$$
 — по определению $a^{\frac{21-1}{2}} = a^{10}$

Квадратичные вычеты 3: {0, 1}

Квадратичные вычеты 7: {0, 1, 2, 4}

a = 2:

$$\left(\frac{2}{21}\right) = -1$$
$$2^{10} mod \ 21 = 16$$

a = 3:

$$\left(\frac{3}{21}\right) = 0$$
$$3^{10} mod \ 21 = 18$$

a = 4:

$$\left(\frac{4}{21}\right) = 1$$

$$4^{10} \bmod 21 = 4$$

a = 5:

$$\left(\frac{5}{21}\right) = 1$$

$$5^{10} \ mod \ 21 = 16$$

a = 6:

$$\left(\frac{6}{21}\right) = 0$$

$$6^{10} \bmod 21 = 15$$

a = 7:

$$\left(\frac{7}{21}\right) = 0$$

$$7^{10} \bmod 21 = 7$$

a = 8:

$$\left(\frac{8}{21}\right) = -1$$
$$8^{10} \bmod 21 = 1$$

a = 9:

$$\left(\frac{9}{21}\right) = 0$$

$$9^{10} \bmod 21 = 9$$

a = 10:

$$\left(\frac{10}{21}\right) = -1$$

$$10^{10} \ mod \ 21 = 4$$

a = 11:

$$\left(\frac{11}{21}\right) = -1$$

$$11^{10} \, mod \, 21 = 4$$

a = 12:

$$\left(\frac{12}{21}\right) = 0$$

$$12^{10} \, mod \, 21 = 9$$

a = 13:

$$\left(\frac{13}{21}\right) = -1$$

$$13^{10} \mod 21 = 1$$

a = 14:

$$\left(\frac{14}{21}\right) = 0$$

$$14^{10} \ mod \ 21 = 7$$

a = 15:

$$\left(\frac{15}{21}\right) = 0$$

$$15^{10} \ mod \ 21 = 15$$

a = 16:

$$\left(\frac{16}{21}\right) = 1$$

$$16^{10} \mod 21 = 16$$

a = 17:

$$\left(\frac{17}{21}\right) = 1$$

$$17^{10} \mod 21 = 4$$

a = 18:

$$\left(\frac{18}{21}\right) = 0$$

$$17^{10} \bmod 21 = 18$$

a = 19:

$$\left(\frac{19}{21}\right) = -1$$

$$19^{10} \ mod \ 21 = 16$$

a = 20:

$$\left(\frac{20}{21}\right) = 1$$

$$20^{10} \mod 21 = 1$$

Таким образом, по второму критерию подходит только 20

#6

$$y^2 = x^3 + 4x + 4$$
, Z_5

Построим вообще все точки их никак не больше 25:

	0	1	2	3	4
y^2	0	1	4	4	1
$x^3 + 4x + 4$	4	4	0	3	4

Найдем такие пары (x, y), где выполняется равенство $y^2 = x^3 + 4x + 4$:

$$\{(0,2), (0,3), (1,2), (1,3), (2,0), (4,2), (4,3)\}$$

Проверим, что это множество является абелевой группой (по доказанному на лекции операция сложения коммутативна и ассоциативна, нейтральный элемент - бесконечно удаленная точка, остается проверить наличие противоположно элемента и замкнутости множества относительно операции сложения):

1) Проверим наличие противоположного элемента и добавим нейтральный элемент - бесконечно удаленную точку Θ :

$$(0,2) + (0,3) = \Theta$$

$$(1,2) + (1,3) = 0$$

$$(2,0) + (2,0) = \Theta$$

$$(4,2) + (4,3) = 0$$

Таким образом, для каждого элемента имеется противоположный элемент, также есть нейтральный.

2) Проверим замыкание множества, относительно операции сложения:

Попарно просуммируем множество точек (различных) по правилу:

При разных точках (не учитывая противоположные):

$$x_3 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)^2 - x_1 - x_2 \mod 5$$

$$y_3 = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x_3 - x_1) \mod 5$$

При одинаковых (не учитывая противоположные):

$$\lambda = \frac{3x_1^2 + 4}{2y_1} \mod 5$$

$$x_3 = \lambda^2 - x_1 - x_1 \mod 5$$

$$y_3 = \lambda(x_1 - x_3) - y_1 mod 5$$

Обозначим Θ , как null

$$(0, 2) + (0, 2) = (1, 2)$$

$$(0, 2) + (0, 3) = null$$

$$(0, 2) + (1, 2) = (4, 3)$$

$$(0, 2) + (1, 3) = (0, 3)$$

$$(0, 2) + (2, 0) = (4, 2)$$

$$(0, 2) + (4, 2) = (1, 3)$$

$$(0, 2) + (4, 3) = (2, 0)$$

$$(0, 3) + (0, 2) = null$$

$$(0,3) + (0,3) = (1,3)$$

$$(0, 3) + (1, 2) = (0, 2)$$

$$(0, 3) + (1, 3) = (4, 2)$$

$$(0, 3) + (2, 0) = (4, 3)$$

$$(0, 3) + (4, 2) = (2, 0)$$

$$(0,3) + (4,3) = (1,2)$$

$$(1, 2) + (0, 2) = (4, 3)$$

$$(1, 2) + (0, 3) = (0, 2)$$

$$(1, 2) + (1, 2) = (2, 0)$$

$$(1, 2) + (1, 3) = null$$

$$(1, 2) + (2, 0) = (1, 3)$$

$$(1, 2) + (4, 2) = (0, 3)$$

$$(1, 2) + (4, 3) = (4, 2)$$

$$(1, 3) + (0, 2) = (0, 3)$$

$$(1, 3) + (0, 3) = (4, 2)$$

$$(1, 3) + (1, 2) = null$$

$$(1, 3) + (1, 3) = (2, 0)$$

$$(1, 3) + (2, 0) = (1, 2)$$

$$(1, 3) + (4, 2) = (4, 3)$$

$$(1, 3) + (4, 3) = (0, 2)$$

$$(2,0) + (0,2) = (4,2)$$

$$(2, 0) + (0, 3) = (4, 3)$$

$$(2,0) + (1,2) = (1,3)$$

$$(2, 0) + (1, 3) = (1, 2)$$

$$(2, 0) + (2, 0) = null$$

$$(2, 0) + (4, 2) = (0, 2)$$

$$(2, 0) + (4, 3) = (0, 3)$$

$$(4, 2) + (0, 2) = (1, 3)$$

$$(4, 2) + (0, 3) = (2, 0)$$

$$(4, 2) + (1, 2) = (0, 3)$$

$$(4, 2) + (1, 3) = (4, 3)$$

$$(4, 2) + (2, 0) = (0, 2)$$

$$(4, 2) + (4, 2) = (1, 2)$$

$$(4, 2) + (4, 3) = null$$

$$(4, 3) + (0, 2) = (2, 0)$$

$$(4, 3) + (0, 3) = (1, 2)$$

$$(4, 3) + (1, 2) = (4, 2)$$

$$(4, 3) + (1, 3) = (0, 2)$$

$$(4, 3) + (2, 0) = (0, 3)$$

$$(4, 3) + (4, 2) = null$$

$$(4, 3) + (4, 3) = (1, 3)$$

Новых точек получено не было, следовательно множество является замкнутым, значит была построена группа:

$$\{(0,2),(0,3),(1,2),(1,3),(2,0),(4,2),(4,3),\Theta\}$$

3) Определим является ли она циклической:

Проверим какая точка является порождающей:

(0, 2) - порождающая

$$(0,2) \to (1,2) \to (4,3) \to (2,0) \to (4,2) \to (1,3) \to (0,3) \to \Theta$$

(0, 3) - порождающая

$$(0,3) \to (1,3) \to (4,2) \to (2,0) \to (4,3) \to (1,2) \to (0,2) \to \Theta$$

(1, 2) - не порождающая

$$(1,2) \to (2,0) \to (1,3) \to \Theta$$

(1, 3) - не порождающая

$$(1,3) \to (2,0) \to (1,2) \to \Theta$$

(2, 0) - не порождающая

$$(2,0) \rightarrow \Theta$$

(4, 2) - порождающая

$$(4,2) \to (1,2) \to (0,3) \to (2,0) \to (0,2) \to (1,3) \to (4,3) \to 0$$

(4, 3) - порождающая

$$(4,3) \to (1,3) \to (0,2) \to (2,0) \to (0,3) \to (1,2) \to (4,2) \to \Theta$$

9 - не порождающая

{(0,2), (0,3), (4,2), (4,3)} - **порождающие точки,** они есть, значит группа **циклическая**