

Assignment 2

Vigenere Cipher

1. A cipher text is suspected to have been encrypted using the Vigenere Cipher. Describe and demonstrate how the Kasiski Examination method can be used for encryption.

Ans : The Kasiski Examination is a cryptographic method used to estimate the length of the key in a Vigenere cipher. The Vigenere cipher is a polyalphabetic cipher that uses a keyword to determine the shifting of each letter in the plain text. The Kasiski help identify repeated patterns in the cipher text, hence revealing the key.

Steps in Kasiski Examination.

1. Identify Repeated Sequences : Search the cipher text for repeated sequences of letters.
2. Measure Distances : Note the distances between the starting points of these repeated sequences.
3. Find common factors : Determine the greatest common divisor (GCD) of these distances.

Example :

Cipher text : ZICVTW4NGRZGVTWAVZUCAYG
LMGT

Repeated sequence : "VTW" at position
3 and 10.

Distance : $10 - 3 = 7$

GCD : If distances are 7, 14, 21
then GCD(7, 14, 21) = 7
likely key = 7

Q2)

Cipher Text : KSMENZBBLKSMEMW

Plain Text : SECRETMESSAGE

Row : - K S M E N Z B B L K S M E M W
S E C R E T M E S S A G E F S E

$$C = P + k \bmod 26$$

$$k = C - P \bmod 26$$

$$k = 10$$

$$S = 18$$

$$k = 10 - 18 \bmod 26 = -8 \bmod 26 = 18 = S$$

S-E	$18 - 4 \bmod 26 = 14 \equiv 0$	The key is
M-C	$12 - 2 \bmod 26 = 10 \equiv k$	
E-R	$14 - 17 \bmod 26 = 13 \equiv N$	
H-E	$13 - 4 \bmod 26 = 3 \equiv D$	
Z-T	$25 - 19 \bmod 26 = 6 \equiv g$	
B-M	$1 - 12 \bmod 26 \equiv 15 \equiv p$	
B-E	$1 - 4 \bmod 26 = 23 \equiv x$	
L-S	$11 - 18 \bmod 26 = 19 \equiv t$	
X-S	$10 - 18 \bmod 26 = 18 \equiv s$	
S-A	$-18 - 0 \bmod 26 = 18 \equiv s$	
M-G	$12 - 6 \bmod 26 = 6 \equiv g$	
E-E	$4 - 4 \bmod 26 = 0 \equiv a$	

Q3) Cipher text: U28SQVU04XMOP VGP0ZPGVSG, ZWSSZOP
F.PESXUDBMET.SXAIZ

describe how frequency analysis can help
break the cipher if the key length is already
known to be 5

splitting the cipher text into groups based on
the key length

Group 1 \rightarrow position $\{1, 6, 11, 16, 21, 26, \dots\} \rightarrow O, O, X, P, \dots$

Group 2 \rightarrow Position $\{2, 7, 12, 17, 22, 27, 32, 37, 42\} \rightarrow Z, V, O, V, P, \dots$

Group 3 \rightarrow Position $\{3, 8, 13, 18, 23, 28, 33, 38, 43\} \rightarrow Q, U, Y, G, S, \dots$

Group 4 \rightarrow Position $\{4, 9, 14, 19, 24, 29, 34, 39, 44\} \rightarrow S, O, M, P, S, \dots$

Groups \rightarrow Position $\rightarrow (5, 10, 15, 20, 25, 30, 35, 40, 45) \rightarrow 0, 0, 0, \text{VV}$

Analyse frequency of each letter in each group

group 1 $\rightarrow V:1, O:1, X:2, P:1, Z:1, G:1, D:1 \rightarrow X \text{ appears 2 times}$

group 2 $\rightarrow Z:2, V:2, O:1, P:1, M:1, E:1 \rightarrow Z \& V \text{ both appear 2 times}$

group 3 $\rightarrow Q:1, U:1, H:1, G:1, E:1, S:1, B:1, T:1 \left[\begin{matrix} \text{No} \\ \text{repetition} \end{matrix} \right]$

group 4 $\rightarrow S:3, O:1, M:1, P:1, W:1, E:1 \rightarrow S \text{ appears 3 times}$

group 5 $\rightarrow O:2, U:1, V:2, Z:1, T:1, A:1 \rightarrow O \& V \text{ both appear 2 times}$

group 1 : The most frequent letter is "X"

$$\text{shift} = X - E = 23 - 4 = 19 \text{ (T)}$$

group 2 : The most frequent letters are "Z" "8" "V"

$$\text{shift "Z"} = 2 - E = 25 - 4 = 21 \text{ (V)}$$

$$\text{shift "V"} = V - E = 17 \text{ (R)}$$

group 3 : ^{Let's assume} The most frequent letters are "S"

$$\text{shift} = S - E = 18 - 4 = 14 \text{ (O)}$$

group 4 : The most frequent letter is "5"

$$\text{shift} = S - E = 18 - 4 = 14 \text{ (O)}$$

group 5 : The most frequent letter is "0"

$$\text{shift} = O - E = 14 - 4 = 10 \text{ (K)}$$

\therefore key : T, V, O, O, K
"TVOKW"

Vigenère cipher vs Caesar cipher

- Caesar cipher: A monoalphabetic cipher with only 25 possible shifts, making it weak. It vulnerable to brute force and frequency analysis.
- Vigenère cipher: A photo poly alphabetic cipher that uses a repeating key. Its strength increases with key length. A short key is vulnerable to frequency analysis, while long key is much stronger. Shortkey length weakens the vigenère cipher because key repetition makes it vulnerable.
 - It improves security, and a key as long as the message provides perfect security
- Exploiting key length:
 - Attackers can split cipher text into groups based on the key length and use frequency analysis to deduce the key.
 - If short key, attackers can try all possible combinations - If the key length is unknown, attackers can use repeated sequences in the cipher text to estimate it.

Affine Cipher :

O/I cipher key = $(7, 17)$
 $0 \leq x, s \leq 25$
 $d_K(y) = xy + s$

$$e_K(x) = (ax + b) \bmod 26$$

$$a = 7, b = 17$$

$$d_K(y) = a^{-1}x(y - b) \bmod 26$$

$$axa^{-1} \equiv 1 \pmod{26}$$

$a = 7$ $7 \times 15 = 105 \equiv 1 \pmod{26}$

$$a^{-1} = 15 \text{ & } b = 17 \text{ Esu bstitution in decryption form}$$

$$d_K(y) = 15x(y - 17) \bmod 26$$

$$d_K(y) = (15xy - 15 \cdot 17) \bmod 26$$

$$15 \times 17 = 255 \text{ and } 255 \bmod 26 = 21$$

$$d_K(y) = (15y - 21) \bmod 26$$

$$d_K(y) = xy + s$$

$$d_K(y) = (15y + 26 - 21) \bmod 26$$

$$d_K(y) = (15y + 5) \bmod 26$$

$$x = 15 \text{ & } s = 21$$

Q2) $k = (71, 25)$

$$ek(x) = (ax + b) \bmod 26$$

$$d_k(x) = a^{-1} (y - b) \bmod 26$$

$$axa^{-1} = 1 \pmod{26}$$

$$a = 11$$

$$11 \cdot a^{-1} = 1 \pmod{25}$$

$$11 \cdot 79 = 209 \text{ and } 209 \bmod 25 = 1$$

$$a^{-1} = 19$$

$$b = 25$$

$$d_k(y) = 19(y - 25) \bmod 26$$

$$d_k(y) = (19y - 475) \bmod 26$$

$$475 \bmod 26 = 7$$

$$d_k(y) = (9y - 7) \bmod 26$$

$$d_k(y) = 9y + s$$

$$d_k(y) = 19y + 26 - 7 \bmod 26$$

$$d_k(y) = (19y + 19) \bmod 26$$

$$r = 19 \text{ and } s = 19$$

Q3) $e(x) = 3x + 10 \bmod 26$

$$d(y) = (cy + d) \bmod 26 \quad [d(y) = a^{-1} \cdot (y - b) \bmod 26]$$

Cipher Text: "VBNUVCHQUBUZHOUVH.XOLPUVOJRDK"

$$a = 3, b = 10.$$

$$3 \cdot a^{-1} = 1 \bmod 25$$

$$3 \cdot 9 = 27 = 1 \pmod{25} \quad a^{-1} = 9$$

$$d(y) = 9(y - 10) \bmod 26$$

$$d(y) = (9y - 90) \bmod 26$$

$$d(y) = (9y + 18) \bmod 26$$

$$U = (9 \times 20 + 18) \bmod 26 = 16 (P)$$

$$B = (9 \times 1 + 18) \bmod 26 = 18 (B); U(7) = (9 \times 7 + 18) \bmod 26 = 3 (D);$$

$$V(2) = (9 \times 21 + 18) \bmod 26 = 25 (Z); L = (9 \times 11 + 18) \bmod 26 = 13 (N)$$

$$G = (9 \times 6 + 18) \bmod 26 = 20 (U); X = (9 \times 23 + 18) \bmod 26 = 17 (R);$$

$$O = (9 \times 14 + 18) \bmod 26 = 19 (O); D = (9 \times 3 + 18) \bmod 26 = 19 (T);$$

$$P = (9 \times 15 + 18) \bmod 26 = 23 (X); S = (9 \times 9 + 18) \bmod 26 = 21 (V);$$

$$R = (9 \times 17 + 18) \bmod 26 = 15 (P); K = (9 \times 10 + 18) \bmod 26 = 4 (E)$$

Final plain text: "QB DZNQVQ BDRD6ZQ DLTNXQ Z0 VPTE"

$$Q4) e(x) = (9x + 3) \bmod 26$$

$$d(y) := a^{-1}(y - b) \bmod 26$$

$$a = 9, b = 3$$

$$9 \cdot a^{-1} \equiv 1 \pmod{26}$$

$$9 \cdot 3 \equiv 27 \equiv 1 \pmod{26}$$

$$a^{-1} = 3$$

$$d(y) = 3(y - 3) \bmod 26$$

$$d(y) = (3y - 9) \bmod 26$$

$$d(y) = (3y + 17) \bmod 26$$

$$S = (3 \times 18 + 17) \bmod 26 = 19 (T)$$

$$A = (3 \times 0 + 17) \bmod 26 = 17 (R)$$

$$B = (3 \times 1 + 17) \bmod 26 = 20 (U)$$

$$N = (3 \times 13 + 17) \bmod 26 = 4 (E)$$

Plain Text: "TRVF"