Linear Optimization of Humanitarian Aid Logistics

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Introduction

The 2004 Indian Ocean earthquake and subsequent tsunami devastated the surrounding countries, with the most severe impact on the west coast of Sumatra (Wiharta et al. 2008, p. 87). As a response, other nations provided swift aid by sending supplies and donations to affected areas. While there was an outpouring of aid, distributing said aid in a timely and efficient manner was another challenge.

In order to explore the problem of aid distribution, we focused on the region of Aceh in Indonesia, one of the main areas directly impacted by the natural disasters, providing a specific location and conditions. Aid distribution in this context refers to the movement of supplies from designated supply nodes (several airports that hold the supplies initially) to demand nodes (city centres which could further distribute aid locally), while fulfilling supply requirements from the demand nodes. Furthermore, since this distribution should be timely, we aim to minimize the distance that the aid travels, in turn minimizing the time travelled.

We referenced the following map from the two-year tsunami progress report by the International Federation of Red Cross and Red Crescent Societies, which highlights demand locations. We used the centres of each province as demand nodes in our analysis.

Figure 1: Map of Demand Locations

We expanded upon the transportation problem by considering multiple supply types, as well as limiting the transport capacity from each supply location. The range of supplies included medical items and construction materials for shelters, and every central town required a different amount of each type of aid depending on the severity of the natural disaster in that area. Furthermore, we decided that for our model of this problem, the method of transportation would be helicopters, getting rid of the need to map distance through roads and instead using the Euclidean distance derived from geographic coordinates.

Definitions and Equations

Given the variables:

- m: total number of supply nodes, $i \in \{1, m\}$
- n: total number of demand nodes, $j \in \{1, n\}$
- r: total number of aid types, $k \in \{1, r\}$
- x_{ijk}: decision variable denoting the amount of aid in kilograms of type k to deliver from supply node i to demand node j
- c_{ij}: cost of delivering a kilogram of any aid type from supply node i to demand node j (Euclidean distance)
- s_{ik}: kilograms of aid of type k available at supply node i
- d_{jk}: kilograms of aid of type k required by demand node j
- t_i: transport capacity for each supply node i (units: kg*km). Intuitively, one interpretation is that this represents the amount of fuel available at each supply location for helicopters based there to use.

Objective Function:

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{r} c_{ij} \cdot x_{ijk} \tag{1}$$

Such that:

$$\text{For every } (i,k), \sum_{j=1}^{n} x_{ijk} \leq s_{ik} \tag{2}$$

$$\text{For every } (j,k), \sum_{i=1}^{m} x_{jk} \geq d_{jk} \tag{3}$$

$$\text{For every } i, \sum_{j=1}^{n}\sum_{k=1}^{r} c_{ij} \cdot x_{ijk} \leq t_i \tag{4}$$

$$\text{For every } (i,j,k), x_{ijk} \geq 0 \tag{5}$$

The optimization problem above models the movement of aid relief from main supply locations to affected areas during a humanitarian catastrophe. We aim to minimize the distance the transporters travel to deliver different aid types to the impacted locations to efficiently distribute the available aid appropriately, represented by the objective function in the transport problem. The constraints map to real-world assumptions − namely that each supply location has limited aid resources available to distribute in the area and each affected area has a minimum quota of aid relief necessary for its stability. They also posit that the transport capacity of each supply location is limited and that the amount of aid must be non-negative values. We account for a limited number of transport vehicles available and calculate the cost of transporting a certain amount of aid to a location.

In our model, we assume that the time it takes to deliver supplies is proportional to the distance travelled. Naturally, we restrict the amount of helicopters at a supply node to a discrete value (i.e. one whole helicopter must travel from a supply node to a demand node), however supply quantities may be continuous (i.e. measured by weight). We also assume there is at least as much supply as demand, and that each route is a return trip between each supply and demand node.

Modeling Process

We formulated the problem as a linear programming optimization using Python and CVXPY, aiming to minimize total transport distance while ensuring that supply constraints, demand requirements, and helicopter transport capacity were met. The optimization framework was first constructed with a basic dataset to establish the model's structure before being expanded with real-world data.

To determine supply locations, we identified key airports in Aceh that served as critical hubs for storing and distributing humanitarian aid during the emergency (see Appendix Table 1). The supply values at each airport were based on the number of helicopters assigned to that region, obtained from historical records of the relief effort. Similarly, demand locations were chosen based on major towns and regencies that required aid shipments (see Appendix Table 2), with their respective demand values estimated using population density data (see population density map).

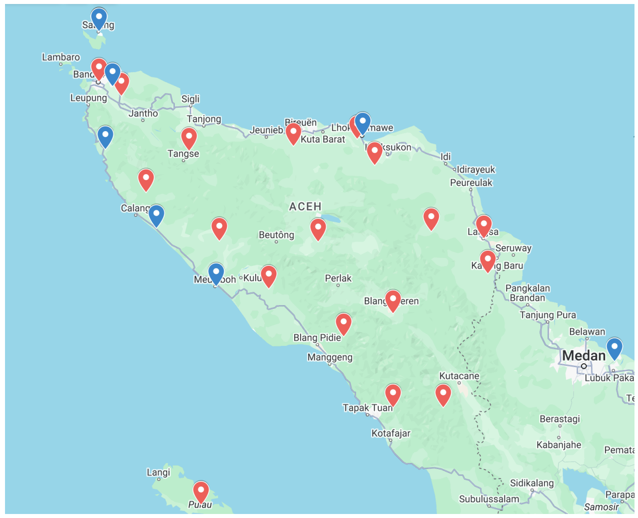

Figure 2: Map of Demand (Red) and Supply (Blue) Locations

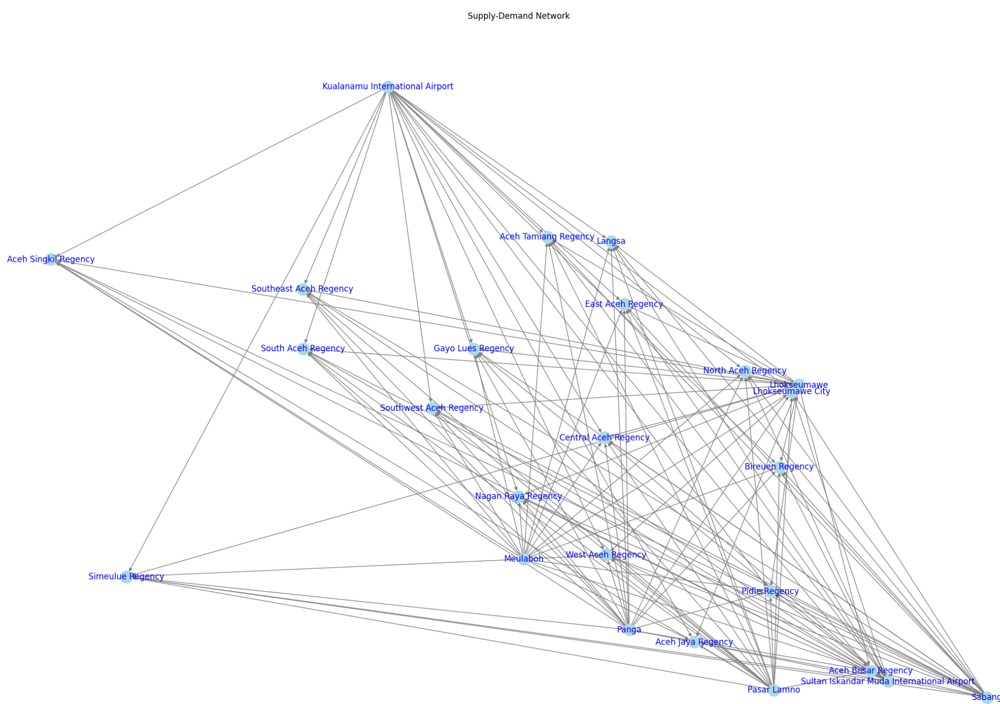

Figure 3: Map of Demand (Red) and Supply (Blue) Locations

Additionally we compiled a tally of transporters used - mainly helicopters - to distribute the goods as outlined in Wiharta et al. We modeled transport costs using Euclidean distance and the transport capacity per helicopter was estimated based on an average payload of 2,500 kg over 500 km, derived from the helicopter models used during the relief efforts in 2004.

Using this setup, we implemented the optimization model, ensuring that aid was distributed efficiently while respecting capacity limits. The final solution provided an optimized allocation of resources, balancing supply availability with demand needs in the affected regions.

The linear programming problem represented in Python code is as follows:

```
# how much aid in kg to deliver along each route
X = cp.Variable((m,n,r))
# minimize total transport distance while meeting constraints
obj = cp.Minimize(cp.sum(cp.multiply(np.dstack([C]*r),X)))
# limit amount of each item each supply location can send
constraint1 = [cp.sum(X[i,:,k]) <= s[i,k] for i in range(m) for k in range(r)]
# limit amount of each item each demand location can receive
constraint2 = [cp.sum(X[:,j,k]) >= d[j,k] for j in range(n) for k in range(r)]
# no sending negative amounts
constraint3 = [X >= 0]
# limit aid deliveries to not exceed transport capacity for each supply location
constraint4 = [cp.sum(cp.multiply(np.dstack([C]*r),X)[i,:,:]) <= t[i] for i in range(m)]

constraints = constraint1 + constraint2 + constraint3 + constraint4
prob = cp.Problem(obj, constraints)
```

Results

Our optimization results show the following key insights.

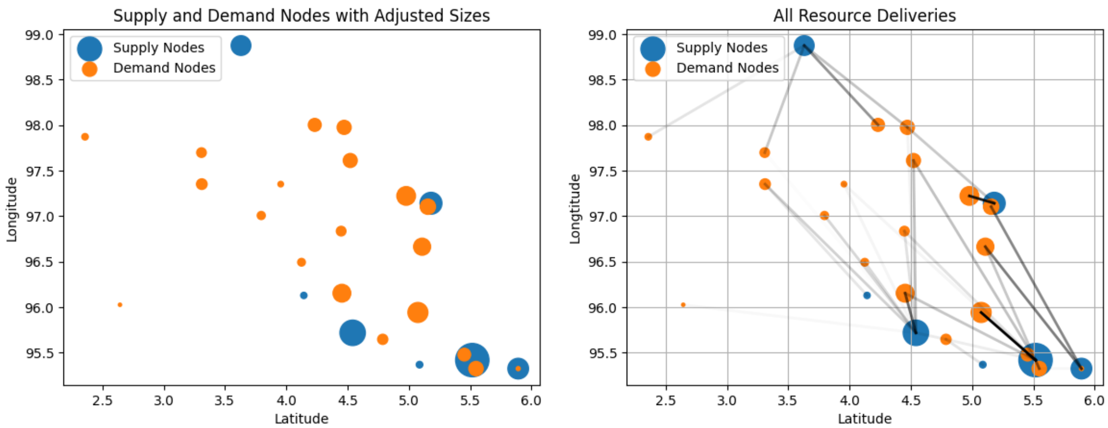

Figure 4: Side-by-side comparison of weighted nodes and ideal paths given by the solver

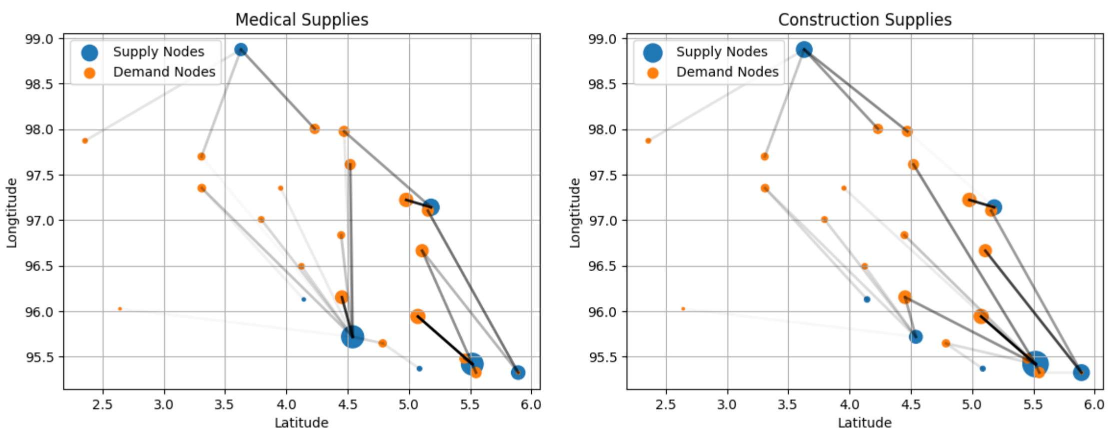

Figure 5: Breakdown of deliveries by resource type

The optimization model successfully minimizes the total transportation cost while satisfying the supply and demand constraints for each node. The objective function value of 84948.92 represents the total minimized transportation cost, which accounts for the Euclidean distances between supply and demand locations, as well as the capacities of the available helicopters. The results demonstrate that certain supply nodes, particularly those with higher transport capacities, are used more heavily to meet the needs of demand nodes with higher aid requirements. Conversely, demand nodes with lower requirements see less supply allocation.

Further Remarks

From our exploration, we found that a further extension could be to explore different modes of transport (sea, land, air) and how that would impact efficiency and resource allocation. Each mode introduces unique considerations such as speed, capacity, cost efficiency, and route constraints. One possible approach would be to introduce multi-modal transport optimization, where aid shipments could transition between different modes based on cost and time trade-offs. For instance, helicopters could be used for urgent deliveries to remote locations, while trucks or boats could handle bulk shipments for efficiency. This would require incorporating additional constraints and variables in our model to account for transfer points, mode-switching costs, and different travel speeds.

Another important extension would be to refine routing strategies. In our current model, helicopters deliver supplies directly from supply nodes to demand nodes. However, allowing helicopters to visit multiple demand nodes in a single trip before returning to a supply node could improve efficiency. This change would alter the problem structure, shifting it from a linear transport optimization to a combinatorial optimization routing problem. Optimizing routes for multi-stop deliveries while considering additional constraints such as fuel limitations, time or priority constraints, while minimizing total distance traveled. By optimizing both route selection and resource allocation, this approach could significantly enhance the effectiveness of humanitarian aid allocation efforts.

Realistically, many organizations – including humanitarian agencies, governments, and military forces from all around the world – collaborate during and after a disaster, each with different resources, transportation capacities and priorities. This is a challenge highlighted in the sources we've used for this research on the 2004 tsunami, where much of the aid either didn't reach the areas that needed it most, was duplicated, or was not the type of aid required. It would be useful to consider how these organizations can coordinate their efforts to avoid duplication of resources, reduce wastage, and ensure efficient aid distribution.

Appendix

Name	Latitude	Longitude	No. helicopters
Sultan Iskandar Muda International Airport	5.51808	95.4173	33
Kualanamu International Airport	3.6312	98.87507	7
Meulaboh	4.14369	96.12811	2
Lhokseumawe	5.18116	97.14132	13
Pasar Lamno	5.0877	95.36713	5
Panga	4.54189	95.71732	12
Sabang	5.8926	95.32376	15
Aceh Offshore	-	-	30

Table 1: Information of Aceh Supply Nodes

Name	Latitude	Longitude	Population Estimates in 2004
Banda Aceh	5.54829	95.32375	192,000
Sabang	5.8926	95.32376	15,300
Aceh Besar Regency	5.45291	95.47778	148,450
East Aceh Regency	4.52241	97.61142	181,170
Bireuen Regency	5.10864	96.66381	269,400
Lhokseumawe City	5.15569	97.10363	217,200
North Aceh Regency	4.97863	97.22214	323,600
Aceh Tamiang Regency	4.23288	98.00288	155,120
Langsa	4.47253	97.97563	183,400
Southeast Aceh Regency	3.30886	97.69822	84,620
Aceh Singkil Regency	2.35894	97.87216	38,620
Simeulue Regency	2.64397	96.02557	10,255
South Aceh Regency	3.3115	97.35165	104,325
Gayo Lues Regency	3.95516	97.35165	28,595
Southwest Aceh Regency	3.79634	97.00683	60,165
Central Aceh Regency	4.44826	96.83509	86,360
Nagan Raya Regency	4.12484	96.49297	54,465
West Aceh Regency	4.45427	96.15269	292,700
Pidie Regency	5.07426	95.94097	370,320
Aceh Jaya Regency	4.78736	95.64579	95,300

Table 2: Information of Aceh Demand Nodes

References

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