1. Kinematics

Decay rate and cross section (Note: $\mathcal{M}$ has a mass dimension of $4 - N_\mathrm{i} - N_\mathrm{f}$.)

$$\text{decay rate (rest frame; } \sqrt{s} = M_0\text{)}: \quad \mathrm{d}\Gamma = \frac{\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}}{2M_0} \left|\mathcal{M}(M_0 \to \{p_1, p_2, \cdots, p_{N_\mathrm{f}}\})\right|^2. \tag{1.1}$$

$$\text{cross section (Lorentz invariant)}: \quad \mathrm{d}\sigma = \frac{\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}}{2E_A 2E_B\, v_{\text{Møl}}} \left|\mathcal{M}(p_A, p_B \to \{p_1, p_2, \cdots, p_{N_\mathrm{f}}\})\right|^2. \tag{1.2}$$

Lorentz-invariant phase space integrals

$$\mathrm{d}\Pi := \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \frac{1}{2E_{\boldsymbol{p}}}, \qquad \overline{\mathrm{d}\Pi^n} := \mathrm{d}\Pi_1\, \mathrm{d}\Pi_2 \cdots \mathrm{d}\Pi_n\, (2\pi)^4\, \delta^{(4)}\left(P_0 - \sum p_n\right). \tag{1.3}$$

Two-body final state (CM frame) With the final momentum $\|\boldsymbol{p}\|$ and solid angle $\Omega = (\cos\theta, \phi)$,

$$\overline{\mathrm{d}\Pi^2}\Big|_{\mathrm{CM}} = \frac{\|\boldsymbol{p}\|}{4\pi\sqrt{s}} \frac{\mathrm{d}\Omega}{4\pi} = \frac{\|\boldsymbol{p}\|}{8\pi\sqrt{s}}\, \mathrm{d}\cos\theta = \frac{\lambda^{(1/2)}(s, m_1^2, m_2^2)}{16\pi s}\, \mathrm{d}\cos\theta \qquad \left(\sqrt{s} = M_0 \text{ or } E_{\mathrm{CM}}\right). \tag{1.4}$$

$$\|\boldsymbol{p}\| = \frac{\sqrt{s}}{2}\, \lambda^{1/2}\left(1; \frac{m_1^2}{s}, \frac{m_2^2}{s}\right), \quad E_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \quad E_2 = \frac{s - m_1^2 + m_2^2}{2\sqrt{s}}, \quad p_1 \cdot p_2 = \frac{s - (m_1^2 + m_2^2)}{2}.$$

2-to-2 cross section:

$$\mathrm{d}\sigma = \frac{\overline{\mathrm{d}\Pi^2}}{4E_A E_B\, v_{\text{Møl}}} |\mathcal{M}|^2 = \frac{\overline{\mathrm{d}\Pi^2}}{2\, \lambda^{(1/2)}(s, m_A^2, m_B^2)} |\mathcal{M}|^2 \overset{\mathrm{CM}}{=} \frac{1}{32\pi s} \sqrt{\frac{\lambda(s, m_1^2, m_2^2)}{\lambda(s, m_A^2, m_B^2)}}\, \mathrm{d}\cos\theta\, |\mathcal{M}|^2 \tag{1.5}$$

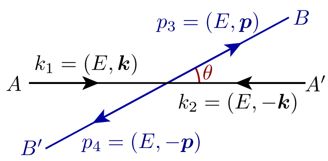
Mandelstam variables For $(k_1, k_2) \to (p_3, p_4)$ collision,

$$\begin{aligned}
&s = (k_1 + k_2)^2 = (p_3 + p_4)^2, && t = (p_3 - k_1)^2 = (p_4 - k_2)^2, && u = (p_3 - k_2)^2 = (p_4 - k_1)^2; \\
&k_1 \cdot k_2 = (s - m_1^2 - m_2^2)/2, && k_1 \cdot p_3 = (m_3^2 + m_1^2 - t)/2, && s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2, \\
&p_3 \cdot p_4 = (s - m_3^2 - m_4^2)/2, && k_1 \cdot p_4 = (m_4^2 + m_1^2 - u)/2; && \\
&(k_1 - k_2)^2 = 2(m_1^2 + m_2^2) - s, && (p_3 - p_4)^2 = 2(m_3^2 + m_4^2) - s. &&
\end{aligned}$$

If the collision is with the "same mass" $(m_A, m_A) \to (m_B, m_B)$,

$$t = m_A^2 + m_B^2 - s/2 + 2kp\cos\theta, \qquad k = \sqrt{s/4 - m_A^2},$$

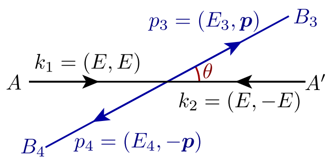
$$u = m_A^2 + m_B^2 - s/2 - 2kp\cos\theta, \qquad p = \sqrt{s/4 - m_B^2}.$$



Instead, if the collision is "initially massless" $(0, 0) \to (m_3, m_4)$,

$$t = (m_3^2 + m_4^2 - s)/2 + p\sqrt{s}\cos\theta, \qquad p = (\sqrt{s}/2)\, \lambda^{1/2}\left(1; m_3^2/s, m_4^2/s\right),$$

$$u = (m_3^2 + m_4^2 - s)/2 - p\sqrt{s}\cos\theta.$$



Three-body final state Mandelstan variables can be defined, for $P \to (p_1, p_2, p_3)$, as

$$s_{ij} = (p_i + p_j)^2; \quad t_{0i} = (P - p_i)^2 = s_{jk}; \quad s_{12} + s_{23} + s_{31} = P^2 + p_1^2 + p_2^2 + p_3^2.$$

For spherically-symmetric processes, the phase-space integral is reduced to, at the center-of-mass frame,

$$\int \overline{\mathrm{d}\Pi^3}_{\text{sph. sym.}}(\sqrt{s}, \mathbf{0}) = \frac{1}{128\pi^3} \frac{1}{s} \int_{(m_2+m_3)^2}^{(\sqrt{s}-m_1)^2} \mathrm{d}s_{23} \int \mathrm{d}s_{13}; \tag{1.6}$$

$$(s_{13})_{\min}^{\max} = \frac{(s + m_3^2 - m_1^2 - m_2^2)^2}{4s_{23}} - \frac{1}{4s_{23}} \left[\lambda^{1/2}(s, m_1^2, s_{23}) \mp \lambda^{1/2}(s_{23}, m_2^2, m_3^2)\right]^2 \tag{1.7}$$

$$= (E_1^* + E_3^*)^2 - \left(\sqrt{E_1^{*2} - m_1^2} \mp \sqrt{E_3^{*2} - m_3^2}\right)^2, \tag{1.8}$$

where $E_1^* = \frac{s - s_{23} - m_1^2}{2\sqrt{s_{23}}}$, and $E_3^* = \frac{s_{23} - m_2^2 + m_3^2}{2\sqrt{s_{23}}}$.

1.1. Fundamentals

Lorentz-invariant phase space:

$$\int \mathrm{d}\Pi = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2E_{\boldsymbol{p}}} = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\frac{1}{2\sqrt{m^2+\|\boldsymbol{p}\|^2}} = \int \frac{\mathrm{d}p_0\,\mathrm{d}^3\boldsymbol{p}}{(2\pi)^4}(2\pi)\,\delta\left(p_0^2-\|\boldsymbol{p}\|^2-m^2\right)\Theta(p_0)$$

Källén function:

$$\lambda(x,y,z) = x^2+y^2+z^2-2xy-2yz-2zx = (x-y-z)^2-4yz;$$

$$\lambda(1;\alpha_1^2,\alpha_2^2) = (1-(\alpha_1+\alpha_2)^2)(1-(\alpha_1-\alpha_2)^2) = (1+\alpha_1+\alpha_2)(1-\alpha_1-\alpha_2)(1+\alpha_1-\alpha_2)(1-\alpha_1+\alpha_2).$$

$$\lambda^{1/2}\left(s;m_1^2,m_2^2\right) = s\,\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right); \qquad \lambda^{1/2}\left(1;\frac{m^2}{s},\frac{m^2}{s}\right) = \sqrt{1-\frac{4m^2}{s}},$$

$$\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right) = \sqrt{1-\frac{2(m_1^2+m_2^2)}{s}+\frac{(m_1^2-m_2^2)^2}{s^2}}, \qquad \lambda^{1/2}\left(1;\frac{m_1^2}{s},0\right) = \frac{s-m_1^2}{s}.$$

Two-body phase space If $f(p_1^\mu,p_2^\mu)$ is Lorentz invariant, $f\equiv f(p_1^2,p_2^2,p_1^\mu p_{2\mu})\equiv f(p_1,p_2,\cos\theta_{12})$. Meanwhile,

$$\int \mathrm{d}\Pi_1\,\mathrm{d}\Pi_2 = \int\frac{\mathrm{d}^3\boldsymbol{p}_1}{(2\pi)^3}\frac{\mathrm{d}^3\boldsymbol{p}_2}{(2\pi)^3}\frac{1}{2E_12E_2} = \int\frac{(4\pi)\,\mathrm{d}p_1\,p_1^2}{(2\pi)^3}\frac{(2\pi)\,\mathrm{d}p_2\,p_2^2\,\mathrm{d}\cos\theta_{12}}{(2\pi)^3}\frac{1}{2E_12E_2} = \int\frac{\mathrm{d}E_+\,\mathrm{d}E_-\,\mathrm{d}s}{128\pi^4}, \tag{1.9}$$

with the replacement of the variables

$$E_\pm = E_1\pm E_2, \qquad s=(p_1+p_2)^2 = m_1^2+m_2^2+2E_1E_2-2\|\boldsymbol{p}_1\|\|\boldsymbol{p}_2\|\cos\theta_{12};$$

$$\left|\frac{\mathrm{d}(E_+,E_-,s)}{\mathrm{d}(p_1,p_2,\cos\theta_{12})}\right| = \frac{4p_1^2p_2^2}{E_1E_2}, \qquad \left|\frac{\mathrm{d}(E_1,E_2,s)}{\mathrm{d}(p_1,p_2,\cos\theta_{12})}\right| = \frac{2p_1^2p_2^2}{E_1E_2}.$$

Therefore,

$$\int\mathrm{d}\Pi_1\,\mathrm{d}\Pi_2 = \frac{1}{128\pi^4}\int_{(m_1+m_2)^2}^\infty \mathrm{d}s\int_{\sqrt{s}}^\infty\mathrm{d}E_+\int_{\min}^{\max}\mathrm{d}E_-, \tag{1.10}$$

where the boundary of E_- is given by

$$\cos\theta_{12} = \frac{E_+^2-E_-^2+2\left(m_1^2+m_2^2-s\right)}{\sqrt{(E_++E_-)^2-4m_1^2}\sqrt{(E_+-E_-)^2-4m_2^2}}\in[-1,1]$$

$$\therefore\quad \left|E_- - \frac{m_1^2-m_2^2}{s}E_+\right| \leqslant \sqrt{E_+^2-s}\cdot\lambda^{1/2}\left(1;\frac{m_1^2}{s},\frac{m_2^2}{s}\right) = 2p\sqrt{\frac{E_+^2-s}{s}}.$$

Two-body phase space with momentum conservation As a general representation in any frame,

$$\overline{\mathrm{d}\Pi^2} = \frac{\mathrm{d}p_1\,\mathrm{d}\Omega\,p_1^2}{16\pi^2}\frac{\delta(E_0-\sqrt{m_1^2+p_1^2}-\sqrt{m_2^2+\|\boldsymbol{P}_0-\boldsymbol{p}_1\|^2})}{E_1E_2} = \frac{1}{8\pi}\,\mathrm{d}\cos\theta_1\frac{p_1^2}{E_0p_1-P_0E_1\cos\theta_1}, \tag{1.11}$$

where the momentum p_1 is given by

$$p_1 = \frac{(E_0^2+m_1^2-m_2^2-P_0^2)P_0\cos\theta_1+E_0\sqrt{\lambda(E_0^2-P_0^2,m_1^2,m_2^2)-4m_1^2P_0^2\sin^2\theta_1}}{2(E_0^2-P_0^2\cos^2\theta_1)}. \tag{1.12}$$

CM frame result is recovered by setting $E_0=\sqrt{s}$ and $P_0=0$.

1.2. Decay rate and Cross section

As $\langle\text{out}|\text{in}\rangle = (2\pi)^4\,\delta^{(4)}(p_\mathrm{i}-p_\mathrm{f})\mathrm{i}\mathcal{M}$ (for in $\neq$ out) and $\langle\boldsymbol{p}|\boldsymbol{p}\rangle = 2E_{\boldsymbol{p}}(2\pi)^3\,\delta^{(3)}(\boldsymbol{0}) = 2E_{\boldsymbol{p}}V$ for one-particle state,

$$\frac{N_\mathrm{ev}}{\prod_\mathrm{in}N_\mathrm{particle}} = \int\mathrm{d}\Pi^\mathrm{out}\frac{|\langle\text{out}|\text{in}\rangle|^2}{\langle\text{in}|\text{in}\rangle} = \int\mathrm{d}\Pi^\mathrm{out}\frac{(2\pi)^8|\mathcal{M}|^2}{\prod_\mathrm{in}(2E)V}\frac{VT}{(2\pi^4)}\,\delta^{(4)}(p_\mathrm{i}-p_\mathrm{f}) = VT\int\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}\frac{|\mathcal{M}|^2}{\prod_\mathrm{in}(2E)V}. \tag{1.13}$$

Therefore, decay rate (at the rest frame) is given by

$$\mathrm{d}\Gamma := \frac{1}{T}\frac{\mathrm{d}N_\mathrm{ev}}{N_\mathrm{particle}} = \frac{1}{T}VT\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}\frac{|\mathcal{M}|^2}{(2E)V} = \frac{1}{2M_0}\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}|\mathcal{M}|^2. \tag{1.14}$$

We also define Lorentz-invariant cross section σ by $N_\mathrm{ev} =: (\rho_Av_\mathrm{Møl}T\sigma)N_B = (\rho_Av_\mathrm{Møl}T\sigma)(\rho_BV)$, or

$$\mathrm{d}\sigma := \frac{\mathrm{d}N_\mathrm{ev}}{\rho_Av_\mathrm{Møl}TN_B} = \frac{V}{v_\mathrm{Møl}T}VT\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}\frac{|\mathcal{M}|^2}{2E_A2E_BV^2} = \frac{1}{2E_A2E_B\,v_\mathrm{Møl}}\overline{\mathrm{d}\Pi^{N_\mathrm{f}}}|\mathcal{M}|^2. \tag{1.15}$$

where the Møller parameter $v_\mathrm{Møl}$ is equal to $v_\mathrm{rel}^\mathrm{NR} = \|\boldsymbol{v}_A-\boldsymbol{v}_B\|$ if $\boldsymbol{v}_A\,/\!\!/\,\boldsymbol{v}_B$ (cf. Ref. [1]). Generally,

$$v_\mathrm{Møl} := \frac{\sqrt{(p_A\cdot p_B)^2-m_A^2m_B^2}}{E_AE_B} = \frac{\sqrt{\lambda(s,m_A^2,m_B^2)}}{2E_AE_B} = \frac{p_A\cdot p_B}{E_AE_B}v_\mathrm{rel} = (1-\boldsymbol{v}_A\cdot\boldsymbol{v}_B)v_\mathrm{rel}, \tag{1.16}$$

where v_rel is the actual relative velocity

$$v_\mathrm{rel} = \sqrt{1-\frac{(1-v_A^2)(1-v_B^2)}{1-(\boldsymbol{v}_A\cdot\boldsymbol{v}_B)^2}} = \frac{\sqrt{\|\boldsymbol{v}_A-\boldsymbol{v}_B\|^2-\|\boldsymbol{v}_A\times\boldsymbol{v}_B\|^2}}{1-\boldsymbol{v}_A\cdot\boldsymbol{v}_B} = \frac{\lambda^{1/2}(s,m_A^2,m_B^2)}{s-(m_A^2+m_B^2)}\neq v_\mathrm{rel}^\mathrm{NR}. \tag{1.17}$$

(Note that $p_A\cdot p_B/E_AE_B = 1$ if $\boldsymbol{p}_A=0$ or $\boldsymbol{p}_B=0$. Also, Each of v_rel, VT, and $E_AE_Bv_\mathrm{Møl}$ is Lorentz invariant.)

1.3. Three body phase space

The phase-space reduction utilizes the identity [2]

$$1 = \int \frac{\mathrm{d}^4 p_{ij}}{(2\pi)^4} (2\pi)^4 \, \delta^{(4)}(p_{ij} - p_i - p_j)\Theta(p_{ij}^0) \tag{1.18}$$

$$= \int \frac{\mathrm{d}^4 p_{ij}}{(2\pi)^4} \left[\int \frac{\mathrm{d}s}{2\pi} (2\pi) \, \delta(s - p_{ij}^2) \right] (2\pi)^4 \, \delta^{(4)}(p_{ij} - p_i - p_j)\Theta(p_{ij}^0) \tag{1.19}$$

$$= \int \frac{\mathrm{d}^3 \boldsymbol{p}_{ij}}{(2\pi)^3} \frac{\mathrm{d}s}{2\pi} \frac{1}{2p_{ij}^0} (2\pi)^4 \, \delta^{(4)}(p_{ij} - p_i - p_j) \Big|_{p_{ij}^0 = \sqrt{s + \|\boldsymbol{p}_{ij}\|^2}}. \tag{1.20}$$

For three-body phase space,

$$\begin{aligned}\overline{\mathrm{d}\Pi^3} = \int \mathrm{d}\Pi_1 \frac{\mathrm{d}^4 p_2 \, \mathrm{d}^4 p_3}{(2\pi)^8} &(2\pi) \, \delta(p_2^2 - m_2^2)(2\pi) \, \delta(p_3^2 - m_3^2)\Theta(p_2^0)\Theta(p_3^0)(2\pi)^4 \, \delta^{(4)} \left(P - p_1 - p_2 - p_3\right) \\ &\times \frac{\mathrm{d}^3 \boldsymbol{p}_{23}}{(2\pi)^3} \frac{\mathrm{d}s_{23}}{2\pi} \frac{1}{2p_{23}^0} (2\pi)^4 \, \delta^{(4)}(p_{23} - p_2 - p_3) \Big|_{p_{23}^0 = \sqrt{s_{23} + \|\boldsymbol{p}_{23}\|^2}}\end{aligned} \tag{1.21}$$

$$\begin{aligned}= \int \frac{\mathrm{d}s_{23}}{2\pi} \int \mathrm{d}\Pi_1 &\frac{\mathrm{d}^3 \boldsymbol{p}_{23}}{(2\pi)^3} \frac{1}{2p_{23}^0} (2\pi)^4 \, \delta^{(4)}(P - p_1 - p_{23}) \Big|_{p_{23}^0 = \sqrt{s_{23} + \|\boldsymbol{p}_{23}\|^2}} \\ &\times \frac{\mathrm{d}^4 p_2 \, \mathrm{d}^4 p_3}{(2\pi)^8} (2\pi) \, \delta(p_2^2 - m_2^2)(2\pi) \, \delta(p_3^2 - m_3^2)\Theta(p_2^0)\Theta(p_3^0)(2\pi)^4 \, \delta^{(4)}(p_{23} - p_2 - p_3).\end{aligned} \tag{1.22}$$

$$= \int \frac{\mathrm{d}s_{23}}{2\pi} \int \mathrm{d}\Pi_1 \frac{\mathrm{d}^3 \boldsymbol{p}_{23}}{(2\pi)^3} \frac{1}{2p_{23}^0} (2\pi)^4 \, \delta^{(4)}(P - p_1 - p_{23}) \Big|_{p_{23}^0 = \sqrt{s_{23} + \|\boldsymbol{p}_{23}\|^2}} \times \overline{\mathrm{d}\Pi^2}(p_{23}^0, \boldsymbol{p}_{23}) \tag{1.23}$$

and $\overline{\mathrm{d}\Pi^2}(p_{23}^0, \boldsymbol{p}_{23})$ is given by Eq. (1.11); explicitly,

$$\overline{\mathrm{d}\Pi^2}(p_{23}^0, \boldsymbol{p}_{23}) = \frac{\mathrm{d}\cos\theta_2}{8\pi} \frac{p_2^2}{p_{23}^0 p_2 - \|\boldsymbol{p}_{23}\| \sqrt{p_2^2 + m_2^2} \cos\theta_2}; \tag{1.24}$$

$$p_2 = \frac{(s_{23} + m_2^2 - m_3^2)\|\boldsymbol{p}_{23}\| \cos\theta_2 + p_{23}^0 \sqrt{\lambda(s_{23}, m_2^2, m_3^2) - 4m_2^2 \|\boldsymbol{p}_{23}\|^2 \sin^2\theta_2}}{2(s_{23} + \|\boldsymbol{p}_{23}\|^2 \sin^2\theta_2)}, \tag{1.25}$$

where θ_2 is the angle between $\boldsymbol{p}_{23}$ and $\boldsymbol{p}_2$ (in the lab frame).

If the matrix element to integrate is spherically symmetric, so as $\overline{\mathrm{d}\Pi^2}(p_{23}^0, \boldsymbol{p}_{23})|\mathcal{M}|^2$, i.e., it is independent of the angle of $\boldsymbol{p}_{23}$. Then one can simply evaluate $\int \mathrm{d}^3 \boldsymbol{p}_{23}$, which leads to, in the center-of-mass frame,

$$\overline{\mathrm{d}\Pi^3}_{\text{sph. sym.}}(\sqrt{s}, \boldsymbol{0}) = \frac{\mathrm{d}s_{23} \, \mathrm{d}\cos\theta_2}{64\pi^3} \frac{p_1}{\sqrt{s}} \frac{p_2^2}{p_2 \sqrt{s_{23} + p_1^2} - p_1 \sqrt{p_2^2 + m_2^2} \cos\theta_2} \Big|_{p_1^2 = \lambda(s, m_1^2, s_{23})/4s} = \frac{s}{128\pi^3} \mathrm{d}x_1 \, \mathrm{d}x_2, \tag{1.26}$$

where we defined $x_i := 2E_i/\sqrt{s}$. Noting that $s_{23} = s + m_1^2 - 2E_1\sqrt{s} = s(1 - x_1) + m_1^2$ etc.,

$$\overline{\mathrm{d}\Pi^3}_{\text{sph. sym.}}(\sqrt{s}, \boldsymbol{0}) = \frac{1}{128\pi^3} \frac{1}{s} \int_{(m_2 + m_3)^2}^{(\sqrt{s} - m_1)^2} \mathrm{d}s_{23} \int \mathrm{d}s_{13}; \tag{1.27}$$

$$(s_{13})_{\text{min}}^{\text{max}} = \frac{(s + m_3^2 - m_1^2 - m_2^2)^2}{4s_{23}} - \frac{1}{4s_{23}} \left[\lambda^{1/2}(s, m_1^2, s_{23}) \mp \lambda^{1/2}(s_{23}, m_2^2, m_3^2) \right]^2. \tag{1.28}$$

This is equal to the PDG-Eq. (47.23) [PDG2018].

2. Gauge theory

SU(2) Fundamental representation $\mathbf{2} = (T^a)_{ij}$, adjoint representation adj. $= (\epsilon^a)^{bc}$.[*1]

$$T_a = \frac{1}{2}\sigma_a, \qquad \mathrm{Tr}(T_a T_b) = \frac{1}{2}\delta_{ab}, \qquad [T_a, T_b] = \mathrm{i}\epsilon^{abc}T^c, \qquad \epsilon^{abc}\epsilon^{ade} = \delta_{bd}\delta_{ce} - \delta_{be}\delta_{cd}$$

Since $\overline{\mathbf{2}} = -(T^a)^*_{ij}$ has identities $-\epsilon T^a \epsilon = -T^{a*}$ and $-\epsilon(-T^{a*})\epsilon = T^a$, we see that $\epsilon^{ab}\mathbf{2}^b$ transforms as $\overline{\mathbf{2}}^a$:

$$\epsilon^{ab}\mathbf{2}^b \to \epsilon^{ab}[\exp(\mathrm{i}g\theta^\alpha T^\alpha)]^{bc}\mathbf{2}^c = \epsilon^{ab}[\exp(\mathrm{i}g\theta^\alpha T^\alpha)]^{bc}(\epsilon^{-1})^{cd}(\epsilon^{de}\mathbf{2}^e) = [\exp(-\mathrm{i}g\theta^\alpha T^{\alpha*})]^{ab}(\epsilon^{bc}\mathbf{2}^c). \tag{2.1}$$

SU(3) Fundamental representation $\mathbf{3} = (\tau^a)_{ij}$, $\overline{\mathbf{3}} = -(\tau^a)^*_{ij}$; adjoint representation adj. $= \mathbf{8} = (f^a)^{bc}$.
Gell-Mann matrices:

$$\lambda_{1\text{–}8} = \begin{pmatrix} 0&1&0\\1&0&0\\0&0&0 \end{pmatrix}, \begin{pmatrix} 0&-\mathrm{i}&0\\ \mathrm{i}&0&0\\0&0&0 \end{pmatrix}, \begin{pmatrix} 1&0&0\\0&-1&0\\0&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&1\\0&0&0\\1&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&-\mathrm{i}\\0&0&0\\ \mathrm{i}&0&0 \end{pmatrix}, \begin{pmatrix} 0&0&0\\0&0&1\\0&1&0 \end{pmatrix}, \begin{pmatrix} 0&0&0\\0&0&-\mathrm{i}\\0&\mathrm{i}&0 \end{pmatrix}, \frac{1}{\sqrt{3}}\begin{pmatrix} 1&0&0\\0&1&0\\0&0&-2 \end{pmatrix}. \tag{2.2}$$

$$\tau_a = \frac{1}{2}\lambda_a, \qquad \mathrm{Tr}(\tau_a\tau_b) = \frac{1}{2}\delta_{ab}, \qquad [\tau_a, \tau_b] = \mathrm{i}f^{abc}\tau^c, \qquad f^{ade}f^{bcd} + f^{bde}f^{cad} + f^{cde}f^{abd} = 0.$$

$$\begin{aligned} \mathbf{3}:\ \phi_a &\to [\exp(\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ab}\phi_b \simeq \phi_a + \mathrm{i}g\theta^\alpha\tau^\alpha_{ab}\phi_b \\ \phi^*_a &\to [\exp(-\mathrm{i}g\theta^\alpha\tau^{\alpha*})]_{ab}\phi^*_b \simeq \phi^*_a - \mathrm{i}g\theta^\alpha\tau^{\alpha*}_{ab}\phi^*_b \\ &= \phi^*_b[\exp(-\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ba} \simeq \phi^*_a - \mathrm{i}g\theta^\alpha\phi^*_b\tau^\alpha_{ba} \end{aligned}$$

$$\begin{aligned} \overline{\mathbf{3}}:\ \phi_a &\to [\exp(-\mathrm{i}g\theta^\alpha\tau^{\alpha*})]_{ab}\phi_b \simeq \phi_a - \mathrm{i}g\theta^\alpha\tau^{\alpha*}_{ab}\phi_b \\ \phi^*_a &\to [\exp(\mathrm{i}g\theta^\alpha\tau^\alpha)]_{ab}\phi^*_b \simeq \phi^*_a + \mathrm{i}g\theta^\alpha\tau^\alpha_{ab}\phi^*_b \end{aligned}$$

[*1] We do not distinguish sub- and superscripts for gauge indices.

A Mathematica code for the Gell-Mann matrices is:

```
GellMann[0] := DiagonalMatrix[{1,1,1}]/Sqrt[3/2]
GellMann[8] := DiagonalMatrix[{1,1,-2}]/Sqrt[3]
GellMann[a:1|2|3|4|5|6|7] := Module[
   {p=Switch[a,1|2|3,{1,2,0},4|5,{1,0,2},6|7,{0,1,2}]},
   Table[If[i*j==0, 0, PauliMatrix[{1,2,3,1,2,1,2}[[a]]][[i,j]]], {i,p}, {j,p}]]
```

3. Spinors

$$(\overline{\psi_1}\psi_2)^* = (\psi_2)^\dagger(\overline{\psi}_1)^\dagger = \overline{\psi_2}\psi_1. \tag{3.1}$$

3.1. Convention ♣TODO: WIP!♣

First we prepare a vector x^μ and a symmetric matrix $\eta^{\mu\nu}$, which we call "contravariant vector" x^μ and the metric $\eta^{\mu\nu}$. Then we perform a Lorentz transformation on x^μ to obtain $(x')^\mu$, with which we can define a matrix $\Lambda(\boldsymbol{v}a, \boldsymbol{\theta})^\mu_\nu$ through $\Lambda^\mu{}_\nu x^\nu = (x')^\mu$.

We then consider Λs for infinitesimal transformations and define $\boldsymbol{S}$, $\boldsymbol{J}$, and $\boldsymbol{K}$ by

$$\Lambda^\mu{}_\nu \simeq \delta^\mu_\nu - \mathrm{i}(\boldsymbol{\theta}\cdot\boldsymbol{J}^\mu{}_\nu + \boldsymbol{\beta}\cdot\boldsymbol{K}^\mu{}_\nu) \simeq \delta^\mu_\nu - \frac{\mathrm{i}}{2}\left[\Lambda^{\alpha\beta}S_{\alpha\beta}\right]^\mu{}_\nu \tag{3.2}$$

Imposing "Lorentz condition" (♣TODO: what?♣), we get the expression for $S = \mathrm{i}(\delta\cdots)$ and $[J^i, J^j] = \cdots$; further, we get $\Lambda^\mu_\nu = \exp(-\mathrm{i}\boldsymbol{\theta}\cdot\boldsymbol{J} - \mathrm{i}\boldsymbol{\xi}\cdot\boldsymbol{K})$, $\boldsymbol{\theta} = (\theta_{23}, \theta_{31}, \theta_{12})$, $\boldsymbol{\xi} = \hat{\boldsymbol{v}}\tanh^{-1}\|\boldsymbol{v}\| = (\theta^{10}, \theta^{20}, \theta^{30})$; $J = (S_{23}, S_{31}, S_{12})$, $K = (S^{01}, S^{02}, S^{03})$....?

Lorentz transformation with a rotation θ around an axis $\hat{\boldsymbol{\theta}}$ and a boost $\boldsymbol{v}$ are given by

$$\Lambda = \exp\left[-\mathrm{i}(\boldsymbol{\theta}\cdot\boldsymbol{J} + \boldsymbol{\beta}\cdot\boldsymbol{K})\right]; \qquad \boldsymbol{\theta} := \theta\hat{\boldsymbol{\theta}}, \quad \boldsymbol{\beta} := \hat{\boldsymbol{v}}\tanh^{-1}\|\boldsymbol{v}\|, \tag{3.3}$$

♣TODO: check!♣

Lorentz transformation (infinitesimal): $\Lambda = \begin{pmatrix} 0 & & \boldsymbol{\beta}^{\mathrm{T}} & \\ & 0 & -\theta_z & \theta_y \\ \boldsymbol{\beta} & \theta_z & 0 & -\theta_x \\ & -\theta_y & \theta_x & 0 \end{pmatrix}$

$[J_{\mu\nu}]^\alpha{}_\beta = \mathrm{i}(\delta^\alpha_\mu\eta_{\nu\beta} - \delta^\alpha_\nu\eta_{\mu\beta})$

Lorentz tensor $M^{\mu_1\mu_2\cdots\mu_n} \propto \bar{\sigma}^{\mu_1\dot{\beta}_1\alpha_1}\cdots M_{\alpha_1\cdots\dot{\beta}_1\cdots}$

Especially $V^\mu =: \frac{1}{2}\bar{\sigma}^{\mu\dot{\beta}\alpha}V_{\alpha\dot{\beta}}$, $V_{\alpha\dot{\beta}} = V^\mu\sigma_{\mu\alpha\dot{\beta}}$; hermite $V_{\alpha\dot{\beta}} \Leftrightarrow real V^\mu$.

$$(V^{\mathrm{T}})_{\alpha\dot{\beta}} = V_{\beta\dot{\alpha}}, \text{♣TODO: (correct? possibly wrong dot-positions?)♣} \tag{3.4}$$

$$(V^*)_{\dot{\alpha}\beta} := (V_{\alpha\dot{\beta}})^*, \tag{3.5}$$

$$(V^\dagger)_{\alpha\dot{\beta}} := (V_{\beta\dot{\alpha}})^* = (V^*)_{\dot{\beta}\alpha} \tag{3.6}$$

♣TODO: anyway not very sure about the reasoning; though my old note says like this...♣

In general, metric is symmetric.

$$(\Lambda^{-1})^\mu{}_\nu = \eta_{\nu\rho}\Lambda^\rho{}_\sigma(\eta^{-1})^{\sigma\mu} =: \Lambda_\nu{}^\mu \tag{3.7}$$

4. Calculation techniques

Polarization sum

$$\text{massless:} \quad \sum_{\pm} \epsilon^*_\mu(k)\epsilon_\nu(k) = -\eta_{\mu\nu} - \frac{n^2 k_\mu k_\nu}{(n\cdot k)^2} + \frac{n_\mu k_\nu + n_\nu k_\mu}{n\cdot k} \quad \Big(\rightsquigarrow -\eta_{\mu\nu} \quad \text{with Ward id.}\Big), \tag{4.1}$$

$$\text{massive:} \quad \sum_{\pm,0} \epsilon^*_\mu(k)\epsilon_\nu(k) = -\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}, \tag{4.2}$$

where $\epsilon\cdot k = 0$ is assumed and n^μ should satisfy $\epsilon\cdot n = 0$, and $k\cdot n \neq 0$ (usually $n^\mu = (1,0,0,0)^\mu$). ♣TODO : derivation♣

5. Loop calculation

Notation follows LoopTools [3]; capital Ms and Ps respectively denote squared masses and momenta.

Passarino–Veltman scalar integrals

$$A_0(M)/M = \Delta_\epsilon + \log\mu^2 + 1 - \log M, \tag{5.1}$$

$$B_0(P, M_0, M_1) = \Delta_\epsilon + \log\mu^2 - \int_0^1 \mathrm{d}x\, \log\left[-x(1-x)P + xM_1 + (1-x)M_0\right] \tag{5.2}$$

$$C_0(P_1, P_2, P_3, M_1, M_2, M_3) = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y\, \frac{x}{Q_1} \tag{5.3}$$

$$= \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y\, \frac{1}{Q_2}; \tag{5.4}$$

$$Q_1 = x(1-x)(1-y)P_2 + x^2y(1-y)P_3 + x(1-x)yP_1 - xyM_1 - (1-x)M_2 - x(1-y)M_3,$$
$$Q_2 = -P_2x^2 - P_1y^2 + (P_1+P_2-P_3)xy + (P_2-M_2+M_3)x + (M_2-M_1+P_3-P_2)y - M_3.$$

Kinematical invariance:

$$B_0(P, M_0, M_1) = B_0(P, M_1, M_0), \quad \begin{aligned} C_0(P_1, P_2, P_3, M_1, M_2, M_3) &= C_0(P_2, P_3, P_1, M_2, M_3, M_1) \\ &= C_0(P_1, P_3, P_2, M_2, M_1, M_3) \end{aligned} \tag{5.5}$$

Special cases:

$$\begin{aligned} C_0(0, P, P, M, M, M') &= \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y\, \frac{-1}{Px^2 - (P-M+M')x + M'}; \\ &= \int_0^1 \mathrm{d}x\, \frac{-x/P}{(x-\alpha)^2 - \lambda(P, M, M')/4P^2}; \qquad \alpha = (P-M+M')/2P. \end{aligned} \tag{5.6}$$

5.1. Passarino–Veltman scalar integrals

See `calculator/loop/PaVeAnalytic.wl` for validation. We use the notation [3,4]

$$\Delta_\epsilon = \frac{2}{4-d} - \gamma + \log 4\pi \equiv \texttt{GetDelta[]} \quad (= 0 \text{ in } \overline{\text{MS}}), \qquad \mu^2 \equiv \texttt{GetMudim[]}, \tag{5.7}$$

where μ is introduced due to the different mass dimension of vector and spinor fields in d-dimensional theory:

$$[A_\mu] = 1 - \frac{4-d}{2}, \quad [\psi] = \frac{3}{2} - \frac{4-d}{2}, \quad [\text{gauge couplings}] = \frac{4-d}{2} \quad \Rightarrow \quad e = (e)_{\text{4-dim.}}\mu^{(4-d)/2}. \tag{5.8}$$

The analytic form of scalar integrals are given in Refs. [4–6].

6. Cosmology

FLRW metric With a scale factor normalized by $a(t_0) = 1$,

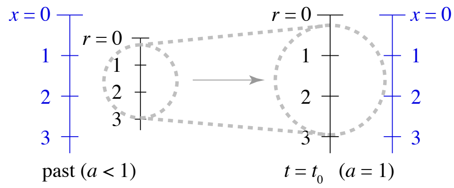
$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left[\frac{\mathrm{d}r^2}{1-Kr^2} + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2\right] \quad (6.1)$$

comoving coordinate $\boldsymbol{r} = (r\sin\theta\cos\phi, r\sin\theta\cos\phi, r\cos\theta)$,

proper coordinate $\boldsymbol{x}(t) = a(t)\boldsymbol{r}$,

comoving distance $\chi_{AB} = \displaystyle\int_{r_A}^{r_B} \frac{\mathrm{d}r}{\sqrt{1-Kr^2}}$,

proper distance $d_{AB}(t) = a(t)\chi_{AB}$.



Ricci tensor and scalar are given by

$$R_{00} = R^0{}_0 = \frac{3\ddot{a}}{a}, \qquad R_{0i} = R_{i0} = R^0{}_i = R^i{}_0 = 0, \qquad R_{ij} \neq 0,$$

$$R^i{}_j = \delta^i_j\left(\frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} + \frac{2K}{a^2}\right); \qquad R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{K}{a^2}\right). \tag{6.2}$$

Particle density For a massless particle, with $L_n^{\pm} = \pm\,\mathtt{PolyLog}(n, \pm\,\mathrm{e}^{\mu/T})$ and arrows denoting $\mu \to 0$,

$$n_{\mathrm{MB}} = \frac{\mathrm{e}^{\mu/T}}{\pi^2}gT^3 \to \frac{1}{\pi^2}gT^3, \qquad \rho_{\mathrm{MB}} = 3Tn_{\mathrm{MB}} \to \frac{3}{\pi^2}gT^4, \tag{6.3}$$

$$n_{\mathrm{BE}} = \frac{L_3^+}{\pi^2}gT^3 \to \frac{\zeta_3}{\pi^2}gT^3, \qquad \rho_{\mathrm{BE}} = \frac{3L_4^+}{\pi^2}gT^4 \to \frac{\pi^2}{30}gT^4, \tag{6.4}$$

$$n_{\mathrm{FD}} = \frac{L_3^-}{\pi^2}gT^3 \to \frac{3}{4}\frac{\zeta_3}{\pi^2}gT^3, \qquad \rho_{\mathrm{FD}} = \frac{3L_4^-}{\pi^2}gT^4 \to \frac{7}{8}\frac{\pi^2}{30}gT^4, \tag{6.5}$$

For massive particle, with $x = m/T$ and $K_n(x) = \mathtt{BesselK}(n, x)$,

$$n_{\mathrm{MB}} = g\,\mathrm{e}^{\mu/T}\cdot\frac{T^3}{2\pi^2}x^2K_2(x) \xrightarrow{x\gg1} g\,\mathrm{e}^{\mu/T}\frac{T^3}{(2\pi)^{3/2}}x^{3/2}\,\mathrm{e}^{-x}, \tag{6.6}$$

$$\rho_{\mathrm{MB}} = \left(3 + \frac{xK_1(x)}{K_2(x)}\right)Tn_{\mathrm{MB}} \xrightarrow{x\gg1} \left(m + \frac{3}{2}T + \frac{15T^2}{8m}\right)n_{\mathrm{MB}}, \qquad p_{\mathrm{MB}} = Tn_{\mathrm{MB}}. \tag{6.7}$$

6.1. FLRW metric

Two conventions are known for FLRW (Фри́дман-Lemaître-Robertson-Walker) metric:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left[\frac{\mathrm{d}r^2}{1-Kr^2} + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2\right] \qquad [r] = \text{(length)}, a \text{ is unitless with } a(t_0)=1 \tag{6.8}$$

$$= \mathrm{d}t^2 - R^2(t)\left[\frac{\mathrm{d}\tilde r^2}{1-\tilde K\tilde r^2} + \tilde r^2\,\mathrm{d}\theta^2 + \tilde r^2\sin^2\theta\,\mathrm{d}\phi^2\right] \qquad [R] = \text{(length)}, \tilde r \text{ is unitless}, \tilde K = \{0,\pm1\} \tag{6.9}$$

related by a rescaling, $R(t)/a(t) = R(t_0) \equiv R_0$, i.e., $r = \tilde r R_0$ and $K = \tilde K/R_0^2$. The curvature radius is given by $6K/a^2$ and a spherical, flat, and hyperspherical universe are respectively given by $K>0$, $K=0$, and $K<0$.

FLRW metric can have several forms. For $\{K>0, K=0, K<0\}$,

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left(\frac{\mathrm{d}r^2}{1-Kr^2} + r^2\,\mathrm{d}\Omega\right) \qquad \mathrm{d}\Omega = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2, \tag{6.10}$$

$$= \mathrm{d}t^2 - a^2(t)\left[\mathrm{d}\boldsymbol{r}^2 + \frac{K(\boldsymbol{r}\cdot\mathrm{d}\boldsymbol{r})^2}{1-K\|\boldsymbol{r}\|^2}\right] \qquad \boldsymbol{r} = (r\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\phi) \tag{6.11}$$

$$= \mathrm{d}t^2 - \left[\frac{a(t)}{1+(K/4)\rho^2}\right]^2(\mathrm{d}\rho^2 + \rho^2\,\mathrm{d}\Omega) \qquad \rho = R_0\tilde\rho := \frac{2r}{1+\sqrt{1-Kr^2}} = \frac{2\tilde r R_0}{1+\sqrt{1-\tilde K\tilde r^2}} \tag{6.12}$$

$$= \mathrm{d}t^2 - \left[\frac{R(t)}{1+(\tilde K/4)\tilde\rho^2}\right]^2(\mathrm{d}\tilde\rho^2 + \tilde\rho^2\,\mathrm{d}\Omega) \tag{6.13}$$

$$= \mathrm{d}t^2 - R^2(t)\Big(\mathrm{d}\tilde\chi^2 + \{\sin\tilde\chi, \tilde\chi, \sinh\tilde\chi\}^2\,\mathrm{d}\Omega\Big) \qquad \mathrm{d}\chi = R_0\,\mathrm{d}\tilde\chi = \frac{\mathrm{d}r'}{\sqrt{1-Kr'^2}} \quad \text{[comoving distance]} \tag{6.14}$$

$$= a^2(t)\Big(\mathrm{d}\eta^2 - \mathrm{d}\chi^2 - R_0^2\{\sin\tilde\chi, \tilde\chi, \sinh\tilde\chi\}^2\,\mathrm{d}\Omega\Big) \qquad \mathrm{d}\eta := \frac{\mathrm{d}t'}{a(t')} \quad \text{[conformal time].} \tag{6.15}$$

Explicitly, χ is given by

$$\chi = \int_0^r \frac{\mathrm{d}r'}{\sqrt{1-Kr'^2}} = \int_0^{\tilde r}\frac{R_0\,\mathrm{d}\tilde r'}{\sqrt{1-\tilde K\tilde r'^2}} = R_0\{\sin^{-1}\tilde r, \tilde r, \sinh^{-1}\tilde r\} = 2R_0\left\{\tan^{-1}\frac{\tilde\rho}{2}, \frac{\tilde\rho}{2}, \tanh^{-1}\frac{\tilde\rho}{2}\right\}. \tag{6.16}$$

The Christofffel symbol, Riemann tensor, Ricci tensor, and Ricci scalar are given by

$$\Gamma^n{}_{ij} = \frac{g^{nk}}{2}(g_{jk,i} + g_{ik,j} - g_{ij,k}), \quad R_{ijk}{}^l = \Gamma^l{}_{jk,i} - \Gamma^l{}_{ik,j} + \Gamma^a{}_{jk}\Gamma^l{}_{ai} - \Gamma^a{}_{ik}\Gamma^l{}_{aj}{}^{*2}, \quad R_{ij} = R_{ikj}{}^k, \quad R = g^{ij}R_{ij}.$$

6.2. Particle cosmology

The particle number density, pressure, and energy density are calculated from distribution functions:

$$f_{\mathrm{MB}}(\boldsymbol{k}) = \frac{g}{\mathrm{e}^{(E-\mu)/T}}, \qquad f_{\mathrm{BE}}(\boldsymbol{k}) = \frac{g}{\mathrm{e}^{(E-\mu)/T}-1}, \qquad f_{\mathrm{FD}}(\boldsymbol{k}) = \frac{g}{\mathrm{e}^{(E-\mu)/T}+1}; \tag{6.17}$$

$$n = \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}f(\boldsymbol{k}), \qquad \rho = \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}Ef(\boldsymbol{k}), \qquad p = \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}k_z v_z f(\boldsymbol{k}) = \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{k^2\cos^2\theta}{E}f(\boldsymbol{k}). \tag{6.18}$$

Note the pressure is (momentum)×(flux per time) on a "wall"; assuming MB, $p=\rho/3$ for $m \ll T$ and $p = T\rho/m$ for $m \gg T$.

A thermal average of a cross section $\sigma(s)$ is schematically given by

$$\langle\sigma v\rangle_{AB\to12\cdots n}(T) = \frac{1}{n_An_B}\int\frac{\mathrm{d}^3\boldsymbol{k}_A}{(2\pi)^3}\frac{\mathrm{d}^3\boldsymbol{k}_B}{(2\pi)^3}(f_Af_B)\Big\{\phi_1\cdots\phi_n\sigma(s)\Big\}v_{\mathrm{M\o l}}; \qquad \phi_X = \mathrm{e}^{(E-\mu)/T}f_X/g, \tag{6.19}$$

Here, the final state statistical factor $\phi_1\cdots\phi_n$ are subject to the phase space integral of the calculation of $\sigma(s)$. They are specifically given by $\phi_{\mathrm{MB}} = 1$, $\phi_{\mathrm{BE}} = 1 + f_{\mathrm{BE}}/g$, and $\phi_{\mathrm{FD}} = 1 - f_{\mathrm{FD}}/g$. Similarly, a thermal averaged decay rate is given by

$$\langle\Gamma\rangle_{A\to12\cdots n} = \frac{1}{n_A}\int\frac{\mathrm{d}^3\boldsymbol{k}_A}{(2\pi)^3}f_A\left\{\phi_1\cdots\phi_n\frac{m_A}{E_A}\Gamma\right\}. \tag{6.20}$$

With MB approximation,

$$\langle\sigma v\rangle = \frac{g_Ag_B}{n_An_B}\mathrm{e}^{(\mu_A+\mu_B)/T}\int\frac{\mathrm{d}^3\boldsymbol{k}_A}{(2\pi)^3}\frac{\mathrm{d}^3\boldsymbol{k}_B}{(2\pi)^3}\mathrm{e}^{-(E_A+E_B)/T}\sigma(s)v_{\mathrm{M\o l}} \tag{6.21}$$

$$= \int\frac{\mathrm{d}s\,\mathrm{d}E_+\,\mathrm{d}E_-}{32m_A^2m_B^2T^2K_2(m_A/T)K_2(m_B/T)}\mathrm{e}^{-E_+/T}4E_AE_B\sigma(s)v_{\mathrm{M\o l}} \quad (\times1/2 \text{ if } A=B) \tag{6.22}$$

$$= \frac{1}{16m_A^2m_B^2TK_2(m_A/T)K_2(m_B/T)}\int\frac{K_1(\sqrt{s}/T)\,\mathrm{d}s}{\sqrt{s}}\sqrt{\lambda(s,m_A^2,m_B^2)}\cdot2E_A2E_Bv_{\mathrm{M\o l}}\sigma(s) \quad (\times1/2), \tag{6.23}$$

$$\langle\Gamma\rangle = \frac{K_1(m_A/T)}{K_2(m_A/T)}\Gamma. \tag{6.24}$$

[*2] Overall sign is convention-dependent.

7. Standard Model

(summary page)

7.1. Particle content and convention

7.2. Lagrangian

7.3. Higgs mechanism

A general expression for composing a Dirac fermion from $\psi_{\mathrm{L}}(T_{3\mathrm{L}}, Y_{\mathrm{L}})$ and $\psi_{\mathrm{R}}(T_{3\mathrm{R}}, Y_{\mathrm{R}})$ is given by

$$\left(g_2 \not{W}_3 T_{3\mathrm{L}} + g_Y \not{B} Y_{\mathrm{L}}\right) P_{\mathrm{L}} + \left(g_2 \not{W}_3 T_{3\mathrm{R}} + g_Y \not{B} Y_{\mathrm{R}}\right) P_{\mathrm{R}} \tag{7.1}$$

$$= \left[\left(|e|\not{A} + g_Z \mathrm{c}_{\mathrm{w}}^2 \not{Z}\right) T_{3\mathrm{L}} + \left(|e|\not{A} - g_Z \mathrm{s}_{\mathrm{w}}^2 \not{Z}\right) Y_{\mathrm{L}}\right] P_{\mathrm{L}} + \text{(right)} \tag{7.2}$$

$$\begin{aligned} &= \frac{T_{3\mathrm{L}} + T_{3\mathrm{R}} + Y_{\mathrm{L}} + Y_{\mathrm{R}}}{2} |e|\not{A} + \frac{T_{3\mathrm{L}}\mathrm{c}_{\mathrm{w}}^2 - Y_{\mathrm{L}}\mathrm{s}_{\mathrm{w}}^2 + T_{3\mathrm{R}}\mathrm{c}_{\mathrm{w}}^2 - Y_{\mathrm{R}}\mathrm{s}_{\mathrm{w}}^2}{2} g_Z \not{Z} \\ &\quad + \frac{-T_{3\mathrm{L}} - Y_{\mathrm{L}} + T_{3\mathrm{R}} + Y_{\mathrm{R}}}{2} |e|\not{A}\gamma_5 + \frac{-\mathrm{c}_{\mathrm{w}}^2 T_{3\mathrm{L}} + \mathrm{s}_{\mathrm{w}}^2 Y_{\mathrm{L}} + \mathrm{c}_{\mathrm{w}}^2 T_{3\mathrm{R}} - \mathrm{s}_{\mathrm{w}}^2 Y_{\mathrm{R}}}{2} g_Z \not{Z}\gamma_5. \end{aligned} \tag{7.3}$$

In the SM, $T_{3\mathrm{L}} + Y_{\mathrm{L}} = Y_{\mathrm{R}} =: Q$ and $T_{3\mathrm{R}} = 0$ lead to

$$Q|e|\not{A} + g_Z \not{Z}\left(T_{3\mathrm{L}} P_{\mathrm{L}} - Q\mathrm{s}_{\mathrm{w}}^2\right). \tag{7.4}$$

7.4. Lagrangian in mass eigenstates

7.5. CKM matrix and Yukawa convention

We use the following convention for the Yukawa interaction terms:

$$\mathcal{L}_{\text{Yukawa}} = \overline{U}Y_u H P_{\mathrm{L}} Q - \overline{D}Y_d H^\dagger P_{\mathrm{L}} Q - \overline{E}Y_e H^\dagger P_{\mathrm{L}} L + \text{h.c.} \tag{7.5}$$

$$= \overline{U_i}Y_{uij}\epsilon^{ab}H^a P_{\mathrm{L}} Q^b_j - \overline{D_i}Y_{dij}H^{a*}P_{\mathrm{L}}Q^a_j - \overline{E_i}Y_{eij}H^{a*}P_{\mathrm{L}}L^a_j + \text{h.c.} \tag{7.6}$$

$$= -\overline{Q^a}Y_u^\dagger\epsilon^{ab}H^{b*}P_{\mathrm{R}}U - \overline{Q^a}Y_d^\dagger H^a P_{\mathrm{R}} D - \overline{L^a}Y_e^\dagger H^a P_{\mathrm{R}} E + \text{h.c.}, \tag{7.7}$$

where the last equality uses $(\overline{\psi_A}P_{\mathrm{L}}\psi_B)^* = \overline{\psi_B}P_{\mathrm{R}}\psi_A$.

These terms are diagonalized by the singular value decomposition $Y = UY^{\text{diag}}V^\dagger$ (see Appendix A.3):

$$\mathcal{L}_{\text{Yukawa}} = \epsilon^{ab}\overline{U}U_u Y_u^{\text{diag}} H^a P_{\mathrm{L}} V_u^\dagger Q^b - \overline{D}U_d Y_d^{\text{diag}} H^{a*} P_{\mathrm{R}} V_d^\dagger Q^a - \overline{E}U_e Y_e^{\text{diag}} H^{a*} P_{\mathrm{R}} V_e^\dagger L^a + \text{h.c.} \tag{7.8}$$

$$\rightsquigarrow -\frac{v}{\sqrt{2}}\overline{U}U_u Y_u^{\text{diag}} V_u^\dagger P_{\mathrm{L}} Q^1 - \frac{v}{\sqrt{2}}\overline{D}U_d Y_d^{\text{diag}} V_d^\dagger P_{\mathrm{L}} Q^2 - \frac{v}{\sqrt{2}}\overline{E}U_e Y_e^{\text{diag}} V_e^\dagger P_{\mathrm{L}} L^2 + \text{h.c.} \tag{7.9}$$

under the EWSB with $v \simeq 246\,\text{GeV}$. Mass eigenstates are

$$\{Q^1, Q^2, L, \overline{U}, \overline{D}, \overline{E}\}^{\text{mass basis}} = \{V_u^\dagger Q^1, V_d^\dagger Q^2, V_e^\dagger L, \overline{U}U_u, \overline{D}U_d, \overline{E}U_e\} \tag{7.10}$$

and, since Q^1 and Q^2 are rotated by different matrices, the weak interaction receives flavor violation.amended as

$$\mathcal{L} \supset \overline{Q}\mathrm{i}\gamma^\mu(-\mathrm{i}g_2 W_\mu)P_{\mathrm{L}}Q \supset \frac{g_2}{\sqrt{2}}\left[\overline{Q^1}\not{W}^+ P_{\mathrm{L}} Q^2 + \overline{Q^2}\not{W}^- P_{\mathrm{L}} Q^1\right] \tag{7.11}$$

$$= \frac{g_2}{\sqrt{2}}\left[\overline{(Q^1)^{\text{mass}}}V_u^\dagger \not{W}^+ P_{\mathrm{L}} V_d (Q^2)^{\text{mass}} + \overline{(Q^2)^{\text{mass}}}V_d^\dagger \not{W}^- P_{\mathrm{L}} V_u (Q^1)^{\text{mass}}\right] \tag{7.12}$$

$$= \frac{g_2}{\sqrt{2}}\left[\overline{(Q^1)^{\text{mass}}}V_{\text{CKM}} \not{W}^+ P_{\mathrm{L}} (Q^2)^{\text{mass}} + \overline{(Q^2)^{\text{mass}}}V_{\text{CKM}}^\dagger \not{W}^- P_{\mathrm{L}} (Q^1)^{\text{mass}}\right], \tag{7.13}$$

where the CKM matrix are defined with positive angles:

$$V_{\text{CKM}} = V_u^\dagger V_d = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & & \\ & \mathrm{c}_{23} & \mathrm{s}_{23} \\ & -\mathrm{s}_{23} & \mathrm{c}_{23} \end{pmatrix}\begin{pmatrix} \mathrm{c}_{13} & & \mathrm{s}_{13}\,\mathrm{e}^{-\mathrm{i}\delta} \\ & 1 & \\ -\mathrm{s}_{13}\,\mathrm{e}^{\mathrm{i}\delta} & & \mathrm{c}_{13} \end{pmatrix}\begin{pmatrix} \mathrm{c}_{12} & \mathrm{s}_{12} & \\ -\mathrm{s}_{12} & \mathrm{c}_{12} & \\ & & 1 \end{pmatrix} \tag{7.14}$$

$$= \begin{pmatrix} \mathrm{c}_{12}\mathrm{c}_{13} & \mathrm{s}_{12}\mathrm{c}_{13} & \mathrm{s}_{13}\,\mathrm{e}^{-\mathrm{i}\delta} \\ -\mathrm{s}_{12}\mathrm{c}_{23} - \mathrm{c}_{12}\mathrm{s}_{23}\mathrm{s}_{13}\,\mathrm{e}^{\mathrm{i}\delta} & \mathrm{c}_{12}\mathrm{c}_{23} - \mathrm{s}_{12}\mathrm{s}_{23}\mathrm{s}_{13}\,\mathrm{e}^{\mathrm{i}\delta} & \mathrm{s}_{23}\mathrm{c}_{13} \\ \mathrm{s}_{12}\mathrm{s}_{23} - \mathrm{c}_{12}\mathrm{c}_{23}\mathrm{s}_{13}\,\mathrm{e}^{\mathrm{i}\delta} & -\mathrm{c}_{12}\mathrm{s}_{23} - \mathrm{s}_{12}\mathrm{c}_{23}\mathrm{s}_{13}\,\mathrm{e}^{\mathrm{i}\delta} & \mathrm{c}_{23}\mathrm{c}_{13} \end{pmatrix} \qquad [\mathrm{s}_{ij} > 0,\ \mathrm{c}_{ij} > 0]. \tag{7.15}$$

Here in the Standard Model, five phases in a unitary matrix (A.17) are removed by rotating fermion phases.

PDG convention [PDG2018, §12] [PDG2020, §12]

$$\mathcal{L} \supset -Y^d_{ij}\overline{Q^I_{Li}}\phi d^I_{Rj} - Y^u_{ij}\overline{Q^I_{Li}}\epsilon\phi^* u^I_{Rj}, \quad Y^{\text{diag}} = V_L Y V_R^\dagger, \quad V_{\text{CKM}} = V_L^u V_L^{d\dagger}. \tag{7.16}$$

So, $Y^u = Y_u^\dagger$, $Y^d = Y_d^\dagger$; $Y^{\text{diag}} = V_R Y^\dagger V_L^\dagger = V_R Y V_L^\dagger$ leads $V_L = V^\dagger$, and the CKM matrix (and components) is in the same convention: $V_{\text{CKM}} = V_u^\dagger V_d = V_{\text{CKM}}$.

SLHA2 convention [7]

$$W \supset \epsilon_{ab}\left[(Y_E)_{ij}H_1^a L_i^b \bar{E}_j + (Y_D)_{ij}H_1^a Q_i^b \bar{D}_j + (Y_U)_{ij}H_2^b Q_i^a \bar{U}_j\right]; \tag{7.17}$$

$$\mathcal{L} \supset -\epsilon_{ab}\left[(Y_E)_{ij}H_1^a \psi^b_{Li}\psi_{\bar{E}j} + (Y_D)_{ij}H_1^a \psi^b_{Qi}\bar{\psi}_{\bar{D}j} + (Y_U)_{ij}H_2^b \psi^a_{Qi}\bar{\psi}_{\bar{U}j}\right] \tag{7.18}$$

$$\rightsquigarrow -\left[\psi_{\bar{E}}v_{\mathrm{d}}Y_E^{\mathrm{T}}\psi_L^2 + \psi_{\bar{D}}v_{\mathrm{d}}Y_D^{\mathrm{T}}\psi_Q^2 + \psi_{\bar{U}}v_{\mathrm{u}}Y_U^{\mathrm{T}}\psi_Q^1\right]; \quad Y^{\text{diag}} = U^\dagger Y^{\mathrm{T}} V, \quad V_{\text{CKM}} = V_u^\dagger V_d. \tag{7.19}$$

Hence, $Y_E = Y_e^{\mathrm{T}}$, $Y_D = Y_d^{\mathrm{T}}$, $Y_U = Y_u^{\mathrm{T}}$; $Y^{\text{diag}} = U^\dagger Y V$, $V = V$ and $V_{\text{CKM}} = V_{\text{CKM}}$.

Wolfenstein parameterization The CKM matrix is precisely written in terms of λ, A, and $\bar{\rho} + \mathrm{i}\bar{\eta}$.

$$\lambda := \mathrm{s}_{12} = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}, \quad A := \frac{\mathrm{s}_{23}}{\lambda^2} = \lambda^{-1}\left|\frac{V_{cb}}{V_{us}}\right|, \quad \bar{\rho} + \mathrm{i}\bar{\eta} := \frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}. \tag{7.20}$$

They are independent of the phase convention and used for SLHA2 input, i.e., `VCKMIN` should contain $(\lambda, A, \bar{\rho}, \bar{\eta})$.

Also, $\bar{\rho} + \mathrm{i}\eta$ is approximately written by

$$R = \rho + \mathrm{i}\eta := \frac{s_{13}\,\mathrm{e}^{\mathrm{i}\delta}}{A\lambda^3} = \frac{V_{ub}^*}{A\lambda^3}\frac{V_{ud}}{|V_{ud}|} = \frac{(\bar{\rho} + \mathrm{i}\bar{\eta})\sqrt{1 - A^2\lambda^4}}{\sqrt{1 - \lambda^2}\,[1 - A^2\lambda^4(\bar{\rho} + \mathrm{i}\bar{\eta})]} = (\bar{\rho} + \mathrm{i}\bar{\eta})\left(1 + \frac{\lambda^2}{2} + \mathcal{O}(\lambda^4)\right), \tag{7.21}$$

with which

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3 R^* \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - R) & -A\lambda^2 & 1 \end{pmatrix}\mathrm{e}^{\mathrm{i}\Theta} + \begin{pmatrix} \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^7) & 0 \\ \mathcal{O}(\lambda^5) & \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^8) \\ \mathcal{O}(\lambda^5) & \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^4) \end{pmatrix}. \tag{7.22}$$

7.6. General Higgs doublet and Nambu–Goldstone bosons

In linear parameterization,

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{i}\sqrt{2}\phi^+ \\ v+h+\mathrm{i}\phi_3 \end{pmatrix}, \qquad D_\mu H = \begin{pmatrix} \mathrm{i}\partial_\mu\phi^+ - \frac{\mathrm{i}g_2}{2}(v+h+\mathrm{i}\phi_3)W^+_\mu + \left(|e|A_\mu + \frac{\mathrm{c_w^2-s_w^2}}{2}g_Z Z_\mu\right)\phi^+ \\ \partial_\mu(h+\mathrm{i}\phi_3)/\sqrt{2} + \frac{\mathrm{i}g_Z}{2}Z_\mu(v+h+\mathrm{i}\phi_3)/\sqrt{2} + g_2 W^-_\mu\phi^+/\sqrt{2} \end{pmatrix}; \tag{7.23}$$

$$\begin{aligned}|D_\mu H|^2 &= \frac{(\partial_\mu h)^2+(\partial_\mu\phi_3)^2}{2} + \partial_\mu\phi^+\partial^\mu\phi^- + \frac{(v+h)^2}{8}(2g_2^2W^{+\mu}W^-_\mu + g_Z^2Z^\mu Z_\mu) \\ &\quad + \frac{\partial^\mu h}{2}\left[g_2W^+_\mu\phi^- + g_2W^-_\mu\phi^+ - g_Z Z_\mu\phi_3\right] + \frac{\partial^\mu\phi_3}{2}\left[g_Z(v+h)Z_\mu + \mathrm{i}g_2(W^+_\mu\phi^- - W^-_\mu\phi^+)\right] \\ &\quad + \left\{\frac{\partial^\mu\phi^+}{2}\left[-g_2(v+h-\mathrm{i}\phi_3)W^-_\mu + (2|e|A_\mu + (\mathrm{c_w^2-s_w^2})g_Z Z_\mu)\mathrm{i}\phi^-\right] + \text{H.c.}\right\} \\ &\quad + \frac{\mathrm{i}g_2(v+h)}{2}(|e|A^\mu - g_Z\mathrm{s_w^2}Z^\mu)(W^-_\mu\phi^+ - W^+_\mu\phi^-) + \frac{g_2\phi_3}{2}(|e|A^\mu - g_Z\mathrm{s_w^2}Z^\mu)(W^-_\mu\phi^+ + W^+_\mu\phi^-) \\ &\quad + \frac{\phi_3^2}{8}\left(2g_2^2W^{+\mu}W^-_\mu + g_Z^2Z^\mu Z_\mu\right) + \frac{\phi^+\phi^-}{4}\left[2g_2^2W^{+\mu}W^-_\mu + \left(2|e|A_\mu + g_Z(\mathrm{c_w^2-s_w^2})Z_\mu\right)^2\right];\end{aligned} \tag{7.24}$$

$$V = \lambda|H|^4 - \mu^2|H|^2 = \frac{\lambda}{4}h^4 + \lambda v h^3 + \frac{2\lambda v^2}{2}h^2 - \frac{\lambda}{4}v^4 + \frac{\lambda}{4}(2\phi^+\phi^- + \phi_3^2)^2 + \frac{\lambda}{2}(h^2+2vh)(2\phi^+\phi^- + \phi_3^2), \tag{7.25}$$

where $v = \mu/\sqrt{\lambda} \sim 246\,\mathrm{GeV}$, $\lambda \sim 0.13$, and $\mu \sim 89\,\mathrm{GeV}$. In exponential parameterization,

$$H = \frac{1}{\sqrt{2}}\exp\left(\frac{\mathrm{i}}{v}\sigma_i\varphi_i\right)\begin{pmatrix}0\\ v+h\end{pmatrix}, \tag{7.26}$$

$$D_\mu H = \frac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\sigma_i\varphi_i/v}\left[\mathrm{i}\sigma_i\partial_\mu\varphi_i\begin{pmatrix}0\\ 1+h/v\end{pmatrix} + \begin{pmatrix}0\\ \partial_\mu h\end{pmatrix}\right] + \frac{1}{\sqrt{2}}\frac{-\mathrm{i}}{2}(g_2\sigma_iW_{i\mu} + g_YB_\mu)\,\mathrm{e}^{\mathrm{i}\sigma_i\varphi_i/v}\begin{pmatrix}0\\ v+h\end{pmatrix} \tag{7.27}$$

$$= \frac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\sigma_i\varphi_i/v}\left[\begin{pmatrix}0\\ \partial_\mu h\end{pmatrix} + \mathrm{i}\left(\sigma_i\partial_\mu\varphi_i - \frac{g_2v}{2}\,\mathrm{e}^{-\mathrm{i}\sigma_j\varphi_j/v}\sigma_i\,\mathrm{e}^{\mathrm{i}\sigma_k\varphi_k/v}W_{i\mu} - \frac{g_Yv}{2}B_\mu\right)\begin{pmatrix}0\\ 1+h/v\end{pmatrix}\right], \tag{7.28}$$

$$V = \lambda|H|^4 - \mu^2|H|^2 = \frac{\lambda}{4}h^4 + \lambda v h^3 + \frac{2\lambda v^2}{2}h^2 - \frac{\lambda}{4}v^4. \tag{7.29}$$

These expressions have gauge degeneracy (i.e., without gauge-fixing terms and ghost terms) and thus not ready for calculations. If we choose the unitarity gauge, $\phi_i(x) = 0$,

$$\mathcal{L}_H = |D_\mu H|^2 - V(H) = \frac{1}{2}(\partial_\mu h)^2 - \frac{2\lambda v^2}{2}h^2 - \frac{\lambda}{4}h^4 - \lambda v h^3 + \frac{(v+h)^2}{8}(2g_2^2W^{+\mu}W^-_\mu + g_Z^2Z^\mu Z_\mu) + \frac{\lambda}{4}v^4. \tag{7.30}$$

7.7. CP-violating $F\tilde{F}$ terms

The Standard Model contains CP-violating terms

$$\mathcal{L}_{\text{gauge},\cancel{CP}} = \frac{g_3^2\Theta_g}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma} + \frac{g_2^2\Theta_W}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}W^a_{\mu\nu}W^a_{\rho\sigma} + \frac{g_Y^2\Theta_B}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}B_{\rho\sigma}. \tag{7.31}$$

We here discuss we can ignore Θ_W and Θ_B, while Θ_g causes the strong CP roblem.

One should first note that the value of Θ_i depends on the basis of the chiral fermions: in Sec. 7.5 fermions are redefined by rotations. These rotations generate these terms and the angles are modified. It is then found that Θ_W can be rotated away. Let us see this explicitly, starting from the mass basis, i.e., $Y_{u,d,e}$ are positive diagonal and $\mathrm{SU}(2)$ interactions are amended by V_{CKM}. As we do not introduce phases in, e.g., W–u–d interaction and fermion mass matrix, the possible rotation is limited to

$$(Q,U,D) \to \mathrm{e}^{\mathrm{i}\theta}(Q,U,D), \qquad (L_i,E_i) \to \mathrm{e}^{\mathrm{i}\theta_i}(L_i,E_i). \tag{7.32}$$

These rotations affect the CP-violating terms (Cf. Fujikawa method):[*3]

$$\Delta\Theta_W \propto 9\theta_Q + \sum\theta_{L_i} = 9\theta + \sum\theta_i \qquad \Delta\Theta_B \propto \frac{1}{2}\theta_Q + \frac{3}{2}\theta_L - (4\theta_U + \theta_D + 3\sum\theta_{E_i}) = -\frac{9}{2}\theta - \frac{1}{2}\sum\theta_i, \tag{7.33}$$

which means either Θ_W or Θ_B can be rotated away. As we discuss below, it is convenient to set $\Theta_W = 0$ and $\Theta_B \neq 0$.

Meanwhile, as $\Delta\Theta_g = 0$, we cannot remove Θ_g.[*4] We define $\Theta_{\mathrm{QCD}} := (\Theta_g$ in the mass basis), which induces CP-violation in the strong sector. However, such CP-violation is not observed yet; this contradiction is called strong CP problem.

The form $\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\mu\nu}$ is a total derivative and the effect is pushed away to the surface.[*5] As discussed in [8, §23], the $\mathrm{U}(1)_Y$ surface term does not do anything (in the simple spacetime) but the $\mathrm{SU}(N)$ surface term corresponds to topologically non-trivial configuration of the gauge fields, labeled by a winding number ν. Such different configuration should be summed up in, e.g., the path integral formalism, and observed as the instanton effect ("sphaleron" for $\mathrm{SU}(2)_W$). If $\Theta_W \neq 0$, the processes $\nu \to \nu \pm 1$ would have different rate and CP would be violated in the processes. As Θ_B is not related to such process, we take $\Theta_W = 0$ and, though $\Theta_B \neq 0$, do not further consider Θ_B.

[*3] Fail-safe memo: chiral transformation $\psi \to \exp[\mathrm{i}\gamma_5\alpha(x)]\psi$ generates $\Delta\mathcal{L} = -(g^2/16\pi^2)\,\mathrm{Tr}[\alpha F\tilde{F}]$ (cf. Weinberg II Eq.(2.2.24) but the overall sign may differ). For a constant (and non-matrix) α, $\Delta\mathcal{L} = -(\alpha g^2/32\pi^2)F^a\tilde{F}^a$. Also, the absence of gauge anomaly means the corresponding gauge transformations do not induce additional Θ-terms.

[*4] If, e.g., u were massless, we can take $\theta_{u_{\mathrm{R}}} \neq \theta_Q$ and rotate Θ_g away.

[*5] Sho thanks to Kyohei Mukaida and Teppei Kitahara for a very useful discussion.

8. Standard Model Values[*6]

Mass and width

e : $0.510\,998\,9461(31)$ MeV

μ : $105.658\,3745(24)$ MeV, $2.196\,9811(22)\,\mu\mathrm{s} = 659\,\mathrm{m}$

τ : $1.776\,86(12)$ GeV, $290.3(5)\times10^{-15}\,\mathrm{s} = 87.0\,\mu\mathrm{m}$

t : $172.76(30)$ GeV[*7], $1.42^{+0.19}_{-0.15}$ GeV

$m_{\nu;\mathrm{tot}} < 0.2$–$0.3$ eV

h : $125.10(14)$ GeV

W : $80.379(12)$ GeV, $2.085(42)$ GeV

Z : $91.1876(21)$ GeV, $2.4952(23)$ GeV

$(u,d,s)^{\overline{\mathrm{MS}}}_{2\,\mathrm{GeV}}$: $(2.16^{+0.49}_{-0.26}, 4.67^{+0.48}_{-0.17}, 93^{+11}_{-5})$ MeV[*8]

$(\frac{u+d}{2}, \frac{u}{d}, \frac{2s}{u+d})^{\overline{\mathrm{MS}}}_{2\,\mathrm{GeV}}$: $(3.45^{+0.55}_{-0.15}\,\mathrm{MeV}, 0.47^{+0.06}_{-0.07}, 27.3^{+0.7}_{-1.3})$

c : $1.27(2)\,\mathrm{GeV}^{\overline{\mathrm{MS}}}_{m_c}$ $(1.67(7)\,\mathrm{GeV}^{\mathrm{pole}})$

b : $4.18^{+0.03}_{-0.02}\,\mathrm{GeV}^{\overline{\mathrm{MS}}}_{m_b}$ $(4.78(6)\,\mathrm{GeV}^{\mathrm{pole}})$

$\pi^\pm$: $139.570\,39(18)$ MeV	$\rho^\pm_{770}$: $775.11(34)$ MeV	$\eta_c(1S)$: $2983.9(5)$ MeV
π^0 : $134.9768(5)$ MeV	ρ^0_{770} : $775.26(25)$ MeV	$J/\psi(1S)$: $3096.900(6)$ MeV
η : $547.862(17)$ MeV	ϕ_{1020} : $1019.461(16)$ MeV	$\eta_b(1S)$: $9398.7(20)$ MeV
η' : $957.78(6)$ MeV	ω_{782} : $782.65(12)$ MeV	$\Upsilon(1S)$: $9460.30(26)$ MeV
$K^\pm$: $493.677(16)$ MeV	$K^{*\pm}_{892}$: $891.66(26)$ MeV	$\Upsilon(2S)$: $10023.26(31)$ MeV
K^0 : $497.611(13)$ MeV	K^{*0}_{892} : $895.55(20)$ MeV	$\Upsilon(3S)$: $10355.2(5)$ MeV
D^0 : $1864.83(5)$ MeV	$B^\pm$: $5279.34(12)$ MeV	$\Upsilon(4S)$: $10579.4(12)$ MeV
$D^\pm$: $1869.65(5)$ MeV	B^0 : $5279.65(12)$ MeV	p : $938.272\,0813(58)$ MeV
$D_s^\pm$: $1968.34(7)$ MeV	B_s : $5366.88(14)$ MeV	n : $939.565\,413(6)$ MeV
	$B_c^\pm$: $6274.9(8)$ MeV	

$\pi^\pm$: $2.6033(5)\times10^{-8}\,\mathrm{s} = 7.80\,\mathrm{m}$	$K^\pm$: $1.2380(20)\times10^{-8}\,\mathrm{s} = 3.71\,\mathrm{m}$	$p \overset{\diamond}{>} 3.6\times10^{29}\,\mathrm{yr}$
π^0 : $8.52(18)\times10^{-17}\,\mathrm{s} = 0.0255\,\mu\mathrm{m}$	K^0_{S} : $0.8954(4)\times10^{-10}\,\mathrm{s} = 26.8\,\mathrm{mm}$	n : $879.4(6)$ s
	K^0_{L} : $5.116(21)\times10^{-8}\,\mathrm{s} = 15.3\,\mathrm{m}$	

$n\,^{2s+1}l_J$	J^{PC}	$I=1$	$I=1/2$	$I=0$		$c\bar{c}$	$b\bar{b}$	charm		bottom		
$1\,^1S_0$	0^{-+}	π	K	η	η'_{958}	$\eta_c(1S)$	$\eta_b(1S)$	D	D_s	B	B_s	B_c
$1\,^1S_1$	1^{--}	ρ_{770}	K^*_{892}	ϕ_{1020}	ω_{782}	$J/\psi(1S)$	$\Upsilon(1S)$	D^*	D_s^*	B^*	B_s^*	

Electric and magnetic moment, important branching ratios, and neutrino property

$a_e = 11\,596\,521.8091(26)\times10^{-10}$	$\mathrm{Br}(\tau\to e,\mu) \simeq 35.2\%$	$\Delta m^2_{21}/\mathrm{eV}^2 = 7.53(18)\times10^{-5}$
$a_\mu = 11\,659\,208.9(54)(33)\times10^{-10}$	$\mathrm{Br}(\tau\to\mathrm{had}) \simeq 64.8\%$	$\Delta m^2_{32}/\mathrm{eV}^2 =$
$a_\tau \overset{**}{\in} [-0.052, 0.013]$	$\mathrm{Br}(\tau;\text{1-prong}) = 85.24(6)\%$	$2.453(34)\times10^{-3}$(NH)
$\mu_p = 2.792\,847\,3446(8)\mu_N$	$\mathrm{Br}(\tau;\text{3-prong}) = 14.55(6)\%$	$-2.546^{+0.034}_{-0.040}\times10^{-3}$(IH)
$\mu_n = -1.913\,0427(5)\mu_N$	$\mathrm{Br}(Z\to\mathrm{had}) = 69.911(56)\%$	$\sin^2\theta_{12} = 0.307(13)$
$d_e \overset{\diamond}{<} 0.11\times10^{-28}\,e\,\mathrm{cm}$	$\mathrm{Br}(Z\to b\bar{b}) = 15.12(5)\%$	$\sin^2\theta_{13} = 0.0218(7)$
$d_\mu \overset{**}{<} 1.8\times10^{-19}\,e\,\mathrm{cm}$	$\mathrm{Br}(Z\to e,\mu,\tau) \simeq 10.10\%$	$\sin^2\theta_{23}(\mathrm{NH}) = 0.545(21)$
$d_p \overset{?}{<} 0.021\times10^{-23}\,e\,\mathrm{cm}$	$\mathrm{Br}(Z\to\mathrm{inv}) = 20.000(55)\%$	$\sin^2\theta_{23}(\mathrm{IH}) = 0.547(21)$
$d_n \overset{\diamond}{<} 0.18\times10^{-25}\,e\,\mathrm{cm}$	$\mathrm{Br}(W\to\mathrm{had}) = 67.41(27)\%$	$\delta = 1.36(17)\pi$

[*6] Data source: **PDG2020**. Confidence levels are shown by the marks $*$, $**$, $***$ (1–3σ), $\diamond$ (90%), and $\diamond\diamond$ (99%).
[*7] Cross section measurement gives $\overline{\mathrm{MS}}$ top mass $162.5^{+2.1}_{-1.5}$ GeV, equivalent to $172.4(7)$ GeV.
[*8] $m^{\overline{\mathrm{MS}}}_{1\,\mathrm{GeV}} = m^{\overline{\mathrm{MS}}}_{2\,\mathrm{GeV}}\times1.35$.

CKM matrix

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97401(11) & 0.22650(48) & 0.00361^{(+11)}_{(-\ 9)} \\ 0.22636(48) & 0.97320(11) & 0.04053^{(+83)}_{(-61)} \\ 0.00854^{(+23)}_{(-16)} & 0.03978^{(+82)}_{(-60)} & 0.999172^{(+24)}_{(-35)} \end{pmatrix}; \quad J = 3.00^{(+15)}_{(-\ 9)} \times 10^{-5}$$

$$(\lambda, A, \bar\rho, \bar\eta) = (0.22650(48), 0.790^{(+17)}_{(-12)}, 0.141^{(+16)}_{(-17)}, 0.357(11))$$

$$(\sin\theta_{12}, \sin\theta_{13}, \sin\theta_{23}, \delta) = (0.22650(48), 0.00361^{(+11)}_{(-\ 9)}, 0.04053^{(+83)}_{(-61)}, 1.196^{(+45)}_{(-43)})$$

Astrophysical

$T_0 = 2.7255(6)\,\mathrm{K}$ | $H_0 = 100h\,\mathrm{km/s/Mpc},\ h = 0.674(5)$ | $M_\odot = 1.98841(4)\times 10^{30}\,\mathrm{kg}$

$n_\gamma = 410.7(3)\hat T_0^3\,\mathrm{cm}^{-3}$ | $\rho_{\rm crit} = 1.053\,672(24)\times 10^{-5} h^2\,\mathrm{GeV/cm}^3$ | $M_\oplus = 5.97217(13)\times 10^{24}\,\mathrm{kg}$

$\rho_\gamma = 0.2606(2)\hat T_0^4\,\mathrm{eV/cm}^3$ | $G_{\rm N} = 6.708\,83(15)\times 10^{-39}\,\mathrm{GeV}^{-2}$ | $R_0 = 8.178(13)(22)\,\mathrm{kpc}$

$s = 2891.2\hat T_0^3\,\mathrm{cm}^{-3}$ | $M_{\rm Pl} = 1.220\,890(14)\times 10^{19}\,\mathrm{GeV}$ | $v_0 = 240(8)\,\mathrm{km/s}$

$\Omega_\gamma h^2 = 2.473\times 10^{-5}\hat T_0^4$ | $M_0 = 2.435\,323(28)\times 10^{18}\,\mathrm{GeV}$ | $\rho_{\rm disk} = 3.7(5)\,\mathrm{GeV/cm}^3$

$[\hat T_0 = T_0/2.7255\,\mathrm{K}]$ | $\eta = n_{\rm b}/n_\gamma \overset{**}{\in} [5.8, 6.5]\times 10^{-10}$

Planck 2018 6-parameter fit to flat ΛCDM cosmology:

$\{\Omega_{\rm b}h^2, \Omega_{\rm CDM}h^2\} = \{0.02237(15), 0.1200(12)\}$ | $(z,t)_{\rm M=R} = 3402(26), 5.11(8)\times 10^4\,\mathrm{yr}$

$\Omega_{\{\rm b,CDM,\Lambda\}} = \{0.0493(6), 0.265(7), 0.685(7)\}$ | $(z,t)_* = 1089.92(25), 3.729(10)\times 10^5\,\mathrm{yr}$

$\Lambda = 1.088(30)\times 10^{-56}\,\mathrm{cm}^{-2}$ | $(z,t)_{\rm i} = 7.7(7), 6.90(90)\times 10^8\,\mathrm{yr}$

$\Omega_K = 0.0007(19)$ | $(z,t)_{\rm q} = 0.636(18), 7.70(10)\times 10^9\,\mathrm{yr}$

$N_{\rm eff} = 2.99(17)$ | $t_0 = 1.3797(23)\times 10^{10}\,\mathrm{yr}$

Standard Model parameter fit

$\alpha_{\rm EM}^{-1}(0) = 137.035\,999\,084(21)$ | $\sin^2\theta^{\overline{\rm MS}}(M_Z) = 0.23121(4)$ | $\alpha_{\rm s}(m_Z) = 0.1179(10)$

$\hat\alpha^{(4)}(m_\tau)^{-1} = 133.472(7)$ | $\sin^2\theta^{\overline{\rm MS}}(0) = 0.23857(5)$ | $G_{\rm F} = 1.166\,378\,7(6)\times 10^{-5}\,\mathrm{GeV}^{-2}$

$\hat\alpha^{(5)}(m_Z)^{-1} = 127.952(9)$ | $\sin^2\theta^{\text{on-shell}} = 0.22337(10)$ | $\overset{\rm tree}{=} g_2^2/(4\sqrt2 m_W^2) = 1/(\sqrt2 v^2)$

$\Delta\alpha_{\rm had}^{(5)}(m_Z) = 0.02766(7)$ | $\overset{\rm tree}{=} (g'/g_Z)^2 = 1-(m_W/m_Z)^2$

$\overline{\rm MS}$ parameters at $Q_0 = 173.1\,\mathrm{GeV}$ based on Ref. [9] (cf. Ref. [10]):

$g_s = 1.161\,8(4\,5)$ | $v = 246.605(12)\,\mathrm{GeV}$ | $\lambda = 0.126\,07(30)$

$g = 0.647\,653(281)$ | $-m^2 = 8612.0(22.8)\,\mathrm{GeV}^2 = (92.80(12)\,\mathrm{GeV})^2$

$g' = 0.358\,542(70)$ | $y_{t,c,u} = \{0.931(4), 0.0341(10), 6.8(1.1)\times 10^{-6}\}$

$|e| = 0.313\,68(18)$ | $y_{b,s,d} = \{0.015\,53(14), 0.000\,293(25), 1.47(10)\times 10^{-5}\}$

$g_Z = 0.740\,27(25)$ | $y_{\tau,\mu,e} = \{0.009\,994\,4(8), 0.000\,588\,38(11), 2.793\,0(2\,6)\times 10^{-6}\}$

9. Neutrino

(summary page)

9.1. Convention

We extend the SM Yukawa (7.5) to include the neutrino mass terms:

$$\mathcal{L}_{\mathrm{Y}+\nu} = \overline{U}Y_u H P_{\mathrm{L}} Q - \overline{D}Y_d H^\dagger P_{\mathrm{L}} Q + \overline{N}Y_n H P_{\mathrm{L}} L - \overline{E}Y_e H^\dagger P_{\mathrm{L}} L - \frac{1}{2}\overline{N}M_N N^{\mathrm{c}} + \text{h.c.} \tag{9.1}$$

$$= -\overline{Q^a}Y_u^\dagger \epsilon^{ab} H^{b*} P_{\mathrm{R}} U - \overline{Q^a}Y_d^\dagger H^a P_{\mathrm{R}} D - \overline{L^a}Y_n^\dagger \epsilon^{ab} H^{b*} P_{\mathrm{R}} N - \overline{L^a}Y_e^\dagger H^a P_{\mathrm{R}} E - \frac{1}{2}\overline{N}M_N N^{\mathrm{c}} + \text{h.c.}, \tag{9.2}$$

where M_N, a *complex symmetric* Majorana mass matrix, may be absent.

As we explicitly describe in Sec. 9.1, we use the standard convention for the PMNS matrix (Pontecorvo–牧–中川–坂田):

$$|\nu_\alpha^{\text{flavor}}\rangle = [U_{\mathrm{PMNS}}]^*_{\alpha i}\,|\nu_i^{\text{mass}}\rangle, \qquad \nu^{\text{flavor}} = U_{\mathrm{PMNS}}\nu^{\text{mass}}, \qquad [\alpha = e, \mu, \tau, \mathrm{r}_1, \mathrm{r}_2, \dots] \tag{9.3}$$

where the “flavor basis” is defined by the charged lepton mass basis for first three elements, while in most cases by the basis that diagonalizes Majorana mass term for the rest.

Dirac neutrino If M_N is absent, neutrinos become Dirac fermions and the discussion goes parallel to the CKM matrix:

$$Y_n = U_n Y_n^{\text{diag}} V_n^\dagger, \qquad \{\nu_{\mathrm{L}}, e_{\mathrm{L}}, \overline{N}, \overline{E}\}^{\text{mass basis}} = \{V_n^\dagger L^1, V_e^\dagger L^2, \overline{N}U_n, \overline{E}U_e\} \tag{9.4}$$

and

$$\mathcal{L} \supset \overline{L}\mathrm{i}\gamma^\mu(-\mathrm{i}g_2 W_\mu)P_{\mathrm{L}} L \supset \frac{g_2}{\sqrt{2}}\left[\overline{L^1}\not{W}^+ P_{\mathrm{L}} L^2 + \overline{L^2}\not{W}^- P_{\mathrm{L}} L^1\right] \tag{9.5}$$

$$= \frac{g_2}{\sqrt{2}}\left[\overline{(L^1)^{\text{mass}}}V_n^\dagger V_e \not{W}^+ P_{\mathrm{L}}(L^2)^{\text{mass}} + \overline{(L^2)^{\text{mass}}}V_e^\dagger V_n \not{W}^- P_{\mathrm{L}}(L^1)^{\text{mass}}\right] \tag{9.6}$$

$$= \frac{g_2}{\sqrt{2}}\left[\overline{\nu_{\mathrm{L}}^{\text{mass}}}[U_{\mathrm{PMNS}}]^\dagger \not{W}^+ e_{\mathrm{L}}^{\text{mass}} + \overline{e_{\mathrm{L}}^{\text{mass}}}[U_{\mathrm{PMNS}}]\not{W}^- \nu_{\mathrm{L}}^{\text{mass}}\right]. \tag{9.7}$$

The mass eigenstate Dirac field is given by

$$\nu^{\text{mass}} = \begin{pmatrix}\nu_{\mathrm{L}}^{\text{mass}} \\ N^{\text{mass}}\end{pmatrix} = \nu_{\mathrm{L}}^{\text{mass}} + N^{\text{mass}} = V_n^\dagger L^1 + U_n^\dagger N \qquad \Big(L^1 = P_{\mathrm{L}} V_n \nu^{\text{mass}},\ N = P_{\mathrm{R}} U_n \nu^{\text{mass}}\Big). \tag{9.8}$$

The PMNS matrix is given in *the opposite manner* to the CKM matrix:

$$U_{\mathrm{PMNS}}^{\text{Dirac}} = V_e^\dagger V_n. \tag{9.9}$$

In general, $Y_n \in \mathbb{C}^{n\times 3}$, $V_n \in \mathbb{U}_{\mathbb{C}}^3$, $U_n \in \mathbb{U}_{\mathbb{C}}^n$, and the Dirac-PMNS matrix is also a 3×3 unitary matrix. Models with $n < 3$ yields $3-n$ massless left-handed neutrinos, while $n > 3$ results in $n-3$ massless right-handed neutrinos.

Majorana neutrino Models with $M_N \neq 0$ generate so-called Majorana neutrino masses. If $n < 3$, the model has $3-n$ massless neutrinos and $2n$ massive neutrinos, while all neutrinos are massive if $n \geqslant 3$.

Using Weyl spinor,

$$L^1 = \begin{pmatrix}\nu_{\mathrm{L}} \\ 0\end{pmatrix}, \quad N = \begin{pmatrix}0 \\ \bar{n}_{\mathrm{R}}\end{pmatrix}, \quad M_{\mathrm{D}} := \frac{v}{\sqrt{2}}Y_n, \quad M_{\mathrm{D}}^{\text{diag}} := U_n M_{\mathrm{D}} Y_n^\dagger, \tag{9.10}$$

$$\mathcal{L}_{\mathrm{Y}+\nu} \supset -\frac{v}{\sqrt{2}}\overline{N}Y_n P_{\mathrm{L}} L^1 - \frac{1}{2}\overline{N}M_N N^{\mathrm{c}} + \text{h.c.} = -\frac{v}{\sqrt{2}}n_{\mathrm{R}} Y_n \nu_{\mathrm{L}} - \frac{1}{2}n_{\mathrm{R}} M_N n_{\mathrm{R}} + \text{h.c.} \tag{9.11}$$

$$= -\frac{1}{2}\begin{pmatrix}\nu_{\mathrm{L}} & n_{\mathrm{R}}\end{pmatrix}\begin{pmatrix}0 & M_{\mathrm{D}}^{\mathrm{T}} \\ M_{\mathrm{D}} & M_N\end{pmatrix}\begin{pmatrix}\nu_{\mathrm{L}} \\ n_{\mathrm{R}}\end{pmatrix} + \text{h.c.} =: -\frac{1}{2}\tilde{\nu}^{\mathrm{T}}\tilde{M}\tilde{\nu} + \text{h.c.} \tag{9.12}$$

As $\tilde{M}$ is a complex symmetric $(3+n)\times(3+n)$ matrix, it can be AT-diagonalized:

$$\tilde{M} = \tilde{R}\tilde{M}^{\text{diag}}\tilde{R}^{\mathrm{T}}; \qquad -\mathcal{L}_{\mathrm{Y}+\nu} \supset \frac{1}{2}\tilde{\nu}^{\mathrm{T}}\tilde{M}\tilde{\nu} = \frac{1}{2}(\tilde{\nu}^{\text{mass}})^{\mathrm{T}}\tilde{M}^{\text{diag}}\tilde{\nu}^{\text{mass}}; \qquad \tilde{\nu}^{\text{mass}} = \tilde{R}^{\mathrm{T}}\tilde{\nu} \tag{9.13}$$

and $\tilde{M}$ gives the neutrino masses. The neutrino mixing is then given by

$$\begin{pmatrix}\nu_{\mathrm{L}} \\ n_{\mathrm{R}}\end{pmatrix} = \tilde{R}^*\begin{pmatrix}\nu_{1\text{–}3} \\ \nu_{4\text{–}}\end{pmatrix} \tag{9.14}$$

and find that the PMNS matrix is now extended to a $3\times n$ matrix $V_e^\dagger[\tilde{R}^*]_{\text{upper}}$.

Usually the discussion should be start from the basis in which Y_e and M_N are positive diagonal with increasing diagonal elements (“charged lepton mass basis” combined with “Majorana mass basis”). Then

$$\tilde{M} = \begin{pmatrix}0 & M_{\mathrm{D}}^{\mathrm{T}} \\ M_{\mathrm{D}} & M_N^{\text{diag}}\end{pmatrix}, \quad \begin{pmatrix}\nu_{\mathrm{L}} \\ n_{\mathrm{R}}\end{pmatrix} = \begin{pmatrix}\nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_{\text{sterile};i}\end{pmatrix} = \tilde{R}^*\begin{pmatrix}\nu_{1\text{–}3} \\ \nu_{4\text{–}}\end{pmatrix}. \tag{9.15}$$

The PMNS matrix appears as a submatrices, which is no longer unitary:

$$\tilde{R}^* =: \begin{pmatrix}R_{11}^* & R_{12}^* \\ R_{21}^* & R_{22}^*\end{pmatrix} =: \begin{pmatrix}U_{\mathrm{PMNS}}^{\text{Majorana}} & U_{\text{active-heavy}} \\ U_{\text{sterile-light}} & U_{\text{sterile-heavy}}\end{pmatrix}. \tag{9.16}$$

Connection of the above two formalism If we apply AT-diagonalization to the Dirac case (with $n = 3$), we obtain

$$\tilde{M} = \frac{1}{2}\begin{pmatrix}0 & (U_n M_{\mathrm{D}}^{\text{diag}} V_n^\dagger)^{\mathrm{T}} \\ U_n M_{\mathrm{D}}^{\text{diag}} V_n^\dagger & 0\end{pmatrix}, \quad \tilde{R} = \begin{pmatrix}V_n^* & \mathrm{i}V_n^* \\ U_n & -\mathrm{i}U_n\end{pmatrix}, \quad \tilde{M}^{\text{diag}} = \begin{pmatrix}M_{\mathrm{D}}^{\text{diag}} & 0 \\ 0 & M_{\mathrm{D}}^{\text{diag}}\end{pmatrix} \tag{9.17}$$

and find that three pairs of degenerate Weyl fermions form three Dirac neutrinos.

PMNS matrix In the Dirac neutrino models the PMNS matrix is unitary, which we parameterize

$$U_{\rm PMNS}^{\rm Dirac} = V_e^\dagger V_n = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}\,e^{-i\delta_{\rm CP}} \\ & 1 & \\ -s_{13}\,e^{i\delta_{\rm CP}} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\,e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}\,e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\,e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\,e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{pmatrix}, \tag{9.18}$$

where $\theta_{ij} \in [0, \pi/2]$ and $\delta_{\rm CP} \in [0, 2\pi]$ as shown in Eq. (A.17) and the five phases are removed as done in Sec. 7.5.

If the Majorana mass terms are present, $U_{\rm PMNS}^{\rm Majorana}$, which is defined by Eq. (9.3), is no longer unitary. However, if $M_N \gg M_{\rm D}$, it is approximately unitary:

$$U_{\rm PMNS}^{\rm Majorana} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \approx U_{\rm PMNS}^{\rm Dirac} \begin{pmatrix} e^{i\eta_1} & & \\ & e^{i\eta_2} & \\ & & 1 \end{pmatrix}. \tag{9.19}$$

Here additional two phases are introduced because we can no longer rotate $n_{\rm R}$; three among (a, b, c, d, e) in Eq. (A.17) are removed by rotating L_i, and two remains.

Current experiments measure the value of the matrix and the above parameterization still works well (cf. Ref. [11]). Thus we hereafter identify the Dirac PMNS matrix as $U_{\rm PMNS}$:

$$U_{\rm PMNS} := U_{\rm PMNS}^{\rm Dirac} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\,e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}\,e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\,e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\,e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{pmatrix}; \tag{9.20}$$

$$U_{\rm PMNS}^{\rm Majorana} = U_{\rm PMNS} \begin{pmatrix} e^{i\eta_1} & & \\ & e^{i\eta_2} & \\ & & 1 \end{pmatrix} + \mathcal{O}\left(\frac{M_{\rm D}}{M_N}\right). \tag{9.21}$$

Accordingly, the components $U_{\alpha i}$ is in general slightly different from (α, i) component of $U_{\rm PMNS}$, e.g.,

$$U_{e1} \approx c_{12}c_{13}$$

with the exactness recovered in the Dirac case.

It should be noted that the discussion in Sec. 7.7 holds. As far as the baryon number is conserved, we can remove the $\Theta_W W\tilde{W}$ term by quark rotation. Hence, the above-discussed models have CP violation only in the CKM and (extended) PMNS matrices.

PDG and NuFIT convention The convention agrees with PDG [PDG2020, §14] and NuFIT [11, v5.0]. Compared with PDG,[*9]

$$\mathcal{L} \supset -\bar\nu_s M_D \nu_L - \frac{1}{2}\bar\nu_s M_N \nu_s^c - \overline{L^2} M_l P_{\rm R} E \quad (14.6)+(14.27),$$

$$\left\{ (V^\nu)^{\rm T} \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix} V^\nu \text{ or } V_R^{\nu\dagger} M_D V^\nu \right\} = \mathrm{diag}(m_i) \quad \text{(Majorana; 14.9)+(Dirac; 14.15)},$$

$$\nu_L = P_{\rm L} V^\nu \nu^{\rm mass} \quad (14.14)\&(14.18), \qquad V^{l\dagger} M_l V_R^l = \mathrm{diag}(m_e, m_\mu, m_\tau) \quad (14.31).$$

These leads to $V_R^\nu = U_n$ and $V^\nu = V_n$ in Dirac case, $M_D = Y_n v/\sqrt{2} = M_{\rm D}$, $M_N = M_N$, and $V^\nu = R^*$ in Majorana case, and $V^l = V_e$ and $V_R^l = U_e$ (note $M_l = Y_e^\dagger v/\sqrt{2}$). So

$$U_{ij} = (\text{diagonal phases}) \times V^{l\dagger} V^\nu \times (\text{diagonal phases}) = (\text{phases}) V_e^\dagger \{V_\nu \text{ or } R^*\} (\text{phases}) = U_{\rm PMNS}^{\rm (Dirac/Majorana)}$$

9.2. Casas–Ibarra parameterization

General basis We start from the neutrino mass matrix

$$-\mathcal{L} \supset \frac{1}{2}\begin{pmatrix} \nu_{\rm L} & n_{\rm R} \end{pmatrix} \begin{pmatrix} 0 & M_{\rm D}^{\rm T} \\ M_{\rm D} & M_N \end{pmatrix} \begin{pmatrix} \nu_{\rm L} \\ n_{\rm R} \end{pmatrix} + \text{h.c.} = \frac{1}{2}\begin{pmatrix} \nu_{{\rm L}i} & n_{{\rm R}a} \end{pmatrix} \begin{pmatrix} 0_{ij} & (M_{\rm D}^{\rm T})_{ib} \\ M_{{\rm D}aj} & M_{Nab} \end{pmatrix} \begin{pmatrix} \nu_{{\rm L}j} \\ n_{{\rm R}b} \end{pmatrix} + \text{h.c.} \tag{9.22}$$

The AT-factorization can be separated into two steps:

$$\tilde M = \begin{pmatrix} 0 & M_{\rm D}^{\rm T} \\ M_{\rm D} & M_N \end{pmatrix} \longrightarrow \begin{pmatrix} M_{\rm L} & 0 \\ 0 & M_{\rm H} \end{pmatrix} \xrightarrow{\rm ATF} \begin{pmatrix} M_{\rm L}^{\rm diag} & 0 \\ 0 & M_{\rm H}^{\rm diag} \end{pmatrix} = \tilde M^{\rm diag}, \tag{9.23}$$

where intermediate matrices $M_{\rm L}$ and $M_{\rm H}$ are complex symmetric. This is expressed with unitary matrices U, U_1, and U_2:

$$\tilde M = U \begin{pmatrix} M_{\rm L} & 0 \\ 0 & M_{\rm H} \end{pmatrix} U_1^{\rm T} = U \begin{pmatrix} U_1 M_{\rm L}^{\rm diag} U_1^{\rm T} & 0 \\ 0 & U_2 M_{\rm H}^{\rm diag} U_2^{\rm T} \end{pmatrix} U^{\rm T}, \tag{9.24}$$

following the convention in Eq. (A.21). The first equality is calculated as an expansion in $M_{\rm D}/M_N$ once we assume the see-saw mechanism:

$$U \simeq \begin{pmatrix} 1 & M_{\rm D}^{\rm T} M_N^{-1} \\ -M_N^{-1} M_{\rm D}^* & 1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} M_{\rm D}^{\rm T} M_N^{-2} M_{\rm D}^* & 0 \\ 0 & M_N^{-1} M_{\rm D}^* M_{\rm D}^{\rm T} M_N^{-1} \end{pmatrix} + \cdots, \tag{9.25}$$

$$M_{\rm L} \simeq -M_{\rm D}^{\rm T} M_N^{-1} M_{\rm D} + \cdots, \tag{9.26}$$

$$M_{\rm H} \simeq M_N + \frac{1}{2}(M_{\rm D} M_{\rm D}^\dagger M_N^{-1} + M_N^{-1} M_{\rm D}^* M_{\rm D}^{\rm T}) + \cdots, \tag{9.27}$$

[*9] Sho thinks Eq. (14.9) of PDG2020 lacks 1/2 in the right-most term.

At the leading order,

$$U_1 M_{\mathrm{L}}^{\text{diag}} U_1^{\mathrm{T}} = [U^\dagger \tilde{M} U^*]_{\text{upper left}} \approx -M_{\mathrm{D}}^{\mathrm{T}} M_N^{-1} M_{\mathrm{D}} \approx -M_{\mathrm{D}}^{\mathrm{T}} M_{\mathrm{H}}^{-1} M_{\mathrm{D}} = -M_{\mathrm{D}}^{\mathrm{T}} (U_2 M_{\mathrm{H}}^{\text{diag}} U_2^{\mathrm{T}})^{-1} M_{\mathrm{D}},$$

or

$$-M_{\mathrm{L}}^{\text{diag}} \approx U_1^\dagger M_{\mathrm{D}}^{\mathrm{T}} U_2^* (M_{\mathrm{H}}^{\text{diag}})^{-1} U_2^\dagger M_{\mathrm{D}} U_1^*.$$

This is decomposed to

$$[\mathrm{i}M_{\mathrm{L}}^{\text{diag}}]^{1/2}[\mathrm{i}M_{\mathrm{L}}^{\text{diag}}]^{1/2} = [(M_{\mathrm{H}}^{\text{diag}})^{-1/2} U_2^\dagger M_{\mathrm{D}} U_1^*]^{\mathrm{T}} [(M_{\mathrm{H}}^{\text{diag}})^{-1/2} U_2^\dagger M_{\mathrm{D}} U_1^*]. \tag{9.28}$$

This is the master equation for Casas-Ibarra parameterization [12].

"Standard" parameterization Let us assume that we started from the above-discussed (Y_e, M_N)-diagonal basis. Then, noting

$$\tilde{R} = U \begin{pmatrix} U_1 & \\ & U_2 \end{pmatrix} \approx \begin{pmatrix} U_1 & M_{\mathrm{D}}^{\mathrm{T}} M_N^{-1} U_2 \\ -M_N^{-1} M_{\mathrm{D}}^* U_1 & U_2 \end{pmatrix}, \tag{9.29}$$

we can identify $U_1^* \approx U_{\mathrm{PMNS}}^{\text{Majorana}}$ and $U_2 \approx 1$; the master equation now becomes

$$[\mathrm{i}M_{\mathrm{L}}^{\text{diag}}]^{1/2}[\mathrm{i}M_{\mathrm{L}}^{\text{diag}}]^{1/2} = [(M_{\mathrm{H}}^{\text{diag}})^{-1/2} M_{\mathrm{D}} U_{\mathrm{PMNS}}^{\text{Majorana}}]^{\mathrm{T}} [(M_{\mathrm{H}}^{\text{diag}})^{-1/2} M_{\mathrm{D}} U_{\mathrm{PMNS}}^{\text{Majorana}}]. \tag{9.30}$$

We will use this parameterization below.

Example: three right-handed neutrinos Let us assume all the neutrinos are massive thanks to three right-handed neutrinos. Then $M_{\mathrm{L}}^{\text{diag}}$ is invertible and

$$R := -\mathrm{i}(M_{\mathrm{H}}^{\text{diag}})^{-1/2} M_{\mathrm{D}} U_{\mathrm{PMNS}}^{\text{Majorana}} (M_{\mathrm{L}}^{\text{diag}})^{-1/2} \quad\Longrightarrow\quad R^{\mathrm{T}} R = 1. \tag{9.31}$$

Conversely, with a matrix R satisfying $R^{\mathrm{T}} R = 1$, the "Yukawa matrix" is given by

$$M_{\mathrm{D}} = \mathrm{i}\sqrt{M_{\mathrm{H}}^{\text{diag}}} R \sqrt{M_{\mathrm{L}}^{\text{diag}}} (U_{\mathrm{PMNS}}^{\text{Majorana}})^\dagger. \tag{9.32}$$

Now we successfully parameterized Y_n by a "complex orthogonal" matrix R. The extended PMNS matrix is given by

$$\approx \begin{pmatrix} U_1^* & M_{\mathrm{D}}^\dagger M_N^{-1} U_2^* \\ -M_N^{-1} M_{\mathrm{D}} U_1^* & U_2^* \end{pmatrix} \approx \begin{pmatrix} U_{\mathrm{PMNS}}^{\text{Majorana}} & -\mathrm{i}U_{\mathrm{PMNS}}^{\text{Majorana}} \sqrt{M_{\mathrm{L}}^{\text{diag}}} R^\dagger (M_{\mathrm{H}}^{\text{diag}})^{-1/2} \\ -\mathrm{i}(M_{\mathrm{H}}^{\text{diag}})^{-1/2} R \sqrt{M_{\mathrm{L}}^{\text{diag}}} & 1 \end{pmatrix}. \tag{9.33}$$

The parameter matrix R is given by[*10]

$$R = \begin{pmatrix} \mathrm{c}_{12}\mathrm{c}_{13} & \mathrm{s}_{12}\mathrm{c}_{13} & \mathrm{s}_{13} \\ -\zeta \mathrm{s}_{12}\mathrm{c}_{23} - \mathrm{c}_{12}\mathrm{s}_{23}\mathrm{s}_{13} & \zeta \mathrm{c}_{12}\mathrm{c}_{23} - \mathrm{s}_{12}\mathrm{s}_{23}\mathrm{s}_{13} & \mathrm{s}_{23}\mathrm{c}_{13} \\ \zeta \mathrm{s}_{12}\mathrm{s}_{23} - \mathrm{c}_{12}\mathrm{c}_{23}\mathrm{s}_{13} & -\zeta \mathrm{c}_{12}\mathrm{s}_{23} - \mathrm{s}_{12}\mathrm{c}_{23}\mathrm{s}_{13} & \mathrm{c}_{23}\mathrm{c}_{13} \end{pmatrix}, \tag{9.34}$$

where $\mathrm{c}_{12} \equiv \cos\theta_{12}$ etc. and

$$\zeta = \pm 1; \qquad (\theta_{12}, \theta_{23}, \theta_{13}) \in \mathbb{C}, \quad |\operatorname{Re}\theta_{12}| \leqslant \pi, \quad |\operatorname{Re}\theta_{23}| \leqslant \pi, \quad |\operatorname{Re}\theta_{13}| \leqslant \frac{\pi}{2}. \tag{9.35}$$

This R satisfies $RR^{\mathrm{T}} = 1$, which however is not general (as in the next example).

Example: two right-handed neutrinos For models with two right-handed neutrinos, one neutrino is massless and $M_{\mathrm{L}}^{\text{diag}}$ is not invertible. However the parameterization

$$M_{\mathrm{D}} = \mathrm{i}\sqrt{M_{\mathrm{H}}^{\text{diag}}} R \sqrt{M_{\mathrm{L}}^{\text{diag}}} (U_{\mathrm{PMNS}}^{\text{Majorana}})^\dagger \tag{9.36}$$

works with[*11]

$$R_{\text{normal hierarchy}} = \begin{pmatrix} 0 & \cos z & \zeta\sin z \\ 0 & -\sin z & \zeta\cos z \end{pmatrix}, \qquad R_{\text{inverse hierarchy}} = \begin{pmatrix} \cos z & \zeta\sin z & 0 \\ -\sin z & \zeta\cos z & 0 \end{pmatrix}, \tag{9.37}$$

where $z \in \mathbb{C}$ and $\zeta = \pm 1$.

[*10] For $w \in \mathbb{C}$, $\sin z_1 = w$ and $\cos z_2 = w$ always have solutions $z_{1,2} \in \mathbb{C}$. Meanwhile, $\tan z = w$ has no solution if and only if $w = \pm\mathrm{i}$. Then, using this fact, one first expresses R_{i3} components by $\zeta_{\mathrm{A,B,C}} = \pm 1$ and $\theta_{\mathrm{A,B}} \in \mathbb{C}$, restricting $0 \leqslant \operatorname{Re}\theta_{\mathrm{A,B}} \leqslant \pi/2$ ($\Leftrightarrow \operatorname{Re}\sin\theta \geqslant 0 \wedge \operatorname{Re}\cos\theta \geqslant 0$), and then gets an expression of R with three angles and six signs. Five signs are absorbed by enlarging $\operatorname{Re}\theta$ and one sign remains, which is ζ.

[*11] See, e.g., Ref. [13]. Sho also thanks Kai Schmitz for his note.

10. Supersymmetry with $\eta = \mathrm{diag}(+,-,-,-)$

Convention Our convention follows DHM (except for D_μ):

$$\eta = \mathrm{diag}(1,-1,-1,-1); \quad \epsilon^{0123} = -\epsilon_{0123} = 1, \quad \epsilon^{12} = \epsilon_{21} = \epsilon^{\dot{1}\dot{2}} = \epsilon_{\dot{2}\dot{1}} = 1 \quad \left(\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = \delta^\alpha_\gamma\right),$$
$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta, \quad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta, \quad \bar\psi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\bar\psi_{\dot\beta}, \quad \bar\psi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\bar\psi^{\dot\beta};$$
$$\sigma^\mu_{\alpha\dot\alpha} := (\mathbf{1},\boldsymbol{\sigma})_{\alpha\dot\alpha}, \quad {\sigma^{\mu\nu}}_\alpha{}^\beta := \frac{\mathrm{i}}{4}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)_\alpha{}^\beta, \ ^{*12} \quad \left(\sigma^\mu_{\alpha\dot\beta} = \epsilon_{\alpha\delta}\epsilon_{\dot\beta\dot\gamma}\bar\sigma^{\mu\dot\gamma\delta}, \quad \bar\sigma^{\mu\dot\alpha\beta} = \epsilon^{\dot\alpha\dot\delta}\epsilon^{\beta\gamma}\sigma^\mu_{\gamma\dot\delta}\right)$$
$$\bar\sigma^{\mu\dot\alpha\alpha} := (\mathbf{1},-\boldsymbol{\sigma})^{\dot\alpha\alpha}, \quad {\bar\sigma^{\mu\nu\dot\alpha}}{}_{\dot\beta} := \frac{\mathrm{i}}{4}(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu)^{\dot\alpha}{}_{\dot\beta}, \ ^{*12}$$
$$(\psi\xi) := \psi^\alpha\xi_\alpha, \quad (\bar\psi\bar\chi) := \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha}; \qquad \frac{\mathrm{d}}{\mathrm{d}\theta^\alpha}(\theta\theta) := \theta_\alpha \quad \text{[left derivative]}.$$

Especially, spinor-index contraction is done as ${}^\alpha{}_\alpha$ and ${}_{\dot\alpha}{}^{\dot\alpha}$ except for ϵ_{ab} (which always comes from left). Noting that complex conjugate reverses spinor order: $(\psi^\alpha\xi^\beta)^* := (\xi^\beta)^*(\psi^\alpha)^*$,

$$\bar\psi^{\dot\alpha} := (\psi^\alpha)^*, \quad \epsilon^{\dot a\dot b} := (\epsilon^{ab})^*, \qquad \left(\psi\chi\right)^* = (\bar\psi\bar\chi),$$
$$\left(\sigma^\mu_{\alpha\dot\beta}\right)^* = \bar\sigma^\mu{}_{\dot\alpha\beta} = \epsilon_{\beta\delta}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\mu\dot\gamma\delta}, \qquad \left(\sigma^{\mu\nu}\right)^{\dagger\alpha}{}_\beta = \bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}, \qquad \left(\sigma^{\mu\nu}{}_\alpha{}^\beta\right)^* = \bar\sigma^{\mu\nu\dot\beta}{}_{\dot\alpha} = \bar\sigma^{\mu\nu}{}_{\dot\alpha}{}^{\dot\beta} = \epsilon_{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\delta}\bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\delta},$$
$$\left(\bar\sigma^{\mu\dot\alpha\beta}\right)^* = \sigma^{\mu\alpha\dot\beta} = \epsilon^{\dot\beta\dot\delta}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\delta}, \qquad \left(\bar\sigma^{\mu\nu}\right)^\dagger{}_{\dot\alpha}{}^{\dot\beta} = \sigma^{\mu\nu}{}_\alpha{}^\beta, \qquad \left(\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}\right)^* = \sigma^{\mu\nu}{}_\beta{}^\alpha = \bar\sigma^{\mu\nu\alpha}{}_\beta = \epsilon_{\beta\delta}\epsilon^{\alpha\gamma}\sigma^{\mu\nu}{}_\gamma{}^\delta.$$

Contraction formulae

$$\theta^\alpha\theta^\beta = -\tfrac12(\theta\theta)\epsilon^{\alpha\beta} \qquad \bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac12(\bar\theta\bar\theta)\epsilon^{\dot\alpha\dot\beta} \qquad (\theta\xi)(\theta\chi) = -\tfrac12(\theta\theta)(\xi\chi) \qquad (\theta\sigma^\nu\bar\theta)\theta^\alpha = \tfrac12(\theta\theta)(\bar\theta\bar\sigma^\nu)^\alpha$$
$$\theta_\alpha\theta_\beta = \tfrac12(\theta\theta)\epsilon_{\alpha\beta} \qquad \bar\theta_{\dot\alpha}\bar\theta_{\dot\beta} = -\tfrac12(\bar\theta\bar\theta)\epsilon_{\dot\alpha\dot\beta} \qquad (\bar\theta\bar\xi)(\bar\theta\bar\chi) = -\tfrac12(\bar\theta\bar\theta)(\bar\xi\bar\chi) \qquad (\theta\sigma^\nu\bar\theta)\bar\theta_{\dot\alpha} = -\tfrac12(\theta\sigma^\nu)_{\dot\alpha}(\bar\theta\bar\theta)$$
$$\theta^\alpha\theta_\beta = \tfrac12(\theta\theta)\delta^\alpha_\beta \qquad \bar\theta_{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac12(\bar\theta\bar\theta)\delta^{\dot\alpha}_{\dot\beta} \qquad (\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta) = \tfrac12(\theta\theta)(\bar\theta\bar\theta)\eta^{\mu\nu}$$
$$(\theta\sigma^\mu\bar\sigma^\nu\theta) = (\theta\theta)\eta^{\mu\nu} \qquad (\bar\theta\bar\sigma^\mu\sigma^\nu\bar\theta) = (\bar\theta\bar\theta)\eta^{\mu\nu} \qquad (\sigma^\mu\bar\theta)_\alpha(\theta\sigma^\nu\bar\theta) = \tfrac12(\bar\theta\bar\theta)(\sigma^\mu\bar\sigma^\nu\theta)_\alpha$$

$$\sigma^\mu\bar\sigma^\nu = \eta^{\mu\nu} - 2\mathrm{i}\sigma^{\mu\nu} \qquad \sigma^\mu\bar\sigma^\nu\sigma^\rho + \sigma^\rho\bar\sigma^\nu\sigma^\mu = 2\left(\sigma^\mu\eta^{\rho\nu} - \sigma^\nu\eta^{\mu\rho} + \sigma^\rho\eta^{\mu\nu}\right)$$
$$\bar\sigma^\mu\sigma^\nu = \eta^{\mu\nu} - 2\mathrm{i}\bar\sigma^{\mu\nu} \qquad \sigma^\mu\bar\sigma^\nu\sigma^\rho - \sigma^\rho\bar\sigma^\nu\sigma^\mu = 2\mathrm{i}\sigma_\sigma\epsilon^{\mu\nu\rho\sigma}$$
$$\mathrm{Tr}\left(\sigma^\mu\bar\sigma^\nu\right) = \mathrm{Tr}\left(\bar\sigma^\mu\sigma^\nu\right) = 2\eta^{\mu\nu} \qquad \bar\sigma^\mu\sigma^\nu\bar\sigma^\rho + \bar\sigma^\rho\sigma^\nu\bar\sigma^\mu = 2\left(\bar\sigma^\mu\eta^{\rho\nu} - \bar\sigma^\nu\eta^{\mu\rho} + \bar\sigma^\rho\eta^{\mu\nu}\right)$$
$$\sigma^\mu_{\alpha\dot\alpha}\bar\sigma_\mu{}^{\dot\beta\beta} = 2\delta^{\dot\beta}_{\dot\alpha}\delta^\alpha_\beta \qquad \bar\sigma^\mu\sigma^\nu\bar\sigma^\rho - \bar\sigma^\rho\sigma^\nu\bar\sigma^\mu = -2\mathrm{i}\bar\sigma_\sigma\epsilon^{\mu\nu\rho\sigma}$$
$$\sigma_{\mu\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = 2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta} \qquad \epsilon_{\dot\beta\dot\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon_{\dot\beta\dot\alpha}\epsilon^{\dot\alpha\dot\gamma}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\gamma} = \epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\beta}$$
$$\bar\sigma_\mu{}^{\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = 2\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta} \qquad \epsilon_{\beta\alpha}\bar\sigma^{\mu\dot\alpha\alpha} = \epsilon_{\beta\alpha}\epsilon^{\dot\alpha\dot\gamma}\epsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot\gamma} = \epsilon^{\dot\alpha\dot\gamma}\sigma^\mu_{\beta\dot\gamma}$$
$$\mathrm{Tr}\left(\sigma^{\mu\nu}\right) = \mathrm{Tr}\left(\bar\sigma^{\mu\nu}\right) = 0 \qquad \mathrm{Tr}\left(\sigma^{\mu\nu}\sigma^{\rho\sigma}\right) = \tfrac12\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right) - \tfrac12\mathrm{i}\epsilon^{\mu\nu\rho\sigma}$$
$$\bar\sigma^{\mu\nu} = -\bar\sigma^{\nu\mu} \qquad \sigma^{\mu\nu} = -\sigma^{\nu\mu} \qquad \mathrm{Tr}\left(\bar\sigma^{\mu\nu}\bar\sigma^{\rho\sigma}\right) = \tfrac12\mathrm{i}\epsilon^{\mu\nu\rho\sigma} + \tfrac12\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right)$$
$$\sigma^{\mu\nu}{}_\alpha{}^\beta\epsilon_{\beta\gamma} = \sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon_{\beta\alpha} \qquad \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} - \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = -2\mathrm{i}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\beta}\epsilon_{\alpha\beta} - 2\mathrm{i}\sigma^{\mu\nu}{}_\alpha{}^\gamma\epsilon_{\gamma\beta}\epsilon_{\dot\alpha\dot\beta}$$
$$\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}\epsilon^{\dot\beta\dot\gamma} = \bar\sigma^{\mu\nu\dot\gamma}{}_{\dot\beta}\epsilon^{\dot\beta\dot\alpha} \qquad \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} + \sigma^\nu_{\alpha\dot\alpha}\sigma^\mu_{\beta\dot\beta} = 4\sigma^{\rho\mu}{}_\alpha{}^\gamma\epsilon_{\gamma\beta}\epsilon_{\dot\alpha\dot\gamma}\bar\sigma^{\sigma\nu\dot\gamma}{}_{\dot\beta}\eta_{\rho\sigma} + \epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\eta^{\mu\nu}$$
$$\bar\sigma_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = -2\mathrm{i}\bar\sigma^{\mu\nu} \qquad \bar\sigma^{\mu\dot\alpha\alpha}\bar\sigma^{\nu\dot\beta\beta} - \bar\sigma^{\nu\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = -2\mathrm{i}\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\gamma}\epsilon^{\dot\gamma\dot\beta}\epsilon^{\alpha\beta} - 2\mathrm{i}\epsilon^{\alpha\gamma}\sigma^{\mu\nu}{}_\gamma{}^\beta\epsilon^{\dot\alpha\dot\beta}$$
$$\sigma_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} = 2\mathrm{i}\sigma^{\mu\nu} \qquad \bar\sigma^{\mu\dot\alpha\alpha}\bar\sigma^{\nu\dot\beta\beta} + \bar\sigma^{\nu\dot\alpha\alpha}\bar\sigma^{\mu\dot\beta\beta} = 4\epsilon^{\alpha\gamma}\sigma^{\sigma\nu}{}_\gamma{}^\beta\bar\sigma^{\rho\mu\dot\alpha}{}_{\dot\gamma}\epsilon^{\dot\gamma\dot\beta}\eta_{\rho\sigma} + \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}\eta^{\mu\nu}$$

$$\bar\xi\bar\sigma^\mu\chi = -\chi\sigma^\mu\bar\xi \qquad \bar\xi\bar\sigma^\mu\sigma^\nu\bar\chi = \bar\chi\bar\sigma^\nu\sigma^\mu\bar\xi \qquad \xi\sigma^\mu\bar\sigma^\nu\chi = \chi\sigma^\nu\bar\sigma^\mu\xi \qquad \bar\xi\bar\sigma^\mu\sigma^\nu\bar\sigma^\rho\chi = -\chi\sigma^\rho\bar\sigma^\nu\sigma^\mu\bar\xi$$
$$\left(\xi\sigma^\mu\bar\chi\right)^* = \chi\sigma^\mu\bar\xi \qquad \left(\bar\xi\bar\sigma^\mu\chi\right)^* = \bar\chi\bar\sigma^\mu\xi \qquad \left(\bar\chi\bar\sigma^\mu\sigma^\nu\bar\xi\right)^* = \xi\sigma^\nu\bar\sigma^\mu\chi \qquad (\xi[\sigma\mathrm{s}]\chi)^* = \bar\chi[\sigma\mathrm{s}_{\text{reversed}}]\bar\xi$$
$$(\xi\chi)\psi^\alpha = -(\psi\xi)\chi^\alpha - (\psi\chi)\xi^\alpha \qquad (\xi\chi)\bar\psi_{\dot\alpha} = \tfrac12(\xi\sigma^\mu\bar\psi)(\chi\sigma_\mu)_{\dot\alpha}$$
$$\mathrm{i}\psi_i\sigma^\mu\partial_\mu\bar\psi_j = -\mathrm{i}\partial_\mu\bar\psi_j\bar\sigma^\mu\psi_i \equiv \mathrm{i}\bar\psi_j\bar\sigma^\mu\partial_\mu\psi_i = -\mathrm{i}\partial_\mu\psi_i\sigma^\mu\bar\psi_j$$

*12 As the definition of $\sigma^{\mu\nu}$ and $\bar\sigma^{\mu\nu}$ are not unified in literature, they are not used in this CheatSheet except for this page.

Superfields

$$\Phi = \phi(x) + \sqrt{2}\theta\psi(x) - \mathrm{i}\partial_\mu\phi(x)(\theta\sigma^\mu\bar\theta) + F(x)\theta^2 + \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu\psi(x)\sigma^\mu\bar\theta)\theta^2 - \frac{\theta^4}{4}\partial^2\phi(x), \tag{10.1}$$

$$\Phi^* = \phi^*(x) + \sqrt{2}\bar\psi(x)\bar\theta + F^*(x)\bar\theta^2 + \mathrm{i}\partial_\mu\phi^*(x)(\theta\sigma^\mu\bar\theta) - \frac{\mathrm{i}}{\sqrt{2}}[\theta\sigma^\mu\partial_\mu\bar\psi(x)]\bar\theta^2 - \frac{\theta^4}{4}\partial^2\phi^*(x), \tag{10.2}$$

$$V = (\bar\theta\bar\sigma^\mu\theta)A_\mu(x) + \bar\theta^2\theta\lambda(x) + \theta^2\bar\theta\bar\lambda(x) + \frac{\theta^4}{2}D(x) \qquad \text{(in Wess-Zumino supergauge).} \tag{10.3}$$

Without gauge symmetries

$$\mathcal{L} = \Phi_i^*\Phi_i\Big|_{\theta^4} + \left(W(\Phi_i)\Big|_{\theta^2} + \text{H.c.}\right); \tag{10.4}$$

$$\Phi_i^*\Phi_i\Big|_{\theta^4} = (\partial_\mu\phi_i^*)(\partial^\mu\phi_i) + \mathrm{i}\bar\psi_i\sigma^\mu\partial_\mu\psi_i + F_i^*F_i, \tag{10.5}$$

$$\begin{aligned} W(\Phi_i)\Big|_{\theta^2} &\rightsquigarrow \Big[\kappa_i\Phi_i + m_{ij}\Phi_i\Phi_j + y_{ijk}\Phi_i\Phi_j\Phi_k\Big]_{\theta^2} \\ &= \kappa_iF_i + m_{ij}\left(-\psi_i\psi_j + F_i\phi_j + \phi_iF_j\right) \\ &\qquad + y_{ijk}\Big[-(\psi_i\psi_j\phi_k + \psi_i\phi_j\psi_k + \phi_i\psi_j\psi_k) + \phi_i\phi_jF_k + \phi_iF_j\phi_k + F_i\phi_j\phi_k\Big]. \end{aligned} \tag{10.6}$$

With a U(1) gauge symmetry [*13]

$$\mathcal{L} = \Phi_i^*\,\mathrm{e}^{2gVQ_i}\Phi_i\Big|_{\theta^4} + \left[\left(\frac{1}{4} - \frac{\mathrm{i}g^2\Theta}{32\pi^2}\right)\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + W(\Phi_i)\Big|_{\theta^2} + \text{H.c.}\right] + \Lambda_{\mathrm{FI}}D; \tag{10.7}$$

$$\Phi_i\,\mathrm{e}^{2gQ_iV}\Phi_i\Big|_{\theta^4} \equiv \mathrm{D}^\mu\phi_i^*\,\mathrm{D}_\mu\phi_i + \mathrm{i}\bar\psi_i\bar\sigma^\mu\,\mathrm{D}_\mu\psi_i + F_i^*F_i - \sqrt{2}gQ_i\phi_i^*\lambda\psi_i - \sqrt{2}gQ_i\bar\psi_i\bar\lambda\phi_i + gQ_i\phi_i^*\phi_iD, \tag{10.8}$$

$$\begin{aligned} \left(\frac{1}{4} - \frac{\mathrm{i}g^2\Theta}{32\pi^2}\right)\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + \text{H.c.} &= \frac{1}{2}\,\mathrm{Re}\,\mathcal{W}\mathcal{W}\big|_{\theta^2} + \frac{g^2\Theta}{16\pi^2}\,\mathrm{Im}\,\mathcal{W}\mathcal{W}\big|_{\theta^2} \\ &\equiv \mathrm{i}\bar\lambda\bar\sigma^\mu\,\mathrm{D}_\mu\lambda + \frac{1}{2}DD - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g^2\Theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}, \end{aligned} \tag{10.9}$$

$$\begin{aligned} &\mathrm{D}_\mu\phi_i = (\partial_\mu - \mathrm{i}gQ_iA_\mu)\phi_i, \qquad \mathrm{D}_\mu\psi_i = (\partial_\mu - \mathrm{i}gQ_iA_\mu)\psi_i, \\ &\mathrm{D}^\mu\phi_i^* = (\partial^\mu + \mathrm{i}gQ_iA^\mu)\phi_i^*, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad \mathrm{D}_\mu\lambda = \partial_\mu\lambda. \end{aligned}$$

$$\{\phi,\psi,F\} \xrightarrow{\text{gauge}} \mathrm{e}^{\mathrm{i}gQ_i\theta}\{\phi,\psi,F\}, \qquad A_\mu \xrightarrow{\text{gauge}} A_\mu + \partial_\mu\theta, \qquad \lambda \xrightarrow{\text{gauge}} \lambda, \qquad D \xrightarrow{\text{gauge}} D. \tag{10.10}$$

[*13] We use the convention with $V \ni \lambda(x)\theta\bar\theta^2$, which corresponds to $\lambda = \mathrm{i}\lambda_{\mathrm{SLHA}}$. In SLHA convention, the scalar-fermion-gaugino interaction is replaced to

$$-\sqrt{2}g\mathrm{i}\lambda^a_{\mathrm{SLHA}}(\phi^*t^a\psi) - \sqrt{2}g(-\mathrm{i}\bar\lambda^a_{\mathrm{SLHA}})(\bar\psi t^a\phi).$$

With an SU(N) gauge symmetry

$$\mathcal{L} = \Phi^* \mathrm{e}^{2gV}\Phi\Big|_{\theta^4} + \left[\left(\frac{1}{2} - \frac{\mathrm{i}g^2\Theta}{16\pi^2}\right)\mathrm{Tr}\,\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + W(\Phi)\Big|_{\theta^2} + \text{H.c.}\right]; \tag{10.11}$$

$$\Phi^* \mathrm{e}^{2gV}\Phi\Big|_{\theta^4} := \Phi_i^*\left[\mathrm{e}^{2gV^a t^a_\Phi}\right]_{ij}\Phi_j\Big|_{\theta^4} \tag{10.12}$$

$$\begin{aligned}&= (\partial_\mu\phi_i^*)(\partial^\mu\phi_i) + \mathrm{i}\bar\psi_i\bar\sigma^\mu\partial_\mu\psi_i + F_i^*F_i - \sqrt{2}g\lambda^a(\phi^*t^a\psi) - \sqrt{2}g\bar\lambda^a(\bar\psi^*t^a\phi)\\ &\qquad + gA^a_\mu\bar\psi\bar\sigma^\mu(t^a\psi) + 2\mathrm{i}gA^a_\mu\phi^*\partial_\mu(t^a\phi) + g^2A^{a\mu}A^b_\mu(\phi^*t^at^b\phi) + gD^a(\phi^*t^a\phi)\end{aligned} \tag{10.13}$$

$$= \mathrm{D}^\mu\phi^*\,\mathrm{D}_\mu\phi + \mathrm{i}\bar\psi_i\bar\sigma^\mu\,\mathrm{D}_\mu\psi_i + F^*F - \sqrt{2}g\lambda^a(\phi^*t^a\psi) - \sqrt{2}g\bar\lambda^a(\bar\psi t^a\phi) + gD^a(\phi^*t^a\phi) \tag{10.14}$$

$$\begin{aligned}\left(\frac{1}{2} - \frac{\mathrm{i}g^2\Theta}{16\pi^2}\right)\mathrm{Tr}\,\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + \text{H.c.} &= \mathrm{Re}\,\mathrm{Tr}\,\mathcal{W}\mathcal{W}\big|_{\theta^2} + \frac{g^2\Theta}{8\pi^2}\,\mathrm{Im}\,\mathrm{Tr}\,\mathcal{W}\mathcal{W}\big|_{\theta^2}\\ &= \mathrm{i}\lambda^a\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a + \frac{1}{2}D^aD^a - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{g^2\Theta}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma};\end{aligned} \tag{10.15}$$

$$\mathrm{D}_\mu\phi_i = \partial_\mu\phi_i - \mathrm{i}gA^a_\mu t^a_{ij}\phi_j, \qquad \mathrm{D}_\mu\psi_i = \partial_\mu\psi_i - \mathrm{i}gA^a_\mu t^a_{ij}\psi_j, \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gA^b_\mu A^c_\nu f^{abc},$$
$$\mathrm{D}^\mu\phi_i^* = \partial^\mu\phi_i^* + \mathrm{i}gA^{a\mu}\phi_j^*t^a_{ji}, \qquad \mathrm{D}_\mu\lambda^a_\alpha = \partial_\mu\lambda^a_\alpha + gf^{abc}A^b_\mu\lambda^c_\alpha.$$

$$\{\phi,\psi,F\} \xrightarrow{\text{gauge}} \mathrm{e}^{\mathrm{i}g\theta^a t^a}\{\phi,\psi,F\},$$
$$A^a_\mu \xrightarrow{\text{gauge}} A^a_\mu + \partial_\mu\theta^a + gf^{abc}A^b_\mu\theta^c + \mathcal{O}(\theta^2), \qquad \lambda^a \xrightarrow{\text{gauge}} \lambda^a + gf^{abc}\lambda^b\theta^c + \mathcal{O}(\theta^2),$$
$$D^a \xrightarrow{\text{gauge}} D^a + gf^{abc}D^b\theta^c + \mathcal{O}(\theta^2), \qquad \bar\lambda^a \xrightarrow{\text{gauge}} \bar\lambda^a + gf^{abc}\bar\lambda^b\theta^c + \mathcal{O}(\theta^2).$$ [*14]

Auxiliary fields and Scalar potential In all of the above three theories,

$$\mathcal{L} \supset F_i^*F_i + F_i\frac{\partial W}{\partial\Phi_i}\Big|_{\text{scalar}} + F_i^*\frac{\partial W^*}{\partial\Phi_i^*}\Big|_{\text{scalar}} + \frac{1}{2}D^aD^a + gD^a(\phi^*t^a\phi); \tag{10.16}$$

$$\langle F_i^*\rangle = -\frac{\partial W}{\partial\Phi_i}\Big|_{\text{scalar}}, \qquad \langle D^a\rangle = -g\phi^*t^a\phi; \tag{10.17}$$

$$\mathcal{L} \supset -V_{\text{SUSY}} = -\left[\langle F_i^*\rangle\langle F_i\rangle + \frac{g^2}{2}(\phi^*t^a\phi)(\phi^*t^a\phi)\right]. \tag{10.18}$$

[*14] ♣TODO: give in non-infinitesimal form♣

10.1. Lorentz symmetry as SU(2)×SU(2)

10.2. Supersymmetry algebra

We define the generators as

$$P_\mu := \mathrm{i}\partial_\mu, \quad \{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_{\dot\alpha}\} = -2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu = -2\sigma^\mu_{\alpha\dot\alpha}P_\mu, \quad \{\mathcal{Q}_\alpha, \mathcal{Q}_\beta\} = \{\bar{\mathcal{Q}}_{\dot\alpha}, \bar{\mathcal{Q}}_{\dot\beta}\} = 0, \tag{10.19}$$

which is realized by

$$\mathcal{Q}_\alpha = \frac{\partial}{\partial\theta^\alpha} + \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \quad \bar{\mathcal{Q}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu, \quad \mathcal{Q}^\alpha = -\frac{\partial}{\partial\theta_\alpha} - \mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\partial_\mu, \quad \bar{\mathcal{Q}}^{\dot\alpha} = \frac{\partial}{\partial\bar\theta_{\dot\alpha}} + \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu,$$

$$\mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} - \mathrm{i}(\sigma^\mu\bar\theta)_\alpha\partial_\mu, \quad \bar{\mathcal{D}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + \mathrm{i}(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu, \quad \mathcal{D}^\alpha = -\frac{\partial}{\partial\theta_\alpha} + \mathrm{i}(\bar\theta\bar\sigma^\mu)^\alpha\partial_\mu, \quad \bar{\mathcal{D}}^{\dot\alpha} = \frac{\partial}{\partial\bar\theta_{\dot\alpha}} - \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu;$$

$\mathcal{D}_\alpha$ etc. works as covariant derivatives because of the commutation relations

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} = +2\mathrm{i}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu, \qquad \{\mathcal{Q}_\alpha, \mathcal{D}_\beta\} = \{\mathcal{Q}_\alpha, \bar{\mathcal{D}}_{\dot\beta}\} = \{\bar{\mathcal{Q}}_{\dot\alpha}, \mathcal{D}_\beta\} = \{\bar{\mathcal{Q}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = \{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = \{\bar{\mathcal{D}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = 0.$$

Derivative formulae

$$\epsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\beta} = -\frac{\partial}{\partial\theta_\alpha} \qquad \frac{\partial}{\partial\theta^\alpha}\theta\theta = 2\theta_\alpha \qquad \frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta_\beta}\theta\theta = -2\delta^\beta_\alpha \qquad \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\beta}}\bar\theta\bar\theta = 2\delta^{\dot\beta}_{\dot\alpha}$$

$$\epsilon_{\alpha\beta}\frac{\partial}{\partial\theta_\beta} = -\frac{\partial}{\partial\theta^\alpha} \qquad \frac{\partial}{\partial\theta_\alpha}\theta\theta = -2\theta^\alpha \qquad \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta_\beta}\theta\theta = 2\epsilon^{\alpha\beta} \qquad \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\beta}}\bar\theta\bar\theta = -2\epsilon^{\dot\alpha\dot\beta}$$

$$\epsilon^{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta^{\dot\beta}} = -\frac{\partial}{\partial\bar\theta_{\dot\alpha}} \qquad \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\bar\theta\bar\theta = 2\bar\theta^{\dot\alpha} \qquad \frac{\partial}{\partial\theta_\alpha}\frac{\partial}{\partial\theta^\beta}\theta\theta = 2\delta^\alpha_\beta \qquad \frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar\theta^{\dot\beta}}\bar\theta\bar\theta = -2\delta^{\dot\alpha}_{\dot\beta}$$

$$\epsilon_{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar\theta_{\dot\beta}} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} \qquad \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\bar\theta\bar\theta = -2\bar\theta_{\dot\alpha} \qquad \frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\theta\theta = -2\epsilon_{\alpha\beta} \qquad \frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta^{\dot\beta}}\bar\theta\bar\theta = 2\epsilon_{\dot\alpha\dot\beta}$$

In addition, we define

$$(y, \theta', \bar\theta') := (x - \mathrm{i}\theta\sigma^\mu\bar\theta, \theta, \bar\theta) : \tag{10.20}$$

$$\bar{\mathcal{D}}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta'^{\dot\alpha}}; \qquad \begin{pmatrix} \frac{\partial}{\partial x^\mu} \\ \frac{\partial}{\partial\theta^\alpha} \\ \frac{\partial}{\partial\bar\theta^{\dot\alpha}} \end{pmatrix} = \begin{pmatrix} \delta^\nu_\mu & 0 & 0 \\ -\mathrm{i}(\sigma^\nu\bar\theta)_\alpha & \delta^\beta_\alpha & 0 \\ \mathrm{i}(\theta\sigma^\nu)_{\dot\alpha} & 0 & \delta^{\dot\beta}_{\dot\alpha} \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial y^\nu} \\ \frac{\partial}{\partial\theta'^\beta} \\ \frac{\partial}{\partial\bar\theta'^{\dot\beta}} \end{pmatrix}, \quad \begin{pmatrix} \frac{\partial}{\partial y^\nu} \\ \frac{\partial}{\partial\theta'^\beta} \\ \frac{\partial}{\partial\bar\theta'^{\dot\beta}} \end{pmatrix} = \begin{pmatrix} \delta^\mu_\nu & 0 & 0 \\ \mathrm{i}(\sigma^\mu\bar\theta)_\beta & \delta^\alpha_\beta & 0 \\ -\mathrm{i}(\theta\sigma^\mu)_{\dot\beta} & 0 & \delta^{\dot\alpha}_{\dot\beta} \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial x^\mu} \\ \frac{\partial}{\partial\theta^\alpha} \\ \frac{\partial}{\partial\bar\theta^{\dot\alpha}} \end{pmatrix}, \tag{10.21}$$

and a function $f : \mathbb{C}^4 \to \mathbb{C}$ (independent of θ' and $\bar\theta'$) is expanded as

$$f(y) = f(x - \mathrm{i}\theta\sigma\bar\theta) = f(x) - \mathrm{i}(\theta\sigma^\mu\bar\theta)\partial_\mu f(x) - \frac{1}{4}\theta^4\partial^2 f(x). \tag{10.22}$$

Note that we differentiate $[f(y)]^*$ and $f^*(y)$:

$$[f(y)]^* = f(x) + \mathrm{i}(\theta\sigma^\mu\bar\theta)\partial_\mu f^*(x) - \frac{1}{4}\theta^4\partial^2 f^*(x) = f^*(y + \mathrm{i}\theta\sigma\bar\theta) = f^*(y^*). \tag{10.23}$$

10.3. Superfields

SUSY-invariant Lagrangian SUSY transformation is induced by $\xi\mathcal{Q} + \bar\xi\bar{\mathcal{Q}} = \xi^\alpha\partial_\alpha + \bar\xi_{\dot\alpha}\partial^{\dot\alpha} + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu$. Therefore, for an object Ψ in the superspace,

$$\big[\Psi\big]_{\theta^4} \xrightarrow{\text{SUSY}} \Big[\Psi + \xi^\alpha\partial_\alpha\Psi + \bar\xi_{\dot\alpha}\partial^{\dot\alpha}\Psi + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu\Psi\Big]_{\theta^4} = \big[\Psi + \mathrm{i}(\xi\sigma^\mu\bar\theta + \bar\xi\bar\sigma^\mu\theta)\partial_\mu\Psi\big]_{\theta^4}, \tag{10.24}$$

which means $\big[\Psi\big]_{\theta^4}$ is SUSY-invariant up to total derivative, i.e., $\int \mathrm{d}^4x\big[\Psi\big]_{\theta^4}$ is SUSY-invariant action. Also,

$$\big[\Psi\big]_{\theta^2} \xrightarrow{\text{SUSY}} \Big[\Psi + \bar\xi_{\dot\alpha}\Big(\partial^{\dot\alpha} + \mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu\Big)\Psi\Big]_{\theta^2} = \Big[\Psi + \bar\xi_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\Psi + 2\mathrm{i}(\bar\sigma^\mu\theta)^{\dot\alpha}\partial_\mu\Psi\Big]_{\theta^2} \tag{10.25}$$

will be SUSY-invariant if $\bar{\mathcal{D}}_{\dot\alpha}\Psi = 0$, i.e., Ψ is a chiral superfield. Therefore, SUSY-invariant Lagrangian is given by

$$\mathcal{L} = \Big[(\text{any real superfield})\Big]_{\theta^4} + \Big[(\text{any chiral superfield})\Big]_{\theta^2} + \Big[(\text{any chiral superfield})^*\Big]_{\bar\theta^2}. \tag{10.26}$$

Chiral superfield A chiral superfield is a superfield that satisfies $\bar{\mathcal{D}}_{\dot\alpha}\Phi = 0$, i.e., we find

$$\Phi = \phi(y) + \sqrt{2}\theta'\psi(y) + \theta'^2F(y) \tag{10.27}$$

$$= \phi(x) + \sqrt{2}\theta\psi(x) - \mathrm{i}\partial_\mu\phi(x)(\theta\sigma^\mu\bar\theta) + F(x)\theta^2 + \frac{\mathrm{i}}{\sqrt{2}}(\partial_\mu\psi(x)\sigma^\mu\bar\theta)\theta^2 - \frac{1}{4}\partial^2\phi(x)\theta^4 \tag{10.28}$$

$$\Phi^* = \phi^*(x) + \sqrt{2}\bar\psi(x)\bar\theta + F^*(x)\bar\theta^2 + \mathrm{i}\partial_\mu\phi^*(x)(\theta\sigma^\mu\bar\theta) - \frac{\mathrm{i}}{\sqrt{2}}[\theta\sigma^\mu\partial_\mu\bar\psi(x)]\bar\theta^2 - \frac{1}{4}\partial^2\phi^*(x)\theta^4; \tag{10.29}$$

their product is expanded as

$$\begin{aligned}\Phi_i^*\Phi_j &= \phi_i^*\phi_j + \sqrt{2}\phi_i^*(\theta\psi_j) + \sqrt{2}(\bar\psi_i\bar\theta)\phi_j + \phi_i^*F_j\theta^2 + 2(\bar\psi_i\bar\theta)(\theta\psi_j) - \mathrm{i}\left(\phi_i^*\partial_\mu\phi_j - \partial_\mu\phi_i^*\phi_j\right)(\theta\sigma^\mu\bar\theta) + F_i^*\phi_j\bar\theta^2\\
&\quad + \left[\sqrt{2}\bar\psi_i\bar\theta F_j - \frac{\mathrm{i}\left(\partial_\mu\phi_i^*\cdot\psi_j\sigma^\mu\bar\theta - \phi_i^*\partial_\mu\psi_j\sigma^\mu\bar\theta\right)}{\sqrt{2}}\right]\theta^2 + \left[\sqrt{2}F_i^*\theta\psi_j + \frac{\mathrm{i}\left(\theta\sigma^\mu\bar\psi_i\partial_\mu\phi_j - \theta\sigma^\mu\partial_\mu\bar\psi_i\phi_j\right)}{\sqrt{2}}\right]\bar\theta^2\\
&\quad + \frac{1}{4}\left(4F_i^*F_j - \phi_i^*\partial^2\phi_j - (\partial^2\phi_i^*)\phi_j + 2(\partial_\mu\phi_i^*)(\partial^\mu\phi_j) + 2\mathrm{i}(\psi_j\sigma^\mu\partial_\mu\bar\psi_i) - 2\mathrm{i}(\partial_\mu\psi_j\sigma^\mu\bar\psi_i)\right)\theta^4
\end{aligned}\tag{10.30}$$

$$\begin{aligned}&\equiv \phi_i^*\phi_j + \sqrt{2}\phi_i^*(\theta\psi_j) + \sqrt{2}(\bar\psi_i\bar\theta)\phi_j + \phi_i^*F_j\theta^2 + 2(\bar\psi_i\bar\theta)(\theta\psi_j) - 2\mathrm{i}\left(\phi_i^*\partial_\mu\phi_j\right)(\theta\sigma^\mu\bar\theta) + F_i^*\phi_j\bar\theta^2\\
&\quad + \sqrt{2}\left(\bar\psi_i\bar\theta F_j + \mathrm{i}\phi_i^*\partial_\mu\psi_j\sigma^\mu\bar\theta\right)\theta^2 + \sqrt{2}\left(F_i^*\theta\psi_j - \mathrm{i}\theta\sigma^\mu\partial_\mu\bar\psi_i\phi_j\right)\bar\theta^2\\
&\quad + \left(F_i^*F_j + (\partial_\mu\phi_i^*)(\partial^\mu\phi_j) + \mathrm{i}\bar\psi_i\bar\sigma^\mu\partial_\mu\psi_j\right)\theta^4
\end{aligned}\tag{10.31}$$

$$\Phi_i\Phi_j\Big|_{\theta^2} = -\psi_i\psi_j + F_i\phi_j + \phi_iF_j \tag{10.32}$$

$$\Phi_i\Phi_j\Phi_k\Big|_{\theta^2} = -(\psi_i\psi_j)\phi_k - (\psi_k\psi_i)\phi_j - (\psi_j\psi_k)\phi_i + \phi_i\phi_jF_k + \phi_k\phi_iF_j + \phi_j\phi_kF_i \tag{10.33}$$

$$\mathrm{e}^{k\Phi} = \mathrm{e}^{k\phi}\left[1 + \sqrt{2}k\theta\psi + \left(kF - \frac{k^2}{2}\psi\psi\right)\theta^2 - \mathrm{i}k\partial_\mu\phi(\theta\sigma^\mu\bar\theta) + \frac{\mathrm{i}k\left(\partial_\mu\psi + k\psi\partial_\mu\phi\right)\sigma^\mu\bar\theta\theta^2}{\sqrt{2}} - \frac{k}{4}\left(\partial^2\phi + k\partial_\mu\phi\partial^\mu\phi\right)\theta^4\right]; \tag{10.34}$$

note that $\Phi_i\Phi_j$, $\Phi_i\Phi_j\Phi_k$, and $\mathrm{e}^{k\Phi}$ are all chiral superfields.

Vector superfield A vector superfield is a superfield V that satisfies $V = V^*$. It is given by real fields $\{C, M, N, D, A_\mu\}$ and Grassmann fields $\{\chi, \lambda\}$ as[*15]

$$\begin{aligned}V(x,\theta,\bar\theta) &= C(x) + \mathrm{i}\theta\chi(x) - \mathrm{i}\bar\theta\bar\chi(x) + \frac{1}{2}\left(M(x) + \mathrm{i}N(x)\right)\theta^2 + \frac{1}{2}\left(M(x) - \mathrm{i}N(x)\right)\bar\theta^2 + (\bar\theta\bar\sigma^\mu\theta)A_\mu(x)\\
&\quad \left(\lambda(x) + \frac{1}{2}\partial_\mu\bar\chi(x)\bar\sigma^\mu\right)\theta\bar\theta^2 + \theta^2\bar\theta\left(\bar\lambda(x) + \frac{1}{2}\bar\sigma^\mu\partial_\mu\chi(x)\right) + \frac{1}{2}\left(D(x) - \frac{1}{2}\partial^2C(x)\right)\theta^4.
\end{aligned}\tag{10.35}$$

With this convention,

$$V \to V - \mathrm{i}\Phi + \mathrm{i}\Phi^* \iff \begin{cases} C \to C - \mathrm{i}\phi + \mathrm{i}\phi^*, & \chi \to \chi - \sqrt{2}\psi, \qquad \lambda \to \lambda,\\ M + \mathrm{i}N \to M + \mathrm{i}N - 2\mathrm{i}F, & A_\mu \to A_\mu + \partial_\mu(\phi + \phi^*), \qquad D \to D.\end{cases} \tag{10.36}$$

The exponential of a vector superfield is also a vector superfield:

$$\begin{aligned}\mathrm{e}^{kV} &= \mathrm{e}^{kC}\Bigg\{1 + \mathrm{i}k(\theta\chi - \bar\theta\bar\chi) + \left(\frac{M + \mathrm{i}N}{2}k + \frac{\chi\chi}{4}k^2\right)\theta^2 + \left(\frac{M - \mathrm{i}N}{2}k + \frac{\bar\chi\bar\chi}{4}k^2\right)\bar\theta^2 + (k^2\theta\chi\bar\theta\bar\chi - k\theta\sigma^\mu\bar\theta A_\mu)\\
&\quad + \left[k\bar\theta\bar\lambda - \mathrm{i}k\bar\theta\bar\chi\left(\frac{M + \mathrm{i}N}{2}k + \frac{\chi\chi}{4}k^2\right) + \frac{1}{2}k\bar\theta\bar\sigma^\mu\left(\partial_\mu\chi - \mathrm{i}k\chi A_\mu\right)\right]\theta^2\\
&\quad + \left[k\theta\lambda + \mathrm{i}k\theta\chi\left(\frac{M - \mathrm{i}N}{2}k + \frac{\bar\chi\bar\chi}{4}k^2\right) - \frac{1}{2}k\theta\sigma^\mu\left(\partial_\mu\bar\chi + \mathrm{i}k\bar\chi A_\mu\right)\right]\bar\theta^2\\
&\quad + \left[\frac{k}{2}\left(D - \frac{1}{2}\partial^2C\right) - \frac{1}{2}\mathrm{i}k^2(\lambda\chi - \bar\lambda\bar\chi) + \left(\frac{M + \mathrm{i}N}{2}k + \frac{\chi\chi}{4}k^2\right)\left(\frac{M - \mathrm{i}N}{2}k + \frac{\bar\chi\bar\chi}{4}k^2\right)\right.\\
&\qquad \left. + \frac{k^3}{4}\bar\chi\bar\sigma^\mu\chi A_\mu + \frac{k^2}{4}\left(\mathrm{i}\bar\chi\bar\sigma^\mu\partial_\mu\chi - \mathrm{i}\partial_\mu\bar\chi\bar\sigma^\mu\chi + A^\mu A_\mu\right)\right]\theta^4\Bigg\}.
\end{aligned}\tag{10.37}$$

Supergauge symmetry The gauge transformation $\phi(x) \to \mathrm{e}^{\mathrm{i}g\theta^a(x)t^a}\phi(x)$ is not closed in the chiral superfield: $\mathrm{e}^{\mathrm{i}g\theta^a(x)t^a}\Phi(x)$ is not a chiral superfield if the parameter $\theta(x)$ has x^μ-dependence. Hence, in supersymmetric theories, it is extended to *supergauge symmetry* parameterized by a chiral superfield $\Omega(x)$, which is given by

$$\Phi \to \mathrm{e}^{2\mathrm{i}g\Omega^a(x)t^a}\Phi, \qquad \Phi^* \to \Phi^*\,\mathrm{e}^{-2\mathrm{i}g\Omega^{*a}(x)t^a} \tag{10.38}$$

for a chiral superfield Φ and an anti-chiral superfield Φ^*. The supergauge-invariant Lagrangian should be

$$\mathcal{L} \sim \Phi^*\cdot(\text{real superfield})\cdot\Phi; \tag{10.39}$$

we parameterize the "real superfield" as $\mathrm{e}^{2gV^a(x)t^a}$:

$$\mathcal{L} = \left[\Phi^*\,\mathrm{e}^{2gV^a(x)t^a}\,\Phi\right]_{\theta^4}; \qquad \mathrm{e}^{2gV^a(x)t^a} \to \mathrm{e}^{2\mathrm{i}g\Omega^{*a}(x)t^a}\,\mathrm{e}^{2gV^a(x)t^a}\,\mathrm{e}^{-2\mathrm{i}g\Omega^a(x)t^a}. \tag{10.40}$$

[*15] Different coordination of "i"s are found in literature. Take care, especially, $\lambda(\text{ours}) = \mathrm{i}\lambda(\text{Wess-Bagger}) = \mathrm{i}\lambda(\text{SLHA})$.

In Abelian case, t^a is replaced by the charge Q of Φ and

$$\mathcal{L} = \Big[\Phi^* \,\mathrm{e}^{2gQV(x)}\Phi\Big]_{\theta^4}; \qquad \Phi \to \mathrm{e}^{2\mathrm{i}gQ\Omega(x)}\Phi, \quad \Phi^* \to \Phi^*\,\mathrm{e}^{-2\mathrm{i}gQ\Omega^*(x)}, \tag{10.41}$$

$$\mathrm{e}^{2gQV(x)} \to \mathrm{e}^{2\mathrm{i}gQ\Omega^*(x)}\,\mathrm{e}^{2gQV(x)}\,\mathrm{e}^{-2\mathrm{i}gQ\Omega(x)} = \mathrm{e}^{2gQ\left(V-\mathrm{i}\Omega+\mathrm{i}\Omega^*\right)}. \tag{10.42}$$

The usual gauge transformation corresponds to the real part of the lowest component of Ω, i.e., $\theta \equiv 2\,\mathrm{Re}\,\phi = \phi + \phi^*$, and we use the other components to fix the supergauge so that C, M, N and χ are eliminated:

$$\text{supergauge fixing:}\quad V(x) \longrightarrow (\bar\theta\bar\sigma^\mu\theta)A_\mu(x) + \bar\theta^2\theta\lambda(x) + \theta^2\bar\theta\bar\lambda(x) + \frac{1}{2}D(x) \qquad \text{(Wess-Zumino gauge);} \tag{10.43}$$

$$\mathrm{e}^{2gQV} \longrightarrow 1 + gQ\left(-2\theta\sigma^\mu\bar\theta A_\mu + 2\theta^2\bar\theta\bar\lambda + 2\bar\theta^2\theta\lambda + D\theta^4\right) + g^2Q^2A^\mu A_\mu\theta^4. \tag{10.44}$$

The gauge transformation is the remnant freedom: $\Theta = \phi(y) = \phi - \mathrm{i}\partial_\mu\phi(\theta\sigma^\mu\bar\theta) - \partial^2\phi\theta^4/4$ with ϕ being real;

$$\Phi_i \to \mathrm{e}^{2\mathrm{i}gQ\Theta}\Phi_i, \qquad \mathrm{e}^{2gQV} \to \mathrm{e}^{2gQ(V-\mathrm{i}\Theta+\mathrm{i}\Theta^*)}. \tag{10.45}$$

Rules for each component is obvious in $(y,\theta,\bar\theta)$-basis and given by

$$\{\phi,\psi,F\} \to \mathrm{e}^{\mathrm{i}gQ\theta}\{\phi,\psi,F\}, \qquad A_\mu \to A_\mu + \partial_\mu\theta, \qquad \lambda\to\lambda, \qquad D\to D. \tag{10.46}$$

For non-Abelian gauges, the supergauge transformation for the real field is evaluated as

$$\begin{aligned}
\mathrm{e}^{2gV} &\to \mathrm{e}^{2\mathrm{i}g\Omega^*}\,\mathrm{e}^{2gV}\,\mathrm{e}^{-2\mathrm{i}g\Omega} && (10.47)\\
&= \left(\mathrm{e}^{2\mathrm{i}g\Omega^*}\,\mathrm{e}^{2gV}\,\mathrm{e}^{-2\mathrm{i}g\Omega^*}\right)\left(\mathrm{e}^{2\mathrm{i}g\Omega^*}\,\mathrm{e}^{-2\mathrm{i}g\Omega}\right) && (10.48)\\
&= \exp\left(\mathrm{e}^{[2\mathrm{i}g\Omega^*,}2gV\right)\mathrm{e}^{2\mathrm{i}g(\Omega^*-\Omega)} + \mathcal{O}(\Omega^2) && (10.49)\\
&= \exp\left(2gV + [2\mathrm{i}g\Omega^*, 2gV]\right)\mathrm{e}^{2\mathrm{i}g(\Omega^*-\Omega)} + \mathcal{O}(\Omega^2); && (10.50)\\
&= \exp\left[2gV + [2\mathrm{i}g\Omega^*, 2gV] + \int_0^1 \mathrm{d}t\, g(\mathrm{e}^{[2gV,})2\mathrm{i}g(\Omega^*-\Omega)] + \mathcal{O}(\Omega^2)\right] && (10.51)\\
&= \exp\left[2gV + [2\mathrm{i}g\Omega^*, 2gV] + \sum_{n=0}^{\infty}\frac{B_n\left([2gV,)^n\right.}{n!}2\mathrm{i}g(\Omega^*-\Omega)]\right] + \mathcal{O}(\Omega^2) && (10.52)\\
&= \exp\left[2g\left(V + \mathrm{i}(\Omega^*-\Omega) - [V,\mathrm{i}g(\Omega^*+\Omega)] + \sum_{n=2}^{\infty}\frac{\mathrm{i}B_n\left([2gV,)^n\right.}{n!}(\Omega^*-\Omega)]\right) + \mathcal{O}(\Omega^2)\right]. && (10.53)
\end{aligned}$$

Here, again we can use the "non-gauge" component of Ω to eliminate the C-term etc., i.e., we fix $\mathrm{i}(\Omega^*-\Omega)$, the second term of the expansion, to remove those terms:

$$V - [V,\mathrm{i}g(\Omega^*+\Omega)] + \left(\mathrm{i} + \sum_{n=2}^{\infty}\frac{\mathrm{i}B_n\left([2gV,)^n\right.}{n!}\right)(\Omega^*-\Omega)] + \mathcal{O}(\Omega^2) = (\bar\theta\bar\sigma^\mu\theta)A_\mu + \bar\theta^2\theta\lambda + \theta^2\bar\theta\bar\lambda + \frac{1}{2}D; \tag{10.54}$$

this defines the Wess-Zumino gauge:

$$\text{supergauge fixing:}\quad V^a(x) \longrightarrow (\bar\theta\bar\sigma^\mu\theta)A^a_\mu(x) + \bar\theta^2\theta\lambda^a(x) + \theta^2\bar\theta\bar\lambda^a(x) + \frac{1}{2}D^a(x), \tag{10.55}$$

$$\mathrm{e}^{2gV^at^a} \longrightarrow 1 + g\left(-2\theta\sigma^\mu\bar\theta A^a_\mu + 2\theta^2\bar\theta\bar\lambda^a + 2\bar\theta^2\theta\lambda^a + D^a\theta^4\right)t^a + g^2A^{a\mu}A^b_\mu\theta^4t^at^b. \tag{10.56}$$

The gauge transformation is given by

$$\Phi \to \mathrm{e}^{2\mathrm{i}g\Theta^at^a}\Phi, \quad \mathrm{e}^{2gV^at^a} \to \mathrm{e}^{2\mathrm{i}g\Theta^bt^b}\,\mathrm{e}^{2gV^at^a}\,\mathrm{e}^{-2\mathrm{i}g\Theta^ct^c}. \tag{10.57}$$

For components in chiral superfields,

$$\{\phi,\psi,F\} \to \mathrm{e}^{\mathrm{i}g\theta^at^a}\{\phi,\psi,F\}, \tag{10.58}$$

while for vector superfield we can express as infinitesimal transformation:

$$\begin{aligned}
V \to V' &\simeq V + \mathrm{i}(\Theta^*-\Theta) - [V,\mathrm{i}g(\Theta^*+\Theta)] + \sum_{n=2}^{\infty}\frac{\mathrm{i}B_n\left([2gV,)^n\right.}{n!}(\Theta^*-\Theta)] && (10.59)\\
&= V + 2(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\phi - \left[V,\mathrm{i}g\left(2\phi - \frac{\theta^4}{2}\partial^2\phi\right)\right] + 2\sum_{n=2}^{\infty}\frac{B_n\left([2gV,)^n\right.}{n!}(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\phi] && (10.60)\\
&= V + 2(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\phi + 2gf^{abc}V^b\phi^ct^a \qquad \text{(Wess-Zumino gauge)} && (10.61)
\end{aligned}$$

$$\begin{aligned}
\therefore\ A^a_\mu &\to A^a_\mu + \partial_\mu\theta^a + gf^{abc}A^b_\mu\theta^c + \mathcal{O}(\theta^2), \quad & \lambda^a &\to \lambda^a + gf^{abc}\lambda^b\theta^c + \mathcal{O}(\theta^2),\\
D^a &\to D^a + gf^{abc}D^b\theta^c + \mathcal{O}(\theta^2), & \bar\lambda^a &\to \bar\lambda^a + gf^{abc}\bar\lambda^b\theta^c + \mathcal{O}(\theta^2).
\end{aligned} \tag{10.62}$$

Gauge-field strength The real superfield e^V is gauge-invariant in Abelian case and a candidate in Lagrangian term, but this is not case in non-Abelian case. We thus define a chiral superfield from e^V:

$$\mathcal{W}_\alpha = \frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\left(\mathrm{e}^{-2gV}\mathcal{D}_\alpha\,\mathrm{e}^{2gV}\right); \tag{10.63}$$

$$\mathcal{W}_\alpha \xrightarrow{\text{gauge}} \mathrm{e}^{2\mathrm{i}g\Omega}\mathcal{W}_\alpha\,\mathrm{e}^{-2\mathrm{i}g\Omega} \quad \left(\mathcal{W}^a_\alpha \xrightarrow{\text{gauge}} [\mathrm{e}^{+2g\tilde f^c\Omega^c}]^{ab}W^b_\alpha \quad \text{with} \quad [\tilde f^c]_{ab} = f^{abc}\right);^{*16} \tag{10.64}$$

it is not supergauge- or Lorentz-invariatn, but $\mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha) = \mathrm{Tr}(\epsilon^{\alpha\beta}\mathcal{W}_\beta\mathcal{W}_\alpha)$ is supergauge- and Lorentz-invariant, and its θ^2-term is SUSY-invariant, which becomes a candidate in SUSY Lagrangian with its Hermitian conjugate.

In Wess-Zumino gauge, it is given by

$$\mathcal{W}_\alpha = \left\{\lambda^a_\alpha(y) + \theta_\alpha D^a(y) + \frac{[\mathrm{i}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)\theta]_\alpha}{4}F^a_{\mu\nu}(y) + \theta^2\left[\mathrm{i}\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a(y^*)\right]_\alpha\right\}t^a \tag{10.65}$$

$$\begin{aligned} &= \left[\lambda^a_\alpha + \theta_\alpha D^a + \frac{\mathrm{i}}{2}(\sigma^\mu\bar\sigma^\nu\theta)_\alpha F^a_{\mu\nu} + \mathrm{i}\theta^2(\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a)_\alpha + \mathrm{i}(\bar\theta\bar\sigma^\mu\theta)\partial_\mu\lambda^a_\alpha - \frac{\theta^4}{4}\partial^2\lambda^a_\alpha\right.\\ &\qquad\left. + \frac{\mathrm{i}\theta^2(\sigma^\mu\bar\theta)_\alpha}{2}\left(\partial_\mu D^a + \mathrm{i}\partial^\nu F^a_{\mu\nu} - gf^{abc}\epsilon_{\mu\nu\rho\sigma}A^{\nu b}\partial^\rho A^{\sigma c}\right)\right]T^a, \end{aligned} \tag{10.66}$$

where, as usual,

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gA^b_\mu A^c_\nu f^{abc}, \qquad \mathrm{D}_\mu\lambda^a_\alpha = \partial_\mu\lambda^a_\alpha + gf^{abc}A^b_\mu\lambda^c_\alpha. \tag{10.67}$$

Also,

$$[\mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha)]_{\theta^2} = \left[\mathrm{i}\lambda^a\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^b + \mathrm{i}\lambda^b\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a + D^aD^b - \frac{1}{4}\left(\mathrm{i}\epsilon^{\sigma\mu\nu\rho} + 2\eta^{\mu\rho}\eta^{\nu\sigma}\right)F^a_{\mu\nu}F^b_{\rho\sigma}\right]\mathrm{Tr}(t^at^b) \tag{10.68}$$

$$= \mathrm{i}\lambda^a\sigma^\mu\,\mathrm{D}_\mu\bar\lambda^a + \frac{1}{2}D^aD^a - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\mathrm{i}}{8}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}, \tag{10.69}$$

$$[\mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha)]_{\theta^4} = \frac{\theta^4}{4}\left(2(\partial^\mu\lambda^a)(\partial_\mu\lambda^b) - \lambda^a\partial^2\lambda^b - (\partial^2\lambda^a)\lambda^b\right)\mathrm{Tr}(t^at^b) = \frac{\theta^4}{4}\left((\partial^\mu\lambda^a)(\partial_\mu\lambda^a) - \lambda^a\partial^2\lambda^a\right). \tag{10.70}$$

For Abelian theory,

$$\mathcal{W}_\alpha = \frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\left(\mathrm{e}^{-2gV}\mathcal{D}_\alpha\,\mathrm{e}^{2gV}\right) = \frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}\mathcal{D}_\alpha(2gV), \tag{10.71}$$

$$\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} = 2\left(\mathrm{i}\lambda\sigma^\mu\,\mathrm{D}_\mu\bar\lambda + \frac{1}{2}DD - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mathrm{i}}{8}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right). \tag{10.72}$$

10.4. Lagrangian blocks

Lagrangian construction The supergauge transformation is summarized as

$$\Phi_i \to [U_\Phi]_{ij}\Phi_j, \qquad \tilde\Phi_j \to \tilde\Phi_i[U_\Phi^{-1}]_{ij}, \qquad \mathcal{W}_\alpha \to U_{\mathcal{W}}\mathcal{W}_\alpha U_{\mathcal{W}}^{-1}, \tag{10.73}$$

where

$$\tilde\Phi^*_j := \Phi^*_i[\mathrm{e}^{2gVt^a_\Phi}]_{ij}, \qquad U_\Phi := \exp(2\mathrm{i}g\Omega^at^a_\Phi), \qquad U_{\mathcal{W}} := \exp(2\mathrm{i}g\Omega^at^a_{\mathcal{W}}), \tag{10.74}$$

t^a_Φ is the representation matrix or U(1) charge for the field Φ, and $t^a_{\mathcal{W}}$ is the representation matrix that is used to define $\mathcal{W}_\alpha$. To construct a Lagrangian, we should composite these ingredients in real and invariant under SUSY, supergauge, and Lorentz transformation. A sufficient condition for SUSY invariance is given by (10.26), so

$$\mathcal{L} = \left[K(\Phi_i,\tilde\Phi^*_j)\right]_{\theta^4} + \left\{\left[f_{ab}(\Phi_i)\mathcal{W}^a\mathcal{W}^b\right]_{\theta^2} + \mathrm{H.c.}\right\} + \left\{\left[W(\Phi_i)\right]_{\theta^2} + \mathrm{H.c.}\right\} + D \tag{10.75}$$

is one possible construction. The Kähler function K should be real and supergauge invariant, the gauge kinetic function f should be holomorphic and supergauge invariant with $\mathcal{W}^a\mathcal{W}^b$, and the superpotential W is holomorphic and supergauge invariant. The last term D (Fayet-Illiopoulos term) comes from V of an U(1) gauge boson; note that its supergauge invariance is due to the intentional definition of V.

One can construct more general Lagrangian; for example, one can introduce a vector superfield that is not associated to a gauge symmetry, but then the supergauge fixing is not available and one has to include C or M fields.

Renormalizable Lagrangian Since $[\Phi]_{\theta^4}$ is a total derivative, renormalizable Lagrangian is limited to

$$\mathcal{L} = \left[\Phi^*_i[\mathrm{e}^{2gVt^a_\Phi}]_{ij}\Phi_j\right]_{\theta^4} + \left\{[\mathcal{W}^a\mathcal{W}^a]_{\theta^2} + [W(\Phi_i)]_{\theta^2} + \mathrm{H.c.}\right\} + D \tag{10.76}$$

up to numeric coefficients. With multiple gauge groups, the Kähler part is extended as $\Phi^*_i[\mathrm{e}^{2gVt^a_\Phi}\,\mathrm{e}^{2g'V't'^a_\Phi}\cdots]_{ij}\Phi_j$, where the inner part is obviously commutable.

[*16] ♣TODO:This equivalence should be checked/explained in gauge-theory section; especially, the sign is not verified and might be opposite.♣

11. Minimal Supersymmetric Standard Model

Gauge symmetry: $\mathrm{SU}(3)_{\mathrm{color}} \times \mathrm{SU}(2)_{\mathrm{weak}} \times \mathrm{U}(1)_Y$

Particle content:

(a) Chiral superfields

	SU(3)	SU(2)	U(1)	B	L	scalar/spinor
Q_i	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$1/3$		$\tilde{q}_{\mathrm{L}}, q_{\mathrm{L}}$ $[\to (u_{\mathrm{L}}, d_{\mathrm{L}})]$
L_i		$\mathbf{2}$	$-1/2$		1	$\tilde{l}_{\mathrm{L}}, l_{\mathrm{L}}$ $[\to (\nu_{\mathrm{L}}, l_{\mathrm{L}})]$
U^{c}_i	$\overline{\mathbf{3}}$		$-2/3$	$-1/3$		$\tilde{u}^{\mathrm{c}}_{\mathrm{R}}, u^{\mathrm{c}}_{\mathrm{R}}$
D^{c}_i	$\overline{\mathbf{3}}$		$1/3$	$-1/3$		$\tilde{d}^{\mathrm{c}}_{\mathrm{R}}, d^{\mathrm{c}}_{\mathrm{R}}$
E^{c}_i			1		-1	$\tilde{e}^{\mathrm{c}}_{\mathrm{R}}, e^{\mathrm{c}}_{\mathrm{R}}$
H_{u}		$\mathbf{2}$	$1/2$			$h_{\mathrm{u}}, \tilde{h}_{\mathrm{u}}$ $[\to (h^+_{\mathrm{u}}, h^0_{\mathrm{u}})]$
H_{d}		$\mathbf{2}$	$-1/2$			$h_{\mathrm{d}}, \tilde{h}_{\mathrm{d}}$ $[\to (h^0_{\mathrm{d}}, h^-_{\mathrm{d}})]$

(b) Vector superfields

	SU(3)	SU(2)	U(1)	ino/boson
g	adj.			$\tilde{g}, g_\mu$
W		adj.		$\tilde{w}, W_\mu$
B				$\tilde{b}, B_\mu$

Here, each of the column groups shows (from left to right) superfield name, charges for the gauge symmetries, other quantum numbers if relevant, and notation for corresponding fields (and $\mathrm{SU}(2)$ decomposition).

"c"-notation For scalars, $\tilde{\phi}^{\mathrm{c}}_{\mathrm{R}} := \phi^*_{\mathrm{R}} = \mathsf{C}\phi_{\mathrm{R}}\mathsf{C}$ (because the intrinsic phase for C is $+1$ for quarks and leptons.) For matter spinors, $\psi^{\mathrm{c}}_{\mathrm{R}} := \psi_{\mathrm{R}}$ (and $\psi_{\mathrm{R}} = \bar{\psi}^{\mathrm{c}}_{\mathrm{R}}$); Dirac spinors are thus

$$\psi_{\mathrm{L}} = \begin{pmatrix}\psi_{\mathrm{L}}\\ 0\end{pmatrix}, \quad \overline{\psi_{\mathrm{L}}} = \begin{pmatrix}0 & \bar{\psi}_{\mathrm{L}}\end{pmatrix}, \quad \psi^{\mathrm{c}}_{\mathrm{R}} := \begin{pmatrix}\psi^{\mathrm{c}}_{\mathrm{R}}\\ 0\end{pmatrix} = C\begin{pmatrix}0\\ \psi_{\mathrm{R}}\end{pmatrix} = C\psi_{\mathrm{R}}, \quad \overline{\psi^{\mathrm{c}}_{\mathrm{R}}} = \begin{pmatrix}0 & \psi_{\mathrm{R}}\end{pmatrix} = \begin{pmatrix}\bar{\psi}_{\mathrm{R}} & 0\end{pmatrix} C = \overline{\psi_{\mathrm{R}}}C.$$

Superpotential and $\cancel{\mathrm{SUSY}}$-terms

$$W_{\mathrm{RPC}} = \mu H_{\mathrm{u}} H_{\mathrm{d}} - y_{uij} U^{\mathrm{c}}_i H_{\mathrm{u}} Q_j + y_{dij} D^{\mathrm{c}}_i H_{\mathrm{d}} Q_j + y_{eij} E^{\mathrm{c}}_i H_{\mathrm{d}} L_j, \tag{11.1}$$

$$W_{\mathrm{RPV}} = -\kappa_i L_i H_{\mathrm{u}} + \frac{1}{2}\lambda_{ijk} L_i L_j E^{\mathrm{c}}_k + \lambda'_{ijk} L_i Q_j D^{\mathrm{c}}_k + \frac{1}{2}\lambda''_{ijk} U^{\mathrm{c}}_i D^{\mathrm{c}}_j D^{\mathrm{c}}_k, \tag{11.2}$$

$$\mathcal{L}_{\cancel{\mathrm{SUSY}}} = -\frac{1}{2}\left(M_3 \tilde{g}_0 \tilde{g}_0 + M_2 \tilde{w}\tilde{w} + M_1 \tilde{b}\tilde{b} + \text{H.c.}\right) - V_{\cancel{\mathrm{SUSY}}}; \tag{11.3}$$

$$\begin{aligned} V^{\mathrm{RPC}}_{\cancel{\mathrm{SUSY}}} = {} & \left(\tilde{q}^*_{\mathrm{L}} m^2_Q \tilde{q}_{\mathrm{L}} + \tilde{l}^*_{\mathrm{L}} m^2_L \tilde{l}_{\mathrm{L}} + \tilde{u}^*_{\mathrm{R}} m^2_{U^{\mathrm{c}}} \tilde{u}_{\mathrm{R}} + \tilde{d}^*_{\mathrm{R}} m^2_{D^{\mathrm{c}}} \tilde{d}_{\mathrm{R}} + \tilde{e}^*_{\mathrm{R}} m^2_{E^{\mathrm{c}}} \tilde{e}_{\mathrm{R}} + m^2_{H_{\mathrm{u}}} |h_{\mathrm{u}}|^2 + m^2_{H_{\mathrm{d}}} |h_{\mathrm{d}}|^2\right) \\ & + \left(-\tilde{u}^*_{\mathrm{R}} h_{\mathrm{u}} a_{\mathrm{u}} \tilde{q}_{\mathrm{L}} + \tilde{d}^*_{\mathrm{R}} h_{\mathrm{d}} a_{\mathrm{d}} \tilde{q}_{\mathrm{L}} + \tilde{e}^*_{\mathrm{R}} h_{\mathrm{d}} a_{\mathrm{e}} \tilde{l}_{\mathrm{L}} + b H_{\mathrm{u}} H_{\mathrm{d}} + \text{H.c.}\right) \\ & + \left(+\tilde{u}^*_{\mathrm{R}} h^*_{\mathrm{d}} c_{\mathrm{u}} \tilde{q}_{\mathrm{L}} + \tilde{d}^*_{\mathrm{R}} h^*_{\mathrm{u}} c_{\mathrm{d}} \tilde{q}_{\mathrm{L}} + \tilde{e}^*_{\mathrm{R}} h^*_{\mathrm{u}} c_{\mathrm{e}} \tilde{l}_{\mathrm{L}} + \text{H.c.}\right), \end{aligned} \tag{11.4}$$

$$\begin{aligned} V^{\mathrm{RPV}}_{\cancel{\mathrm{SUSY}}} = {} & \left(-b_i \tilde{l}_{\mathrm{L}i} H_{\mathrm{u}} + \frac{1}{2} T_{ijk} \tilde{l}_{\mathrm{L}i} \tilde{l}_{\mathrm{L}j} \tilde{e}^*_{\mathrm{R}k} + T'_{ijk} \tilde{l}_{\mathrm{L}i} \tilde{q}_{\mathrm{L}j} \tilde{d}^*_{\mathrm{R}k} + \frac{1}{2} T''_{ijk} \tilde{u}^*_{\mathrm{R}i} \tilde{d}^*_{\mathrm{R}j} \tilde{d}^*_{\mathrm{R}k} + \tilde{l}^*_{\mathrm{L}i} M^2_{Li} H_{\mathrm{d}} + \text{H.c.}\right) \\ & + \left(C^1_{ijk} \tilde{l}^*_{\mathrm{L}i} \tilde{q}_{\mathrm{L}j} \tilde{u}^*_{\mathrm{R}k} + C^2_i h^*_{\mathrm{u}} h_{\mathrm{d}} \tilde{e}^*_{\mathrm{R}i} + C^3_{ijk} \tilde{d}_{\mathrm{R}i} \tilde{u}^*_{\mathrm{R}j} \tilde{e}^*_{\mathrm{R}k} + \frac{1}{2} C^4_{ijk} \tilde{d}_{\mathrm{R}i} \tilde{q}_{\mathrm{L}j} \tilde{q}_{\mathrm{L}k} + \text{H.c.}\right), \end{aligned} \tag{11.5}$$

($\lambda_{ijk} = -\lambda_{jik}$, $\lambda''_{ijk} = -\lambda''_{ikj}$, and $C^4_{ijk} = C^4_{ikj}$.)

11.1. Notation

Our notation in this section (and the previous section) follows DHM [14, PhysRept] and Martin [15, v7] (but note that Martin uses $(-,+,+,+)$-metric) for RPC part and SLHA2 convention for RPV part. In particular, the sign of gauge bosons are fixed by $\mathrm{D}_\mu\phi = \partial_\mu\phi - \mathrm{i}gA^a_\mu t^a_{ij}\phi_j$, and the phase of gauginos are by $\mathcal{L} \ni \sqrt{2}g(\phi^* t^a\psi\lambda^a)$. Phases of ϕ and ψ in chiral superfields are not yet specified; they are later used to remove $F\tilde{F}$ terms and diagonalize Yukawa matrices.

11.2. Lagrangian construction

The most generic form of the Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\mathrm{matter}} + \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{super}} + \mathcal{L}_{\mathrm{FI}} + \mathcal{L}_{\cancel{\mathrm{SUSY}}}; \tag{11.6}$$

$$\mathcal{L}_{\mathrm{matter}} = \Phi_Q^* \exp\left(2g_Y(\tfrac{1}{6})V_B + 2g_2V_W^aT^a + 2g_3V_g^a\tau^a\right)\Phi_Q\Big|_{\theta^4} + \cdots; \tag{11.7}$$

$$\mathcal{L}_{\mathrm{gauge}} = \left[\frac{1}{4}\left(1-\frac{\mathrm{i}g_Y^2\Theta_B}{8\pi^2}\right)\mathcal{W}_B\mathcal{W}_B + \frac{1}{4}\left(1-\frac{\mathrm{i}g_2^2\Theta_W}{8\pi^2}\right)\mathcal{W}_W^a\mathcal{W}_W^a + \frac{1}{4}\left(1-\frac{\mathrm{i}g_3^2\Theta_g}{8\pi^2}\right)\mathcal{W}_g^a\mathcal{W}_g^a\right]_{\theta^2} + \text{H.c.}; \tag{11.8}$$

$$\mathcal{L}_{\mathrm{super}} = W(\Phi)\Big|_{\theta^2} + \text{H.c.}, \tag{11.9}$$

$$W(\Phi) = W_{\mathrm{RPC}} + W_{\mathrm{RPV}}, \tag{11.10}$$

$$W_{\mathrm{RPC}} = \mu H_{\mathrm{u}}H_{\mathrm{d}} - y_{uij}U^{\mathrm{c}}_iH_{\mathrm{u}}Q_j + y_{dij}D^{\mathrm{c}}_iH_{\mathrm{d}}Q_j + y_{eij}E^{\mathrm{c}}_iH_{\mathrm{d}}L_j, \tag{11.11}$$

$$W_{\mathrm{RPV}} = -\kappa_iL_iH_{\mathrm{u}} + \frac{1}{2}\lambda_{ijk}L_iL_jE^{\mathrm{c}}_k + \lambda'_{ijk}L_iQ_jD^{\mathrm{c}}_k + \frac{1}{2}\lambda''_{ijk}U^{\mathrm{c}}_iD^{\mathrm{c}}_jD^{\mathrm{c}}_k; \tag{11.12}$$

$$\mathcal{L}_{\mathrm{FI}} = \Lambda_{\mathrm{FI}}D_B; \tag{11.13}$$

$$\mathcal{L}_{\cancel{\mathrm{SUSY}}} = -\frac{1}{2}\left(M_3\tilde{g}_0\tilde{g}_0 + M_2\tilde{w}\tilde{w} + M_1\tilde{b}\tilde{b} + \text{H.c.}\right) - \left(V^{\mathrm{RPC}}_{\cancel{\mathrm{SUSY}}} + V^{\mathrm{RPV}}_{\cancel{\mathrm{SUSY}}}\right), \tag{11.14}$$

$$\begin{aligned} V^{\mathrm{RPC}}_{\cancel{\mathrm{SUSY}}} = {}& \left(\tilde{q}_{\mathrm{L}}^*m_Q^2\tilde{q}_{\mathrm{L}} + \tilde{l}_{\mathrm{L}}^*m_L^2\tilde{l}_{\mathrm{L}} + \tilde{u}_{\mathrm{R}}^*m_{U^{\mathrm{c}}}^2\tilde{u}_{\mathrm{R}} + \tilde{d}_{\mathrm{R}}^*m_{D^{\mathrm{c}}}^2\tilde{d}_{\mathrm{R}} + \tilde{e}_{\mathrm{R}}^*m_{E^{\mathrm{c}}}^2\tilde{e}_{\mathrm{R}} + m_{H_{\mathrm{u}}}^2|h_{\mathrm{u}}|^2 + m_{H_{\mathrm{d}}}^2|h_{\mathrm{d}}|^2\right) \\ & + \left(-\tilde{u}_{\mathrm{R}}^*h_{\mathrm{u}}a_{\mathrm{u}}\tilde{q}_{\mathrm{L}} + \tilde{d}_{\mathrm{R}}^*h_{\mathrm{d}}a_{\mathrm{d}}\tilde{q}_{\mathrm{L}} + \tilde{e}_{\mathrm{R}}^*h_{\mathrm{d}}a_{\mathrm{e}}\tilde{l}_{\mathrm{L}} + bH_{\mathrm{u}}H_{\mathrm{d}} + \text{H.c.}\right) \\ & + \left(\tilde{u}_{\mathrm{R}}^*h_{\mathrm{d}}^*c_{\mathrm{u}}\tilde{q}_{\mathrm{L}} + \tilde{d}_{\mathrm{R}}^*h_{\mathrm{u}}^*c_{\mathrm{d}}\tilde{q}_{\mathrm{L}} + \tilde{e}_{\mathrm{R}}^*h_{\mathrm{u}}^*c_{\mathrm{e}}\tilde{l}_{\mathrm{L}} + \text{H.c.}\right), \end{aligned} \tag{11.15}$$

$$\begin{aligned} V^{\mathrm{RPV}}_{\cancel{\mathrm{SUSY}}} = {}& \left(-b_i\tilde{l}_{\mathrm{L}i}H_{\mathrm{u}} + \frac{1}{2}T_{ijk}\tilde{l}_{\mathrm{L}i}\tilde{l}_{\mathrm{L}j}\tilde{e}^*_{\mathrm{R}k} + T'_{ijk}\tilde{l}_{\mathrm{L}i}\tilde{q}_{\mathrm{L}j}\tilde{d}^*_{\mathrm{R}k} + \frac{1}{2}T''_{ijk}\tilde{u}^*_{\mathrm{R}i}\tilde{d}^*_{\mathrm{R}j}\tilde{d}^*_{\mathrm{R}k} + \tilde{l}^*_{\mathrm{L}i}M^2_{Li}H_{\mathrm{d}} + \text{H.c.}\right) \\ & + \left(C^1_{ijk}\tilde{l}^*_{\mathrm{L}i}\tilde{q}_{\mathrm{L}j}\tilde{u}^*_{\mathrm{R}k} + C^2_ih^*_{\mathrm{u}}h_{\mathrm{d}}\tilde{e}^*_{\mathrm{R}i} + C^3_{ijk}\tilde{d}_{\mathrm{R}i}\tilde{u}^*_{\mathrm{R}j}\tilde{e}^*_{\mathrm{R}k} + \frac{1}{2}C^4_{ijk}\tilde{d}_{\mathrm{R}i}\tilde{q}_{\mathrm{L}j}\tilde{q}_{\mathrm{L}k} + \text{H.c.}\right). \end{aligned} \tag{11.16}$$

Hereafter we do not consider Θ_W and Θ_B as in the Standard Model (Sec. 7.7)[*17], while the SU(3) angle Θ_g forms QCD phase Θ_{QCD} together with the phases from Yukawa matrices. Also we assume the absence of Fayet-Illiopoulos term: $\Lambda_{\mathrm{FI}} = 0$. Then,

$$\mathcal{L}_{\mathrm{matter}} = \sum_{\mathrm{matters}}\left[\mathrm{D}^\mu\phi^*\,\mathrm{D}_\mu\phi + \mathrm{i}\bar{\psi}\bar{\sigma}^\mu\,\mathrm{D}_\mu\psi - \sqrt{2}\sum_{\mathrm{gauge}} g\left(\lambda^a(\phi^*t^a\psi) + \bar{\lambda}^a(\bar{\psi}t^a\phi)\right)\right] + (F\text{-terms}), \tag{11.17}$$

$$\mathcal{L}_{\mathrm{gauge}} = \sum_{\mathrm{gauges}}\left(-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \mathrm{i}\bar{\lambda}^a\bar{\sigma}^\mu\,\mathrm{D}_\mu\lambda^a\right) + \frac{g_3^2\Theta_g}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma} + (D\text{-terms}), \tag{11.18}$$

$$\begin{aligned} \mathcal{L}_{\mathrm{super}} = {}& \epsilon^{ab}\Big(-\mu\tilde{h}^a_{\mathrm{u}}\tilde{h}^b_{\mathrm{d}} - y_{dij}h^a_{\mathrm{d}}d^{\mathrm{c}x}_{\mathrm{R}i}q^{bx}_{\mathrm{L}j} - y_{dij}\tilde{d}^{x*}_{\mathrm{R}i}\tilde{h}^a_{\mathrm{d}}q^{bx}_{\mathrm{L}j} + y_{dji}\tilde{q}^{ax}_{\mathrm{L}i}\tilde{h}^b_{\mathrm{d}}d^{\mathrm{c}x}_{\mathrm{R}j} \\ & -y_{eij}\tilde{e}^*_{\mathrm{R}i}\tilde{h}^a_{\mathrm{d}}l^b_{\mathrm{L}j} - y_{eij}h^a_{\mathrm{d}}e^{\mathrm{c}}_{\mathrm{R}i}l^b_{\mathrm{L}j} + y_{eji}\tilde{l}^a_{\mathrm{L}i}\tilde{h}^b_{\mathrm{d}}e^{\mathrm{c}}_{\mathrm{R}j} + y_{uij}h^a_{\mathrm{u}}u^{\mathrm{c}x}_{\mathrm{R}i}q^{bx}_{\mathrm{L}j} + y_{uij}\tilde{u}^{x*}_{\mathrm{R}i}\tilde{h}^a_{\mathrm{u}}q^{bx}_{\mathrm{L}j} - y_{uji}\tilde{q}^{ax}_{\mathrm{L}i}\tilde{h}^b_{\mathrm{u}}u^{\mathrm{c}x}_{\mathrm{R}j} \\ & -\kappa_i\tilde{h}^a_{\mathrm{u}}l^b_{\mathrm{L}i} - \lambda_{ikj}\tilde{l}^a_{\mathrm{L}i}e^{\mathrm{c}}_{\mathrm{R}j}l^b_{\mathrm{L}k} - \frac{1}{2}\lambda_{jki}\tilde{e}^*_{\mathrm{R}i}l^a_{\mathrm{L}j}l^b_{\mathrm{L}k} - \lambda'_{ikj}\tilde{l}^a_{\mathrm{L}i}d^{\mathrm{c}x}_{\mathrm{R}j}q^{bx}_{\mathrm{L}k} + \lambda'_{kij}\tilde{q}^{ax}_{\mathrm{L}i}d^{\mathrm{c}x}_{\mathrm{R}j}l^b_{\mathrm{L}k} + \lambda'_{kji}\tilde{d}^{x*}_{\mathrm{R}i}q^{ax}_{\mathrm{L}j}l^b_{\mathrm{L}k}\Big) \\ & - \frac{1}{2}\epsilon^{xyz}\lambda''_{ijk}\tilde{u}^{x*}_{\mathrm{R}i}d^{\mathrm{c}y}_{\mathrm{R}j}d^{\mathrm{c}z}_{\mathrm{R}k} + \epsilon^{xyz}\lambda''_{jik}\tilde{d}^{x*}_{\mathrm{R}i}u^{\mathrm{c}y}_{\mathrm{R}j}d^{\mathrm{c}z}_{\mathrm{R}k} + \text{H.c.} + (F\text{-terms}), \end{aligned} \tag{11.19}$$

$$\mathcal{L}_{\cancel{\mathrm{SUSY}}} = -\frac{1}{2}\left(M_3\tilde{g}_0\tilde{g}_0 + M_2\tilde{w}\tilde{w} + M_1\tilde{b}\tilde{b} + \text{H.c.}\right) - \left(V^{\mathrm{RPC}}_{\cancel{\mathrm{SUSY}}} + V^{\mathrm{RPV}}_{\cancel{\mathrm{SUSY}}}\right), \tag{11.20}$$

and the F- and D-terms form the supersymmetric scalar potential

$$V_{\mathrm{SUSY}} = F_i^*F_i + \frac{1}{2}D^aD^a; \qquad F_i = -W_i^* = -\frac{\delta W^*}{\delta\phi_i^*}, \qquad D^a = -g(\phi^*t^a\phi), \tag{11.21}$$

$$V = V_{\mathrm{SUSY}} + V^{\mathrm{RPC}}_{\cancel{\mathrm{SUSY}}} + V^{\mathrm{RPV}}_{\cancel{\mathrm{SUSY}}}, \tag{11.22}$$

where t_a corresponds to the gauge-symmetry generator relevant for each ϕ.

[*17]The rotations to remove Θ_W may generate phases in the RPV terms. In other words, we define the RPV terms in the $\Theta_W = 0$ basis.

Each auxiliary term is given by

$$-F^*_{h^a_\mathrm{u}} = \epsilon^{ab}\left(-\tilde u^{x*}_\mathrm{R} y_u \tilde q^{bx}_\mathrm{L} + \mu h^b_\mathrm{d} + \kappa_i \tilde l^b_{\mathrm{L}i}\right), \tag{11.23}$$

$$-F^*_{h^a_\mathrm{d}} = \epsilon^{ab}\left(\tilde e^*_\mathrm{R} y_e \tilde l^b_\mathrm{L} + \tilde d^{x*}_\mathrm{R} y_d \tilde q^{bx}_\mathrm{L} - \mu h^b_\mathrm{u}\right), \tag{11.24}$$

$$-F^*_{\tilde q^{ax}_{\mathrm{L}i}} = \epsilon^{ab}\left(-y_{dji} h^b_\mathrm{d} \tilde d^{x*}_{\mathrm{R}j} + y_{uji} h^b_\mathrm{u} \tilde u^{x*}_{\mathrm{R}j} - \lambda'_{kij} \tilde d^{x*}_{\mathrm{R}j} \tilde l^b_{\mathrm{L}k}\right), \tag{11.25}$$

$$-F^*_{\tilde u^{x*}_{\mathrm{R}i}} = -y_{uij} h_\mathrm{u} \tilde q^x_{\mathrm{L}j} + \frac{1}{2}\epsilon^{xyz}\lambda''_{ijk} \tilde d^{y*}_{\mathrm{R}j} \tilde d^{z*}_{\mathrm{R}k}, \tag{11.26}$$

$$-F^*_{\tilde d^{x*}_{\mathrm{R}i}} = y_{dij} h_\mathrm{d} \tilde q^x_{\mathrm{L}j} + \lambda'_{jki} \tilde l_{\mathrm{L}j} \tilde q^x_{\mathrm{L}k} - \lambda''_{jik}\epsilon^{xyz} \tilde u^{y*}_{\mathrm{R}j} \tilde d^{z*}_{\mathrm{R}k}, \tag{11.27}$$

$$-F^*_{\tilde l^a_{\mathrm{L}i}} = \epsilon^{ab}\left(-y_{eji} \tilde e^*_{\mathrm{R}j} h^b_\mathrm{d} - \kappa_i h^b_\mathrm{u} + \lambda_{ikj} \tilde e^*_{\mathrm{R}j} \tilde l^b_{\mathrm{L}k} + \lambda'_{ikj} \tilde d^{x*}_{\mathrm{R}j} \tilde q^{bx}_{\mathrm{L}k}\right), \tag{11.28}$$

$$-F^*_{\tilde e^*_{\mathrm{R}i}} = y_{eij} h_\mathrm{d} \tilde l_{\mathrm{L}j} + \frac{1}{2}\lambda_{jki} \tilde l_{\mathrm{L}j} \tilde l_{\mathrm{L}k}. \tag{11.29}$$

$$D^\alpha_{\mathrm{SU}(3)} = -g_3 \sum_{i=1}^3 \left(\sum_{a=1,2} \tilde q^{a*}_{\mathrm{L}i} \tau^\alpha \tilde q^a_{\mathrm{L}i} - \tilde u^*_{\mathrm{R}i} \tau^\alpha \tilde u_{\mathrm{R}i} - \tilde d^*_{\mathrm{R}i} \tau^\alpha \tilde d_{\mathrm{R}i}\right), \tag{11.30}$$

$$D^\alpha_{\mathrm{SU}(2)} = -g_2 \left[\sum_{i=1}^3 \left(\sum_{x=1}^3 \tilde q^{x*}_{\mathrm{L}i} T^\alpha \tilde q^x_{\mathrm{L}i} + \tilde l^*_{\mathrm{L}i} T^\alpha \tilde l_{\mathrm{L}i}\right) + h^*_\mathrm{u} T^\alpha h_\mathrm{u} + h^*_\mathrm{d} T^\alpha h_\mathrm{d}\right], \tag{11.31}$$

$$D_{\mathrm{U}(1)} = -g_1 \left(\frac{1}{6}|\tilde q_\mathrm{L}|^2 - \frac{1}{2}|\tilde l_\mathrm{L}|^2 - \frac{2}{3}|\tilde u_\mathrm{R}|^2 + \frac{1}{3}|\tilde d_\mathrm{R}|^2 + |\tilde e_\mathrm{R}|^2 + \frac{1}{2}|h_\mathrm{u}|^2 - \frac{1}{2}|h_\mathrm{d}|^2\right). \tag{11.32}$$

11.3. Full Lagrangian

Here the Lagrangian $\mathcal{L} = \mathcal{L}_\mathrm{vector} + \mathcal{L}_\mathrm{fermions} + \mathcal{L}_\mathrm{SFG} + \mathcal{L}_\mathrm{scalar}$ is explicitly given:

$$\mathcal{L}_\mathrm{vector} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{g_3^2\Theta_g}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}, \tag{11.33}$$

$$\mathcal{L}_\mathrm{fermions} = \mathrm{i}\bar\psi\bar\sigma^\mu \mathrm{D}_\mu\psi + \mathrm{i}\bar\lambda^a\bar\sigma^\mu \mathrm{D}_\mu\lambda^a - \frac{1}{2}\left(M_3\tilde g_0\tilde g_0 + M_2\tilde w\tilde w + M_1\tilde b\tilde b + \text{H.c.}\right) + \mathcal{L}_\mathrm{super}\big|_{\text{no }F\text{-terms}}, \tag{11.34}$$

$$\mathcal{L}_\mathrm{SFG} = -\sqrt{2}g\lambda^a(\phi^* t^a\psi) - \sqrt{2}g\bar\lambda^a(\bar\psi t^a\phi), \tag{11.35}$$

$$\mathcal{L}_\mathrm{scalar} = \mathrm{D}^\mu\phi^* \mathrm{D}_\mu\phi - V. \tag{11.36}$$

11.3.1. Vector part

$$\begin{aligned}\mathcal{L}_\mathrm{vector} = &-\frac{1}{2}(\partial_\mu B_\nu - \partial_\nu B_\mu)\partial^\mu B^\nu - \frac{1}{2}(\partial_\mu g^a_\nu - \partial_\nu g^a_\mu)\partial^\mu g^{a\nu} - \frac{1}{2}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)\partial^\mu W^{a\nu}\\ &- g_2\epsilon^{abc}W^b_\mu W^c_\nu\partial^\mu W^{a\nu} - \frac{g_2^2}{4}\epsilon^{abe}\epsilon^{cde}W^a_\mu W^b_\nu W^{c\mu}W^{d\nu}\\ &- g_3 f^{abc}g^b_\mu g^c_\nu\partial^\mu g^{a\nu} - \frac{g_3^2}{4}f^{cde}f^{abe}g^a_\mu g^b_\nu g^{c\mu}g^{d\nu} + \frac{g_3^2\Theta_g}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma},\end{aligned} \tag{11.37}$$

$$\begin{aligned}= &\text{(gluons)} - \frac{1}{2}(\partial_\mu A_\nu - \partial_\nu A_\mu)\partial^\mu A^\nu - (\partial_\mu W^-_\nu - \partial_\nu W^-_\mu)\partial_\mu W^{+\nu} - \frac{1}{2}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)\partial^\mu Z^\nu\\ &+ \mathrm{i}g_2\mathrm{c_w}\left[(W^-_\mu Z_\nu - W^-_\nu Z_\mu)\partial^\mu W^{+\nu} - (W^+_\mu Z_\nu - W^+_\nu Z_\mu)\partial^\mu W^{-\nu} + (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu)\partial^\mu Z^\nu\right]\\ &+ \mathrm{i}|e|\left[(W^-_\mu A_\nu - W^-_\nu A_\mu)\partial^\mu W^{+\nu} - (W^+_\mu A_\nu - W^+_\nu A_\mu)\partial^\mu W^{-\nu} + (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu)\partial^\mu A^\nu\right]\\ &+ \frac{g_2^2}{2}W^{+\mu}W^+_\mu W^{-\nu}W^-_\nu - \frac{g_2^2}{2}W^{+\mu}W^{+\nu}W^-_\mu W^-_\nu - g_2^2W^{+\mu}W^-_\mu Z^\nu Z_\nu + g_2^2W^{+\mu}W^{-\nu}Z_\mu Z_\nu\\ &- e^2W^{+\mu}W^-_\mu A^\nu A_\nu + e^2W^{+\mu}W^-_\mu Z^\nu Z_\nu + e^2W^{+\mu}W^{-\nu}A_\mu A_\nu - e^2W^{+\mu}W^{-\nu}Z_\mu Z_\nu\\ &- 2g_2^2\mathrm{c_w s_w}W^{+\mu}W^-_\mu A^\nu Z_\nu + g_2^2\mathrm{c_w s_w}W^{+\mu}W^{-\nu}A_\mu Z_\nu + g_2^2\mathrm{c_w s_w}W^{+\mu}W^{-\nu}A_\nu Z_\mu,\end{aligned} \tag{11.38}$$

where

$$W^1_\mu = \frac{W^+_\mu + W^-_\mu}{\sqrt{2}},\quad W^2_\mu = \frac{\mathrm{i}(W^+_\mu - W^-_\mu)}{\sqrt{2}};\qquad W^\pm_\mu = \frac{W^1_\mu \mp \mathrm{i}W^2_\mu}{\sqrt{2}};$$

$$\begin{pmatrix}W^3_\mu\\ B_\mu\end{pmatrix} = \begin{pmatrix}\mathrm{c_w} & \mathrm{s_w}\\ -\mathrm{s_w} & \mathrm{c_w}\end{pmatrix}\begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix};\qquad \begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix} = \begin{pmatrix}\mathrm{c_w} & -\mathrm{s_w}\\ \mathrm{s_w} & \mathrm{c_w}\end{pmatrix}\begin{pmatrix}W^3_\mu\\ B_\mu\end{pmatrix};$$

$$|e| = g_2\mathrm{s_w} = g_Y\mathrm{c_w} = g_Z\mathrm{s_w c_w},\quad g_Z = g_2/\mathrm{c_w} = g_Y/\mathrm{s_w};\qquad g_Y = |e|/\mathrm{c_w} = g_Z\mathrm{s_w} = g_2\mathrm{t_w},\quad g_2 = |e|/\mathrm{s_w} = g_Z\mathrm{c_w}.$$

11.3.2. Fermion part

$$
\begin{aligned}
&\mathcal{L}_{\text{fermions}} \\
&= \mathrm{i}\bar{q}_{\mathrm{L}}\bar{\sigma}^{\mu}\left(\partial_{\mu} - \mathrm{i}g_3 g_{\mu}^{a}\tau^{a} - \mathrm{i}g_2 W_{\mu}^{a}T^{a} - \tfrac{1}{6}\mathrm{i}g_Y B_{\mu}\right)q_{\mathrm{L}} \\
&\quad + \mathrm{i}\bar{u}_{\mathrm{R}}^{\mathrm{c}}\bar{\sigma}^{\mu}\left(\partial_{\mu} + \mathrm{i}g_3 g_{\mu}^{a}\tau^{a*} + \tfrac{2}{3}\mathrm{i}g_Y B_{\mu}\right)u_{\mathrm{R}}^{\mathrm{c}} + \mathrm{i}\bar{d}_{\mathrm{R}}^{\mathrm{c}}\bar{\sigma}^{\mu}\left(\partial_{\mu} + \mathrm{i}g_3 g_{\mu}^{a}\tau^{a*} - \tfrac{1}{3}\mathrm{i}g_Y B_{\mu}\right)d_{\mathrm{R}}^{\mathrm{c}} \\
&\quad + \mathrm{i}\bar{l}_{\mathrm{L}}\bar{\sigma}^{\mu}\left(\partial_{\mu} - \mathrm{i}g_2 W_{\mu}^{a}T^{a} + \tfrac{1}{2}\mathrm{i}g_Y B_{\mu}\right)l_{\mathrm{L}} + \mathrm{i}\bar{e}_{\mathrm{R}}^{\mathrm{c}}\bar{\sigma}^{\mu}\left(\partial_{\mu} - \mathrm{i}g_Y B_{\mu}\right)e_{\mathrm{R}}^{\mathrm{c}} \\
&\quad + \mathrm{i}\bar{\tilde{h}}_{\mathrm{u}}\bar{\sigma}^{\mu}\left(\partial_{\mu} - \mathrm{i}g_2 W_{\mu}^{a}T^{a} - \tfrac{1}{2}\mathrm{i}g_Y B_{\mu}\right)\tilde{h}_{\mathrm{u}} + \mathrm{i}\bar{\tilde{h}}_{\mathrm{d}}\bar{\sigma}^{\mu}\left(\partial_{\mu} - \mathrm{i}g_2 W_{\mu}^{a}T^{a} + \tfrac{1}{2}\mathrm{i}g_Y B_{\mu}\right)\tilde{h}_{\mathrm{d}} \\
&\quad + \mathrm{i}\bar{\tilde{g}}_0^{a}\bar{\sigma}^{\mu}\left(\partial_{\mu}\tilde{g}_0^{a} + g_3 f^{abc}g_{\mu}^{b}\tilde{g}_0^{c}\right) + \mathrm{i}\bar{\tilde{w}}^{a}\bar{\sigma}^{\mu}\left(\partial_{\mu}\tilde{w}^{a} + g_2\epsilon^{abc}W_{\mu}^{b}\tilde{w}^{c}\right) + \mathrm{i}\bar{\tilde{b}}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{b} \\
&\quad - \frac{1}{2}\left(M_3\tilde{g}_0^{a}\tilde{g}_0^{a} + M_2\tilde{w}^{a}\tilde{w}^{a} + M_1\tilde{b}\tilde{b} + \text{H.c.}\right) + \mathcal{L}_{\text{super}}\big|_{\text{no }F\text{-terms}}
\end{aligned} \tag{11.39}
$$

$$
\begin{aligned}
&= \mathrm{i}\bar{\tilde{b}}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{b} - \frac{1}{2}\left(M_1\tilde{b}\tilde{b} + M_1^{*}\bar{\tilde{b}}^{a}\bar{\tilde{b}}^{a}\right) + \mathrm{i}\bar{\tilde{g}}_0^{a}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{g}_0^{a} - \frac{1}{2}\left(M_3\tilde{g}_0^{a}\tilde{g}_0^{a} + M_3^{*}\bar{\tilde{g}}_0^{a}\bar{\tilde{g}}_0^{a}\right) - \mathrm{i}g_3 f^{abc}(\bar{\tilde{g}}_0^{a}\bar{\sigma}^{\mu}\tilde{g}_0^{b})g_{\mu}^{c} \\
&\quad + \mathrm{i}\bar{\tilde{w}}^{+}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{w}^{+} + \mathrm{i}\bar{\tilde{w}}^{-}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{w}^{-} + \mathrm{i}\bar{\tilde{w}}^{3}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{w}^{3} - \left(M_2\tilde{w}^{+}\tilde{w}^{-} + M_2^{*}\bar{\tilde{w}}^{+}\bar{\tilde{w}}^{-}\right) - \frac{1}{2}\left(M_2\tilde{w}^{3}\tilde{w}^{3} + M_2^{*}\bar{\tilde{w}}^{3}\bar{\tilde{w}}^{3}\right) \\
&\quad + g_2(\bar{\tilde{w}}^{3}\bar{\sigma}^{\mu}\tilde{w}^{-} - \bar{\tilde{w}}^{+}\bar{\sigma}^{\mu}\tilde{w}^{3})W_{\mu}^{+} - g_2(\bar{\tilde{w}}^{3}\bar{\sigma}^{\mu}\tilde{w}^{+} - \bar{\tilde{w}}^{-}\bar{\sigma}^{\mu}\tilde{w}^{3})W_{\mu}^{-} + g_2(\bar{\tilde{w}}^{+}\bar{\sigma}^{\mu}\tilde{w}^{+} - \bar{\tilde{w}}^{-}\bar{\sigma}^{\mu}\tilde{w}^{-})(\mathrm{c_w}Z_{\mu} + \mathrm{s_w}A_{\mu}) \\
&\quad + \bar{u}_{\mathrm{L}}\bar{\sigma}^{\mu}(\mathrm{i}\partial_{\mu} + g_3\tau^{a}g_{\mu}^{a})u_{\mathrm{L}} + \bar{u}_{\mathrm{R}}^{\mathrm{c}}\bar{\sigma}^{\mu}(\mathrm{i}\partial_{\mu} - g_3\tau^{a*}g_{\mu}^{a})u_{\mathrm{R}}^{\mathrm{c}} + \mathrm{i}\bar{\nu}_{\mathrm{L}}\bar{\sigma}^{\mu}\partial_{\mu}\nu_{\mathrm{L}} \\
&\quad + \bar{d}_{\mathrm{L}}\bar{\sigma}^{\mu}(\mathrm{i}\partial_{\mu} + g_3\tau^{a}g_{\mu}^{a})d_{\mathrm{L}} + \bar{d}_{\mathrm{R}}^{\mathrm{c}}\bar{\sigma}^{\mu}(\mathrm{i}\partial_{\mu} - g_3\tau^{a*}g_{\mu}^{a})d_{\mathrm{R}}^{\mathrm{c}} + \mathrm{i}\bar{e}_{\mathrm{L}}\bar{\sigma}^{\mu}\partial_{\mu}e_{\mathrm{L}} + \mathrm{i}\bar{e}_{\mathrm{R}}^{\mathrm{c}}\bar{\sigma}^{\mu}\partial_{\mu}e_{\mathrm{R}}^{\mathrm{c}} \\
&\quad + \mathrm{i}\bar{\tilde{h}}_{\mathrm{d}}^{-}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{h}_{\mathrm{d}}^{-} + \mathrm{i}\bar{\tilde{h}}_{\mathrm{d}}^{0}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{h}_{\mathrm{d}}^{0} + \mathrm{i}\bar{\tilde{h}}_{\mathrm{u}}^{+}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{h}_{\mathrm{u}}^{+} + \mathrm{i}\bar{\tilde{h}}_{\mathrm{u}}^{0}\bar{\sigma}^{\mu}\partial_{\mu}\tilde{h}_{\mathrm{u}}^{0} \\
&\quad + \frac{g_2}{\sqrt{2}}\left(\bar{u}_{\mathrm{L}}\bar{\sigma}^{\mu}d_{\mathrm{L}} + \bar{\nu}_{\mathrm{L}}\bar{\sigma}^{\mu}e_{\mathrm{L}} + \bar{\tilde{h}}_{\mathrm{u}}^{+}\bar{\sigma}^{\mu}\tilde{h}_{\mathrm{u}}^{0} + \bar{\tilde{h}}_{\mathrm{d}}^{0}\bar{\sigma}^{\mu}\tilde{h}_{\mathrm{d}}^{-}\right)W_{\mu}^{+} + \frac{g_2}{\sqrt{2}}\left(\bar{d}_{\mathrm{L}}\bar{\sigma}^{\mu}u_{\mathrm{L}} + \bar{e}_{\mathrm{L}}\bar{\sigma}^{\mu}\nu_{\mathrm{L}} + \bar{\tilde{h}}_{\mathrm{u}}^{0}\bar{\sigma}^{\mu}\tilde{h}_{\mathrm{u}}^{+} + \bar{\tilde{h}}_{\mathrm{d}}^{-}\bar{\sigma}^{\mu}\tilde{h}_{\mathrm{d}}^{0}\right)W_{\mu}^{-} \\
&\quad + \frac{g_Z(3 - 4\mathrm{s_w^2})}{6}\bar{u}_{\mathrm{L}}\bar{\sigma}^{\mu}u_{\mathrm{L}}Z_{\mu} + \frac{g_Z\mathrm{s_w^2}}{3}\bar{u}_{\mathrm{R}}^{\mathrm{c}}\bar{\sigma}^{\mu}u_{\mathrm{R}}^{\mathrm{c}}Z_{\mu} + \frac{g_Z(2\mathrm{s_w^2} - 3)}{6}\bar{d}_{\mathrm{L}}\bar{\sigma}^{\mu}d_{\mathrm{L}}Z_{\mu} - \frac{g_Z\mathrm{s_w^2}}{3}\bar{d}_{\mathrm{R}}^{\mathrm{c}}\bar{\sigma}^{\mu}d_{\mathrm{R}}^{\mathrm{c}}Z_{\mu} \\
&\quad + \frac{g_Z}{2}\bar{\nu}_{\mathrm{L}}\bar{\sigma}^{\mu}\nu_{\mathrm{L}}Z_{\mu} + \frac{g_Z(2\mathrm{s_w^2} - 1)}{2}\bar{e}_{\mathrm{L}}\bar{\sigma}^{\mu}e_{\mathrm{L}}Z_{\mu} - g_Z\mathrm{s_w^2}\bar{e}_{\mathrm{R}}^{\mathrm{c}}\bar{\sigma}^{\mu}e_{\mathrm{R}}^{\mathrm{c}}Z_{\mu} \\
&\quad + \frac{g_Z(1 - 2\mathrm{s_w^2})}{2}\bar{\tilde{h}}_{\mathrm{u}}^{+}\bar{\sigma}^{\mu}\tilde{h}_{\mathrm{u}}^{+}Z_{\mu} + \frac{g_Z}{2}\bar{\tilde{h}}_{\mathrm{u}}^{0}\bar{\sigma}^{\mu}\tilde{h}_{\mathrm{u}}^{0}Z_{\mu} + \frac{g_Z(2\mathrm{s_w^2} - 1)}{2}\bar{\tilde{h}}_{\mathrm{d}}^{-}\bar{\sigma}^{\mu}\tilde{h}_{\mathrm{d}}^{-}Z_{\mu} + \frac{g_Z}{2}\bar{\tilde{h}}_{\mathrm{d}}^{0}\bar{\sigma}^{\mu}\tilde{h}_{\mathrm{d}}^{0}Z_{\mu} \\
&\quad + \frac{2|e|}{3}(\bar{u}_{\mathrm{L}}\bar{\sigma}^{\mu}u_{\mathrm{L}} - \bar{u}_{\mathrm{R}}^{\mathrm{c}}\bar{\sigma}^{\mu}u_{\mathrm{R}}^{\mathrm{c}})A_{\mu} - \frac{|e|}{3}(\bar{d}_{\mathrm{L}}\bar{\sigma}^{\mu}d_{\mathrm{L}} - \bar{d}_{\mathrm{R}}^{\mathrm{c}}\bar{\sigma}^{\mu}d_{\mathrm{R}}^{\mathrm{c}})A_{\mu} - |e|(\bar{e}_{\mathrm{L}}\bar{\sigma}^{\mu}e_{\mathrm{L}} - \bar{e}_{\mathrm{R}}^{\mathrm{c}}\bar{\sigma}^{\mu}e_{\mathrm{R}}^{\mathrm{c}})A_{\mu} \\
&\quad + |e|\bar{\tilde{h}}_{\mathrm{u}}^{+}\bar{\sigma}^{\mu}\tilde{h}_{\mathrm{u}}^{+}A_{\mu} - |e|\bar{\tilde{h}}_{\mathrm{d}}^{-}\bar{\sigma}^{\mu}\tilde{h}_{\mathrm{d}}^{-}A_{\mu} + \mathcal{L}_{\text{super}}\big|_{\text{no }F\text{-terms}};
\end{aligned} \tag{11.40}
$$

here,

$$
\begin{aligned}
\mathcal{L}_{\text{super}}\big|_{\text{no F-terms}} &= -\mu\tilde{h}_{\mathrm{u}}^{+}\tilde{h}_{\mathrm{d}}^{-} + \mu\tilde{h}_{\mathrm{u}}^{0}\tilde{h}_{\mathrm{d}}^{0} + y_{uij}h_{\mathrm{u}}^{+}u_{\mathrm{R}i}^{\mathrm{c}}d_{\mathrm{L}j} - y_{uij}h_{\mathrm{u}}^{0}u_{\mathrm{R}i}^{\mathrm{c}}u_{\mathrm{L}j} + y_{uij}\tilde{d}_{\mathrm{L}j}\tilde{h}_{\mathrm{u}}^{+}u_{\mathrm{R}i}^{\mathrm{c}} - y_{uij}\tilde{u}_{\mathrm{L}j}\tilde{h}_{\mathrm{u}}^{0}u_{\mathrm{R}i}^{\mathrm{c}} \\
&\quad + y_{uji}\tilde{u}_{\mathrm{R}j}^{*}\tilde{h}_{\mathrm{u}}^{+}d_{\mathrm{L}i} - y_{uji}\tilde{u}_{\mathrm{R}j}^{*}\tilde{h}_{\mathrm{u}}^{0}u_{\mathrm{L}i} + y_{dij}h_{\mathrm{d}}^{-}d_{\mathrm{R}i}^{\mathrm{c}}u_{\mathrm{L}j} - y_{dij}h_{\mathrm{d}}^{0}d_{\mathrm{R}i}^{\mathrm{c}}d_{\mathrm{L}j} - y_{dij}\tilde{d}_{\mathrm{L}j}\tilde{h}_{\mathrm{d}}^{0}d_{\mathrm{R}i}^{\mathrm{c}} \\
&\quad + y_{dij}\tilde{u}_{\mathrm{L}j}\tilde{h}_{\mathrm{d}}^{-}d_{\mathrm{R}i}^{\mathrm{c}} + y_{dji}\tilde{d}_{\mathrm{R}j}^{*}\tilde{h}_{\mathrm{d}}^{-}u_{\mathrm{L}i} - y_{dji}\tilde{d}_{\mathrm{R}j}^{*}\tilde{h}_{\mathrm{d}}^{0}d_{\mathrm{L}i} + y_{eij}h_{\mathrm{d}}^{-}e_{\mathrm{R}i}^{\mathrm{c}}\nu_{\mathrm{L}j} - y_{eij}h_{\mathrm{d}}^{0}e_{\mathrm{R}i}^{\mathrm{c}}e_{\mathrm{L}j} \\
&\quad - y_{eij}\tilde{e}_{\mathrm{L}j}\tilde{h}_{\mathrm{d}}^{0}e_{\mathrm{R}i}^{\mathrm{c}} + y_{eij}\tilde{\nu}_{\mathrm{L}j}\tilde{h}_{\mathrm{d}}^{-}e_{\mathrm{R}i}^{\mathrm{c}} + y_{eji}\tilde{e}_{\mathrm{R}j}^{*}\tilde{h}_{\mathrm{d}}^{-}\nu_{\mathrm{L}i} - y_{eji}\tilde{e}_{\mathrm{R}j}^{*}\tilde{h}_{\mathrm{d}}^{0}e_{\mathrm{L}i} \\
&\quad - \kappa_i\tilde{h}_{\mathrm{u}}^{+}e_{\mathrm{L}i} + \kappa_i\tilde{h}_{\mathrm{u}}^{0}\nu_{\mathrm{L}i} - \lambda_{ijk}\tilde{e}_{\mathrm{R}k}^{*}\nu_{\mathrm{L}i}e_{\mathrm{L}j} - \lambda_{jki}\tilde{e}_{\mathrm{L}k}e_{\mathrm{R}i}^{\mathrm{c}}\nu_{\mathrm{L}j} + \lambda_{jki}\tilde{\nu}_{\mathrm{L}k}e_{\mathrm{R}i}^{\mathrm{c}}e_{\mathrm{L}j} \\
&\quad - \lambda'_{jik}\tilde{d}_{\mathrm{R}k}^{*}d_{\mathrm{L}i}\nu_{\mathrm{L}j} + \lambda'_{jik}\tilde{d}_{\mathrm{R}k}^{*}u_{\mathrm{L}i}e_{\mathrm{L}j} - \lambda'_{jki}\tilde{d}_{\mathrm{L}k}d_{\mathrm{R}i}^{\mathrm{c}}\nu_{\mathrm{L}j} + \lambda'_{jki}\tilde{u}_{\mathrm{L}k}d_{\mathrm{R}i}^{\mathrm{c}}e_{\mathrm{L}j} + \lambda'_{kji}\tilde{e}_{\mathrm{L}k}d_{\mathrm{R}i}^{\mathrm{c}}u_{\mathrm{L}j} \\
&\quad - \lambda'_{kji}\tilde{\nu}_{\mathrm{L}k}d_{\mathrm{R}i}^{\mathrm{c}}d_{\mathrm{L}j} - \epsilon^{xyz}\lambda''_{ijk}\tilde{d}_{\mathrm{R}k}^{x*}u_{\mathrm{R}i}^{\mathrm{c}y}d_{\mathrm{R}j}^{\mathrm{c}z} - \frac{1}{2}\epsilon^{xyz}\lambda''_{kij}\tilde{u}_{\mathrm{R}k}^{x*}d_{\mathrm{R}i}^{\mathrm{c}y}d_{\mathrm{R}j}^{\mathrm{c}z} + \text{H.c.}
\end{aligned} \tag{11.41}
$$

11.3.3. Scalar-fermion-gaugino interaction

$$
\begin{aligned}
\mathcal{L}_{\mathrm{SFG}} = {} & -g_2\tilde{u}_{\mathrm{L}}^{*}d_{\mathrm{L}}\tilde{w}^{+} - g_2\tilde{u}_{\mathrm{L}}\bar{d}_{\mathrm{L}}\bar{\tilde{w}}^{+} - g_2\tilde{d}_{\mathrm{L}}^{*}u_{\mathrm{L}}\tilde{w}^{-} - g_2\tilde{d}_{\mathrm{L}}\bar{u}_{\mathrm{L}}\bar{\tilde{w}}^{-} \\
& - \sqrt{2}g_3\tilde{u}_{\mathrm{L}}^{*}\tau^{a}u_{\mathrm{L}}\tilde{g}_0^{a} + \sqrt{2}g_3\tilde{u}_{\mathrm{R}}^{*}\tau^{a}\bar{u}_{\mathrm{R}}^{\mathrm{c}}\bar{\tilde{g}}_0^{a} - \sqrt{2}g_3\tilde{u}_{\mathrm{L}}\tau^{a*}\bar{u}_{\mathrm{L}}\bar{\tilde{g}}_0^{a} + \sqrt{2}g_3\tilde{u}_{\mathrm{R}}\tau^{a*}u_{\mathrm{R}}^{\mathrm{c}}\tilde{g}_0^{a} \\
& - \frac{g_2}{\sqrt{2}}\tilde{u}_{\mathrm{L}}^{*}u_{\mathrm{L}}\tilde{w}^{3} - \frac{g_2}{\sqrt{2}}\tilde{u}_{\mathrm{L}}\bar{u}_{\mathrm{L}}\bar{\tilde{w}}^{3} - \frac{g_Y}{3\sqrt{2}}\tilde{u}_{\mathrm{L}}^{*}u_{\mathrm{L}}\tilde{b} + \frac{2\sqrt{2}g_Y}{3}\tilde{u}_{\mathrm{R}}^{*}\bar{u}_{\mathrm{R}}^{\mathrm{c}}\bar{\tilde{b}} - \frac{g_Y}{3\sqrt{2}}\tilde{u}_{\mathrm{L}}\bar{u}_{\mathrm{L}}\bar{\tilde{b}} + \frac{2\sqrt{2}g_Y}{3}\tilde{u}_{\mathrm{R}}u_{\mathrm{R}}^{\mathrm{c}}\tilde{b} \\
& - \sqrt{2}g_3\tilde{d}_{\mathrm{L}}^{*}\tau^{a}d_{\mathrm{L}}\tilde{g}_0^{a} + \sqrt{2}g_3\tilde{d}_{\mathrm{R}}^{*}\tau^{a}\bar{d}_{\mathrm{R}}^{\mathrm{c}}\bar{\tilde{g}}_0^{a} - \sqrt{2}g_3\tilde{d}_{\mathrm{L}}\tau^{a*}\bar{d}_{\mathrm{L}}\bar{\tilde{g}}_0^{a} + \sqrt{2}g_3\tilde{d}_{\mathrm{R}}\tau^{a*}d_{\mathrm{R}}^{\mathrm{c}}\tilde{g}_0^{a} \\
& + \frac{g_2}{\sqrt{2}}\tilde{d}_{\mathrm{L}}^{*}d_{\mathrm{L}}\tilde{w}^{3} + \frac{g_2}{\sqrt{2}}\tilde{d}_{\mathrm{L}}\bar{d}_{\mathrm{L}}\bar{\tilde{w}}^{3} - \frac{g_Y}{3\sqrt{2}}\tilde{d}_{\mathrm{L}}^{*}d_{\mathrm{L}}\tilde{b} - \frac{\sqrt{2}g_Y}{3}\tilde{d}_{\mathrm{R}}^{*}\bar{d}_{\mathrm{R}}^{\mathrm{c}}\bar{\tilde{b}} - \frac{g_Y}{3\sqrt{2}}\tilde{d}_{\mathrm{L}}\bar{d}_{\mathrm{L}}\bar{\tilde{b}} - \frac{\sqrt{2}g_Y}{3}\tilde{d}_{\mathrm{R}}d_{\mathrm{R}}^{\mathrm{c}}\tilde{b} \\
& - g_2\tilde{e}_{\mathrm{L}}\bar{\nu}_{\mathrm{L}}\bar{\tilde{w}}^{-} - g_2\tilde{\nu}_{\mathrm{L}}\bar{e}_{\mathrm{L}}\bar{\tilde{w}}^{+} - g_2\tilde{e}_{\mathrm{L}}^{*}\nu_{\mathrm{L}}\tilde{w}^{-} - g_2\tilde{\nu}_{\mathrm{L}}^{*}e_{\mathrm{L}}\tilde{w}^{+} \\
& - \frac{g_2}{\sqrt{2}}\tilde{\nu}_{\mathrm{L}}^{*}\nu_{\mathrm{L}}\tilde{w}^{3} + \frac{g_Y}{\sqrt{2}}\tilde{\nu}_{\mathrm{L}}^{*}\nu_{\mathrm{L}}\tilde{b} - \frac{g_2}{\sqrt{2}}\tilde{\nu}_{\mathrm{L}}\bar{\nu}_{\mathrm{L}}\bar{\tilde{w}}^{3} + \frac{g_Y}{\sqrt{2}}\tilde{\nu}_{\mathrm{L}}\bar{\nu}_{\mathrm{L}}\bar{\tilde{b}} \\
& + \frac{g_2}{\sqrt{2}}\tilde{e}_{\mathrm{L}}^{*}e_{\mathrm{L}}\tilde{w}^{3} + \frac{g_Y}{\sqrt{2}}\tilde{e}_{\mathrm{L}}^{*}e_{\mathrm{L}}\tilde{b} - \sqrt{2}g_Y\tilde{e}_{\mathrm{R}}^{*}\bar{e}_{\mathrm{R}}^{\mathrm{c}}\bar{\tilde{b}} + \frac{g_2}{\sqrt{2}}\tilde{e}_{\mathrm{L}}\bar{e}_{\mathrm{L}}\bar{\tilde{w}}^{3} + \frac{g_Y}{\sqrt{2}}\tilde{e}_{\mathrm{L}}\bar{e}_{\mathrm{L}}\bar{\tilde{b}} - \sqrt{2}g_Y\tilde{e}_{\mathrm{R}}e_{\mathrm{R}}^{\mathrm{c}}\tilde{b} \\
& - g_2h_{\mathrm{u}}^{+*}\tilde{h}_{\mathrm{u}}^{0}\tilde{w}^{+} - g_2h_{\mathrm{d}}^{0}\bar{\tilde{h}}_{\mathrm{d}}^{-}\bar{\tilde{w}}^{+} - g_2h_{\mathrm{d}}^{-*}\tilde{h}_{\mathrm{d}}^{0}\tilde{w}^{-} - g_2h_{\mathrm{u}}^{0}\bar{\tilde{h}}_{\mathrm{u}}^{+}\bar{\tilde{w}}^{-} \\
& - g_2h_{\mathrm{d}}^{0*}\tilde{h}_{\mathrm{d}}^{-}\tilde{w}^{+} - g_2h_{\mathrm{u}}^{0*}\tilde{h}_{\mathrm{u}}^{+}\tilde{w}^{-} - g_2h_{\mathrm{u}}^{+}\bar{\tilde{h}}_{\mathrm{u}}^{0}\bar{\tilde{w}}^{+} - g_2h_{\mathrm{d}}^{-}\bar{\tilde{h}}_{\mathrm{d}}^{0}\bar{\tilde{w}}^{-} \\
& - \frac{g_2}{\sqrt{2}}h_{\mathrm{u}}^{+}\bar{\tilde{h}}_{\mathrm{u}}^{+}\bar{\tilde{w}}^{3} - \frac{g_Y}{\sqrt{2}}h_{\mathrm{u}}^{+}\bar{\tilde{h}}_{\mathrm{u}}^{+}\bar{\tilde{b}} - \frac{g_2}{\sqrt{2}}h_{\mathrm{u}}^{+*}\tilde{h}_{\mathrm{u}}^{+}\tilde{w}^{3} - \frac{g_Y}{\sqrt{2}}h_{\mathrm{u}}^{+*}\tilde{h}_{\mathrm{u}}^{+}\tilde{b} + \frac{g_2}{\sqrt{2}}h_{\mathrm{d}}^{-*}\tilde{h}_{\mathrm{d}}^{-}\tilde{w}^{3} + \frac{g_Y}{\sqrt{2}}h_{\mathrm{d}}^{-*}\tilde{h}_{\mathrm{d}}^{-}\tilde{b} \\
& + \frac{g_2}{\sqrt{2}}h_{\mathrm{d}}^{-}\bar{\tilde{h}}_{\mathrm{d}}^{-}\bar{\tilde{w}}^{3} + \frac{g_Y}{\sqrt{2}}h_{\mathrm{d}}^{-}\bar{\tilde{h}}_{\mathrm{d}}^{-}\bar{\tilde{b}} + \frac{g_2}{\sqrt{2}}h_{\mathrm{u}}^{0*}\tilde{h}_{\mathrm{u}}^{0}\tilde{w}^{3} - \frac{g_Y}{\sqrt{2}}h_{\mathrm{u}}^{0*}\tilde{h}_{\mathrm{u}}^{0}\tilde{b} - \frac{g_2}{\sqrt{2}}h_{\mathrm{d}}^{0*}\tilde{h}_{\mathrm{d}}^{0}\tilde{w}^{3} + \frac{g_Y}{\sqrt{2}}h_{\mathrm{d}}^{0*}\tilde{h}_{\mathrm{d}}^{0}\tilde{b} \\
& + \frac{g_2}{\sqrt{2}}h_{\mathrm{u}}^{0}\bar{\tilde{h}}_{\mathrm{u}}^{0}\bar{\tilde{w}}^{3} - \frac{g_Y}{\sqrt{2}}h_{\mathrm{u}}^{0}\bar{\tilde{h}}_{\mathrm{u}}^{0}\bar{\tilde{b}} - \frac{g_2}{\sqrt{2}}h_{\mathrm{d}}^{0}\bar{\tilde{h}}_{\mathrm{d}}^{0}\bar{\tilde{w}}^{3} + \frac{g_Y}{\sqrt{2}}h_{\mathrm{d}}^{0}\bar{\tilde{h}}_{\mathrm{d}}^{0}\bar{\tilde{b}}
\end{aligned}
\tag{11.42}
$$

11.3.4. Scalar part

$$
\begin{aligned}
\mathcal{L}_{\text{scalar}} = {} & (\partial_\mu \tilde{u}_{\rm L}^* + \mathrm{i} g_3 \tilde{u}_{\rm L}^* \tau^a g_\mu^a)(\partial^\mu \tilde{u}_{\rm L} - \mathrm{i} g_3 g^{b\mu} \tau^b \tilde{u}_{\rm L}) + (\partial_\mu \tilde{u}_{\rm R} - \mathrm{i} g_3 \tilde{u}_{\rm R} \tau^{a*} g_\mu^a)(\partial^\mu \tilde{u}_{\rm R}^* + \mathrm{i} g_3 g_\mu^b \tau^{b*} \tilde{u}_{\rm R}^*) \\
& + (\partial_\mu \tilde{d}_{\rm L}^* + \mathrm{i} g_3 \tilde{d}_{\rm L}^* \tau^a g_\mu^a)(\partial^\mu \tilde{d}_{\rm L} - \mathrm{i} g_3 g^{b\mu} \tau^b \tilde{d}_{\rm L}) + (\partial_\mu \tilde{d}_{\rm R} - \mathrm{i} g_3 \tilde{d}_{\rm R}^y \tau^{a*} g_\mu^a)(\partial^\mu \tilde{d}_{\rm R}^* + \mathrm{i} g_3 g_\mu^b \tau^{b*} \tilde{d}_{\rm R}^*) \\
& + \sqrt{2} g_2 g_3 \tilde{u}_{\rm L}^* \tau^a \tilde{d}_{\rm L} W^{+\mu} g_\mu^a + \sqrt{2} g_2 g_3 \tilde{d}_{\rm L}^* \tau^a \tilde{u}_{\rm L} W^{-\mu} g_\mu^a + \frac{4}{3} g_3 |e| (\tilde{u}_{\rm L}^* \tau^a \tilde{u}_{\rm L} + \tilde{u}_{\rm R}^* \tau^a \tilde{u}_{\rm R}) g^{a\mu} A_\mu \\
& - \frac{2}{3} g_3 |e| (\tilde{d}_{\rm L}^* \tau^a \tilde{d}_{\rm L} + \tilde{d}_{\rm R}^* \tau^a \tilde{d}_{\rm R}) g^{a\mu} A_\mu + \frac{(3 - 4\mathrm{s}_{\rm w}^2) g_Z}{3} g_3 \tilde{u}_{\rm L}^* \tau^a \tilde{u}_{\rm L} g^{a\mu} Z_\mu - \frac{4\mathrm{s}_{\rm w}^2 g_Z}{3} g_3 \tilde{u}_{\rm R}^* \tau^a \tilde{u}_{\rm R} g^{a\mu} Z_\mu \\
& + \frac{(2\mathrm{s}_{\rm w}^2 - 3) g_Z}{3} g_3 \tilde{d}_{\rm L}^* \tau^a \tilde{d}_{\rm L} g^{a\mu} Z_\mu + \frac{2\mathrm{s}_{\rm w}^2 g_Z}{3} g_3 \tilde{d}_{\rm R}^* \tau^a \tilde{d}_{\rm R} g^{a\mu} Z_\mu \\
& + \frac{\mathrm{i} g_2}{\sqrt{2}} W_\mu^+ \left(\tilde{u}_{\rm L}^* \partial^\mu \tilde{d}_{\rm L} - \tilde{d}_{\rm L} \partial^\mu \tilde{u}_{\rm L}^* \right) - \frac{\mathrm{i} g_2}{\sqrt{2}} W_\mu^- \left(\tilde{u}_{\rm L} \partial^\mu \tilde{d}_{\rm L}^* - \tilde{d}_{\rm L}^* \partial^\mu \tilde{u}_{\rm L} \right) \\
& + \frac{2\mathrm{i}}{3} |e| A_\mu \left(\tilde{u}_{\rm L}^* \partial^\mu \tilde{u}_{\rm L} - \tilde{u}_{\rm L} \partial^\mu \tilde{u}_{\rm L}^* + \tilde{u}_{\rm R}^* \partial^\mu \tilde{u}_{\rm R} - \tilde{u}_{\rm R} \partial^\mu \tilde{u}_{\rm R}^* \right) - \frac{\mathrm{i}}{3} |e| A_\mu \left(\tilde{d}_{\rm L}^* \partial^\mu \tilde{d}_{\rm L} - \tilde{d}_{\rm L} \partial^\mu \tilde{d}_{\rm L}^* + \tilde{d}_{\rm R}^* \partial^\mu \tilde{d}_{\rm R} - \tilde{d}_{\rm R} \partial^\mu \tilde{d}_{\rm R}^* \right) \\
& + \frac{\mathrm{i}(4\mathrm{s}_{\rm w}^2 - 3) g_Z}{6} Z_\mu \left(\tilde{u}_{\rm L} \partial^\mu \tilde{u}_{\rm L}^* - \tilde{u}_{\rm L}^* \partial^\mu \tilde{u}_{\rm L} \right) + \frac{\mathrm{i}(2\mathrm{s}_{\rm w}^2 - 3) g_Z}{6} Z_\mu \left(\tilde{d}_{\rm L}^* \partial^\mu \tilde{d}_{\rm L} - \tilde{d}_{\rm L} \partial^\mu \tilde{d}_{\rm L}^* \right) \\
& - \frac{2\mathrm{i}\mathrm{s}_{\rm w}^2 g_Z}{3} Z_\mu \left(\tilde{u}_{\rm R}^* \partial^\mu \tilde{u}_{\rm R} - \tilde{u}_{\rm R} \partial^\mu \tilde{u}_{\rm R}^* \right) + \frac{\mathrm{i}\mathrm{s}_{\rm w}^2 g_Z}{3} Z_\mu \left(\tilde{d}_{\rm R}^* \partial^\mu \tilde{d}_{\rm R} - \tilde{d}_{\rm R} \partial^\mu \tilde{d}_{\rm R}^* \right) \\
& + \frac{g_2^2}{2} \left(|\tilde{u}_{\rm L}|^2 + |\tilde{d}_{\rm L}|^2 \right) W^{+\mu} W_\mu^- - \frac{\mathrm{s}_{\rm w}^2 g_2 g_Z}{3\sqrt{2}} \tilde{u}_{\rm L}^* \tilde{d}_{\rm L} W^{+\mu} Z_\mu - \frac{\mathrm{s}_{\rm w}^2 g_2 g_Z}{3\sqrt{2}} \tilde{d}_{\rm L}^* \tilde{u}_{\rm L} W^{-\mu} Z_\mu \\
& + \frac{(3 - 4\mathrm{s}_{\rm w}^2)^2 g_Z^2}{36} |\tilde{u}_{\rm L}|^2 Z^\mu Z_\mu + \frac{4\mathrm{s}_{\rm w}^4 g_Z^2}{9} |\tilde{u}_{\rm R}|^2 Z^\mu Z_\mu + \frac{(3 - 2\mathrm{s}_{\rm w}^2)^2 g_Z^2}{36} |\tilde{d}_{\rm L}|^2 Z^\mu Z_\mu + \frac{\mathrm{s}_{\rm w}^4 g_Z^2}{9} |\tilde{d}_{\rm R}|^2 Z^\mu Z_\mu \\
& + \frac{4}{9} e^2 \left(|\tilde{u}_{\rm L}|^2 + |\tilde{u}_{\rm R}|^2 \right) A^\mu A_\mu + \frac{1}{9} e^2 \left(|\tilde{d}_{\rm L}|^2 + |\tilde{d}_{\rm R}|^2 \right) A^\mu A_\mu + \frac{|e| g_2}{3\sqrt{2}} \tilde{u}_{\rm L}^* \tilde{d}_{\rm L} W^{+\mu} A_\mu + \frac{|e| g_2}{3\sqrt{2}} \tilde{d}_{\rm L}^* \tilde{u}_{\rm L} W^{-\mu} A_\mu \\
& + \frac{2(3 - 4\mathrm{s}_{\rm w}^2) g_Z |e|}{9} |\tilde{u}_{\rm L}|^2 A^\mu Z_\mu - \frac{8\mathrm{s}_{\rm w}^2 g_Z |e|}{9} |\tilde{u}_{\rm R}|^2 A^\mu Z_\mu + \frac{(3 - 2\mathrm{s}_{\rm w}^2) g_Z |e|}{9} |\tilde{d}_{\rm L}|^2 A^\mu Z_\mu - \frac{2\mathrm{s}_{\rm w}^2 g_Z |e|}{9} |\tilde{d}_{\rm R}|^2 A^\mu Z_\mu \\[1ex]
& + \partial_\mu \tilde{e}_{\rm R} \partial^\mu \tilde{e}_{\rm R}^* + \partial_\mu \tilde{e}_{\rm L}^* \partial^\mu \tilde{e}_{\rm L} + \partial_\mu \tilde{\nu}_{\rm L}^* \partial^\mu \tilde{\nu}_{\rm L} + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^+ (\tilde{\nu}_{\rm L}^* \partial^\mu \tilde{e}_{\rm L} - \tilde{e}_{\rm L} \partial^\mu \tilde{\nu}_{\rm L}^*) + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^- (\tilde{e}_{\rm L}^* \partial^\mu \tilde{\nu}_{\rm L} - \tilde{\nu}_{\rm L} \partial^\mu \tilde{e}_{\rm L}^*) \\
& - \frac{\mathrm{i}(1 - 2\mathrm{s}_{\rm w}^2) g_Z}{2} Z_\mu (\tilde{e}_{\rm L}^* \partial^\mu \tilde{e}_{\rm L} - \tilde{e}_{\rm L} \partial^\mu \tilde{e}_{\rm L}^*) + \frac{\mathrm{i} g_Z}{2} Z_\mu (\tilde{\nu}_{\rm L}^* \partial^\mu \tilde{\nu}_{\rm L} - \tilde{\nu}_{\rm L} \partial^\mu \tilde{\nu}_{\rm L}^*) + \mathrm{i}\mathrm{s}_{\rm w}^2 g_Z Z_\mu (\tilde{e}_{\rm R}^* \partial^\mu \tilde{e}_{\rm R} - \tilde{e}_{\rm R} \partial^\mu \tilde{e}_{\rm R}^*) \\
& + \mathrm{i} |e| A_\mu (\tilde{e}_{\rm L} \partial^\mu \tilde{e}_{\rm L}^* - \tilde{e}_{\rm L}^* \partial^\mu \tilde{e}_{\rm L} + \tilde{e}_{\rm R} \partial^\mu \tilde{e}_{\rm R}^* - \tilde{e}_{\rm R}^* \partial^\mu \tilde{e}_{\rm R}) \\
& + \frac{g_2^2}{2} (|\tilde{\nu}_{\rm L}|^2 + |\tilde{e}_{\rm L}|^2) W^{+\mu} W_\mu^- + \frac{g_2 g_Z \mathrm{s}_{\rm w}^2}{\sqrt{2}} \left(\tilde{e}_{\rm L}^* \tilde{\nu}_{\rm L} W_\mu^- Z^\mu + \tilde{\nu}_{\rm L}^* \tilde{e}_{\rm L} W_\mu^+ Z^\mu \right) - \frac{g_2 |e|}{\sqrt{2}} \left(\tilde{\nu}_{\rm L}^* \tilde{e}_{\rm L} W_\mu^+ A^\mu + \tilde{e}_{\rm L}^* \tilde{\nu}_{\rm L} W_\mu^- A^\mu \right) \\
& + \frac{(1 - 2\mathrm{s}_{\rm w}^2)^2 g_Z^2}{4} |\tilde{e}_{\rm L}|^2 Z^\mu Z_\mu + \frac{g_Z^2}{4} |\tilde{\nu}_{\rm L}|^2 Z^\mu Z_\mu + g_Z^2 \mathrm{s}_{\rm w}^4 |\tilde{e}_{\rm R}|^2 Z^\mu Z_\mu \\
& + e^2 (|\tilde{e}_{\rm L}|^2 + |\tilde{e}_{\rm R}|^2) A^\mu A_\mu + (1 - 2\mathrm{s}_{\rm w}^2) |e| g_Z |\tilde{e}_{\rm L}|^2 A^\mu Z_\mu - 2\mathrm{s}_{\rm w}^2 g_Z |e| |\tilde{e}_{\rm R}|^2 A^\mu Z_\mu \\[1ex]
& + \partial_\mu h_{\rm d}^{-*} \partial^\mu h_{\rm d}^- + \partial_\mu h_{\rm d}^{0*} \partial^\mu h_{\rm d}^0 + \partial_\mu h_{\rm u}^{+*} \partial^\mu h_{\rm u}^+ + \partial_\mu h_{\rm u}^{0*} \partial^\mu h_{\rm u}^0 + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^+ \left(h_{\rm u}^{+*} \partial^\mu h_{\rm u}^0 - h_{\rm u}^0 \partial^\mu h_{\rm u}^{+*} \right) \\
& + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^- \left(h_{\rm u}^{0*} \partial^\mu h_{\rm u}^+ - h_{\rm u}^+ \partial^\mu h_{\rm u}^{0*} \right) + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^+ \left(h_{\rm d}^{0*} \partial^\mu h_{\rm d}^- - h_{\rm d}^- \partial^\mu h_{\rm d}^{0*} \right) + \mathrm{i} \frac{g_2}{\sqrt{2}} W_\mu^- \left(h_{\rm d}^{-*} \partial^\mu h_{\rm d}^0 - h_{\rm d}^0 \partial^\mu h_{\rm d}^{-*} \right) \\
& + \frac{\mathrm{i}(1 - 2\mathrm{s}_{\rm w}^2) g_Z}{2} Z_\mu \left(h_{\rm u}^{+*} \partial^\mu h_{\rm u}^+ - h_{\rm u}^+ \partial^\mu h_{\rm u}^{+*} \right) + \frac{\mathrm{i}(2\mathrm{s}_{\rm w}^2 - 1) g_Z}{2} Z_\mu \left(h_{\rm d}^{-*} \partial^\mu h_{\rm d}^- - h_{\rm d}^- \partial^\mu h_{\rm d}^{-*} \right) \\
& - \frac{\mathrm{i} g_Z}{2} Z_\mu \left(h_{\rm u}^{0*} \partial^\mu h_{\rm u}^0 - h_{\rm u}^0 \partial^\mu h_{\rm u}^{0*} \right) + \frac{\mathrm{i} g_Z}{2} Z_\mu \left(h_{\rm d}^{0*} \partial^\mu h_{\rm d}^0 - h_{\rm d}^0 \partial^\mu h_{\rm d}^{0*} \right) \\
& - \mathrm{i} |e| A_\mu \left(h_{\rm d}^{-*} \partial^\mu h_{\rm d}^- - h_{\rm d}^- \partial^\mu h_{\rm d}^{-*} \right) + \mathrm{i} |e| A_\mu \left(h_{\rm u}^{+*} \partial^\mu h_{\rm u}^+ - h_{\rm u}^+ \partial^\mu h_{\rm u}^{+*} \right) \\
& + \frac{g_2^2}{2} \left(|h_{\rm u}^+|^2 + |h_{\rm u}^0|^2 + |h_{\rm d}^0|^2 + |h_{\rm d}^-|^2 \right) W^{+\mu} W_\mu^- + \frac{\mathrm{s}_{\rm w}^2 g_Z g_2}{\sqrt{2}} \left(h_{\rm d}^{0*} h_{\rm d}^- - h_{\rm u}^{+*} h_{\rm u}^0 \right) W_\mu^+ Z^\mu \\
& + \frac{\mathrm{s}_{\rm w}^2 g_Z g_2}{\sqrt{2}} \left(h_{\rm d}^{-*} h_{\rm d}^0 - h_{\rm u}^{0*} h_{\rm u}^+ \right) W_\mu^- Z^\mu + \frac{g_2 |e|}{\sqrt{2}} \left(h_{\rm u}^{+*} h_{\rm u}^0 - h_{\rm d}^{0*} h_{\rm d}^- \right) W_\mu^+ A^\mu + \frac{g_2 |e|}{\sqrt{2}} \left(h_{\rm u}^{0*} h_{\rm u}^+ - h_{\rm d}^{-*} h_{\rm d}^0 \right) W_\mu^- A^\mu \\
& + \frac{(1 - 2\mathrm{s}_{\rm w}^2)^2 g_Z^2}{4} \left(|h_{\rm u}^+|^2 + |h_{\rm d}^-|^2 \right) Z^\mu Z_\mu + \frac{g_Z^2}{4} \left(|h_{\rm u}^0|^2 + |h_{\rm d}^0|^2 \right) Z^\mu Z_\mu + e^2 \left(|h_{\rm u}^+|^2 + |h_{\rm d}^-|^2 \right) A^\mu A_\mu \\
& + (1 - 2\mathrm{s}_{\rm w}^2) |e| g_Z \left(|h_{\rm u}^+|^2 + |h_{\rm d}^-|^2 \right) A^\mu Z_\mu \\
& - (V_{\rm SUSY} + V_{\cancel{\rm SUSY}}) \,,
\end{aligned}
$$

(11.43)

where the scalar potential is given by

$$
\begin{aligned}
V_{\mathrm{SUSY}} = {} & |h_{\mathrm{u}}|^2 \left(|\mu|^2 + \sum_i |\kappa_i|^2 \right) + |\mu|^2 |h_{\mathrm{d}}|^2 + \left(\kappa_i^* \mu \tilde{l}_{\mathrm{L}i}^* h_{\mathrm{d}} + \mathrm{H.c.} \right) + \kappa_i^* \kappa_j \tilde{l}_{\mathrm{L}i}^* \tilde{l}_{\mathrm{L}j} \\
& + \Big[- y_{uij} \mu^* h_{\mathrm{d}}^* \tilde{u}_{\mathrm{R}i}^* \tilde{q}_{\mathrm{L}j} - y_{uij} \kappa_k^* \tilde{u}_{\mathrm{R}i}^* \tilde{q}_{\mathrm{L}j} \tilde{l}_{\mathrm{L}k}^* - \left(y_{dij} \mu^* + \lambda'_{kji} \kappa_k^* \right) h_{\mathrm{u}}^* \tilde{d}_{\mathrm{R}i}^* \tilde{q}_{\mathrm{L}j} \\
& \quad + y_{eij} \kappa_j^* \tilde{e}_{\mathrm{R}i}^* h_{\mathrm{u}}^* h_{\mathrm{d}} + \left(\lambda_{jki} \kappa_k^* - y_{eij} \mu^* \right) h_{\mathrm{u}}^* \tilde{e}_{\mathrm{R}i}^* \tilde{l}_{\mathrm{L}j} + \mathrm{H.c.} \Big] \\
& + \frac{1}{8} \left(g_2^2 + g_Y^2 \right) |h_{\mathrm{d}}|^4 + \frac{1}{8} \left(g_2^2 + g_Y^2 \right) |h_{\mathrm{u}}|^4 + \left(-\frac{g_2^2}{4} |h_{\mathrm{d}}|^2 |h_{\mathrm{u}}|^2 - \frac{g_Y^2}{4} |h_{\mathrm{d}}|^2 |h_{\mathrm{u}}|^2 + \frac{g_2^2}{2} |h_{\mathrm{d}}^* h_{\mathrm{u}}|^2 \right) \\
& + \left(-\frac{g_2^2}{4} |h_{\mathrm{u}}|^2 |\tilde{q}_{\mathrm{L}}|^2 + \frac{g_Y^2}{12} |h_{\mathrm{u}}|^2 |\tilde{q}_{\mathrm{L}}|^2 + \frac{g_2^2}{2} |h_{\mathrm{u}}^b \tilde{q}_{\mathrm{L}i}|^2 + \epsilon^{ac} \epsilon^{bd} (y_u^\dagger y_u)_{ji} h_{\mathrm{u}}^a h_{\mathrm{u}}^{b*} \tilde{q}_{\mathrm{L}i}^c \tilde{q}_{\mathrm{L}j}^{d*} \right) \\
& + \left(-\frac{g_2^2}{4} |h_{\mathrm{d}}|^2 |\tilde{q}_{\mathrm{L}}|^2 - \frac{g_Y^2}{12} |h_{\mathrm{d}}|^2 |\tilde{q}_{\mathrm{L}}|^2 + \frac{g_2^2}{2} |h_{\mathrm{d}}^* \tilde{q}_{\mathrm{L}i}|^2 + \epsilon^{ac} \epsilon^{bd} (y_d^\dagger y_d)_{ji} h_{\mathrm{d}}^a h_{\mathrm{d}}^{b*} \tilde{q}_{\mathrm{L}i}^c \tilde{q}_{\mathrm{L}j}^{d*} \right) \\
& + \left(-\frac{g_2^2}{4} |h_{\mathrm{u}}|^2 |\tilde{l}_{\mathrm{L}}|^2 - \frac{g_Y^2}{4} |h_{\mathrm{u}}|^2 |\tilde{l}_{\mathrm{L}}|^2 + \frac{g_2^2}{2} |h_{\mathrm{u}}^* \tilde{l}_{\mathrm{L}i}|^2 \right) \\
& + \left(-\frac{g_2^2}{4} |h_{\mathrm{d}}|^2 |\tilde{l}_{\mathrm{L}}|^2 + \frac{g_Y^2}{4} |h_{\mathrm{d}}|^2 |\tilde{l}_{\mathrm{L}}|^2 + \frac{g_2^2}{2} |h_{\mathrm{d}}^* \tilde{l}_{\mathrm{L}i}|^2 + \epsilon^{ac} \epsilon^{bd} (y_e^\dagger y_e)_{ji} h_{\mathrm{d}}^a h_{\mathrm{d}}^{b*} \tilde{l}_{\mathrm{L}i}^c \tilde{l}_{\mathrm{L}j}^{d*} \right) \\
& + |h_{\mathrm{u}}|^2 \left(-\frac{g_Y^2}{3} |\tilde{u}_{\mathrm{R}}|^2 + (y_u y_u^\dagger)_{ij} \tilde{u}_{\mathrm{R}i}^* \tilde{u}_{\mathrm{R}j} \right) + \frac{g_Y^2}{3} |h_{\mathrm{d}}|^2 |\tilde{u}_{\mathrm{R}}|^2 \\
& + \frac{g_Y^2}{6} |h_{\mathrm{u}}|^2 |\tilde{d}_{\mathrm{R}}|^2 + |h_{\mathrm{d}}|^2 \left(-\frac{g_Y^2}{6} |\tilde{d}_{\mathrm{R}}|^2 + (y_d y_d^\dagger)_{ij} \tilde{d}_{\mathrm{R}i}^* \tilde{d}_{\mathrm{R}j} \right) - \left[(y_u y_d^\dagger)_{ij} \tilde{u}_{\mathrm{R}i}^* \tilde{d}_{\mathrm{R}j} (h_{\mathrm{d}}^* h_{\mathrm{u}}) + \mathrm{H.c.} \right] \\
& + \frac{g_Y^2}{2} |h_{\mathrm{u}}|^2 |\tilde{e}_{\mathrm{R}}|^2 + |h_{\mathrm{d}}|^2 \left(-\frac{g_Y^2}{2} |\tilde{e}_{\mathrm{R}}|^2 + (y_e y_e^\dagger)_{ij} \tilde{e}_{\mathrm{R}i}^* \tilde{e}_{\mathrm{R}j} \right) \\
& + \Big[-\frac{1}{2} \epsilon^{ab} \epsilon^{xyz} y_{ulk} \lambda''^*_{lij} h_{\mathrm{u}}^a \tilde{d}_{\mathrm{R}i}^x \tilde{d}_{\mathrm{R}j}^y \tilde{q}_{\mathrm{L}k}^{bz} + \epsilon^{ab} \epsilon^{xyz} y_{dlk} \lambda''^*_{ijl} h_{\mathrm{d}}^a \tilde{u}_{\mathrm{R}i}^x \tilde{d}_{\mathrm{R}j}^y \tilde{q}_{\mathrm{L}k}^{bz} - y_{uil} \lambda'^*_{klj} h_{\mathrm{u}}^a \tilde{u}_{\mathrm{R}i}^* \tilde{d}_{\mathrm{R}j} \tilde{l}_{\mathrm{L}k}^{a*} \\
& \quad + y_{dil} \lambda'^*_{klj} h_{\mathrm{d}}^a \tilde{d}_{\mathrm{R}i}^* \tilde{d}_{\mathrm{R}j} \tilde{l}_{\mathrm{L}k}^{a*} - \epsilon^{ab} \epsilon^{cd} y_{dli} \lambda'^*_{kjl} h_{\mathrm{d}}^a \tilde{q}_{\mathrm{L}i}^b \tilde{q}_{\mathrm{L}j}^{c*} \tilde{l}_{\mathrm{L}k}^{d*} + y_{eil} \lambda^*_{klj} h_{\mathrm{d}}^a \tilde{e}_{\mathrm{R}i}^* \tilde{e}_{\mathrm{R}j} \tilde{l}_{\mathrm{L}k}^{a*} \\
& \quad - y_{ejl} \lambda'^*_{lki} h_{\mathrm{d}}^a \tilde{e}_{\mathrm{R}j}^* \tilde{d}_{\mathrm{R}i} \tilde{q}_{\mathrm{L}k}^{a*} + \frac{1}{2} \epsilon^{ab} \epsilon^{cd} y_{eli} \lambda^*_{jkl} h_{\mathrm{d}}^a \tilde{l}_{\mathrm{L}i}^b \tilde{l}_{\mathrm{L}j}^{c*} \tilde{l}_{\mathrm{L}k}^{d*} + \mathrm{H.c.} \Big] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{72} - \frac{g_2^2}{8} \right) |\tilde{q}_{\mathrm{L}}|^4 + \frac{g_2^2}{4} \tilde{q}_{\mathrm{L}i}^{ax} \tilde{q}_{\mathrm{L}j}^{by} \tilde{q}_{\mathrm{L}i}^{bx*} \tilde{q}_{\mathrm{L}j}^{ay*} + \frac{g_3^2}{4} \tilde{q}_{\mathrm{L}i}^{ax} \tilde{q}_{\mathrm{L}j}^{by} \tilde{q}_{\mathrm{L}i}^{ay*} \tilde{q}_{\mathrm{L}j}^{bx*} \right] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{2 g_Y^2}{9} \right) |\tilde{u}_{\mathrm{R}}|^4 + \frac{g_3^2}{4} \tilde{u}_{\mathrm{R}i}^{x*} \tilde{u}_{\mathrm{R}j}^{y*} \tilde{u}_{\mathrm{R}i}^y \tilde{u}_{\mathrm{R}j}^x \right] \\
& + \left[\left(-\frac{g_3^2}{12} + \frac{g_Y^2}{18} \right) |\tilde{d}_{\mathrm{R}}|^4 + \frac{g_3^2}{4} \tilde{d}_{\mathrm{R}i}^{x*} \tilde{d}_{\mathrm{R}j}^{y*} \tilde{d}_{\mathrm{R}i}^y \tilde{d}_{\mathrm{R}j}^x + \frac{1}{2} \lambda''_{mij} \lambda''^*_{mkl} \tilde{d}_{\mathrm{R}i}^{x*} \tilde{d}_{\mathrm{R}j}^{y*} \tilde{d}_{\mathrm{R}k}^x \tilde{d}_{\mathrm{R}l}^y \right] \\
& + \left[\left(\frac{g_3^2}{6} - \frac{g_Y^2}{9} \right) |\tilde{u}_{\mathrm{R}}|^2 |\tilde{q}_{\mathrm{L}}|^2 - \frac{g_3^2}{2} \tilde{u}_{\mathrm{R}i}^{x*} \tilde{q}_{\mathrm{L}j}^{ax} \tilde{u}_{\mathrm{R}i}^y \tilde{q}_{\mathrm{L}j}^{ay*} + y_{uik} y_{ujl}^* \tilde{u}_{\mathrm{R}i}^{x*} \tilde{u}_{\mathrm{R}j}^y \tilde{q}_{\mathrm{L}k}^{ax} \tilde{q}_{\mathrm{L}l}^{ay*} \right] \\
& + \left[\left(\frac{g_3^2}{6} + \frac{g_Y^2}{18} \right) |\tilde{d}_{\mathrm{R}}|^2 |\tilde{q}_{\mathrm{L}}|^2 - \frac{g_3^2}{2} \tilde{d}_{\mathrm{R}i}^{x*} \tilde{q}_{\mathrm{L}j}^{ax} \tilde{d}_{\mathrm{R}i}^y \tilde{q}_{\mathrm{L}j}^{ay*} + \left(y_{dik} y_{djl}^* + \lambda'_{mki} \lambda'^*_{mlj} \right) \tilde{d}_{\mathrm{R}i}^{x*} \tilde{q}_{\mathrm{L}k}^{ax} \tilde{d}_{\mathrm{R}j}^y \tilde{q}_{\mathrm{L}l}^{ay*} \right] \\
& + \left[- \left(\frac{g_3^2}{6} + \frac{2 g_Y^2}{9} \right) |\tilde{d}_{\mathrm{R}}|^2 |\tilde{u}_{\mathrm{R}}|^2 + \left(\frac{g_3^2}{2} - \lambda''_{ikm} \lambda''^*_{ljm} \right) \tilde{u}_{\mathrm{R}i}^{x*} \tilde{d}_{\mathrm{R}j}^x \tilde{d}_{\mathrm{R}j}^{y*} \tilde{u}_{\mathrm{R}i}^y + \lambda''_{ikm} \lambda''^*_{jlm} \tilde{u}_{\mathrm{R}i}^{x*} \tilde{u}_{\mathrm{R}j}^x \tilde{d}_{\mathrm{R}k}^{y*} \tilde{d}_{\mathrm{R}l}^y \right] \\
& + \left[- \left(\frac{g_2^2}{4} + \frac{g_Y^2}{12} \right) |\tilde{l}_{\mathrm{L}}|^2 |\tilde{q}_{\mathrm{L}}|^2 + \frac{g_2^2}{2} \tilde{q}_{\mathrm{L}i}^{ax} \tilde{q}_{\mathrm{L}i}^{bx*} \tilde{l}_{\mathrm{L}j}^b \tilde{l}_{\mathrm{L}j}^{a*} + \epsilon^{ac} \epsilon^{bd} \lambda'_{kim} \lambda'^*_{ljm} \tilde{q}_{\mathrm{L}i}^{ax} \tilde{q}_{\mathrm{L}j}^{bx*} \tilde{l}_{\mathrm{L}k}^c \tilde{l}_{\mathrm{L}l}^{d*} \right] + \frac{g_Y^2}{6} |\tilde{e}_{\mathrm{R}}|^2 |\tilde{q}_{\mathrm{L}}|^2 \\
& + \left(-\frac{g_Y^2}{6} |\tilde{d}_{\mathrm{R}}|^2 |\tilde{l}_{\mathrm{L}}|^2 + \lambda'_{lmi} \lambda'^*_{kmj} \tilde{d}_{\mathrm{R}i}^* \tilde{d}_{\mathrm{R}j} \tilde{l}_{\mathrm{L}k}^* \tilde{l}_{\mathrm{L}l} \right) + \frac{g_Y^2}{3} |\tilde{u}_{\mathrm{R}}|^2 |\tilde{l}_{\mathrm{L}}|^2 - \frac{2 g_Y^2}{3} |\tilde{u}_{\mathrm{R}}|^2 |\tilde{e}_{\mathrm{R}}|^2 + \frac{g_Y^2}{3} |\tilde{d}_{\mathrm{R}}|^2 |\tilde{e}_{\mathrm{R}}|^2 \\
& + \left[\left(-\frac{g_2^2}{8} + \frac{g_Y^2}{8} \right) |\tilde{l}_{\mathrm{L}}|^4 + \frac{g_2^2}{4} \tilde{l}_{\mathrm{L}i}^a \tilde{l}_{\mathrm{L}j}^b \tilde{l}_{\mathrm{L}i}^{b*} \tilde{l}_{\mathrm{L}j}^{a*} + \frac{1}{4} \epsilon^{ab} \epsilon^{cd} \lambda_{ijm} \lambda^*_{klm} \tilde{l}_{\mathrm{L}i}^a \tilde{l}_{\mathrm{L}j}^b \tilde{l}_{\mathrm{L}k}^{c*} \tilde{l}_{\mathrm{L}l}^{d*} \right] \\
& + \frac{g_Y^2}{2} |\tilde{e}_{\mathrm{R}}|^4 + \left[-\frac{g_Y^2}{2} |\tilde{e}_{\mathrm{R}}|^2 |\tilde{l}_{\mathrm{L}}|^2 + \left(y_{eik} y_{ejl}^* + \lambda_{kmi} \lambda^*_{lmj} \right) \tilde{e}_{\mathrm{R}i}^* \tilde{e}_{\mathrm{R}j} \tilde{l}_{\mathrm{L}k} \tilde{l}_{\mathrm{L}l}^* \right] \\
& + \left[\left(y_{dik} y_{ejl}^* - \lambda'_{mki} \lambda^*_{lmj} \right) \tilde{d}_{\mathrm{R}i}^{x*} \tilde{q}_{\mathrm{L}k}^{ax} \tilde{e}_{\mathrm{R}j} \tilde{l}_{\mathrm{L}l}^{a*} - \epsilon^{ab} \epsilon^{xyz} \lambda'_{lkm} \lambda''^*_{ijm} \tilde{l}_{\mathrm{L}l}^b \tilde{q}_{\mathrm{L}k}^{az} \tilde{d}_{\mathrm{R}j}^y \tilde{u}_{\mathrm{R}i}^x + \mathrm{H.c.} \right] .
\end{aligned}
\tag{11.44}
$$

11.4. Higgs mechanism and fermion composition

The scalar potential includes

$$\begin{aligned} V_{\mathrm{SUSY}} \supset\, & |h_{\mathrm{u}}|^2\Bigl(|\mu|^2 + \sum|\kappa_i|^2\Bigr) + |\mu|^2|h_{\mathrm{d}}|^2 + \frac{g_Z^2}{8}\left(|h_{\mathrm{u}}|^2 - |h_{\mathrm{d}}|^2\right)^2 + \frac{g_2^2}{2}|h_{\mathrm{d}}^* h_{\mathrm{u}}|^2 \\ & + \Bigl(\kappa_i^*\mu\tilde{l}_{\mathrm{L}i}^* h_{\mathrm{d}} + \text{H.c.}\Bigr) + \kappa_i^*\kappa_j\tilde{l}_{\mathrm{L}i}^*\tilde{l}_{\mathrm{L}j} \end{aligned} \tag{11.45}$$

$$V_{\cancel{\mathrm{SUSY}}} \supset m_{H_{\mathrm{u}}}^2|h_{\mathrm{u}}|^2 + m_{H_{\mathrm{d}}}^2|h_{\mathrm{d}}|^2 + \epsilon^{ab}\left(bh_{\mathrm{u}}^a h_{\mathrm{d}}^b + b^* h_{\mathrm{u}}^{a*}h_{\mathrm{d}}^{b*} - b_i\tilde{l}_{\mathrm{L}i}^a h_{\mathrm{u}}^b - b_i^*\tilde{l}_{\mathrm{L}i}^{a*}h_{\mathrm{u}}^{b*}\right) + \tilde{l}_{\mathrm{L}i}^* M_{Li}^2 h_{\mathrm{d}} + \tilde{l}_{\mathrm{L}i} M_{Li}^{2*} h_{\mathrm{d}}^*; \tag{11.46}$$

the Higgs mass term is given by

$$V \supset \begin{pmatrix} h_{\mathrm{u}} & h_{\mathrm{d}}^* & \tilde{l}_{\mathrm{L}i}^* \end{pmatrix} \begin{pmatrix} |\mu|^2 + m_{H_{\mathrm{u}}}^2 + \sum|\kappa_i|^2 & b & -b_j \\ b^* & |\mu|^2 + m_{H_{\mathrm{d}}}^2 & \kappa_j\mu^* + M_{Lj}^{2*} \\ -b_i^* & \kappa_i^*\mu + M_{Li}^2 & (m_L^2)_{ij} + \kappa_i^*\kappa_j \end{pmatrix} \begin{pmatrix} h_{\mathrm{u}}^* \\ h_{\mathrm{d}} \\ \tilde{l}_{\mathrm{L}j} \end{pmatrix} \tag{11.47}$$

while corresponding fermion terms are

$$\mathcal{L} \supset \epsilon^{ab}\left(-\mu\tilde{h}_{\mathrm{u}}^a\tilde{h}_{\mathrm{d}}^b - \kappa_i\tilde{h}_{\mathrm{u}}^a l_{\mathrm{L}i}^b\right). \tag{11.48}$$

If the R-parity is not conserved, we redefine (H_{d}, L) superfields so that the mass matrix is block-diagonal, which corresponds to $\mathrm{U}(4)_{H_{\mathrm{d}},L} \to \mathrm{U}(3)_L \times \mathrm{U}(1)_{H_{\mathrm{d}}}$ (DOF counting: $16 \to 9 + 1$ to remove b_i'). Then lepton and $\tilde{h}_{\mathrm{d}}$ are mixed.[*18] With R-parity conservation, we do not suffer from these mixings.

11.4.1. Higgs potential and induced mass in R-parity conserved case

We perform "SU(2)-notation fixing", i.e., use the freedom associated to T_1 and T_2 of SU(2), so that $\langle h_{\mathrm{u}}^+\rangle = 0$. Then $\langle h_{\mathrm{d}}^-\rangle = 0$ and effectively

$$V_{\mathrm{pot}} = (|\mu|^2 + m_{H_{\mathrm{u}}}^2)|h_{\mathrm{u}}^0|^2 + (|\mu|^2 + m_{H_{\mathrm{d}}}^2)|h_{\mathrm{d}}^0|^2 + \frac{g_Z^2}{8}\left(|h_{\mathrm{u}}^0|^2 - |h_{\mathrm{d}}^0|^2\right)^2 - \left(bh_{\mathrm{u}}^0 h_{\mathrm{d}}^0 + \text{H.c.}\right). \tag{11.49}$$

We redefine H_{d} superfield so that $b > 0$.[*19] Then $\arg\langle h_{\mathrm{u}}^0\rangle = -\arg\langle h_{\mathrm{d}}^0\rangle$ and, with T_3-rotation, $\langle h_{\mathrm{u}}^0\rangle > 0$ and $\langle h_{\mathrm{d}}^0\rangle > 0$:

$$\langle h_{\mathrm{u}}^0\rangle =: v_{\mathrm{u}} =: \frac{v_{\mathrm{SM}}}{\sqrt{2}}\sin\beta, \qquad \langle h_{\mathrm{d}}^0\rangle =: v_{\mathrm{d}} =: \frac{v_{\mathrm{SM}}}{\sqrt{2}}\cos\beta; \tag{11.50}$$

$$V_{\mathrm{pot}} = \frac{1}{2}(|\mu|^2 + m_{H_{\mathrm{u}}}^2)v_{\mathrm{SM}}^2\sin^2\beta + \frac{1}{2}(|\mu|^2 + m_{H_{\mathrm{d}}}^2)v_{\mathrm{SM}}^2\cos^2\beta + \frac{g_Z^2}{32}v_{\mathrm{SM}}^4\cos^2 2\beta - \frac{1}{2}v_{\mathrm{SM}}^2 b\sin 2\beta. \tag{11.51}$$

This potential can have two minima; one with $0 < \beta \leqslant \pi/4$ and the other with $\pi/4 \leqslant \beta < \pi/2$:

$$\tan\beta = \frac{B \mp \sqrt{B^2 - 4b^2}}{2b} \quad \left(\cos 2\beta = \pm\frac{\sqrt{B^2 - 4b^2}}{B}\right), \qquad m_Z^2 := \frac{g_Z^2}{4}v_{\mathrm{SM}}^2 = \left(\pm\frac{m_{H_{\mathrm{d}}}^2 - m_{H_{\mathrm{u}}}^2}{\sqrt{B^2 - 4b^2}} - 1\right)B, \tag{11.52}$$

where $B := 2|\mu|^2 + m_{H_{\mathrm{u}}}^2 + m_{H_{\mathrm{d}}}^2 > 2b > 0$ and m_Z is the Z-boson tree-level mass. Also,

$$\sin 2\beta = \frac{2b}{2|\mu|^2 + m_{H_{\mathrm{u}}}^2 + m_{H_{\mathrm{d}}}^2}, \qquad m_Z^2 = \frac{-(m_{H_{\mathrm{d}}}^2 - m_{H_{\mathrm{u}}}^2)}{\cos 2\beta} - \left(2|\mu|^2 + m_{H_{\mathrm{u}}}^2 + m_{H_{\mathrm{d}}}^2\right) \tag{11.53}$$

are satisfied in both solutions.

Higgs sector The Nambu-Goldstone–Higgs mixings and the mass terms for the charged Higgs bosons are given by

$$\begin{aligned} \mathcal{L} \supset\, & \partial_\mu h_{\mathrm{d}}^{-*}\partial^\mu h_{\mathrm{d}}^- + \partial_\mu h_{\mathrm{u}}^{+*}\partial^\mu h_{\mathrm{u}}^+ + \left(-b - \frac{1}{2}g_2^2 v_{\mathrm{u}}v_{\mathrm{d}}\right)(h_{\mathrm{u}}^+ h_{\mathrm{d}}^- + h_{\mathrm{u}}^{+*}h_{\mathrm{d}}^{-*}) \\ & + \left[\frac{g_Y^2(v_{\mathrm{u}}^2 - v_{\mathrm{d}}^2) - g_2^2(v_{\mathrm{u}}^2 + v_{\mathrm{d}}^2)}{4} - |\mu|^2 - m_{H_{\mathrm{d}}}^2\right]|h_{\mathrm{d}}^-|^2 + \left[\frac{g_Y^2(v_{\mathrm{d}}^2 - v_{\mathrm{u}}^2) - g_2^2(v_{\mathrm{u}}^2 + v_{\mathrm{d}}^2)}{4} - |\mu|^2 - m_{H_{\mathrm{u}}}^2\right]|h_{\mathrm{u}}^+|^2 \\ & + \frac{\mathrm{i}g_2}{\sqrt{2}}W_\mu^-\partial^\mu\left(v_{\mathrm{u}}h_{\mathrm{u}}^+ - v_{\mathrm{u}}h_{\mathrm{u}}^{+*} - v_{\mathrm{d}}h_{\mathrm{d}}^{-*} + v_{\mathrm{d}}h_{\mathrm{d}}^-\right) \end{aligned} \tag{11.54}$$

and those for the neutral Higgs bosons are

$$\begin{aligned} \mathcal{L} \supset\, & \partial_\mu h_{\mathrm{d}}^{0*}\partial^\mu h_{\mathrm{d}}^0 + \partial_\mu h_{\mathrm{u}}^{0*}\partial^\mu h_{\mathrm{u}}^0 - \frac{g_Z^2 v_{\mathrm{d}}^2}{8}(h_{\mathrm{d}}^0 h_{\mathrm{d}}^0 + h_{\mathrm{d}}^{0*}h_{\mathrm{d}}^{0*}) - \frac{g_Z^2 v_{\mathrm{u}}^2}{8}(h_{\mathrm{u}}^0 h_{\mathrm{u}}^0 + h_{\mathrm{u}}^{0*}h_{\mathrm{u}}^{0*}) \\ & + \left(b + \frac{g_Z^2 v_{\mathrm{u}}v_{\mathrm{d}}}{4}\right)(h_{\mathrm{u}}^0 h_{\mathrm{d}}^0 + h_{\mathrm{u}}^{0*}h_{\mathrm{d}}^{0*}) + \frac{g_Z^2 v_{\mathrm{u}}v_{\mathrm{d}}}{4}(h_{\mathrm{u}}^0 h_{\mathrm{d}}^{0*} + h_{\mathrm{u}}^{0*}h_{\mathrm{d}}^0) \\ & + \left(\frac{g_Z^2(v_{\mathrm{u}}^2 - 2v_{\mathrm{d}}^2)}{4} - |\mu|^2 - m_{H_{\mathrm{d}}}^2\right)|h_{\mathrm{d}}^0|^2 + \left(\frac{g_Z^2(v_{\mathrm{d}}^2 - 2v_{\mathrm{u}}^2)}{4} - |\mu|^2 - m_{H_{\mathrm{u}}}^2\right)|h_{\mathrm{d}}^0|^2 \\ & + \frac{\mathrm{i}g_Z}{2}Z_\mu\partial^\mu\left(v_{\mathrm{d}}h_{\mathrm{d}}^0 - v_{\mathrm{d}}h_{\mathrm{d}}^{0*} - v_{\mathrm{u}}h_{\mathrm{u}}^0 + v_{\mathrm{u}}h_{\mathrm{u}}^{0*}\right). \end{aligned} \tag{11.55}$$

[*18] If we separated leptons and $\tilde{h}_{\mathrm{d}}$ first, sleptons would acquire VEVs and lepton-gaugino mixings would be induced.

[*19] Note that T_3-rotation induces $h_{\mathrm{u}}^0 \to \mathrm{e}^{\mathrm{i}\theta/2}h_{\mathrm{u}}^0$ and $h_{\mathrm{d}}^0 \to \mathrm{e}^{-\mathrm{i}\theta/2}h_{\mathrm{d}}^0$; it cannot remove the phase of b.

Therefore, with $m_W := c_w m_Z$ and

$$\begin{pmatrix} h_u^+ \\ h_d^{-*} \end{pmatrix} = \begin{pmatrix} s_\beta & c_\beta \\ -c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} -iG^+ \\ H^+ \end{pmatrix}, \qquad \begin{pmatrix} h_u^0 \\ h_d^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_u \\ \phi_d \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} s_\beta & c_\beta \\ -c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}, \tag{11.56}$$

we have

$$\begin{aligned} \mathcal{L} \supset{}& \partial_\mu G^{+*}\partial^\mu G^+ + \partial_\mu H^{+*}\partial^\mu H^+ + m_W(W_\mu^-\partial^\mu G^+ + W_\mu^+\partial^\mu G^{+*}) + \left(\frac{m_{H_d}^2 - m_{H_u}^2}{\cos 2\beta} + m_Z^2 s_w^2\right)|H^+|^2 \\ &+ \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 + \frac{1}{2}(\partial_\mu A^0)^2 + \frac{1}{2}(\partial_\mu G^0)^2 + m_Z Z_\mu \partial^\mu G^0 - \frac{B}{2}A_0^2 \\ &- \frac{1}{4}\left(B + m_Z^2 + (B - m_Z^2)\cos 2\beta\right)\phi_u^2. - \frac{1}{4}\left(B + m_Z^2 - (B - m_Z^2)\cos 2\beta\right)\phi_d^2 + \frac{1}{2}(B + m_Z^2)(\sin 2\beta)\phi_u\phi_d. \end{aligned} \tag{11.57}$$

In particular, the tree-level masses are

$$m_{A_0}^2 = B = 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2, \tag{11.58}$$

$$m_{H^+}^2 = m_{A_0}^2 + m_W^2, \tag{11.59}$$

$$m_{h,H} = \frac{1}{2}\left(m_{A_0}^2 + m_Z^2 \mp \sqrt{\left(m_{A_0}^2 - m_Z^2\right)^2 + 4m_{A_0}^2 m_Z^2 \sin^2 2\beta}\right) \tag{11.60}$$

with

$$\begin{pmatrix} \phi_d \\ \phi_u \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}, \qquad \frac{\tan 2\alpha}{\tan 2\beta} = \frac{m_{A_0}^2 + m_Z^2}{m_{A_0}^2 - m_Z^2}. \tag{11.61}$$

The mixing α is stored in `ALPHA` block of SLHA, while `HMIX` stores

$$\mu = \mu, \quad \tan\beta = \tan\beta, \quad v = v_{SM}(\sim 246\,\mathrm{GeV}), \quad m_A^2 = \frac{2b}{\sin 2\beta}, \tag{11.62}$$

at the scale specified. The above discussion holds even with CP-violation, but quantum corrections mix the three Higgs bosons; such information should be stored in `(IM)VCHMIX`. ♣TODO:discuss when needed♣

Mass terms in the Lagrangian The other mass terms are given by

$$\begin{aligned} \mathcal{L} \supset{}& m_W^2 W^{+\mu}W_\mu^- + \frac{1}{2}m_Z^2 Z_\mu Z^\mu - \frac{1}{2}M_3\tilde{g}_0^a\tilde{g}_0^a - \frac{1}{2}M_3^*\bar{\tilde{g}}_0^a\bar{\tilde{g}}_0^a \\ &+ \left(-\frac{1}{2}M_1\tilde{b}\tilde{b} - \frac{1}{2}M_2\tilde{w}^3\tilde{w}^3 + \mu\tilde{h}_u^0\tilde{h}_d^0 + c_\beta m_Z s_w\tilde{h}_d^0\tilde{b} - c_w c_\beta m_Z\tilde{h}_d^0\tilde{w}^3 - m_Z s_w s_\beta\tilde{h}_u^0\tilde{b} + c_w m_Z s_\beta\tilde{h}_u^0\tilde{w}^3 + \text{h.c.}\right) \\ &- M_2\tilde{w}^+\tilde{w}^- - \mu\tilde{h}_u^+\tilde{h}_d^- - M_2^*\bar{\tilde{w}}^+\bar{\tilde{w}}^- - \mu^*\bar{\tilde{h}}_u^+\bar{\tilde{h}}_d^- - \sqrt{2}m_W\left(c_\beta\tilde{h}_d^-\tilde{w}^+ + s_\beta\tilde{h}_u^+\tilde{w}^- + c_\beta\bar{\tilde{h}}_d^-\bar{\tilde{w}}^+ + s_\beta\bar{\tilde{h}}_u^+\bar{\tilde{w}}^-\right) \\ &- v_u y_{uij}u_{Ri}^c u_{Lj} - v_d y_{dij}d_{Ri}^c d_{Lj} - v_d y_{eij}e_{Ri}^c e_{Lj} - v_u y_{uij}^*\bar{u}_{Ri}^c\bar{u}_{Lj} - v_d y_{dij}^*\bar{d}_{Ri}^c\bar{d}_{Lj} - v_d y_{eij}^*\bar{e}_{Ri}^c\bar{e}_{Lj} \\ &- \tilde{u}_L^*\left(m_Q^2 + v_u^2 y_u^\dagger y_u + \frac{3 - 4s_w^2}{6}c_{2\beta}m_Z^2\right)\tilde{u}_L - \tilde{u}_R^*\left(m_{U^c}^2 + v_u^2 y_u y_u^\dagger + \frac{4s_w^2}{6}c_{2\beta}m_Z^2\right)\tilde{u}_R \\ &- v_u a_{uij}\tilde{u}_{Ri}^*\tilde{u}_{Lj} + v_d\mu^* y_{uij}\tilde{u}_{Ri}^*\tilde{u}_{Lj} - v_u a_{uij}^*\tilde{u}_{Ri}\tilde{u}_{Lj}^* + v_d\mu y_{uij}^*\tilde{u}_{Ri}\tilde{u}_{Lj}^* \\ &- \tilde{d}_L^*\left(m_Q^2 + v_d^2 y_d^\dagger y_d + \frac{-3 + 2s_w^2}{6}c_{2\beta}m_Z^2\right)\tilde{d}_L - \tilde{d}_R^*\left(m_{D^c}^2 + v_d^2 y_d y_d^\dagger + \frac{-2s_w^2}{6}c_{2\beta}m_Z^2\right)\tilde{d}_R \\ &- v_d a_{dij}\tilde{d}_{Ri}^*\tilde{d}_{Lj} + v_u\mu^* y_{dij}\tilde{d}_{Ri}^*\tilde{d}_{Lj} - v_d a_{dij}^*\tilde{d}_{Ri}\tilde{d}_{Lj}^* + v_u\mu y_{dij}^*\tilde{d}_{Ri}\tilde{d}_{Lj}^* \\ &- \tilde{\nu}_L^*\left(m_L^2 + \frac{1}{2}c_{2\beta}m_Z^2\right)\tilde{\nu}_L \\ &- \tilde{e}_L^*\left(m_L^2 + v_d^2 y_e^\dagger y_e + \frac{-1 + 2s_w^2}{2}c_{2\beta}m_Z^2\right) - \tilde{e}_R^*\left(m_{E^c}^2 + v_d^2 y_e y_e^\dagger + (-s_w^2)c_{2\beta}m_Z^2\right)\tilde{e}_R \\ &- v_d a_{eij}\tilde{e}_{Ri}^*\tilde{e}_{Lj} + v_u\mu^* y_{eij}\tilde{e}_{Ri}^*\tilde{e}_{Lj} - v_d a_{eij}^*\tilde{e}_{Ri}\tilde{e}_{Lj}^* + v_u\mu y_{eij}^*\tilde{e}_{Ri}\tilde{e}_{Lj}^*, \end{aligned} \tag{11.63}$$

where, at the tree level, the gauge boson mass m_W and m_Z, the gluino mass M_3, and matter-fermion masses $v_u y_u$, $v_d y_d$, and $v_d y_e$ are given with the “correct” sign (as far as $M_3 > 0$, etc.).

Neutralinos and charginos The mass matrices for neutralinos and charginos are given by

$$\begin{aligned} -\mathcal{L} \supset{}& \frac{1}{2}\begin{pmatrix} \tilde{b} \\ \tilde{w}^3 \\ \tilde{h}_d^0 \\ \tilde{h}_u^0 \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} M_1 & 0 & -c_\beta s_w m_Z & +s_\beta s_w m_Z \\ 0 & M_2 & +c_\beta c_w m_Z & -s_\beta c_w m_Z \\ -c_\beta s_w m_Z & +c_\beta c_w m_Z & 0 & -\mu \\ +s_\beta s_w m_Z & -s_\beta c_w m_Z & -\mu & 0 \end{pmatrix} \begin{pmatrix} \tilde{b} \\ \tilde{w}^3 \\ \tilde{h}_d^0 \\ \tilde{h}_u^0 \end{pmatrix} + \text{h.c.} \\ &+ \begin{pmatrix} \tilde{w}^- & \tilde{h}_d^- \end{pmatrix} \begin{pmatrix} M_2 & \sqrt{2}s_\beta m_W \\ \sqrt{2}c_\beta m_W & \mu \end{pmatrix} \begin{pmatrix} \tilde{w}^+ \\ \tilde{h}_u^+ \end{pmatrix} + \begin{pmatrix} \bar{\tilde{w}}^- & \bar{\tilde{h}}_d^- \end{pmatrix} \begin{pmatrix} M_2^* & \sqrt{2}s_\beta m_W \\ \sqrt{2}c_\beta m_W & \mu^* \end{pmatrix} \begin{pmatrix} \bar{\tilde{w}}^+ \\ \bar{\tilde{h}}_u^+ \end{pmatrix}. \end{aligned} \tag{11.64}$$

Note that the mass matrices themselves are the same as those in SLHA convention, $\mathcal{M}_{\tilde{\psi}^0}$ and $\mathcal{M}_{\tilde{\psi}^+}$, while the fields are in different convention. Therefore, we continue our discussion based only on the mass matrices so that the discussion is free from the choice of field convention.

As $\mathcal{M}_{\tilde{\psi}^0}$ is a complex symmetric matrix, there is a unitary matrix $\tilde{N}$ such that $M_{\tilde{\psi}^0} = \tilde{N}^*\mathcal{M}_{\tilde{\psi}^0}\tilde{N}^\dagger$, where $M_{\tilde{\psi}^0}$ is a *positive* diagonal matrix whose elements are (non-negative) singular values of $\mathcal{M}_{\tilde{\psi}^0}$ and in increasing order (Autonne-Takagi factorization). In SLHA2 convention with CP-violation, this matrix $\tilde{N}$ is stored as the `(IM)NMIX` blocks and the (positive) masses are stored in the `MASS` block. Meanwhile, if M_1, M_2 and μ are real, $\mathcal{M}_{\tilde{\psi}^0}$ is a real symmetric matrix and there is a real orthogonal matrix $\hat{N}$ such that $\hat{M}_{\tilde{\psi}^0} = \hat{N}^*\mathcal{M}_{\tilde{\psi}^0}\hat{N}^\dagger = \hat{N}\mathcal{M}_{\tilde{\psi}^0}\hat{N}^{\mathrm{T}}$, where $\hat{M}_{\tilde{\psi}^0}$ is a *real* diagonal matrix whose elements are the eigenvalues of $\mathcal{M}_{\tilde{\psi}^0}$ and in absolute-value-increasing order (spectral theorem). This matrix $\hat{N}$ is the `NMIX` block of SLHA convention and $\hat{M}_{ii}$ is stored in the `MASS` block, hence `MASS` block may have negative values for neutralinos.

The chargino mass matrix $\mathcal{M}_{\tilde{\psi}^+}$ is decomposed as $M_{\tilde{\psi}^+} = U^*\mathcal{M}_{\tilde{\psi}^+}V^\dagger$, where U and V are unitary matrices and the elements of the diagonal matrix $M_{\tilde{\psi}^+}$ are singular values of $\mathcal{M}_{\tilde{\psi}^+}$ (thus non-negative) and sorted in increasing order (singular value decomposition). These U and V are stored in `(IM)UMIX` and `(IM)VMIX`, and the singular values are stored in `MASS` block. Because the SVD theorem is closed in $\mathbb{R}$, if M_2 and μ are real, U and V can be real, and the `IM`-blocks are omitted.

In summary,

$$M_{\tilde{\psi}^0} = \tilde{N}^*\mathcal{M}_{\tilde{\psi}^0}\tilde{N}^\dagger, \qquad \tilde{N} = \texttt{(IM)NMIX}, \qquad (\texttt{MASS}) = [M_{\tilde{\psi}^0}]_{ii} \geqslant 0 \quad \text{(singular values)}; \tag{11.65}$$

$$\hat{M}_{\tilde{\psi}^0} = \hat{N}\mathcal{M}_{\tilde{\psi}^0}\hat{N}^{\mathrm{T}}, \qquad \hat{N} = \texttt{NMIX}, \qquad (\texttt{MASS}) = [\hat{M}_{\tilde{\psi}^0}]_{ii} \in \mathbb{R} \quad \text{(eigenvalues)}; \tag{11.66}$$

$$M_{\tilde{\psi}^+} = U^*\mathcal{M}_{\tilde{\psi}^+}V^\dagger, \qquad U = \texttt{(IM)UMIX}, \quad V = \texttt{(IM)VMIX}, \qquad (\texttt{MASS}) = [M_{\tilde{\psi}^+}]_{ii} \geqslant 0 \quad \text{(singular values)}. \tag{11.67}$$

Note that the singular values are equal to absolute values of the eigenvalues, which guarantees consistency of the two decomposition.

We then define matrix N by[*20]

$$N = \begin{cases} \tilde{N} \\ \mathrm{diag}(\varphi_i)\cdot\hat{N} \end{cases} = \mathrm{diag}(\varphi_i)\cdot\Big((\texttt{NMIX}) + \mathrm{i}(\texttt{IMNMIX})\Big); \qquad \varphi_i = \begin{cases} 1 & \text{if } (\texttt{MASS})_i \geqslant 0, \\ \mathrm{i} & \text{if } (\texttt{MASS})_i < 0. \end{cases} \tag{11.68}$$

It gives the proper mass diagonalization in both of the NMIX convention:

$$N^*\mathcal{M}_{\tilde{\psi}^0}N^\dagger = \begin{cases} \tilde{N}^*\mathcal{M}_{\tilde{\psi}^0}\tilde{N}^\dagger = M_{\tilde{\psi}^0}, \\ \mathrm{diag}(\varphi_i^*)\hat{N}^*\mathcal{M}_{\tilde{\psi}^0}\hat{N}^\dagger\,\mathrm{diag}(\varphi_i^*) = \mathrm{diag}(\varphi_i^*)\hat{M}_{\tilde{\psi}^0}\,\mathrm{diag}(\varphi_i^*) \end{cases} = M_{\tilde{\psi}^0} \quad \text{(neutralino masses} \geqslant 0). \tag{11.69}$$

Noting that the discussion up here is irrelevant of the convention, we have the neutralino/chargino mass eigenstates,

$$\tilde{\chi}^0_i = N_{ij}\begin{pmatrix} \tilde{b} \\ \tilde{w}^3 \\ \tilde{h}^0_{\mathrm{d}} \\ \tilde{h}^0_{\mathrm{u}} \end{pmatrix}_j, \quad \tilde{\chi}^+_i = V_{ij}\begin{pmatrix} \tilde{w}^+ \\ \tilde{h}^+_{\mathrm{u}} \end{pmatrix}_j, \quad \tilde{\chi}^-_i = U_{ij}\begin{pmatrix} \tilde{w}^- \\ \tilde{h}^-_{\mathrm{d}} \end{pmatrix}_j, \tag{11.70}$$

in our convention and the mass terms are now

$$-\mathcal{L} \supset \frac{1}{2}(\tilde{\chi}^0)^{\mathrm{T}}M_{\tilde{\psi}^0}\tilde{\chi}^0 + (\tilde{\chi}^-)^{\mathrm{T}}M_{\tilde{\psi}^+}\tilde{\chi}^+ + \text{h.c.} \tag{11.71}$$

Quarks, leptons, and super-CKM basis We here take the super-CKM basis. In the "original" Lagrangian,

$$-\mathcal{L} \supset u^{\mathrm{c}}_{\mathrm{R}}(v_{\mathrm{u}}y_u)u_{\mathrm{L}} + d^{\mathrm{c}}_{\mathrm{R}}(v_{\mathrm{d}}y_d)d_{\mathrm{L}} + e^{\mathrm{c}}_{\mathrm{R}}(v_{\mathrm{d}}y_e)e_{\mathrm{L}} + \text{h.c.} \tag{11.72}$$

$$= u^{\mathrm{c}}_{\mathrm{R}}(v_{\mathrm{u}}U_u y_u^{\mathrm{diag}}V_u^\dagger)u_{\mathrm{L}} + d^{\mathrm{c}}_{\mathrm{R}}(v_{\mathrm{d}}U_d y_d^{\mathrm{diag}}V_d^\dagger)d_{\mathrm{L}} + e^{\mathrm{c}}_{\mathrm{R}}(v_{\mathrm{d}}U_e y_e^{\mathrm{diag}}V_e^\dagger)e_{\mathrm{L}} + \text{h.c.}, \tag{11.73}$$

so the super-CKM basis is given by

$$\left[Q^1, Q^2, L, U^{\mathrm{c}}, D^{\mathrm{c}}, E^{\mathrm{c}}\right]_{\text{super-CKM}} = \left[V_u^\dagger Q^1, V_d^\dagger Q^2, V_e^\dagger L, U^{\mathrm{c}}U_u, D^{\mathrm{c}}U_d, E^{\mathrm{c}}U_e\right]_{\text{“original”}}. \tag{11.74}$$

Then the CKM mixings appear as, for example,

$$\left[\bar{u}_{\mathrm{L}}\bar{\sigma}^\mu d_{\mathrm{L}}W^+_\mu + \bar{d}_{\mathrm{L}}\bar{\sigma}^\mu u_{\mathrm{L}}W^-_\mu\right]_{\text{“original”}} = \left[\bar{u}_{\mathrm{L}}V_u^\dagger V_d\bar{\sigma}^\mu d_{\mathrm{L}}W^+_\mu + \bar{d}_{\mathrm{L}}V_d^\dagger V_u\bar{\sigma}^\mu u_{\mathrm{L}}W^-_\mu\right]_{\text{super-CKM}}; \tag{11.75}$$

i.e., defining $V_{\mathrm{CKM}} = V_u^\dagger V_d$ as in Sec. 7.5, the Lagrangian is amended as, e.g., $\bar{u}_{\mathrm{L}}d_{\mathrm{L}} \to \bar{u}_{\mathrm{L}}V_{\mathrm{CKM}}d_{\mathrm{L}}$, $\tilde{d}^*_{\mathrm{L}}\tilde{u}_{\mathrm{L}} \to \tilde{d}^*_{\mathrm{L}}V^\dagger_{\mathrm{CKM}}\tilde{u}_{\mathrm{L}}$.

[*20] The sign of φ_i is arbitrary and (should be) unphysical.

Squark masses in super-CKM basis Finally, the squark masses are given by

$$
\begin{aligned}
-\mathcal{L} \supset{}& \tilde{u}_{\mathrm{L}}^* \left(m_Q^2 + m_u^2 + \frac{3-4\mathrm{s}_{\mathrm{w}}^2}{6}\mathrm{c}_{2\beta}m_Z^2\right)\tilde{u}_{\mathrm{L}} + \tilde{u}_{\mathrm{R}}^* \left(m_{U^c}^2 + m_u^2 + \frac{4\mathrm{s}_{\mathrm{w}}^2}{6}\mathrm{c}_{2\beta}m_Z^2\right)\tilde{u}_{\mathrm{R}}\\
&+ \tilde{u}_{\mathrm{R}}^*(v_{\mathrm{u}}a_{\mathrm{u}} - \mu^* m_u \cot\beta)\tilde{u}_{\mathrm{L}} + \tilde{u}_{\mathrm{L}}^*(v_{\mathrm{u}}a_{\mathrm{u}}^\dagger - \mu m_u\cot\beta)\tilde{u}_{\mathrm{R}}\\
&+ \tilde{d}_{\mathrm{L}}^* \left(V_d^\dagger(V_u m_Q^2 V_u^\dagger)V_d + m_d^2 + \frac{-3+2\mathrm{s}_{\mathrm{w}}^2}{6}\mathrm{c}_{2\beta}m_Z^2\right)\tilde{d}_{\mathrm{L}} + \tilde{d}_{\mathrm{R}}^* \left(m_{D^c}^2 + m_d^2 + \frac{-2\mathrm{s}_{\mathrm{w}}^2}{6}\mathrm{c}_{2\beta}m_Z^2\right)\tilde{d}_{\mathrm{R}}\\
&+ \tilde{d}_{\mathrm{R}}^*(v_{\mathrm{d}}a_d - \mu^* m_d\tan\beta)\tilde{d}_{\mathrm{L}} + \tilde{d}_{\mathrm{L}}^*(v_{\mathrm{d}}a_d^\dagger - \mu m_d\tan\beta)\tilde{d}_{\mathrm{R}}\\
&+ \tilde{\nu}_{\mathrm{L}}^*\left(m_L^2 + \frac{1}{2}\mathrm{c}_{2\beta}m_Z^2\right)\tilde{\nu}_{\mathrm{L}}\\
&+ \tilde{e}_{\mathrm{L}}^*\left(m_L^2 + m_e^2 + \frac{-1+2\mathrm{s}_{\mathrm{w}}^2}{2}\mathrm{c}_{2\beta}m_Z^2\right) + \tilde{e}_{\mathrm{R}}^*\left(m_{E^c}^2 + m_e^2 + (-\mathrm{s}_{\mathrm{w}}^2)\mathrm{c}_{2\beta}m_Z^2\right)\tilde{e}_{\mathrm{R}}\\
&+ \tilde{e}_{\mathrm{R}}^*(v_{\mathrm{d}}a_e - \mu^* m_e\tan\beta)\tilde{e}_{\mathrm{L}} + \tilde{e}_{\mathrm{L}}^*(v_{\mathrm{d}}a_e^\dagger - \mu m_e\tan\beta)\tilde{e}_{\mathrm{R}},
\end{aligned}
\tag{11.76}
$$

where the sfermion soft masses, yukawas, and a-terms are rewritten in super-CKM basis:

$$[m_Q^2, m_{U^c}^2, m_{D^c}^2, m_L^2, m_{E^c}^2]_{\text{super-CKM}} = [V_d^\dagger m_Q^2 V_d, U_u^\dagger m_{U^c}^2 U_u, U_d^\dagger m_{D^c}^2 U_d, V_e^\dagger m_L^2 V_e, U_e^\dagger m_{E^c}^2 U_e]_{\text{“original”}}, \tag{11.77}$$

$$[a_u, a_d, a_e]_{\text{super-CKM}} = [U_u^\dagger a_u V_u, U_d^\dagger a_d V_d, U_e^\dagger a_e V_e]_{\text{“original”}} \tag{11.78}$$

(note that m_Q^2 is diagonalied for down-type; not for up-type). In matrix form,

$$
\begin{aligned}
-\mathcal{L} \supset{}& \begin{pmatrix}\tilde{u}_{\mathrm{L}i}^*\\ \tilde{u}_{\mathrm{R}i}^*\end{pmatrix}^{\mathrm{T}} \begin{pmatrix} [V_{\mathrm{CKM}}m_Q^2V_{\mathrm{CKM}}^\dagger]_{ij} + \left(m_u^2 + \frac{3-4\mathrm{s}_{\mathrm{w}}^2}{6}\mathrm{c}_{2\beta}m_Z^2\right)\delta_{ij} & v_u[a_u^\dagger]_{ij} - (\mu m_u\cot\beta)\delta_{ij}\\ v_u[a_u]_{ij} - (\mu^* m_u\cot\beta)\delta_{ij} & [m_{U^c}^2]_{ij} + \left(m_u^2 + \frac{2\mathrm{s}_{\mathrm{w}}^2}{3}\mathrm{c}_{2\beta}m_Z^2\right)\delta_{ij}\end{pmatrix}\begin{pmatrix}\tilde{u}_{\mathrm{L}j}\\ \tilde{u}_{\mathrm{R}j}\end{pmatrix}\\
&+ \begin{pmatrix}\tilde{d}_{\mathrm{L}i}^*\\ \tilde{d}_{\mathrm{R}i}^*\end{pmatrix}^{\mathrm{T}} \begin{pmatrix} [m_Q^2]_{ij} + \left(m_d^2 + \frac{-3+2\mathrm{s}_{\mathrm{w}}^2}{6}\mathrm{c}_{2\beta}m_Z^2\right)\delta_{ij} & v_d[a_d^\dagger]_{ij} - (\mu m_d\tan\beta)\delta_{ij}\\ v_d[a_d]_{ij} - (\mu^* m_d\tan\beta)\delta_{ij} & [m_{D^c}^2]_{ij} + \left(m_d^2 - \frac{\mathrm{s}_{\mathrm{w}}^2}{3}\mathrm{c}_{2\beta}m_Z^2\right)\delta_{ij}\end{pmatrix}\begin{pmatrix}\tilde{d}_{\mathrm{L}j}\\ \tilde{d}_{\mathrm{R}j}\end{pmatrix}\\
&+ \tilde{\nu}_{\mathrm{L}i}^*\Big([m_L^2]_{ij} + \left(\tfrac{1}{2}\mathrm{c}_{2\beta}m_Z^2\right)\delta_{ij}\Big)\tilde{\nu}_{\mathrm{L}j}\\
&+ \begin{pmatrix}\tilde{e}_{\mathrm{L}i}^*\\ \tilde{e}_{\mathrm{R}i}^*\end{pmatrix}^{\mathrm{T}} \begin{pmatrix} [m_L^2]_{ij} + \left(m_e^2 + \frac{-1+2\mathrm{s}_{\mathrm{w}}^2}{2}\mathrm{c}_{2\beta}m_Z^2\right)\delta_{ij} & v_d[a_e^\dagger]_{ij} - (\mu m_e\tan\beta)\delta_{ij}\\ v_d[a_e]_{ij} - (\mu^* m_e\tan\beta)\delta_{ij} & [m_{E^c}^2]_{ij} + \left(m_e^2 - \mathrm{s}_{\mathrm{w}}^2\mathrm{c}_{2\beta}m_Z^2\right)\delta_{ij}\end{pmatrix}\begin{pmatrix}\tilde{e}_{\mathrm{L}j}\\ \tilde{e}_{\mathrm{R}j}\end{pmatrix}
\end{aligned}
\tag{11.79}
$$

$$= \begin{pmatrix}\tilde{u}_{\mathrm{L}i}^*\\ \tilde{u}_{\mathrm{R}i}^*\end{pmatrix}^{\mathrm{T}}\mathcal{M}_u\begin{pmatrix}\tilde{u}_{\mathrm{L}j}\\ \tilde{u}_{\mathrm{R}j}\end{pmatrix} + \begin{pmatrix}\tilde{d}_{\mathrm{L}i}^* & \tilde{d}_{\mathrm{R}i}^*\end{pmatrix}^{\mathrm{T}}\mathcal{M}_d\begin{pmatrix}\tilde{d}_{\mathrm{L}j}\\ \tilde{d}_{\mathrm{R}j}\end{pmatrix} + \tilde{\nu}_{\mathrm{L}}^*\mathcal{M}_\nu\tilde{\nu}_{\mathrm{L}} + \left(\tilde{e}_{\mathrm{L}i}^*\tilde{e}_{\mathrm{R}i}^*\right)^{\mathrm{T}}\mathcal{M}_e\begin{pmatrix}\tilde{e}_{\mathrm{L}j}\\ \tilde{e}_{\mathrm{R}j}\end{pmatrix} \tag{11.80}$$

The sfermion mass matrices are diagonalized by unitary matrices as

$$\mathcal{M} = R\mathcal{M}^{\mathrm{diag}}R^\dagger;\quad \tilde{f}_i = R_{ij}\begin{pmatrix}\tilde{f}_{\mathrm{L}}\\ \tilde{f}_{\mathrm{R}}\end{pmatrix}_j = \begin{pmatrix}R_{ij}^{\mathrm{L}} & R_{ij}^{\mathrm{R}}\end{pmatrix}\begin{pmatrix}\tilde{f}_{\mathrm{L}j}\\ \tilde{f}_{\mathrm{R}j}\end{pmatrix};\qquad \tilde{f}_{\mathrm{L}i} = [R^{\mathrm{L}\dagger}]_{ij}\tilde{f}_i,\quad \tilde{f}_{\mathrm{R}i} = [R^{\mathrm{R}\dagger}]_{ij}\tilde{f}_i. \tag{11.81}$$

where R_{ij} is 6×6 and $R_{ij}^{\mathrm{L,R}}$ are 3×6 matrices (except for sneutrinos).

These R-matrices are the same as DSQMIX etc. of SLHA2 format, but note that our notation for the other parameters is slightly different from SLHA's:

$$m_{\tilde{Q},\tilde{L}}^2 = m_{Q,L}^2|_{\text{“orig”}},\qquad m_{\tilde{u},\tilde{d},\tilde{e}}^2 = (m_{U^c,D^c,E^c}^2|_{\text{“orig”}})^{\mathrm{T}},\qquad T_{U,D,E} = (a_{u,d,e}|_{\text{“orig”}})^{\mathrm{T}}, \tag{11.82}$$

$$\hat{m}_{\tilde{Q},\tilde{L}}^2 = m_{Q,L}^2|_{\mathrm{sCKM}},\qquad \hat{m}_{\tilde{u},\tilde{d},\tilde{e}}^2 = U^\dagger T^{\mathrm{T}}V = m_{U^c,D^c,E^c}^2|_{\mathrm{sCKM}},\qquad \hat{T}_{U,D,E} = U^\dagger T^{\mathrm{T}}V = a_{u,d,e}|_{\mathrm{sCKM}}, \tag{11.83}$$

together with $Y_{u,d,e} = (y_{u,d,e})^{\mathrm{T}}$. Anyway, the SLHA2 blocks corresponds to the variable in our convention as

$$\texttt{(IM)MSX2(IN)} = m_{Q,L,U^c,D^c,E^c}^2\big|_{\mathrm{sCKM};\overline{\mathrm{DR}}},\quad \texttt{(IM)TX(IN)} = a_{u,d,e}\big|_{\mathrm{sCKM};\overline{\mathrm{DR}}},\quad \texttt{YX} = y_{u,d,e}\big|_{\mathrm{sCKM};\overline{\mathrm{DR}}}. \tag{11.84}$$

The sfermion mass matrices above are in super-CKM basis, so their off-diagonal entries immediately induce flavor violation or sfermion left-right mixing. In old SLHA format, we assume that flavor- and CP-violation is absent and left-right mixing is ignorable except for third generation, which leads

$$\mathcal{M}_d = \begin{pmatrix} \tilde{m}_{dL11}^2 &&&&& \\ & \tilde{m}_{dL22}^2 &&&& \\ && \tilde{m}_{dL33}^2 &&& v_d a_{d33} - \mu m_b\tan\beta \\ &&& \tilde{m}_{dR11}^2 && \\ &&&& \tilde{m}_{dR22}^2 & \\ && v_d a_{d33} - \mu m_b\tan\beta &&& \tilde{m}_{dR33}^2 \end{pmatrix};\quad \begin{pmatrix}\tilde{d}_{\mathrm{L}}\\ \tilde{s}_{\mathrm{L}}\\ \tilde{b}_1\\ \tilde{d}_{\mathrm{R}}\\ \tilde{s}_{\mathrm{R}}\\ \tilde{b}_2\end{pmatrix} = \begin{pmatrix} 1 &&&&& \\ & 1 &&&& \\ && F_{11} &&& F_{12} \\ &&& 1 && \\ &&&& 1 & \\ && F_{21} &&& F_{22}\end{pmatrix}\begin{pmatrix}\tilde{d}_{\mathrm{L}j}\\ \tilde{d}_{\mathrm{R}j}\end{pmatrix} \tag{11.85}$$

and these F_{ij} are stored in SBOTMIX etc.

Squark masses in super-CKM basis The gluino mass and phase is given by

$$m_{\tilde{g}} = |M_3|;\quad M_3 = m_{\tilde{g}}\,\mathrm{e}^{\mathrm{i}\theta_3} = m_{\tilde{g}}\varphi_{\tilde{g}}^{-2};\quad \tilde{g}_0 = \varphi_{\tilde{g}}\tilde{g}. \tag{11.86}$$

11.4.2. Fermion composition

Now we show the fermion-related Lagrangian terms verbosely. In super-CKM basis, the interaction terms are

$$
\begin{aligned}
&\mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{SFG}} = \mathrm{i}\bar{\tilde\chi}^0_i\bar\sigma^\mu\partial_\mu\tilde\chi^0_i + \mathrm{i}\bar{\tilde\chi}^-_i\bar\sigma^\mu\partial_\mu\tilde\chi^-_i + \mathrm{i}\bar{\tilde\chi}^+_i\bar\sigma^\mu\partial_\mu\tilde\chi^+_i + \mathrm{i}\bar{\tilde g}^a\bar\sigma^\mu\partial_\mu\tilde g^a + \mathrm{i}\bar u_{\mathrm{L}i}\bar\sigma^\mu\partial_\mu u_{\mathrm{L}i} + \mathrm{i}\bar u^{\mathrm{c}}_{\mathrm{R}i}\bar\sigma^\mu\partial_\mu u^{\mathrm{c}}_{\mathrm{R}i}\\
&\quad + \mathrm{i}\bar d_{\mathrm{L}i}\bar\sigma^\mu\partial_\mu d_{\mathrm{L}i} + \mathrm{i}\bar d^{\mathrm{c}}_{\mathrm{R}i}\bar\sigma^\mu\partial_\mu d^{\mathrm{c}}_{\mathrm{R}i} + \mathrm{i}\bar\nu_{\mathrm{L}i}\bar\sigma^\mu\partial_\mu\nu_{\mathrm{L}i} + \mathrm{i}\bar e_{\mathrm{L}i}\bar\sigma^\mu\partial_\mu e_{\mathrm{L}i} + \mathrm{i}\bar e^{\mathrm{c}}_{\mathrm{R}i}\bar\sigma^\mu\partial_\mu e^{\mathrm{c}}_{\mathrm{R}i} - \frac{m_{\tilde g}}{2}(\tilde g^a\tilde g^a + \text{h.c.})\\
&\quad - m_{\tilde\chi^\pm_i}\left(\tilde\chi^-_i\tilde\chi^+_i + \bar{\tilde\chi}^-_i\bar{\tilde\chi}^+_i\right) - m_{di}\left(d^{\mathrm{c}}_{\mathrm{R}i}d_{\mathrm{L}i} + \bar d^{\mathrm{c}}_{\mathrm{R}i}\bar d^x_{\mathrm{L}i}\right) - m_{ui}\left(u^{\mathrm{c}}_{\mathrm{R}i}u_{\mathrm{L}j} + \bar u^{\mathrm{c}}_{\mathrm{R}i}\bar u_{\mathrm{L}i}\right) - m_{ei}\left(e^{\mathrm{c}}_{\mathrm{R}i}e_{\mathrm{L}i} + \bar e^{\mathrm{c}}_{\mathrm{R}i}\bar e_{\mathrm{L}i}\right)\\
&\quad + \Big[\sqrt2 g_3\varphi_{\tilde g}\tilde g^a\left(\tau^{yxa*}\tilde d^y_{\mathrm{R}}d^{\mathrm{c}x}_{\mathrm{R}} + \tau^{yxa*}\tilde u^y_{\mathrm{R}}u^{\mathrm{c}x}_{\mathrm{R}} - \tau^{yxa}\tilde d^{y*}_{\mathrm{L}}d^x_{\mathrm{L}} - \tau^{yxa}\tilde u^{y*}_{\mathrm{L}}u^x_{\mathrm{L}}\right) + \text{h.c.}\Big] - \mathrm{i}g_3f^{abc}g^a_\mu(\bar{\tilde g}^b\bar\sigma^\mu\tilde g^c)\\
&\quad + g_3g^a_\mu\left(\bar d_{\mathrm{L}}\bar\sigma^\mu\tau^a d_{\mathrm{L}} + \bar u_{\mathrm{L}}\bar\sigma^\mu\tau^a u_{\mathrm{L}} - \bar d^{\mathrm{c}}_{\mathrm{R}}\bar\sigma^\mu\tau^{a*}d^{\mathrm{c}}_{\mathrm{R}} - \bar u^{\mathrm{c}}_{\mathrm{R}}\bar\sigma^\mu\tau^{a*}u^{\mathrm{c}}_{\mathrm{R}}\right) + A_\mu|e|\Big[\frac23\bar u_{\mathrm{L}}\bar\sigma^\mu u_{\mathrm{L}} - \frac23\bar u^{\mathrm{c}}_{\mathrm{R}}\bar\sigma^\mu u^{\mathrm{c}}_{\mathrm{R}} - \frac13\bar d_{\mathrm{L}}\bar\sigma^\mu d_{\mathrm{L}}\\
&\quad + \frac13\bar d^{\mathrm{c}}_{\mathrm{R}}\bar\sigma^\mu d^{\mathrm{c}}_{\mathrm{R}} - \bar e_{\mathrm{L}}\bar\sigma^\mu e_{\mathrm{L}} + \bar e^{\mathrm{c}}_{\mathrm{R}}\bar\sigma^\mu e^{\mathrm{c}}_{\mathrm{R}} + \bar{\tilde\chi}^+_i\bar\sigma^\mu\tilde\chi^+_i - \bar{\tilde\chi}^-_i\bar\sigma^\mu\tilde\chi^-_i\Big] + g_ZZ_\mu\Big[\frac{3-4\mathrm{s}^2_{\mathrm{w}}}{6}\bar u_{\mathrm{L}}\bar\sigma^\mu u_{\mathrm{L}} + \frac{2\mathrm{s}^2_{\mathrm{w}}}{3}\bar u^{\mathrm{c}}_{\mathrm{R}}\bar\sigma^\mu u^{\mathrm{c}}_{\mathrm{R}}\\
&\quad - \frac{3-2\mathrm{s}^2_{\mathrm{w}}}{6}\bar d_{\mathrm{L}}\bar\sigma^\mu d_{\mathrm{L}} - \frac{\mathrm{s}^2_{\mathrm{w}}}{3}\bar d^{\mathrm{c}}_{\mathrm{R}}\bar\sigma^\mu d^{\mathrm{c}}_{\mathrm{R}} + \frac12\bar\nu_{\mathrm{L}}\bar\sigma^\mu\nu_{\mathrm{L}} + \frac{2\mathrm{s}^2_{\mathrm{w}}-1}{2}\bar e_{\mathrm{L}}\bar\sigma^\mu e_{\mathrm{L}} - \mathrm{s}^2_{\mathrm{w}}\bar e^{\mathrm{c}}_{\mathrm{R}}\bar\sigma^\mu e^{\mathrm{c}}_{\mathrm{R}} + \frac{N_{j3}N^*_{i3} - N_{j4}N^*_{i4}}{2}\bar{\tilde\chi}^0_j\bar\sigma^\mu\tilde\chi^0_i\\
&\quad + \left(\mathrm{c}^2_{\mathrm{w}}V_{j1}V^*_{i1} + \frac{V_{j2}V^*_{i2}(\mathrm{c}^2_{\mathrm{w}} - \mathrm{s}^2_{\mathrm{w}})}{2}\right)\bar{\tilde\chi}^+_j\bar\sigma^\mu\tilde\chi^+_i + \left(\frac{U_{j2}U^*_{i2}(\mathrm{s}^2_{\mathrm{w}} - \mathrm{c}^2_{\mathrm{w}})}{2} - \mathrm{c}^2_{\mathrm{w}}U_{j1}U^*_{i1}\right)\bar{\tilde\chi}^-_j\bar\sigma^\mu\tilde\chi^-_i\Big]\\
&\quad + \frac{g_2}{\sqrt2}W^+_\mu\Big[\bar u_{\mathrm{L}}\bar\sigma^\mu V_{\mathrm{CKM}}d_{\mathrm{L}} + \bar\nu_{\mathrm{L}}\bar\sigma^\mu e_{\mathrm{L}} + \left(V_{j2}N^*_{i4} - \sqrt2V_{j1}N^*_{i2}\right)\bar{\tilde\chi}^+_j\bar\sigma^\mu\tilde\chi^0_i + \left(\sqrt2N_{j2}U^*_{i1} + N_{j3}U^*_{i2}\right)\bar{\tilde\chi}^0_j\bar\sigma^\mu\tilde\chi^-_i\Big]\\
&\quad + \frac{g_2}{\sqrt2}W^-_\mu\Big[\bar d_{\mathrm{L}}\bar\sigma^\mu V^\dagger_{\mathrm{CKM}}u_{\mathrm{L}} + \bar e_{\mathrm{L}}\bar\sigma^\mu\nu_{\mathrm{L}} + \left(N_{j4}V^*_{i2} - \sqrt2N_{j2}V^*_{i1}\right)\bar{\tilde\chi}^0_j\bar\sigma^\mu\tilde\chi^+_i + \left(\sqrt2U_{j1}N^*_{i2} + U_{j2}N^*_{i3}\right)\bar{\tilde\chi}^-_j\bar\sigma^\mu\tilde\chi^0_i\Big]\\
&\quad + \phi_{\mathrm{u}}\left(-\frac{y_{ui}}{\sqrt2}u^{\mathrm{c}}_{\mathrm{R}i}u_{\mathrm{L}i} + \frac{N^*_{i4}(g_2N^*_{j2} - g_YN^*_{j1})}{2}\tilde\chi^0_i\tilde\chi^0_j - \frac{g_2U^*_{i1}V^*_{j2}}{\sqrt2}\tilde\chi^-_i\tilde\chi^+_j + \text{h.c.}\right)\\
&\quad + \phi_{\mathrm{d}}\left(-\frac{y_{di}}{\sqrt2}d^{\mathrm{c}}_{\mathrm{R}i}d_{\mathrm{L}i} - \frac{y_{ei}}{\sqrt2}e^{\mathrm{c}}_{\mathrm{R}i}e_{\mathrm{L}i} + \frac{N^*_{i3}(g_YN^*_{j1} - g_2N^*_{j2})}{2}\tilde\chi^0_i\tilde\chi^0_j - \frac{g_2U^*_{i2}V^*_{j1}}{\sqrt2}\tilde\chi^-_i\tilde\chi^+_j + \text{h.c.}\right)\\
&\quad + \frac{A^0}{\sqrt2}\Big[-\mathrm{i}\mathrm{c}_{\beta0}y_{ui}u^{\mathrm{c}}_{\mathrm{R}i}u_{\mathrm{L}i} - \mathrm{i}\mathrm{s}_{\beta0}y_{di}d^{\mathrm{c}}_{\mathrm{R}i}d_{\mathrm{L}i} - \mathrm{i}\mathrm{s}_{\beta0}y_{ei}e^{\mathrm{c}}_{\mathrm{R}i}e_{\mathrm{L}i} - \frac{\mathrm{i}(\mathrm{s}_{\beta0}N^*_{i3} - \mathrm{c}_{\beta0}N^*_{i4})(g_YN^*_{j1} - g_2N^*_{j2})}{\sqrt2}\tilde\chi^0_i\tilde\chi^0_j\\
&\quad + \mathrm{i}g_2\left(\mathrm{c}_{\beta0}U^*_{i1}V^*_{j2} + \mathrm{s}_{\beta0}U^*_{i2}V^*_{j1}\right)\tilde\chi^-_i\tilde\chi^+_j + \text{h.c.}\Big] + \frac{G^0}{\sqrt2}\Big[-\mathrm{i}\mathrm{s}_{\beta0}y_{ui}u^{\mathrm{c}}_{\mathrm{R}i}u_{\mathrm{L}i} + \mathrm{i}\mathrm{c}_{\beta0}y_{di}d^{\mathrm{c}}_{\mathrm{R}i}d_{\mathrm{L}i} + \mathrm{i}\mathrm{c}_{\beta0}y_{ei}e^{\mathrm{c}}_{\mathrm{R}i}e_{\mathrm{L}i}\\
&\quad + \frac{\mathrm{i}(g_YN^*_{j1} - g_2N^*_{j2})(\mathrm{c}_{\beta0}N^*_{i3} + \mathrm{s}_{\beta0}N^*_{i4})}{\sqrt2}\tilde\chi^0_i\tilde\chi^0_j - \mathrm{i}g_2(\mathrm{c}_{\beta0}U^*_{i2}V^*_{j1} - \mathrm{s}_{\beta0}U^*_{i1}V^*_{j2})\tilde\chi^-_i\tilde\chi^+_j + \text{h.c.}\Big]\\
&\quad + \Big[\left(\frac{U^*_{i2}(g_YN^*_{j1} + g_2N^*_{j2})}{\sqrt2} - g_2U^*_{i1}N^*_{j3}\right)\mathrm{s}_{\beta+}H^+\tilde\chi^-_i\tilde\chi^0_j - \left(g_2V^*_{i1}N^*_{j4} + \frac{V^*_{i2}\left(g_YN^*_{j1} + g_2N^*_{j2}\right)}{\sqrt2}\right)\mathrm{c}_{\beta+}H^-\tilde\chi^+_i\tilde\chi^0_j\\
&\quad + \mathrm{c}_{\beta+}(u^{\mathrm{c}}_{\mathrm{R}}y_uV_{\mathrm{CKM}}d_{\mathrm{L}})H^+ + \mathrm{s}_{\beta+}(d^{\mathrm{c}}_{\mathrm{R}}y_dV^\dagger_{\mathrm{CKM}}u_{\mathrm{L}})H^- + \mathrm{s}_{\beta+}(e^{\mathrm{c}}_{\mathrm{R}}y_e\nu_{\mathrm{L}})H^- + \text{h.c.}\Big]\\
&\quad + \Big[\mathrm{i}G^+\mathrm{c}_{\beta+}\left(\frac{U^*_{i2}(g_YN^*_{j1} + g_2N^*_{j2})}{\sqrt2} - g_2U^*_{i1}N^*_{j3}\right)\tilde\chi^-_i\tilde\chi^0_j - \mathrm{i}G^-\mathrm{s}_{\beta+}\left(g_2V^*_{i1}N^*_{j4} + \frac{V^*_{i2}(g_YN^*_{j1} + g_2N^*_{j2})}{\sqrt2}\right)\tilde\chi^+_i\tilde\chi^0_j\\
&\quad - \mathrm{i}\mathrm{s}_{\beta+}(u^{\mathrm{c}}_{\mathrm{R}}y_uV_{\mathrm{CKM}}d_{\mathrm{L}})G^+ - \mathrm{i}\mathrm{c}_{\beta+}(d^{\mathrm{c}}_{\mathrm{R}}y_dV^\dagger_{\mathrm{CKM}}u_{\mathrm{L}})G^- - \mathrm{i}\mathrm{c}_{\beta+}(e^{\mathrm{c}}_{\mathrm{R}}y_e\nu_{\mathrm{L}})G^- + \text{h.c.}\Big]\\
&\quad + \Big[-\frac{\tilde u^*_{\mathrm{L}}u_{\mathrm{L}}\tilde\chi^0_j\left(g_YN^*_{j1} + 3g_2N^*_{j2}\right)}{3\sqrt2} + \frac23\sqrt2g_Y\tilde u_{\mathrm{R}}u^{\mathrm{c}}_{\mathrm{R}}\tilde\chi^0_jN^*_{j1} - y_{ui}u^{\mathrm{c}}_{\mathrm{R}i}\tilde u_{\mathrm{L}k}\tilde\chi^0_jN^*_{j4} - y_{ui}\tilde u^*_{\mathrm{R}i}u_{\mathrm{L}i}\tilde\chi^0_jN^*_{j4}\\
&\quad - \frac{\tilde d^*_{\mathrm{L}i}d_{\mathrm{L}i}\tilde\chi^0_j\left(g_YN^*_{j1} - 3g_2N^*_{j2}\right)}{3\sqrt2} - \frac13\sqrt2g_Y\tilde d_{\mathrm{R}i}d^{\mathrm{c}}_{\mathrm{R}i}\tilde\chi^0_jN^*_{j1} - y_{di}\tilde d_{\mathrm{L}i}d^{\mathrm{c}}_{\mathrm{R}i}\tilde\chi^0_jN^*_{j3} - y_{di}\tilde d^*_{\mathrm{R}i}d_{\mathrm{L}i}\tilde\chi^0_jN^*_{j3}\\
&\quad + \frac{\tilde\nu^*_{\mathrm{L}i}\nu_{\mathrm{L}i}\tilde\chi^0_j\left(g_YN^*_{j1} - g_2N^*_{j2}\right)}{\sqrt2} - \sqrt2g_Y\tilde e_{\mathrm{R}i}e^{\mathrm{c}}_{\mathrm{R}i}\tilde\chi^0_jN^*_{j1} + \frac{\tilde e^*_{\mathrm{L}i}e_{\mathrm{L}i}\tilde\chi^0_j\left(g_YN^*_{j1} + g_2N^*_{j2}\right)}{\sqrt2}\\
&\quad - y_{ei}\tilde e_{\mathrm{L}i}e^{\mathrm{c}}_{\mathrm{R}i}\tilde\chi^0_jN^*_{j3} - y_{ei}\tilde e^*_{\mathrm{R}i}e_{\mathrm{L}i}\tilde\chi^0_jN^*_{j3} + \text{h.c.}\Big] + \Big[V^*_{i2}(u^{\mathrm{c}}_{\mathrm{R}}y_uV_{\mathrm{CKM}}\tilde d_{\mathrm{L}})\tilde\chi^+_i + V^*_{i2}(\tilde u^*_{\mathrm{R}}y_uV_{\mathrm{CKM}}d_{\mathrm{L}})\tilde\chi^+_i\\
&\quad + U^*_{j2}(d^{\mathrm{c}}_{\mathrm{R}}y_dV^\dagger_{\mathrm{CKM}}\tilde u_{\mathrm{L}})\tilde\chi^-_j + U^*_{j2}(\tilde d^*_{\mathrm{R}}y_dV^\dagger_{\mathrm{CKM}}u_{\mathrm{L}})\tilde\chi^-_j - U^*_{j1}g_2(\tilde d^*_{\mathrm{L}}V^\dagger_{\mathrm{CKM}}u_{\mathrm{L}})\tilde\chi^-_j - V^*_{i1}g_2(\tilde u^*_{\mathrm{L}}V_{\mathrm{CKM}}d_{\mathrm{L}})\tilde\chi^+_i\\
&\quad - V^*_{i1}g_2\tilde\nu^*_{\mathrm{L}}e_{\mathrm{L}}\tilde\chi^+_i - U^*_{j1}g_2\tilde e^*_{\mathrm{L}}\nu_{\mathrm{L}}\tilde\chi^-_j + U^*_{j2}y_{ei}\tilde\nu_{\mathrm{L}i}e^{\mathrm{c}}_{\mathrm{R}i}\tilde\chi^-_j + U^*_{j2}y_{ei}\tilde e^*_{\mathrm{R}i}\nu_{\mathrm{L}i}\tilde\chi^-_j + \text{h.c.}\Big].\\
&\quad - \epsilon^{xyz}\left(\lambda''_{ijk}\tilde d^{x*}_{\mathrm{R}k}u^{\mathrm{c}y}_{\mathrm{R}i}d^{\mathrm{c}z}_{\mathrm{R}j} + \frac12\lambda''_{kij}\tilde u^{x*}_{\mathrm{R}k}d^{\mathrm{c}y}_{\mathrm{R}i}d^{\mathrm{c}z}_{\mathrm{R}j}\right) + \lambda_{ijk}\left(-\tilde e^*_{\mathrm{R}k}\nu_{\mathrm{L}i}e_{\mathrm{L}j} - \tilde e_{\mathrm{L}j}e^{\mathrm{c}}_{\mathrm{R}k}\nu_{\mathrm{L}i} + \tilde\nu_{\mathrm{L}j}e^{\mathrm{c}}_{\mathrm{R}k}e_{\mathrm{L}i}\right)\\
&\quad + \lambda'_{ijk}\left(-\tilde d^*_{\mathrm{R}k}d_{\mathrm{L}j}\nu_{\mathrm{L}i} + \tilde d^*_{\mathrm{R}k}u_{\mathrm{L}j}e_{\mathrm{L}i} - \tilde d_{\mathrm{L}j}d^{\mathrm{c}}_{\mathrm{R}k}\nu_{\mathrm{L}i} + \tilde u_{\mathrm{L}j}d^{\mathrm{c}}_{\mathrm{R}k}e_{\mathrm{L}i} + \tilde e_{\mathrm{L}i}d^{\mathrm{c}}_{\mathrm{R}k}u_{\mathrm{L}j} - \tilde\nu_{\mathrm{L}i}d^{\mathrm{c}}_{\mathrm{R}k}d_{\mathrm{L}j}\right) + \text{h.c.},
\end{aligned}
\tag{11.87}
$$

where R-parity violating terms are also shown as a reference; they are also redefined in super-CKM basis.

The full Lagrangian is given then by

$$\mathcal{L} = (\mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{SFG}}) + \mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{scalar}}, \tag{11.88}$$

where $\mathcal{L}_{\text{vector}}$ is given by Eq. (11.38), which should be amended with $m_W^2 W^{+\mu} W^-_\mu + (m_Z^2/2) Z^\mu Z_\mu$, mass and kinetic terms of $\mathcal{L}_{\text{scalar}}$ should be read from this section, and the interaction terms of $\mathcal{L}_{\text{scalar}}$ should be read from Eq. (11.43) and Eq. (11.44), replacing Higgs fields by VEVs, Higgs bosons, and Nambu-Goldstone bosons.

These Weyl fermions are combined to Dirac and Majorana fermions:

$$\tilde{\chi}^0_i = \begin{pmatrix} \tilde{\chi}^0_i \\ \bar{\tilde{\chi}}^0_i \end{pmatrix}, \quad \tilde{\chi}^+_i = \begin{pmatrix} \tilde{\chi}^+_i \\ \bar{\tilde{\chi}}^-_i \end{pmatrix}, \quad f = \begin{pmatrix} f_{\mathrm{L}} \\ \bar{f}^{\mathrm{c}}_{\mathrm{R}} \end{pmatrix}; \quad \overline{\tilde{\chi}^0_i} = \begin{pmatrix} \tilde{\chi}^0_i & \bar{\tilde{\chi}}^0_i \end{pmatrix}, \quad \overline{\tilde{\chi}^+_i} = \begin{pmatrix} \tilde{\chi}^-_i & \bar{\tilde{\chi}}^+_i \end{pmatrix}, \quad \overline{f} = \begin{pmatrix} f^{\mathrm{c}}_{\mathrm{R}} & \bar{f}_{\mathrm{L}} \end{pmatrix}. \tag{11.89}$$

For example,

$$\overline{u} P_{\mathrm{L}} d = u^{\mathrm{c}}_{\mathrm{R}} d_{\mathrm{L}}, \quad \overline{u} P_{\mathrm{R}} d = \bar{u}_{\mathrm{L}} \bar{d}^{\mathrm{c}}_{\mathrm{R}}, \quad u^{\mathrm{c}}_{\mathrm{R}} d_{\mathrm{L}} + \text{h.c.} = \overline{u} P_{\mathrm{L}} d + \overline{d} P_{\mathrm{R}} u, \quad \overline{u}\gamma^\mu P_{\mathrm{L}} d = \bar{u}_{\mathrm{L}} \bar{\sigma}^\mu d_{\mathrm{L}}, \quad \overline{u}\gamma^\mu P_{\mathrm{R}} d = -\bar{d}^{\mathrm{c}}_{\mathrm{R}} \bar{\sigma}^\mu u^{\mathrm{c}}_{\mathrm{R}};$$

$$\tilde{\chi}^0 u_{\mathrm{L}} + \text{h.c.} = \overline{\tilde{\chi}^0} P_{\mathrm{L}} u + \overline{u} P_{\mathrm{R}} \tilde{\chi}^0, \quad \tilde{\chi}^0 u^{\mathrm{c}}_{\mathrm{R}} + \text{h.c.} = \overline{u} P_{\mathrm{L}} \tilde{\chi}^0 + \overline{\tilde{\chi}^0} P_{\mathrm{R}} u,$$

$$\overline{u}\gamma^\mu P_{\{\mathrm{L,R}\}} \tilde{\chi}^0 = \{\bar{u}_{\mathrm{L}} \bar{\sigma}^\mu \tilde{\chi}^0, -\bar{\tilde{\chi}}^0 \bar{\sigma}^\mu u^{\mathrm{c}}_{\mathrm{R}}\}, \quad \overline{\tilde{\chi}^0}\gamma^\mu P_{\{\mathrm{L,R}\}} u = \{\bar{\tilde{\chi}}^0 \bar{\sigma} u_{\mathrm{L}}, -\bar{u}^{\mathrm{c}}_{\mathrm{R}} \bar{\sigma} \tilde{\chi}^0\},$$

$$\bar{\tilde{\chi}}^0_i \bar{\sigma} \tilde{\chi}^0_j = \overline{\tilde{\chi}^0_i} \gamma^\mu P_{\mathrm{L}} \tilde{\chi}^0_j = -\overline{\tilde{\chi}^0_j} \gamma^\mu P_{\mathrm{R}} \tilde{\chi}^0_i.$$

With abbreviations

$$\begin{aligned}
(\mathcal{N}_{\phi_{\mathrm{u}}})_{ij} &:= N^*_{i4}(g_2 N^*_{j2} - g_Y N^*_{j1}) P_{\mathrm{L}} + N_{i4}(g_2 N_{j2} - g_Y N_{j1}) P_{\mathrm{R}}, \\
(\mathcal{N}_{\phi_{\mathrm{d}}})_{ij} &:= N^*_{i3}(g_Y N^*_{j1} - g_2 N^*_{j2}) P_{\mathrm{L}} + N_{i3}(g_Y N_{j1} - g_2 N_{j2}) P_{\mathrm{R}}, \\
(\mathcal{N}_{A^0})_{ij} &:= -(\mathrm{s}_{\beta 0} N^*_{i3} - \mathrm{c}_{\beta 0} N^*_{i4})(g_Y N^*_{j1} - g_2 N^*_{j2}) P_{\mathrm{L}} + (\mathrm{s}_{\beta 0} N_{i3} - \mathrm{c}_{\beta 0} N_{i4})(g_Y N_{j1} - g_2 N_{j2}) P_{\mathrm{R}}, \\
(\mathcal{N}_{G^0})_{ij} &:= (g_Y N^*_{j1} - g_2 N^*_{j2})(\mathrm{c}_{\beta 0} N^*_{i3} + \mathrm{s}_{\beta 0} N^*_{i4}) P_{\mathrm{L}} - (g_Y N_{j1} - g_2 N_{j2})(\mathrm{c}_{\beta 0} N_{i3} + \mathrm{s}_{\beta 0} N_{i4}) P_{\mathrm{R}}, \\
(\mathcal{C}_{\phi_{\mathrm{u}}})_{ij} &:= -U^*_{i1} V^*_{j2} P_{\mathrm{L}} - U_{i1} V_{j2} P_{\mathrm{R}}, \\
(\mathcal{C}_{\phi_{\mathrm{d}}})_{ij} &:= -U^*_{i2} V^*_{j1} P_{\mathrm{L}} - U_{i2} V_{j1} P_{\mathrm{R}}, \\
(\mathcal{C}_{A^0})_{ij} &:= (\mathrm{c}_{\beta 0} U^*_{i1} V^*_{j2} + \mathrm{s}_{\beta 0} U^*_{i2} V^*_{j1}) P_{\mathrm{L}} - (\mathrm{c}_{\beta 0} U_{i1} V_{j2} + \mathrm{s}_{\beta 0} U_{i2} V_{j1}) P_{\mathrm{R}}, \\
(\mathcal{C}_{G^0})_{ij} &:= -(\mathrm{c}_{\beta 0} U^*_{i2} V^*_{j1} - \mathrm{s}_{\beta 0} U^*_{i1} V^*_{j2}) P_{\mathrm{L}} + (\mathrm{c}_{\beta 0} U_{i2} V_{j1} - \mathrm{s}_{\beta 0} U_{i1} V_{j2}) P_{\mathrm{R}}, \\
(\mathcal{C}_{H^-})_{ij} &:= g_2 V^*_{i1} N^*_{j4} + \frac{V^*_{i2}(g_Y N^*_{j1} + g_2 N^*_{j2})}{\sqrt{2}} \\
,(\mathcal{C}_{H^+})_{ij} &:= \frac{U^*_{i2}(g_Y N^*_{j1} + g_2 N^*_{j2})}{\sqrt{2}} - g_2 U^*_{i1} N^*_{j3}
\end{aligned}$$

,

we have, noting that squarks and sleptons are in super-CKM basis and not in mass eigenstates,

$$
\begin{aligned}
&\mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{SFG}} \\
&= \frac{1}{2}\overline{\tilde{g}^a}\left[(\mathrm{i}\not{\partial} - m_{\tilde{g}})\delta^{ab} - \mathrm{i}g_3 f^{cab}\not{g}^c\right]\tilde{g}^b + \frac{1}{2}\overline{\tilde{\chi}^0_i}\left[\left(\mathrm{i}\not{\partial} - m_{\tilde{\chi}^0_i}\right)\delta_{ij} + g_Z\left(N_{i3}N^*_{j3} - N_{i4}N^*_{j4}\right)\not{Z}P_{\mathrm{L}}\right]\tilde{\chi}^0_j \\
&+ \overline{\tilde{\chi}^+_i}\left[\left(\mathrm{i}\not{\partial} - m_{\tilde{\chi}^\pm_i} + |e|\not{A}\right)\delta_{ij} + g_Z\not{Z}\left(\frac{2\mathrm{c}^2_{\mathrm{w}}V_{i1}V^*_{j1} + \mathrm{c}_{2\mathrm{w}}V_{i2}V^*_{j2}}{2}P_{\mathrm{L}} + \frac{\mathrm{c}_{2\mathrm{w}}U_{i2}U^*_{j2} - 2\mathrm{c}^2_{\mathrm{w}}U_{i1}U^*_{j1}}{2}P_{\mathrm{R}}\right)\right]\tilde{\chi}^+_j \\
&+ \overline{u_i}\left[\mathrm{i}\not{\partial} - m_{ui} + g_3\tau^a\not{g}^a + \frac{2|e|}{3}\not{A} + g_Z\not{Z}\left(\frac{3-4\mathrm{s}^2_{\mathrm{w}}}{6}P_{\mathrm{L}} - \frac{2\mathrm{s}^2_{\mathrm{w}}}{3}P_{\mathrm{R}}\right)\right]u_i \\
&+ \overline{d_i}\left[\mathrm{i}\not{\partial} - m_{di} + g_3\tau^a\not{g}^a - \frac{|e|}{3}\not{A} + g_Z\not{Z}\left(-\frac{3-2\mathrm{s}^2_{\mathrm{w}}}{6}P_{\mathrm{L}} + \frac{\mathrm{s}^2_{\mathrm{w}}}{3}P_{\mathrm{R}}\right)\right]d_i \\
&+ \overline{\nu_i}\left(\mathrm{i}\not{\partial} + \frac{g_Z}{2}\not{Z}P_{\mathrm{L}}\right)\nu_i + \overline{e_i}\left[\mathrm{i}\not{\partial} - m_{ei} - |e|\not{A} + g_Z\not{Z}\left(\frac{2\mathrm{s}^2_{\mathrm{w}} - 1}{2}P_{\mathrm{L}} + \mathrm{s}^2_{\mathrm{w}}P_{\mathrm{R}}\right)\right]e_i \\
&+ \frac{g_2}{\sqrt{2}}W^+_\mu\left[\overline{u}\gamma^\mu V_{\mathrm{CKM}}P_{\mathrm{L}}d + \overline{\nu}\gamma^\mu P_{\mathrm{L}}e + \overline{\tilde{\chi}^+_j}\gamma^\mu\left(\left(-\sqrt{2}V_{j1}N^*_{i2} + V_{j2}N^*_{i4}\right)P_{\mathrm{L}} + \left(-\sqrt{2}N_{i2}U^*_{j1} - N_{i3}U^*_{j2}\right)P_{\mathrm{R}}\right)\tilde{\chi}^0_i\right] \\
&+ \frac{g_2}{\sqrt{2}}W^-_\mu\left[\overline{d}\gamma^\mu V^\dagger_{\mathrm{CKM}}P_{\mathrm{L}}u + \overline{e}\gamma^\mu P_{\mathrm{L}}\nu + \overline{\tilde{\chi}^0_i}\gamma^\mu\left(\left(-\sqrt{2}N_{i2}V^*_{j1} + N_{i4}V^*_{j2}\right)P_{\mathrm{L}} + \left(\sqrt{2}U_{j1}N^*_{i2} + U_{j2}N^*_{i3}\right)P_{\mathrm{R}}\right)\tilde{\chi}^+_j\right] \\
&+ \sqrt{2}g_3\Big[\overline{\tilde{g}^a}(\varphi^*_{\tilde{g}}\tilde{u}^*_{\mathrm{R}}P_{\mathrm{R}} - \varphi_{\tilde{g}}\tilde{u}^*_{\mathrm{L}}P_{\mathrm{L}})\tau^a u + \overline{u}\tau^a(\varphi_{\tilde{g}}\tilde{u}_{\mathrm{R}}P_{\mathrm{L}} - \varphi^*_{\tilde{g}}\tilde{u}_{\mathrm{L}}P_{\mathrm{R}})\tilde{g}^a \\
&\qquad + \overline{d}\tau^a(\varphi_{\tilde{g}}\tilde{d}_{\mathrm{R}}P_{\mathrm{L}} - \varphi^*_{\tilde{g}}\tilde{d}_{\mathrm{L}}P_{\mathrm{R}})\tilde{g}^a + \overline{\tilde{g}^a}(\varphi^*_{\tilde{g}}\tilde{d}^*_{\mathrm{R}}P_{\mathrm{R}} - \varphi_{\tilde{g}}\tilde{d}^*_{\mathrm{L}}P_{\mathrm{L}})\tau^a d\Big] + \frac{y_{ui}}{\sqrt{2}}\overline{u_i}\left(-\phi_{\mathrm{u}} + \mathrm{i}\gamma_5\mathrm{c}_{\beta0}A^0 + \mathrm{i}\gamma_5\mathrm{s}_{\beta0}G^0\right)u_i \\
&+ \frac{y_{di}}{\sqrt{2}}\overline{d_i}\left(-\phi_{\mathrm{d}} + \mathrm{i}\gamma_5\mathrm{s}_{\beta0}A^0 - \mathrm{i}\gamma_5\mathrm{c}_{\beta0}G^0\right)d_i + \frac{y_{ei}}{\sqrt{2}}\overline{e_i}\left(-\phi_{\mathrm{d}} + \mathrm{i}\gamma_5\mathrm{s}_{\beta0}A^0 - \mathrm{i}\gamma_5\mathrm{c}_{\beta0}G^0\right)e_i \\
&+ \frac{1}{2}\overline{\tilde{\chi}^0_i}\left(\phi_{\mathrm{u}}\mathcal{N}_{\phi_{\mathrm{u}}} + \phi_{\mathrm{d}}\mathcal{N}_{\phi_{\mathrm{d}}} + \mathrm{i}A^0\mathcal{N}_{A^0} + \mathrm{i}G^0\mathcal{N}_{G^0}\right)_{ij}\tilde{\chi}^0_j + \frac{g_2}{\sqrt{2}}\overline{\tilde{\chi}^+_i}\left(\phi_{\mathrm{u}}\mathcal{C}_{\phi_{\mathrm{u}}} + \phi_{\mathrm{d}}\mathcal{C}_{\phi_{\mathrm{d}}} + \mathrm{i}A_0\mathcal{C}_{A^0} + \mathrm{i}G_0\mathcal{C}_{G^0}\right)_{ij}\tilde{\chi}^+_j \\
&- (H^-\mathrm{c}_{\beta+} + \mathrm{i}G^-\mathrm{s}_{\beta+})(\mathcal{C}_{H^-})_{ij}\overline{\tilde{\chi}^0_j}P_{\mathrm{L}}\tilde{\chi}^+_i - (H^+\mathrm{c}_{\beta+} - \mathrm{i}G^+\mathrm{s}_{\beta+})(\mathcal{C}_{H^-})^*_{ij}\overline{\tilde{\chi}^+_i}P_{\mathrm{R}}\tilde{\chi}^0_j \\
&+ (H^+\mathrm{s}_{\beta+} + \mathrm{i}G^+\mathrm{c}_{\beta+})(\mathcal{C}_{H^+})_{ij}\overline{\tilde{\chi}^+_i}P_{\mathrm{L}}\tilde{\chi}^0_j + (H^-\mathrm{s}_{\beta+} - \mathrm{i}G^-\mathrm{c}_{\beta+})(\mathcal{C}_{H^+})^*_{ij}\overline{\tilde{\chi}^0_j}P_{\mathrm{R}}\tilde{\chi}^+_i \\
&+ (H^-\mathrm{s}_{\beta+} - \mathrm{i}G^-\mathrm{c}_{\beta+})\overline{d}y_dV^\dagger_{\mathrm{CKM}}P_{\mathrm{L}}u + (H^-\mathrm{c}_{\beta+} + \mathrm{i}G^-\mathrm{s}_{\beta+})\overline{d}V^\dagger_{\mathrm{CKM}}y_uP_{\mathrm{R}}u + (H^-\mathrm{s}_{\beta+} - \mathrm{i}G^-\mathrm{c}_{\beta+})\overline{e}y_eP_{\mathrm{L}}\nu \\
&+ (H^+\mathrm{c}_{\beta+} - \mathrm{i}G^+\mathrm{s}_{\beta+})\overline{u}y_uV_{\mathrm{CKM}}P_{\mathrm{L}}d + (H^+\mathrm{s}_{\beta+} + \mathrm{i}G^+\mathrm{c}_{\beta+})\overline{u}V_{\mathrm{CKM}}y_dP_{\mathrm{R}}d + (H^+\mathrm{s}_{\beta+} + \mathrm{i}G^+\mathrm{c}_{\beta+})\overline{\nu}y_eP_{\mathrm{R}}e \\
&+ \left(-\frac{g_YN^*_{j1} + 3g_2N^*_{j2}}{3\sqrt{2}}\tilde{u}^*_{\mathrm{L}i} - y_{ui}N^*_{j4}\tilde{u}^*_{\mathrm{R}i}\right)\overline{\tilde{\chi}^0_j}P_{\mathrm{L}}u_i + \left(\frac{2\sqrt{2}g_Y}{3}N_{j1}\tilde{u}^*_{\mathrm{R}i} - y_{ui}N_{j4}\tilde{u}^*_{\mathrm{L}i}\right)\overline{\tilde{\chi}^0_j}P_{\mathrm{R}}u_i \\
&+ \left(\frac{2\sqrt{2}g_Y}{3}N^*_{j1}\tilde{u}_{\mathrm{R}i} - y_{ui}N^*_{j4}\tilde{u}_{\mathrm{L}i}\right)\overline{u_i}P_{\mathrm{L}}\tilde{\chi}^0_j + \left(-\frac{g_YN_{j1} + 3g_2N_{j2}}{3\sqrt{2}}\tilde{u}_{\mathrm{L}i} - y_{ui}N_{j4}\tilde{u}_{\mathrm{R}i}\right)\overline{u_i}P_{\mathrm{R}}\tilde{\chi}^0_j \\
&+ \left(-\frac{g_YN^*_{j1} - 3g_2N^*_{j2}}{3\sqrt{2}}\tilde{d}^*_{\mathrm{L}i} - y_{di}N^*_{j3}\tilde{d}^*_{\mathrm{R}i}\right)\overline{\tilde{\chi}^0_j}P_{\mathrm{L}}d_i + \left(-\frac{\sqrt{2}g_Y}{3}N_{j1}\tilde{d}^*_{\mathrm{R}i} - y_{di}N_{j3}\tilde{d}^*_{\mathrm{L}i}\right)\overline{\tilde{\chi}^0_j}P_{\mathrm{R}}d_i \\
&+ \left(-\frac{\sqrt{2}g_Y}{3}N^*_{j1}\tilde{d}_{\mathrm{R}i} - y_{di}N^*_{j3}\tilde{d}_{\mathrm{L}i}\right)\overline{d_i}P_{\mathrm{L}}\tilde{\chi}^0_j + \left(-\frac{g_YN_{j1} - 3g_2N_{j2}}{3\sqrt{2}}\tilde{d}_{\mathrm{L}i} - y_{di}N_{j3}\tilde{d}_{\mathrm{R}i}\right)\overline{d_i}P_{\mathrm{R}}\tilde{\chi}^0_j \\
&+ \left(\frac{g_YN^*_{j1} + g_2N^*_{j2}}{\sqrt{2}}\tilde{e}^*_{\mathrm{L}i} - y_{ei}N^*_{j3}\tilde{e}^*_{\mathrm{R}i}\right)\overline{\tilde{\chi}^0_j}P_{\mathrm{L}}e_i + \left(-\sqrt{2}g_YN_{j1}\tilde{e}^*_{\mathrm{R}i} - y_{ei}N_{j3}\tilde{e}^*_{\mathrm{L}i}\right)\overline{\tilde{\chi}^0_j}P_{\mathrm{R}}e_i \\
&+ \left(-\sqrt{2}g_YN^*_{j1}\tilde{e}_{\mathrm{R}i} - y_{ei}N^*_{j3}\tilde{e}_{\mathrm{L}i}\right)\overline{e_i}P_{\mathrm{L}}\tilde{\chi}^0_j + \left(+\frac{g_YN_{j1} + g_2N_{j2}}{\sqrt{2}}\tilde{e}_{\mathrm{L}i} - y_{ei}N_{j3}\tilde{e}_{\mathrm{R}i}\right)\overline{e_i}P_{\mathrm{R}}\tilde{\chi}^0_j \\
&+ \frac{g_YN_{j1} - g_2N_{j2}}{\sqrt{2}}\tilde{\nu}_{\mathrm{L}i}\overline{\nu_i}P_{\mathrm{R}}\tilde{\chi}^0_j + \frac{g_YN^*_{j1} - g_2N^*_{j2}}{\sqrt{2}}\tilde{\nu}^*_{\mathrm{L}i}\overline{\tilde{\chi}^0_j}P_{\mathrm{L}}\nu_i \\
&+ \left(U^*_{i2}(\tilde{d}^*_{\mathrm{R}}y_dV^\dagger_{\mathrm{CKM}})_j - U^*_{i1}g_2(\tilde{d}^*_{\mathrm{L}}V^\dagger_{\mathrm{CKM}})_j\right)\overline{\tilde{\chi}^+_i}P_{\mathrm{L}}u_j + V_{i2}(\tilde{d}^*_{\mathrm{L}}V^\dagger_{\mathrm{CKM}}y_u)_j\overline{\tilde{\chi}^+_i}P_{\mathrm{R}}u_j \\
&+ V^*_{i2}(y_uV_{\mathrm{CKM}}\tilde{d}_{\mathrm{L}})_j\overline{u_j}P_{\mathrm{L}}\tilde{\chi}^+_i + \left(U_{i2}(V_{\mathrm{CKM}}y_d\tilde{d}_{\mathrm{R}})_j - U_{i1}g_2(V_{\mathrm{CKM}}\tilde{d}_{\mathrm{L}})_j\right)\overline{u_j}P_{\mathrm{R}}\tilde{\chi}^+_i \\
&+ \left(-V^*_{i1}g_2(\tilde{u}^*_{\mathrm{L}}V_{\mathrm{CKM}})_j + V^*_{i2}(\tilde{u}^*_{\mathrm{R}}y_uV_{\mathrm{CKM}})_j\right)\overline{\tilde{\chi}^-_i}P_{\mathrm{L}}d_j + U_{i2}(\tilde{u}^*_{\mathrm{L}}V_{\mathrm{CKM}}y_d)_j\overline{\tilde{\chi}^-_i}P_{\mathrm{R}}d_j \\
&+ U^*_{i2}(y_dV^\dagger_{\mathrm{CKM}}\tilde{u}_{\mathrm{L}})_j\overline{d_j}P_{\mathrm{L}}\tilde{\chi}^-_i + \left(-V_{i1}g_2(V^\dagger_{\mathrm{CKM}}\tilde{u}_{\mathrm{L}})_j + V_{i2}(V^\dagger_{\mathrm{CKM}}y_u\tilde{u}_{\mathrm{R}})_j\right)\overline{d_j}P_{\mathrm{R}}\tilde{\chi}^-_i \\
&+ \left(-U^*_{i1}g_2\tilde{e}^*_{\mathrm{L}} + U^*_{i2}y_{ei}\tilde{e}^*_{\mathrm{R}i}\right)\overline{\tilde{\chi}^+_i}P_{\mathrm{L}}\nu + \left(-U_{i1}g_2\tilde{e}_{\mathrm{L}} + U_{i2}y_{ei}\tilde{e}_{\mathrm{R}i}\right)\overline{\nu}P_{\mathrm{R}}\tilde{\chi}^+_i \\
&+ \overline{\tilde{\chi}^-_i}\left(-V^*_{i1}g_2\tilde{\nu}^*_{\mathrm{L}}P_{\mathrm{L}} + U_{i2}y_{ej}\tilde{\nu}^*_{\mathrm{L}j}P_{\mathrm{R}}\right)e_j + \overline{e_j}\left(U^*_{i2}y_{ej}\tilde{\nu}_{\mathrm{L}j}P_{\mathrm{L}} - V_{i1}g_2\tilde{\nu}_{\mathrm{L}}P_{\mathrm{R}}\right)\tilde{\chi}^-_i + \text{(RPV part).}
\end{aligned}
\tag{11.90}
$$

11.5. SLHA convention

The SLHA convention [16] is different from our notation; the reinterpretation rules for the MSSM parameters are given in the right table (**magenta color** for objects in other conventions), while

$\mu, b, m^2_{Q,L,H_\mathrm{u},H_\mathrm{d}}$, RPV-trilinears ($\lambda$s and Ts)

are in common.

SLHA		our notation	Martin/DHM
(H_1, H_2)	$=$	$(H_\mathrm{d}, H_\mathrm{u})$	
$Y_{\mathrm{u,d,e}}$	$=$	$(y_{\mathrm{u,d,e}})^\mathrm{T}$	
$T_{\mathrm{u,d,e}}$	$=$	$(a_{\mathrm{u,d,e}})^\mathrm{T}$	
$A_{\mathrm{u,d,e}}$	$=$	$(A_{\mathrm{u,d,e}})^\mathrm{T}$	
$m^2_{U^\mathrm{c},D^\mathrm{c},E^\mathrm{c}}$	$=$	$(m^2_{U^\mathrm{c},D^\mathrm{c},E^\mathrm{c}})^\dagger$	
$M_{1,2,3}$	$=$	$-M_{1,2,3}$	
m_3^2	$=$	b	
m_A^2	$=$	$m^2_{A_0}$ (tree)	
		κ_i	$= -\mu'_i$ (rarely used)
D_i	$=$	b_i	
$m^2_{\tilde{L}_i H_1}$	$=$	M^2_{Li}	

In particular, the chargino/neutralino mass terms in RPC case are given by

$$\mathcal{L} \supset \left[\frac{1}{2} M_1 \tilde{b}\tilde{b} + \frac{1}{2} M_2 \tilde{w}\tilde{w} - \mu \tilde{h}_{\mathrm{u}} \tilde{h}_{\mathrm{d}} - \frac{g_Y}{2\sqrt{2}} \left(h_{\mathrm{u}}^* \tilde{h}_{\mathrm{u}} - h_{\mathrm{d}}^* \tilde{h}_{\mathrm{d}}\right) \tilde{b} - \sqrt{2} g_2 \left(h_{\mathrm{u}}^* T^a \tilde{h}_{\mathrm{u}} + h_{\mathrm{d}}^* T_a \tilde{h}_{\mathrm{d}}\right) \tilde{w}\right] + \text{H.c.} \tag{11.91}$$

$$\to \frac{1}{2} \begin{pmatrix} \tilde{b} \\ \tilde{w} \\ h_{\mathrm{u}}^0 \\ h_{\mathrm{d}}^0 \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} -M_1 & 0 & -m_Z \mathrm{c}_\beta \mathrm{s}_w & m_Z \mathrm{s}_\beta \mathrm{s}_w \\ 0 & -M_2 & m_Z \mathrm{c}_\beta \mathrm{c}_w & -m_Z \mathrm{s}_\beta \mathrm{c}_w \\ -m_Z \mathrm{c}_\beta \mathrm{s}_w & m_Z \mathrm{c}_\beta \mathrm{c}_w & 0 & -\mu \\ m_Z \mathrm{s}_\beta \mathrm{s}_w & -m_Z \mathrm{s}_\beta \mathrm{c}_w & -\mu & 0 \end{pmatrix} \begin{pmatrix} \tilde{b} \\ \tilde{w} \\ h_{\mathrm{u}}^0 \\ h_{\mathrm{d}}^0 \end{pmatrix} \tag{11.92}$$

11.6. GMSB formulae

$$\Gamma^{-1}(\tilde{l} \to l\tilde{G}) = \frac{1}{48\pi M^2} \frac{m_l^5}{m_{\tilde{G}}^2} \times \text{(phase space)} \approx 5.9 \times 10^{-7}\,\mathrm{s}\frac{(m_{3/2}/\mathrm{MeV})^2}{(m_{\tilde{l}}/\mathrm{TeV})^5} \tag{11.93}$$

$$= \frac{3m_l^5}{48\pi F_{\mathrm{tot}}^2} \times \text{(phase space)} \approx 3.3 \times 10^{-9}\,\mathrm{s}\frac{(F_{\mathrm{tot}}/10^{12}\,\mathrm{GeV}^2)^2}{(m_{\tilde{l}}/100\,\mathrm{GeV})^5} = 1.0\,\mathrm{m}\frac{(F_{\mathrm{tot}}/10^{12}\,\mathrm{GeV}^2)^2}{(m_{\tilde{l}}/100\,\mathrm{GeV})^5} \tag{11.94}$$

$$\Gamma^{-1}(\tilde{B} \to \gamma\tilde{G}) = \frac{\mathrm{c}_{\mathrm{w}}^2 m_{\tilde{B}}^5}{48\pi M^2 m_{\tilde{G}}^2}(1-r)^3(1+3r) \qquad \text{where} \quad r := \left(\frac{m_{\tilde{G}}}{m_{\tilde{B}}}\right)^2. \tag{11.95}$$

(cf. [17])

A. Mathematics

A.1. Matrix exponential

Excerpted from §2 and §5 of Hall 2015 [18]:

$$\mathrm{e}^X := \sum_{m=0}^{\infty} \frac{X^m}{m!} \text{ (converges for any } X\text{)}, \quad \log X := \sum_{m=1}^{\infty} (-1)^{m+1} \frac{(A-1)^m}{m} \text{ (conv. if } \|A-I\| < 1\text{)}. \tag{A.1}$$

$$\mathrm{e}^{\log A} = A \ \text{ (if } \|A-I\| < 1\text{)}, \quad \log \mathrm{e}^X = X \text{ and } \|\mathrm{e}^X - 1\| < 1 \ \text{ (if } \|X\| < \log 2\text{)}. \tag{A.2}$$

$$\text{Hilbert-Schmidt norm}: \ \|X\|^2 := \sum_{i,j} |X_{ij}|^2 = \operatorname{Tr} X^\dagger X. \tag{A.3}$$

Properties:

$$\mathrm{e}^{(X^\mathrm{T})} = (\mathrm{e}^X)^\mathrm{T}, \quad \mathrm{e}^{(X^*)} = (\mathrm{e}^X)^*, \quad (\mathrm{e}^X)^{-1} = \mathrm{e}^{-X}, \quad \mathrm{e}^{YXY^{-1}} = Y\,\mathrm{e}^X Y^{-1},$$

$$\det\exp X = \exp\operatorname{Tr} X, \quad \mathrm{e}^{(\alpha+\beta)X} = \mathrm{e}^{\alpha X}\,\mathrm{e}^{\beta X} \text{ for } \alpha,\beta \in \mathbb{C};$$

Baker-Campbell-Hausdorff:

$$\mathrm{e}^X Y\,\mathrm{e}^{-X} = Y + [X,Y] + \frac{1}{2!}[X,[X,Y]] + \frac{1}{3!}[X,[X,[X,Y]]] + \cdots = \mathrm{e}^{[X,]}Y; \tag{A.4}$$

$$\mathrm{e}^X\,\mathrm{e}^Y\,\mathrm{e}^{-X} = \sum_{n=0}^{\infty} \frac{1}{n!}(\mathrm{e}^X Y\,\mathrm{e}^{-X})^n = \exp\left(\mathrm{e}^{[X,]}Y\right); \tag{A.5}$$

$$\log(\mathrm{e}^X\,\mathrm{e}^Y) = X + \int_0^1 \mathrm{d}t\, g(\mathrm{e}^{[X,}\,\mathrm{e}^{t[Y,})Y \qquad \left[g(z) = \frac{\log z}{1-z^{-1}} = 1 - \sum_{n=1}^{\infty} \frac{(1-z)^n}{n(n+1)}; \quad g(\mathrm{e}^y) = \sum_{n=0}^{\infty} \frac{B_n y^n}{n!}\right] \tag{A.6}$$

$$= X + Y + \frac{1}{2}[X,Y] + \frac{1}{12}[X,[X,Y]] - \frac{1}{12}[Y,[X,Y]] + \cdots \quad \text{(Baker-Campbell-Hausdorff)}. \tag{A.7}$$

$$\log(\mathrm{e}^X\,\mathrm{e}^Y) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k}\left(\sum_{m,n=0}^{\infty} \frac{X^m Y^n}{m!n!} - 1\right)^k = \sum_{k=1}^{\infty} \sum_{m_1+n_1>0} \cdots \sum_{m_k+n_k>0} \frac{(-1)^{k-1}}{k} \frac{X^{m_1}Y^{n_1}\cdots X^{m_k}Y^{n_k}}{m_1!n_1!\cdots m_k!n_k!} \tag{A.8}$$

$$\log(\mathrm{e}^X\,\mathrm{e}^Y) = \sum_{k=1}^{\infty} \sum_{m_1+n_1>0} \cdots \sum_{m_k+n_k>0} \frac{(-1)^{k-1}}{k\sum_{i=1}^{k}(m_i+n_i)} \frac{\left([X,\right)^{m_1}\left([Y,\right)^{n_1}\cdots\left([X,\right)^{m_k}\left([Y,\right)^{n_k}]\cdots]}{m_1!n_1!\cdots m_k!n_k!} \tag{A.9}$$

with $[X]$ being X.

Derivative:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{tX} = X\,\mathrm{e}^{tX} = \mathrm{e}^{tX}X \tag{A.10}$$

$$\mathrm{e}^{-X(t)}\left(\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{X(t)}\right) = \frac{I - \mathrm{e}^{-\mathrm{ad}_X}}{\mathrm{ad}_X}\left(\frac{\mathrm{d}X}{\mathrm{d}t}\right) = X' + \frac{[-X,X']}{2!} + \frac{[-X,[-X,X']]}{3!} + \cdots \tag{A.11}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{X(t)}\right)\mathrm{e}^{-X(t)} = X' + \frac{[X,X']}{2!} + \frac{[X,[X,X']]}{3!} + \cdots \tag{A.12}$$

where $X' = \mathrm{d}X/\mathrm{d}t$ and $\mathrm{ad}_X(Y) = [X,Y]$ is the adjoint action of a Lie algebra. Thus, explicitly,

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{aX(t)} = \mathrm{e}^{aX}\left\{\sum_{n=0}^{\infty} \frac{a^{n+1}}{(n+1)!}\left([-X,\right)^n X']\right\} = \left\{\sum_{n=0}^{\infty} \frac{a^{n+1}}{(n+1)!}\left([X,\right)^n X']\right\}\mathrm{e}^{aX} \tag{A.13}$$

Component: If matrices t^a satisfies $[t^a,t^b] = \mathrm{i}f^{abc}t^c$ with totally-antisymmetric $f^{abc} \in \mathbb{R}$,

$$\left[\mathrm{e}^{\theta^a t^a}t_b\,\mathrm{e}^{-\theta^c t^c}\right]_{ij} = \left[\mathrm{e}^{\theta^a[t^a,}t_b\right]_{ij} = \left[\mathrm{e}^{\mathrm{i}\theta^a f^a}\right]^{bc} t^c_{ij} \tag{A.14}$$

holds for $\theta^a \in \mathbb{C}$, where $[f^a]_{bc} = f^{abc}$. ♣TODO: needs verification, generalization/restriction, and a nice proof or reference.♣

A.2. General unitary matrix

$$U_2 = \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\alpha} \end{pmatrix}\begin{pmatrix} \mathrm{c}_\theta & \mathrm{s}_\theta \\ -\mathrm{s}_\theta & \mathrm{c}_\theta \end{pmatrix}\begin{pmatrix} \mathrm{e}^{\mathrm{i}\beta} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\gamma} \end{pmatrix} = \begin{pmatrix} \mathrm{c}_\theta\,\mathrm{e}^{\mathrm{i}\beta} & \mathrm{s}_\theta\,\mathrm{e}^{\mathrm{i}\gamma} \\ -\mathrm{s}_\theta\,\mathrm{e}^{\mathrm{i}(\alpha+\beta)} & \mathrm{c}_\theta\,\mathrm{e}^{\mathrm{i}(\alpha+\gamma)} \end{pmatrix}, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}, \quad \alpha,\beta,\gamma \in \mathbb{R}; \tag{A.15}$$

$$U_3 = \begin{pmatrix} 1 & & \\ & e^{ia} & \\ & & e^{ib} \end{pmatrix} \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}\, e^{-i\delta} \\ & 1 & \\ -s_{13}\, e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} e^{ic} & & \\ & e^{id} & \\ & & e^{ie} \end{pmatrix} \tag{A.16}$$

$$= \begin{pmatrix} 1 & & \\ & e^{ia} & \\ & & e^{ib} \end{pmatrix} \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\, e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\, e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\, e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\, e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\, e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{ic} & & \\ & e^{id} & \\ & & e^{ie} \end{pmatrix} \tag{A.17}$$

with $0 \leqslant \theta_{ij} \leqslant \pi/2$ and $a, b, c, d, e, \delta \in \mathbb{R}$ (see, e.g., Ref. [19]).

```
U3 = Dot[
  DiagonalMatrix[Exp[I {0, a, b}]],
  RotationMatrix[\[Theta]23, {-1, 0, 0}],
  DiagonalMatrix[Exp[I {0,0,+\[Delta]}]],
  RotationMatrix[\[Theta]13, {0, 1, 0}],
  DiagonalMatrix[Exp[I {0,0,-\[Delta]}]],
  RotationMatrix[\[Theta]12, {0, 0, -1}],
  DiagonalMatrix[Exp[I {c, d, e}]]
]
```

A.3. Matrix diagonalization

In this section, $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$ and $\mathbb{U}^n_{\mathbb{K}} \subset \mathbb{K}^{n\times n}$ is the set of the unitary matrices.

Diagonalization A matrix $M \in \mathbb{K}^{n\times n}$ is called diagonalizable if $\exists P$ and $\exists D$ s.t.

$$M = PDP^{-1}; \qquad P \in \mathbb{K}^{n\times n}, \quad D : \text{diagonal matrix } (D_{ii} \in \mathbb{K}). \tag{A.18}$$

In particular,

$$M \text{ is normal} \overset{\text{def}}{\iff} M^\dagger M = MM^\dagger \iff \exists P \in \mathbb{U}^n_{\mathbb{K}} \text{ s.t. } M = PDP^{-1}. \tag{A.19}$$

Singular value decomposition Any $M \in \mathbb{K}^{m\times n}$ can be singular-value decomposed as

$$M = UDV^\dagger; \qquad U \in \mathbb{U}^m_{\mathbb{K}}, \quad V \in \mathbb{U}^n_{\mathbb{K}}, \quad D : \text{non-negative real diagonal matrix } (D_{ii} \geqslant 0). \tag{A.20}$$

Here, the matrix U (V) diagonalizes $MM^\dagger$ ($M^\dagger M$) and $(D_{ii})^2$ are the eigenvalues of $MM^\dagger$ (and $M^\dagger M$).

The calculation on Mathematica is straightforward for this convention:

```
{u, d, v} = SingularValueDecomposition[M]
```

Autonne-Takagi factorization If $M \in \mathbb{C}^{n\times n}$ is symmetric, it can be decomposed as

$$M = RDR^{\mathrm{T}}; \qquad R \in \mathbb{U}^n_{\mathbb{C}}, \quad D : \text{non-negative real diagonal matrix } (D_{ii} \geqslant 0). \tag{A.21}$$

Real symmetric matrices are normal and thus do not need this factorization; we can apply the above "diagonalization" method.

Sample Mathematica code to calculate $\{D, R\}$ (with ordering, if specified) is:

```
AutonneTakagi[M_, order_ : None] := Module[{v0, v, p, ord, R, D},
  ord = If[order === None, Range[Length[M]], order];
  v0 = Eigenvectors[Conjugate[M].M];
  v = Eigenvectors[v0.M.Transpose[v0]].v0; (*resolve degenerate eigenvalues*)
  p = DiagonalMatrix[If[Abs[#] > 0, (#/Abs[#])^(-1/2), 1] & /@ Diagonal[v.M.Transpose[v]]];
  R = ConjugateTranspose[Reverse[p.v][[ord]] // Orthogonalize];
  D = ConjugateTranspose[R].M.Conjugate[R];
  {D, R}];
```

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