SLHA convention compared against References

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1 Comparison

1.1 Higgs potential

1.1.1 SLHA convention

The SLHA [1] is based on Gunion–Haber's notation [2]:

$$\begin{aligned} W &\supset -\epsilon_{ab}\mu H_1^a H_2^b \\ &= -\mu(H_1^1 H_2^2 - H_1^2 H_2^1) \\ &\equiv \mu(-H_1^0 H_2^0 + H_1^- H_2^+), \end{aligned} \tag{SLHA:3}$$

$$\begin{aligned} V_2 &\supset m_{H_1}^2|H_1|^2 + m_{H_2}^2|H_2|^2 - (m_3^2\epsilon_{ab}H_1^a H_2^b + \text{h.c.}), \\ &= m_{H_1}^2|H_1|^2 + m_{H_2}^2|H_2|^2 + \left[m_3^2(-H_1^0 H_2^0 + H_1^- H_2^+) + \text{h.c.}\right]. \end{aligned} \tag{SLHA:7}$$

where $\epsilon_{12} = \epsilon^{12} = +1$. The parameter m_A^2, set by `EXTPAR 24`, is then defined as

$$m_A^2 = \frac{2m_3^2}{\sin 2\beta}. \tag{SLHA:8}$$

The Higgs vacuum expectation value is defined as

$$\langle H_i^0\rangle = \frac{v_i}{\sqrt{2}}; \qquad m_Z^2 = \frac{1}{4}(g'^2 + g^2)(v_1^2 + v_2^2) = \frac{1}{4}(g'^2 + g^2)v^2, \tag{1}$$

i.e., $v \simeq 246\,\text{GeV}$ is given in the `HMIX 3`.

1.1.2 Comparison to GH convention

In Gunion–Haber [2], the definitions are as follows, with their errata applied:

$$W \supset -\epsilon_{ab}\mu H_1^a H_2^b, \tag{GH:3.3–3.4}$$

$$V_{\text{soft}} \supset m_1^2|H_1|^2 + m_2^2|H_2|^2 - (m_{12}^2\epsilon_{ab}H_1^a H_2^b + \text{h.c.}), \tag{GH:3.9}$$

where $\bar{E} \equiv \tilde{R}$ in their notation, with the same definition $\epsilon_{12} = 1$ (found below Eq. (3.2)). So, with the identification $(m_1^2, m_2^2, m_{12}^2) \equiv (m_{H_1}^2, m_{H_2}^2, m_3^2)$, this is identical to SLHA convention. Also we note the difference of the vacuum expectation values:

$$\langle H_1\rangle = \begin{pmatrix}\bar{v}_1\\ 0\end{pmatrix}, \qquad \langle H_2\rangle = \begin{pmatrix}0\\ \bar{v}_2\end{pmatrix}, \tag{GH:3.7}$$

with $\bar{v}_1 > 0$, $\bar{v}_2 > 0$ (3.24), and $\tan\beta = \bar{v}_2/\bar{v}_1 = v_2/v_1$ (2.8). Here we use bars to denote vs under this definition.

We can then use their results, which say

$$m_{H_1}^2 = -|\mu|^2 + 2\lambda_1\bar{v}_2^2 - m_Z^2/2, \tag{GH:3.21c}$$

$$m_{H_2}^2 = -|\mu|^2 + 2\lambda_1\bar{v}_1^2 - m_Z^2/2, \tag{GH:3.21d}$$

$$m_{H_1}^2 + m_{H_2}^2 + 2|\mu|^2 = m_3^2(\tan\beta + \cot\beta) = \frac{2m_3^2}{\sin 2\beta}, \tag{GH:3.22}$$

and, noting that

$$m_Z^2 = (g^2 + g'^2)\bar{v}^2/2, \qquad m_W^2 = g^2\bar{v}^2/2, \tag{2}$$

which are found below Eq. (3.19),

$$\lambda_1 = \frac{g^2 + g'^2}{4} + \frac{m_3^2}{2\bar{v}_1\bar{v}_2} = \frac{m_Z^2}{2(\bar{v}_1^2 + \bar{v}_2^2)} + \frac{m_3^2}{2\bar{v}_1\bar{v}_2}; \tag{3}$$

combining them, we have

$$m_{H_1}^2 = -|\mu|^2 - \frac{m_Z^2}{2}\cos 2\beta + m_3^2\tan\beta, \tag{4}$$

$$m_{H_2}^2 = -|\mu|^2 + \frac{m_Z^2}{2}\cos 2\beta + m_3^2\cot\beta. \tag{5}$$

Also the Higgs parameters are given by

$$m_{H^\pm}^2 = (4\lambda_1 - g'^2)\bar{v}^2 = m_W^2 + m_A^2, \tag{GH:3.16}$$

$$m_{H_3^0}^2 = m_{H^\pm}^2 - m_W^2 = m_A^2, \tag{GH:3.17}$$

$$m_{H_1^0,H_2^0}^2 = \frac{1}{2}\left[m_{H_3^0}^2 + m_Z^2 \pm \sqrt{(m_{H_3^0}^2 + m_Z^2)^2 - 4m_Z^2 m_{H_3^0}^2\cos^2 2\beta}\right], \tag{GH:3.18}$$

$$\tan 2\alpha = \frac{m_{H_3^0}^2 + m_Z^2}{m_{H_3^0}^2 - m_Z^2}\tan 2\beta. \tag{GH:3.19}$$

1.1.3 Comparison to SUSY Primer convention

We here compare the SLHA/GH notation with Martin's SUSY primer [3]. The potentials are given by

$$W_{\mathrm{MSSM}} \supset \mu H_{\mathrm{u}} H_{\mathrm{d}} \equiv \mu(H_2^+ H_1^- - H_1^0 H_2^0), \tag{SP:6.1.1–3}$$

$$-\mathcal{L}_{\mathrm{soft}} \supset m_{H_{\mathrm{d}}}^2|H_{\mathrm{d}}|^2 + m_{H_{\mathrm{u}}}^2|H_{\mathrm{u}}|^2 + (b\epsilon^{ab}H_{\mathrm{u}}^a H_{\mathrm{d}}^b + \text{h.c.}), \tag{SP:6.3.1}$$

$$\bar{v}_u = \langle H_{\mathrm{u}}^0\rangle, \quad \bar{v}_d = \langle H_{\mathrm{d}}^0\rangle, \tag{SP:8.1.5}$$

where $\epsilon^{12} = +1$ (2.13). So the parameters are identified by the replacement

$$\mu = \mu, \tag{6}$$

$$m_3^2 = b, \tag{7}$$

$$m_{H_1}^2 = m_{H_{\mathrm{d}}}^2, \tag{8}$$

$$m_{H_2}^2 = m_{H_{\mathrm{u}}}^2, \tag{9}$$

where the LHS are the SLHA parameters and the RHS are those in SUSY primer. With this identification, we can confirm that the above-shown formulae agree with SUSY Primer's equations (8.1.8)–(8.1.11) and (8.1.19)–(8.1.22).

1.2 Interaction terms

Here, to simplify the notation, we omit the SU(2) indices with assuming

$$AB = -BA \equiv A^1B^2 - B^1A^2.$$

1.2.1 SLHA convention

The SLHA [1] convention for the interection terms are

$$\begin{aligned}
W &= -H_2QY_U\bar{U} + H_1QY_D\bar{D} + H_1LY_E\bar{E}, && \text{(SLHA:3)}\\
V_3 &= -H_2\tilde{Q}T_U\tilde{u}^* + H_1\tilde{Q}T_D\tilde{d}^* + H_1\tilde{L}T_E\tilde{e}^* && \text{(SLHA:5)}\\
V_2 &= \tilde{Q}^*m_Q^2\tilde{Q} + \tilde{L}^*m_L^2\tilde{L} + \tilde{u}m_u^2\tilde{u}^* + \tilde{d}m_d^2\tilde{d}^* + \tilde{e}m_e^2\tilde{e}^* && \text{(SLHA:7)}\\
\mathcal{L}_G &= \frac{M_1}{2}\tilde{b}\tilde{b} + \frac{M_2}{2}\tilde{w}\tilde{w} + \frac{M_3}{2}\tilde{g}\tilde{g} + \text{h.c.} && \text{(SLHA:9)}
\end{aligned}$$

The neutralino and charginos are defined as

$$\tilde{\psi}^0 = (-\mathrm{i}\tilde{b}, -\mathrm{i}\tilde{w}^3, \tilde{h}_1, \tilde{h}_2)^{\mathrm{T}}, \qquad \tilde{\psi}^+ = (-\mathrm{i}\tilde{w}^+, \tilde{h}_2^+), \qquad \tilde{\psi}^- = (-\mathrm{i}\tilde{w}^-, \tilde{h}_1^-). \qquad \text{(SLHA:10+13)}$$

1.2.2 Comparison to SUSY Primer convention

In SUSY primer [3], the interaction terms are defined by

$$\begin{aligned}
W_{\text{MSSM}} &\supset \bar{U}y_uQH_{\text{u}} - \bar{d}y_dQH_{\text{d}} - \bar{e}y_eLH_{\text{d}} && \text{(SP:6.1.1)}\\
&= -H_2Qy_u^{\mathrm{T}}\bar{U} + H_1Qy_d^{\mathrm{T}}\bar{d} + H_1Ly_e^{\mathrm{T}}\bar{e}\\
V_3 &= \tilde{u}^*a_u\tilde{Q}H_{\text{u}} - \tilde{d}^*a_d\tilde{Q}H_{\text{d}} - \tilde{e}^*a_e\tilde{L}H_{\text{d}}\\
&= -H_2\tilde{Q}a_u^{\mathrm{T}}\tilde{u}^* + H_1\tilde{Q}a_d^{\mathrm{T}}\tilde{d}^* + H_1\tilde{L}a_e^{\mathrm{T}}\tilde{e}^*\\
V_2 &\supset \tilde{Q}^*m_Q^2\tilde{Q} + \tilde{L}^*m_L^2\tilde{L} + \tilde{u}^*m_u^2\tilde{u} + \tilde{d}^*m_d^2\tilde{d} + \tilde{e}^*m_e^2\tilde{e}\\
&= \tilde{Q}^*m_Q^2\tilde{Q} + \tilde{L}^*m_L^2\tilde{L} + \tilde{u}(m_u^2)^{\mathrm{T}}\tilde{u}^* + \tilde{d}(m_d^2)^{\mathrm{T}}\tilde{d}^* + \tilde{e}(m_e^2)^{\mathrm{T}}\tilde{e}^*\\
\mathcal{L} &\supset -\frac{M_1}{2}\tilde{b}\tilde{b} - \frac{M_2}{2}\tilde{w}\tilde{w} - \frac{M_3}{2}\tilde{g}\tilde{g} + \text{h.c.}, && \text{(SP:8.1.1)}
\end{aligned}$$

and the neutralinos and charginos are

$$\tilde{\psi}^0 = (\tilde{b}, \tilde{w}^3, \tilde{h}_1, \tilde{h}_2)^{\mathrm{T}}, \qquad \tilde{\psi}^+ = (\tilde{w}^+, \tilde{h}_2^+), \qquad \tilde{\psi}^- = (\tilde{w}^-, \tilde{h}_1^-). \qquad \text{(SP:8.2.1+15)}$$

So the notations can be matched with

$$\begin{aligned}
Y_U, Y_D, Y_E &= y_u^{\mathrm{T}}, y_d^{\mathrm{T}}, y_e^{\mathrm{T}} && (10)\\
T_U, T_D, T_E &= a_u^{\mathrm{T}}, a_d^{\mathrm{T}}, y_e^{\mathrm{T}} && (11)\\
m_Q^2, m_L^2 &= m_Q^2, m_L^2 && (12)\\
m_u^2, m_d^2, m_e^2 &= (m_u^2)^{\mathrm{T}}, (m_d^2)^{\mathrm{T}}, (m_e^2)^{\mathrm{T}}, && (13)\\
M_1, M_2, M_3 &= M_1, M_2, M_3, && (14)
\end{aligned}$$

where the LHS are the SLHA parameters and the RHS are those in SUSY primer.

2 Reverting sfermion rotation

In SLHA2 convention [4], the sfermion mass basis $\tilde{f}_i$ and super-CKM/PMNS basis $\tilde{\boldsymbol{f}}$ are related as

$$\tilde{f}_i = R_{iA}\tilde{f}_A, \tag{15}$$

where R is the rotation matrix (e.g. `USQMIX`), capital (small) indices are used to denote super-CKM/PMNS (mass) eigenstates, and the super-CKM/PMNS eigenstates are defined as, e.g.,

$$\tilde{\boldsymbol{u}} = (\tilde{u}_{\mathrm{L}}, \tilde{c}_{\mathrm{L}}, \tilde{t}_{\mathrm{L}}, \tilde{u}_{\mathrm{R}}, \tilde{c}_{\mathrm{R}}, \tilde{t}_{\mathrm{R}})^{\mathrm{T}}. \tag{16}$$

If the model has no flavor (and CP) violation and the left–right mixings in the first and second generation are negligible, we can convert the SLHA2 output to SLHA1 output. The SLHA1 mixing matrix R' is obtained as

$$R' = \begin{pmatrix} R_{a3} & R_{a6} \\ R_{b3} & R_{b6} \end{pmatrix} \tag{17}$$

with all elements are non-zero and $a<b$, and thus $\tilde{f}_a \equiv \tilde{f}_1^{\mathrm{3rd}}$ and $\tilde{f}_b \equiv \tilde{f}_2^{\mathrm{3rd}}$. The first and second generation fermions, $\tilde{f}_A$, are identified with $\tilde{f}_i$, where $|R_{iA}| > |R_{jA}|$ for all $j \neq i$.

References

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[2] J. F. Gunion and H. E. Haber, *Higgs Bosons in Supersymmetric Models (I)*, Nucl. Phys. **B272** (1986) 1–76 *[Erratum ibid.* ***B402*** *(1993) 567–568]*.

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