SLHA convention compared against References

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1 Comparison

1.1 Higgs potential

1.1.1 SLHA convention

The SLHA [1] is based on Gunion–Haber's notation [2]:

$$\begin{aligned} W &\supset -\epsilon_{ab}\mu H_1^a H_2^b \\ &= -\mu(H_1^1 H_2^2 - H_1^2 H_2^1) \\ &\equiv \mu(-H_1^0 H_2^0 + H_1^- H_2^+), \end{aligned} \tag{SLHA:3}$$

$$\begin{aligned} V_2 &\supset m_{H_1}^2|H_1|^2 + m_{H_2}^2|H_2|^2 - (m_3^2\epsilon_{ab}H_1^a H_2^b + \text{h.c.}), \\ &= m_{H_1}^2|H_1|^2 + m_{H_2}^2|H_2|^2 + \left[m_3^2(-H_1^0H_2^0 + H_1^-H_2^+) + \text{h.c.}\right]. \end{aligned} \tag{SLHA:7}$$

where $\epsilon_{12} = \epsilon^{12} = +1$. The parameter m_A^2, set by `EXTPAR 24`, is then defined as

$$m_A^2 = \frac{2m_3^2}{\sin 2\beta}. \tag{SLHA:8}$$

The Higgs vacuum expectation value is defined as

$$\langle H_i^0 \rangle = \frac{v_i}{\sqrt{2}}; \qquad m_Z^2 = \frac{1}{4}(g'^2 + g^2)(v_1^2 + v_2^2) = \frac{1}{4}(g'^2 + g^2)v^2, \tag{1}$$

i.e., $v \simeq 246\,\text{GeV}$ is given in the `HMIX 3`.

1.1.2 Comparison to GH convention

In Gunion–Haber [2], the definitions are as follows, with their errata applied:

$$W \supset -\epsilon_{ab}\mu H_1^a H_2^b, \tag{GH:3.3–3.4}$$

$$V_{\text{soft}} \supset m_1^2|H_1|^2 + m_2^2|H_2|^2 - (m_{12}^2\epsilon_{ab}H_1^a H_2^b + \text{h.c.}), \tag{GH:3.9}$$

where $\bar{E} \equiv \tilde{R}$ in their notation, with the same definition $\epsilon_{12} = 1$ (found below Eq. (3.2)). So, with the identification $(m_1^2, m_2^2, m_{12}^2) \equiv (m_{H_1}^2, m_{H_2}^2, m_3^2)$, this is identical to SLHA convention. Also we note the difference of the vacuum expectation values:

$$\langle H_1 \rangle = \begin{pmatrix} \bar{v}_1 \\ 0 \end{pmatrix}, \qquad \langle H_2 \rangle = \begin{pmatrix} 0 \\ \bar{v}_2 \end{pmatrix}, \tag{GH:3.7}$$

with $\bar v_1 > 0$, $\bar v_2 > 0$ (3.24), and $\tan\beta = \bar v_2/\bar v_1 = v_2/v_1$ (2.8). Here we use bars to denote vs under this definition.

We can then use their results, which say

$$m_{H_1}^2 = -|\mu|^2 + 2\lambda_1\bar v_2^2 - m_Z^2/2, \tag{GH:3.21c}$$
$$m_{H_2}^2 = -|\mu|^2 + 2\lambda_1\bar v_1^2 - m_Z^2/2, \tag{GH:3.21d}$$
$$m_{H_1}^2 + m_{H_2}^2 + 2|\mu|^2 = m_3^2(\tan\beta + \cot\beta) = \frac{2m_3^2}{\sin 2\beta}, \tag{GH:3.22}$$

and, noting that

$$m_Z^2 = (g^2 + g'^2)\bar v^2/2, \qquad m_W^2 = g^2\bar v^2/2, \tag{2}$$

which are found below Eq. (3.19),

$$\lambda_1 = \frac{g^2 + g'^2}{4} + \frac{m_3^2}{2\bar v_1\bar v_2} = \frac{m_Z^2}{2(\bar v_1^2 + \bar v_2^2)} + \frac{m_3^2}{2\bar v_1\bar v_2}; \tag{3}$$

combining them, we have

$$m_{H_1}^2 = -|\mu|^2 - \frac{m_Z^2}{2}\cos 2\beta + m_3^2\tan\beta, \tag{4}$$
$$m_{H_2}^2 = -|\mu|^2 + \frac{m_Z^2}{2}\cos 2\beta + m_3^2\cot\beta. \tag{5}$$

Also the Higgs parameters are given by

$$m_{H^\pm}^2 = (4\lambda_1 - g'^2)\bar v^2 = m_W^2 + m_A^2, \tag{GH:3.16}$$
$$m_{H_3^0}^2 = m_{H^\pm}^2 - m_W^2 = m_A^2, \tag{GH:3.17}$$
$$m_{H_1^0,H_2^0}^2 = \frac{1}{2}\left[m_{H_3^0}^2 + m_Z^2 \pm \sqrt{(m_{H_3^0}^2 + m_Z^2)^2 - 4m_Z^2 m_{H_3^0}^2\cos^2 2\beta}\right], \tag{GH:3.18}$$
$$\tan 2\alpha = \frac{m_{H_3^0}^2 + m_Z^2}{m_{H_3^0}^2 - m_Z^2}\tan 2\beta. \tag{GH:3.19}$$

1.1.3 Comparison to SUSY Primer convention

We here compare the SLHA/GH notation with Martin's SUSY primer [3]. The potentials are given by

$$W_{\rm MSSM} \supset \mu H_{\rm u} H_{\rm d} \equiv \mu(H_2^+ H_1^- - H_1^0 H_2^0), \tag{SP:6.1.1–3}$$
$$-\mathcal{L}_{\rm soft} \supset m_{H_{\rm d}}^2|H_{\rm d}|^2 + m_{H_{\rm u}}^2|H_{\rm u}|^2 + (b\epsilon^{ab}H_{\rm u}^a H_{\rm d}^b + {\rm h.c.}), \tag{SP:6.3.1}$$
$$\bar v_u = \langle H_{\rm u}^0\rangle, \quad \bar v_d = \langle H_{\rm d}^0\rangle, \tag{SP:8.1.5}$$

where $\epsilon^{12} = +1$ (2.13). So the parameters are identified by the replacement

$$\mu = \mu, \tag{6}$$
$$m_3^2 = b, \tag{7}$$
$$m_{H_1}^2 = m_{H_{\rm d}}^2, \tag{8}$$
$$m_{H_2}^2 = m_{H_{\rm u}}^2, \tag{9}$$

where the LHS are the SLHA parameters and the RHS are those in SUSY primer. With this identification, we can confirm that the above-shown formulae agree with SUSY Primer's equations (8.1.8)–(8.1.11) and (8.1.19)–(8.1.22).

1.2 Interaction terms

Here, to simplify the notation, we omit the SU(2) indices with assuming

$$AB = -BA \equiv A^1B^2 - B^1A^2.$$

1.2.1 SLHA convention

The SLHA [1] convention for the interection terms are

$$W = -H_2QY_U\bar{U} + H_1QY_D\bar{D} + H_1LY_E\bar{E}, \tag{SLHA:3}$$

$$V_3 = -H_2\tilde{Q}T_U\tilde{u}^* + H_1\tilde{Q}T_D\tilde{d}^* + H_1\tilde{L}T_E\tilde{e}^* \tag{SLHA:5}$$

$$V_2 = \tilde{Q}^*m_Q^2\tilde{Q} + \tilde{L}^*m_L^2\tilde{L} + \tilde{u}m_u^2\tilde{u}^* + \tilde{d}m_d^2\tilde{d}^* + \tilde{e}m_e^2\tilde{e}^* \tag{SLHA:7}$$

$$\mathcal{L}_G = \frac{M_1}{2}\tilde{b}\tilde{b} + \frac{M_2}{2}\tilde{w}\tilde{w} + \frac{M_3}{2}\tilde{g}\tilde{g} + \text{h.c.} \tag{SLHA:9}$$

The neutralino and charginos are defined as

$$\tilde{\psi}^0 = (-\mathrm{i}\tilde{b}, -\mathrm{i}\tilde{w}^3, \tilde{h}_1, \tilde{h}_2)^{\mathrm{T}}, \qquad \tilde{\psi}^+ = (-\mathrm{i}\tilde{w}^+, \tilde{h}_2^+), \qquad \tilde{\psi}^- = (-\mathrm{i}\tilde{w}^-, \tilde{h}_1^-). \tag{SLHA:10+13}$$

1.2.2 Comparison to SUSY Primer convention

In SUSY primer [3], the interaction terms are defined by

$$\begin{aligned}
W_{\mathrm{MSSM}} &\supset \bar{U}y_uQH_{\mathrm{u}} - \bar{d}y_dQH_{\mathrm{d}} - \bar{e}y_eLH_{\mathrm{d}} && \text{(SP:6.1.1)}\\
&= -H_2Qy_u^{\mathrm{T}}\bar{U} + H_1Qy_d^{\mathrm{T}}\bar{d} + H_1Ly_e^{\mathrm{T}}\bar{e}\\
V_3 &= \tilde{u}^*a_u\tilde{Q}H_{\mathrm{u}} - \tilde{d}^*a_d\tilde{Q}H_{\mathrm{d}} - \tilde{e}^*a_e\tilde{L}H_{\mathrm{d}}\\
&= -H_2\tilde{Q}a_u^{\mathrm{T}}\tilde{u}^* + H_1\tilde{Q}a_d^{\mathrm{T}}\tilde{d}^* + H_1\tilde{L}a_e^{\mathrm{T}}\tilde{e}^*\\
V_2 &\supset \tilde{Q}^*m_Q^2\tilde{Q} + \tilde{L}^*m_L^2\tilde{L} + \tilde{u}^*m_u^2\tilde{u} + \tilde{d}^*m_d^2\tilde{d} + \tilde{e}^*m_e^2\tilde{e}\\
&= \tilde{Q}^*m_Q^2\tilde{Q} + \tilde{L}^*m_L^2\tilde{L} + \tilde{u}(m_u^2)^{\mathrm{T}}\tilde{u}^* + \tilde{d}(m_d^2)^{\mathrm{T}}\tilde{d}^* + \tilde{e}(m_e^2)^{\mathrm{T}}\tilde{e}^*\\
\mathcal{L} &\supset -\frac{M_1}{2}\tilde{b}\tilde{b} - \frac{M_2}{2}\tilde{w}\tilde{w} - \frac{M_3}{2}\tilde{g}\tilde{g} + \text{h.c.}, && \text{(SP:8.1.1)}
\end{aligned}$$

and the neutralinos and charginos are

$$\tilde{\psi}^0 = (\tilde{b}, \tilde{w}^3, \tilde{h}_1, \tilde{h}_2)^{\mathrm{T}}, \qquad \tilde{\psi}^+ = (\tilde{w}^+, \tilde{h}_2^+), \qquad \tilde{\psi}^- = (\tilde{w}^-, \tilde{h}_1^-). \tag{SP:8.2.1+15}$$

So the notations can be matched with

$$Y_U, Y_D, Y_E = y_u^{\mathrm{T}}, y_d^{\mathrm{T}}, y_e^{\mathrm{T}} \tag{10}$$

$$T_U, T_D, T_E = a_u^{\mathrm{T}}, a_d^{\mathrm{T}}, y_e^{\mathrm{T}} \tag{11}$$

$$m_Q^2, m_L^2 = m_Q^2, m_L^2 \tag{12}$$

$$m_u^2, m_d^2, m_e^2 = (m_u^2)^{\mathrm{T}}, (m_d^2)^{\mathrm{T}}, (m_e^2)^{\mathrm{T}}, \tag{13}$$

$$M_1, M_2, M_3 = M_1, M_2, M_3, \tag{14}$$

where the LHS are the SLHA parameters and the RHS are those in SUSY primer.

References

[1] P. Z. Skands *et al.*, *SUSY Les Houches accord: Interfacing SUSY spectrum calculators, decay packages, and event generators*, JHEP **07** (2004) 036 [hep-ph/0311123].

[2] J. F. Gunion and H. E. Haber, *Higgs Bosons in Supersymmetric Models (I)*, Nucl. Phys. **B272** (1986) 1–76 *[Erratum ibid.* ***B402*** *(1993) 567–568]*.

[3] S. P. Martin, *A Supersymmetry primer*. hep-ph/9709356. Collected in Adv. Ser. Direct. High Energy Phys. **18** (1998) 1–98 and ibid. **21** (2010) 1–153.