SLHA convention compared against References

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0.1 Higgs potential

0.1.1 SLHA convention

The SLHA [?] is based on Gunion–Haber's notation [?]:

$$\begin{aligned} W &\supset -\epsilon_{ab}\mu H_1^a H_2^b \\ &= -\mu(H_1^1 H_2^2 - H_1^2 H_2^1) \\ &\equiv \mu(-H_1^0 H_2^0 + H_1^- H_2^+), \end{aligned} \tag{SLHA:3}$$

$$\begin{aligned} V_2 &\supset m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 - (m_3^2 \epsilon_{ab} H_1^a H_2^b + \text{h.c.}), \\ &= m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + \left[m_3^2(-H_1^0 H_2^0 + H_1^- H_2^+) + \text{h.c.}\right]. \end{aligned} \tag{SLHA:7}$$

where $\epsilon_{12} = \epsilon^{12} = +1$. The parameter m_A^2, set by `EXTPAR 24`, is then defined as

$$m_A^2 = \frac{2m_3^2}{\sin 2\beta}. \tag{SLHA:8}$$

0.1.2 Comparison to GH convention

In Gunion–Haber [?], the definitions are as follows, with their errata applied:

$$W \supset -\epsilon_{ab}\mu H_1^a H_2^b, \tag{GH:3.3–3.4}$$

$$V_{\text{soft}} \supset m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - (m_{12}^2 \epsilon_{ab} H_1^a H_2^b + \text{h.c.}), \tag{GH:3.9}$$

where $\bar{E} \equiv \tilde{R}$ in their notation, with the same definition $\epsilon_{12} = 1$ (found below Eq. (3.2)). So, with the identification $(m_1^2, m_2^2, m_{12}^2) \equiv (m_{H_1}^2, m_{H_2}^2, m_3^2)$, this is identical to SLHA convention. Also we note

$$\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \tag{GH:3.7}$$

with $v_1 > 0$, $v_2 > 0$ (3.24), and $\tan\beta = v_2/v_1$ (2.8).

We can then use their results, which say

$$m_{H_1}^2 = -|\mu|^2 + 2\lambda_1 v_2^2 - m_Z^2/2, \tag{GH:3.21c}$$

$$m_{H_2}^2 = -|\mu|^2 + 2\lambda_1 v_1^2 - m_Z^2/2, \tag{GH:3.21d}$$

$$m_{H_1}^2 + m_{H_2}^2 + 2|\mu|^2 = m_3^2(\tan\beta + \cot\beta) = \frac{2m_3^2}{\sin 2\beta}, \tag{GH:3.22}$$

and, noting that

$$m_Z^2 = (g^2 + g'^2)(v_1^2 + v_2^2)/2, \qquad m_W^2 = g^2(v_1^2 + v_2^2)/2, \tag{1}$$

which are found below Eq. (3.19),

$$\lambda_1 = \frac{g^2 + g'^2}{4} + \frac{m_3^2}{2v_1 v_2} = \frac{m_Z^2}{2(v_1^2 + v_2^2)} + \frac{m_3^2}{2v_1 v_2}; \tag{2}$$

combining them, we have

$$m_{H_1}^2 = -|\mu|^2 - \frac{m_Z^2}{2}\cos 2\beta + m_3^2 \tan\beta, \tag{3}$$

$$m_{H_2}^2 = -|\mu|^2 + \frac{m_Z^2}{2}\cos 2\beta + m_3^2 \cot\beta. \tag{4}$$

Also the Higgs parameters are given by

$$m_{H^\pm}^2 = (4\lambda_1 - g'^2)(v_1^2 + v_2^2) = m_W^2 + m_A^2, \tag{GH:3.16}$$

$$m_{H_3^0}^2 = m_{H^\pm}^2 - m_W^2 = m_A^2, \tag{GH:3.17}$$

$$m_{H_1^0,H_2^0}^2 = \frac{1}{2}\left[m_{H_3^0}^2 + m_Z^2 \pm \sqrt{(m_{H_3^0}^2 + m_Z^2)^2 - 4m_Z^2 m_{H_3^0}^2 \cos^2 2\beta}\right], \tag{GH:3.18}$$

$$\tan 2\alpha = \frac{m_{H_3^0}^2 + m_Z^2}{m_{H_3^0}^2 - m_Z^2}\tan 2\beta. \tag{GH:3.19}$$

0.1.3 Comparison to SUSY Primer convention

We here compare the SLHA/GH notation with Martin's SUSY primer. The potentials are given by

$$W_{\mathrm{MSSM}} \supset \mu H_{\mathrm{u}} H_{\mathrm{d}} \equiv \mu(H_2^+ H_1^- - H_1^0 H_2^0), \tag{SP:6.1.1–3}$$

$$-\mathcal{L}_{\mathrm{soft}} \supset m_{H_{\mathrm{d}}}^2 |H_{\mathrm{d}}|^2 + m_{H_{\mathrm{u}}}^2 |H_{\mathrm{u}}|^2 + (b\epsilon^{ab} H_{\mathrm{u}}^a H_{\mathrm{d}}^b + \mathrm{h.c.}), \tag{SP:6.3.1}$$

where $\epsilon^{12} = +1$ (2.13). So the parameters are identified by the replacement

$$\mu = \mu, \tag{5}$$

$$m_3^2 = b, \tag{6}$$

$$m_{H_1}^2 = m_{H_{\mathrm{d}}}^2, \tag{7}$$

$$m_{H_2}^2 = m_{H_{\mathrm{u}}}^2, \tag{8}$$

where the LHS are the SLHA parameters and the RHS are those in SUSY primer. With this identification, we can confirm that the above-shown formulae agree with SUSY Primer's equations (8.1.8)–(8.1.11) and (8.1.19)–(8.1.22).

0.2 Interaction terms

Here, to simplify the notation, we omit the SU(2) indices with assuming

$$AB = -BA \equiv A^1 B^2 - B^1 A^2.$$

0.2.1 SLHA convention

The SLHA [?] convention for the interection terms are

$$W = -H_2 Q Y_U \bar{U} + H_1 Q Y_D \bar{D} + H_1 L Y_E \bar{E}, \tag{SLHA:3}$$
$$V_3 = -H_2 \tilde{Q} T_U \tilde{u}^* + H_1 \tilde{Q} T_D \tilde{d}^* + H_1 \tilde{L} T_E \tilde{e}^* \tag{SLHA:5}$$
$$V_2 = \tilde{Q}^* m_Q^2 \tilde{Q} + \tilde{L}^* m_L^2 \tilde{L} + \tilde{u} m_u^2 \tilde{u}^* + \tilde{d} m_d^2 \tilde{d}^* + \tilde{e} m_e^2 \tilde{e}^* \tag{SLHA:7}$$
$$\mathcal{L}_G = \frac{M_1}{2}\tilde{b}\tilde{b} + \frac{M_2}{2}\tilde{w}\tilde{w} + \frac{M_3}{2}\tilde{g}\tilde{g} + \text{h.c.} \tag{SLHA:9}$$

0.2.2 Comparison to SUSY Primer convention

In SUSY primer, the interaction terms are defined by

$$\begin{aligned} W_{\text{MSSM}} &\supset \bar{U} y_u Q H_{\text{u}} - \bar{d} y_d Q H_{\text{d}} - \bar{e} y_e L H_{\text{d}} && \text{(SP:6.1.1)} \\ &= -H_2 Q y_u^{\text{T}} \bar{U} + H_1 Q y_d^{\text{T}} \bar{d} + H_1 L y_e^{\text{T}} \bar{e} \\ V_3 &= \tilde{u}^* a_u \tilde{Q} H_{\text{u}} - \tilde{d}^* a_d \tilde{Q} H_{\text{d}} - \tilde{e}^* a_e \tilde{L} H_{\text{d}} \\ &= -H_2 \tilde{Q} a_u^{\text{T}} \tilde{u}^* + H_1 \tilde{Q} a_d^{\text{T}} \tilde{d}^* + H_1 \tilde{L} a_e^{\text{T}} \tilde{e}^* \\ V_2 &\supset \tilde{Q}^* m_Q^2 \tilde{Q} + \tilde{L}^* m_L^2 \tilde{L} + \tilde{u}^* m_u^2 \tilde{u} + \tilde{d}^* m_d^2 \tilde{d} + \tilde{e}^* m_e^2 \tilde{e} \\ &= \tilde{Q}^* m_Q^2 \tilde{Q} + \tilde{L}^* m_L^2 \tilde{L} + \tilde{u}(m_u^2)^{\text{T}} \tilde{u}^* + \tilde{d}(m_d^2)^{\text{T}} \tilde{d}^* + \tilde{e}(m_e^2)^{\text{T}} \tilde{e}^* \\ \mathcal{L} &\supset -\frac{M_1}{2}\tilde{b}\tilde{b} - \frac{M_2}{2}\tilde{w}\tilde{w} - \frac{M_3}{2}\tilde{g}\tilde{g} + \text{h.c.}. && \text{(SP:8.1.1)} \end{aligned}$$

So the notations can be matched with

$$Y_U, Y_D, Y_E = y_u^{\text{T}}, y_d^{\text{T}}, y_e^{\text{T}} \tag{9}$$
$$T_U, T_D, T_E = a_u^{\text{T}}, a_d^{\text{T}}, y_e^{\text{T}} \tag{10}$$
$$m_Q^2, m_L^2 = m_Q^2, m_L^2 \tag{11}$$
$$m_u^2, m_d^2, m_e^2 = (m_u^2)^{\text{T}}, (m_d^2)^{\text{T}}, (m_e^2)^{\text{T}}, \tag{12}$$
$$M_1, M_2, M_3 = -M_1, -M_2, -M_3, \tag{13}$$

where the LHS are the SLHA parameters and the RHS are those in SUSY primer.