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No. XS-1

Date . .

Review of Crosssection

CM-frame

$$p_1^\mu = \begin{pmatrix} E_1 \\ p \end{pmatrix} \quad p_2^\mu = \begin{pmatrix} E_2 \\ -p \end{pmatrix} \quad \therefore \quad s = (E_1+E_2)^2 \quad v = \frac{E_1+E_2}{E_1E_2}p$$

$\sigma = f \times$ Lor. inv.

$$v_1 = \begin{pmatrix} 0 \\ 0 \\ p/E_1 \end{pmatrix} \quad v_2 = \begin{pmatrix} 0 \\ 0 \\ -p/E_2 \end{pmatrix} \qquad f = \frac{1}{4E_1E_2\|v_{rel}\|} = \frac{1}{4p(E_1+E_2)}$$

$$\Lambda = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix}$$

"Lab"-frame

$$\bar{p}_1^\mu = \Lambda^\mu{}_\nu p_1^\nu = \begin{pmatrix} \bar{E}_1 \\ \bar{p}_1 \end{pmatrix} = \begin{pmatrix} \gamma(E_1+\beta p) \\ \gamma(p+\beta E_1) \end{pmatrix} \Rightarrow s = (E_1+E_2)^2$$

$$\bar{p}_2^\mu = \begin{pmatrix} \bar{E}_2 \\ \bar{p}_2 \end{pmatrix} = \begin{pmatrix} \gamma(E_2-\beta p) \\ \gamma(-p+E_2\beta) \end{pmatrix}$$

$$\bar{v}_1^z = \frac{p+E_1\beta}{E_1+p\beta} = \frac{v_1^z+\beta}{1+v_1^z\beta}$$

$$\bar{v}_2^z = \frac{-p+\beta E_2}{E_2-p\beta} = \frac{v_2^z+\beta}{1+v_2^z\beta}$$

$$\Downarrow$$

$$\bar{v} = \frac{E_1+E_2}{\bar{E}_1\bar{E}_2}p = \frac{E_1E_2}{\bar{E}_1\bar{E}_2}v$$

Since $n_1 n_2 \sigma v$ is Lorentz invariant,
(thus $n_1 n_2 f v$)

$$\bar{f} = \frac{n_1n_2v}{\bar{n}_1\bar{n}_2\bar{v}}f = \frac{1}{\frac{\bar{E}_1}{E_1}\frac{\bar{E}_2}{E_2}\frac{E_1E_2}{\bar{E}_1\bar{E}_2}}f$$

$$= \frac{1}{4p(E_1+E_2)}$$

$p_1 - p_2 = 2p$

$\bar{p}_1 - \bar{p}_2 = 2\gamma p + \beta\gamma(E_1+E_2)$

$$= \frac{1}{4\sqrt{(\bar{p}_1\cdot\bar{p}_2)^2 - m_1^2m_2^2}}$$

$$\therefore \sigma_{Lab} = \frac{1}{4\sqrt{(\bar{p}_1\cdot\bar{p}_2)^2-m_1^2m_2^2}}\int dLips \times |M|^2$$

$$\left(\sigma_{CM} = \frac{1}{4\sqrt{(p_1\cdot p_2)^2-m_1^2m_2^2}}\int \cdots\right)$$

$$n_1 = \frac{E_1}{m_1}n_1^0, \quad n_2 = \frac{E_2}{m}n_2^0$$

$$\bar{n}_1 = \frac{\bar{E}_1}{m_1}n_1^0 = \frac{\bar{E}_1}{E_1}n_1$$

$$\underbrace{\boxed{n_1}}_{\Delta T v_1} S \quad \underbrace{\boxed{n_2}}_{\Delta T v_2}$$

n. of collision for one ②:

$S n_1 \Delta T v_1$

n of coll of $S n_1 v_1 \Delta T \times n_2 \Delta V$

$$\tilde\Lambda = \begin{pmatrix}\gamma' & \beta'\gamma' & 0 & 0\\ \beta'\gamma' & \gamma' & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

General Frame

$$\tilde p_1^{\mu} = \tilde\Lambda^{\mu}{}_{\nu}\,\bar p_1 = \begin{pmatrix}\gamma\gamma'(E_1+\beta p)\\ \gamma\gamma'\beta'(E_1+\beta p)\\ 0\\ \gamma(p+\beta E_1)\end{pmatrix} \qquad \tilde p_2^{\mu} = \begin{pmatrix}\gamma\gamma'(E_2-\beta p)\\ \gamma\gamma'\beta'(E_2-\beta p)\\ 0\\ -\gamma(p-E_2\beta)\end{pmatrix}$$

$$\tilde v_1 = \begin{pmatrix}\beta'\\ 0\\ \dfrac{p+E_1\beta}{\gamma'(E_1+\beta p)}\end{pmatrix} \qquad \tilde v_2 = \begin{pmatrix}\beta'\\ 0\\ -\dfrac{p-E_2\beta}{\gamma'(E_2-\beta p)}\end{pmatrix}$$

$$\|\tilde v_1 - \tilde v_2\| = \frac{E_1E_2}{\gamma'\bar E_1\bar E_2}\|v_1-v_2\| = \frac{\gamma' E_1 E_2}{\tilde E_1\tilde E_2}\|v_1-v_2\|$$

$$\tilde f = \frac{1}{\frac{\tilde E_1}{E_1}\,\frac{\tilde E_2}{E_2}\,\frac{\gamma' E_1E_2}{\tilde E_1\tilde E_2}}\, f = \frac{1}{4\gamma'\sqrt{(\tilde p_1\cdot\tilde p_2)^2 - m_1^2m_2^2}}$$

$$= \frac{1}{4(\tilde E_1+\tilde E_2)}\sqrt{\frac{(\tilde E_1+\tilde E_2)^2-(\tilde p_{x1}+\tilde p_{x2})^2}{(\tilde p_1\cdot\tilde p_2)^2-m_1^2m_2^2}}$$

$$\tilde f\,\|\tilde v_1-\tilde v_2\| = \frac{f}{\frac{\tilde E_1\tilde E_2}{E_1E_2}}\|v_1-v_2\| = \frac{1}{4\tilde E_1\tilde E_2}$$