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Appendix A

$K = (\Phi^\dagger)_i Z_{ij} \Phi_j$

$Z = R D R^\dagger$

$= (R D^{\frac{1}{2}} R^\dagger)(R D^{\frac{1}{2}} R^\dagger)$

$\Rightarrow \sqrt{Z} = R D^{\frac{1}{2}} R^\dagger$

$\sqrt{Z} = \sqrt{Z}^\dagger$ for positive eigenvalues

$\begin{cases} \Phi_r = \sqrt{Z}\,\Phi \\ \Phi_r^\dagger = \Phi^\dagger \sqrt{Z} \end{cases}$

$C^2 = |\xi_{11}|^2 + |\xi_{12}|^2$

∘ Toy model

$$W = X D \bar{D} + (y^0 D + Y^0 H) l e$$
$$\equiv X \phi_1 \bar{D} + y^0_i \phi_i l e \quad @ \Lambda$$
$$K = \text{canonical Kähler} \quad @ \Lambda$$

$$\phi_r(\mu)_i = \sqrt{Z(\mu)}_{ij}\,\phi_j(\Lambda) \quad \leftrightarrow \quad \phi_i(\Lambda) \equiv \xi_{ij}(\mu)\,\phi_r(\mu)_j$$
$$l_r(\mu) = \sqrt{Z_l(\mu)}\; l(\Lambda)$$
$$e_r(\mu) = \sqrt{Z_e(\mu)}\; e(\Lambda)$$

running from Λ to μ

At messenger scale μ_X,

$$W \supset k X \phi_1 \bar{D}$$
$$= \underset{\substack{\uparrow \\ \text{NON RENORM. theorem}}}{k} \frac{1}{\sqrt{Z_X(\mu_X)}} X_r(\mu_X) \underbrace{\overbrace{\left[(Z^{\frac{1}{2}})^{-1}\right]_{1j}}^{\xi_{1j}} \phi_{rj}(\mu_X)}_{\text{MASSIVE}} \frac{1}{\sqrt{Z_{\bar{D}}(\mu_X)}} \bar{D}_r(\mu_X)$$

$$\underset{\substack{\text{MASSIVE EIGEN} \\ \text{HEAVY @ } \mu_X}}{\tilde{\phi}_{r1}} \propto \phi_1(\Lambda) \Longrightarrow \frac{1}{C}(\xi_{11}\phi_{r1} + \xi_{12}\phi_{r2})$$

$$\hat{\phi}_{r2} = \frac{1}{C}(-\xi^*_{12}\phi_{r1} + \xi_{11}\phi_{r2})$$

$$W = \frac{k}{\sqrt{Z_X Z_{\bar{D}}}} C X \tilde{\phi}_{r1} \bar{D}$$
$$+ \tilde{\phi}_{r1} l_r e_r \frac{C}{\sqrt{Z_l Z_e}} \left[y^0_1 + \frac{1}{C^2}(\xi_{11}\xi_{21} + \xi^*_{12}\xi_{22}) y^0_2 \right]$$
$$+ \tilde{\phi}_{r2} l_r e_r \frac{1}{C\sqrt{Z_l Z_e}} (\xi_{11}\xi_{22} - \xi_{12}\xi_{21}) y^0_2$$

$$\gamma \equiv \frac{\partial \ln Z}{\partial \ln \mu}$$

$$\begin{pmatrix} \cos\alpha & e^{i\phi}\sin\alpha \\ -e^{-i\phi}\sin\alpha & \cos\alpha \end{pmatrix}$$

Approximating at the 1-loop level,

$$\tilde{y}_1(M) = \frac{C}{\sqrt{Z_\ell Z_e}}(M)\left[y_1^0 + \frac{\xi_{11}\xi_{21} + \xi_{12}^*\xi_{22}}{C^2}(M)\, y_2^0 \right]$$

$$\approx \frac{1}{\sqrt{Z_\ell Z_e Z_{11}}}(M)\, y_1^0 + ?$$

$$\tilde{y}_2(M) \approx \frac{1}{\sqrt{Z_\ell Z_e Z_{22}}}(M)\, y_2^0$$

$$\begin{array}{ccl} y_1^0(\Lambda) & y_2^0(\Lambda) & \leftarrow \phi_1 \text{ defined as that coupling to } X \\ \downarrow & \downarrow & \text{running} \\ & & \text{mass eigenstates} \\ \tilde{y}_{r1}(M) & \tilde{y}_{r2}(M) & \end{array}$$

M

$$Y(M)$$
$$\downarrow$$
$$Y(\mu)$$