Continuous Variable Quantum Computing

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Here I practice Continuous Variable Quantum Computing (CVQC).

I. BASICS

Relevant papers: [1], [2], [3].

$$\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{iyx}\, dy, \qquad \int_{-\infty}^{\infty} f(x)\,\delta(x)\,dx = f(0) \tag{1}$$

$$a = \frac{1}{\sqrt{2}}\left(q + ip\right), \qquad a^\dagger = \frac{1}{\sqrt{2}}\left(q - ip\right), \tag{2}$$

$$\hat{q}\left|s\right\rangle_q = s\left|s\right\rangle_q, \quad \hat{p}\left|s\right\rangle_p = s\left|s\right\rangle_p, \qquad \left\langle r\right|_{p/q}\left|s\right\rangle_{p/q} = \delta(r-s) \tag{3}$$

$$\left\langle r\right|_p\left|s\right\rangle_q = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dt\, e^{-its}\left\langle r\right|_p\left|t\right\rangle_p = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dt\, e^{-its}\delta(r-t) = \frac{e^{-irs}}{\sqrt{2\pi}} \tag{4}$$

$$X(s) = e^{-is\hat{p}}, \quad Z(s) = e^{is\hat{q}}, \quad \left|s\right\rangle_q = X(s)\left|0\right\rangle_q, \quad \left|s\right\rangle_p = Z(s)\left|0\right\rangle_p \tag{5}$$

$$\left|s\right\rangle_p = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dr\, e^{irs}\left|r\right\rangle_q = \mathcal{F}\left|s\right\rangle_q, \tag{6}$$

$$\left|s\right\rangle_q = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dr\, e^{-irs}\left|r\right\rangle_p = \mathcal{F}^\dagger\left|s\right\rangle_p. \tag{7}$$

$$\mathcal{F}\left|\phi\right\rangle = \mathcal{F}\int \mathrm{d}s\,\phi_q(s)\left|s\right\rangle_q = \frac{1}{\sqrt{2\pi}}\int \mathrm{d}s\,\mathrm{d}r\,\phi_q(s)e^{irs}\left|r\right\rangle_q = \int \mathrm{d}s\,\phi_q(s)\left|s\right\rangle_p \tag{8}$$

$$C_Z^{ij} = e^{i\hat{q}_i\hat{q}_j} \tag{9}$$

Stabilizers:

$$K_i(s) = X_i(s)\prod_{j\in N(i)} Z_j(s), \quad i = 1,\ldots,n \tag{10}$$

Nullifiers:

$$H_i = \hat{p}_i - \sum_{j\in N(i)}\hat{q}_j, \quad i = 1,\ldots,n \tag{11}$$

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Finitely squeezed vacuum states:

$$
\begin{aligned}
|0,\Omega\rangle_p &:= \left(\pi\Omega^2\right)^{-1/4}\int dp\, e^{\frac{-p^2}{2\Omega^2}}\,|p\rangle_p \\
&= \left(\pi\Omega^2\right)^{-1/4}\frac{1}{\sqrt{2\pi}}\int dp\,dr\, e^{\frac{-p^2}{2\Omega^2}}e^{irp}\,|r\rangle_q \\
&= \left(\pi\Omega^2\right)^{-1/4}\frac{1}{\sqrt{2\pi}}\sqrt{2\pi}\Omega\int dr\, e^{-\frac{1}{2}r^2\Omega^2}\,|r\rangle_q \\
&= \left(\pi\Omega^{-2}\right)^{-1/4}\int dr\, e^{-\frac{1}{2}r^2\Omega^2}\,|r\rangle_q
\end{aligned}
\tag{12}
$$

$$
\langle\hat{p}^2\rangle = \frac{\Omega^2}{2}, \quad |0,\Omega\rangle_p = |0,\Omega^{-1}\rangle_q \tag{13}
$$

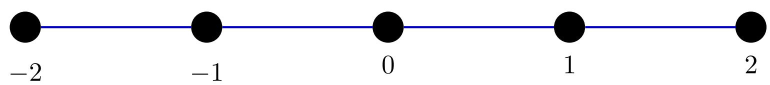
II. MEASUREMENTS ON LINEAR CLUSTER STATES

With nullifier formalism, one is allowed to

1. Replace any nullifier with the linear combination of nullifiers;
2. For any i make following replacements for all p_i and q_i: $p_i \to q_i$ and $q_i \to -p_i$;
3. For any i make following replacements for all p_i and q_i: $p_i \to -p_i$ and $q_i \to -q_i$;

The latter two are just single-qumode operations that do not destroy the entanglement.

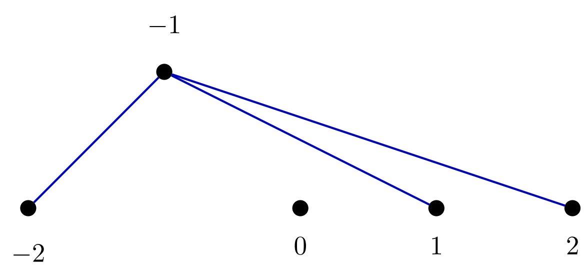
Let's consider a 5-qumode linear cluster state:



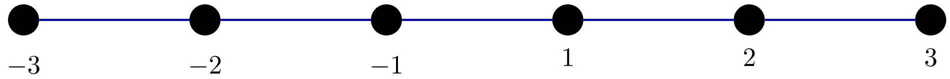
I am labeling qumodes from -2 to 2. This way it is easier to observe the syummetries. The nullifying set is

$$\textit{nullifying set} = \left\{ \begin{array}{l} \hat{p}_{-2} - \hat{q}_{-1} \\ \hat{p}_{-1} - \hat{q}_{-2} - \hat{q}_0 \\ \hat{p}_0 - \hat{q}_{-1} - \hat{q}_1 \\ \hat{p}_1 - \hat{q}_0 - \hat{q}_2 \\ \hat{p}_2 - \hat{q}_1 \end{array} \right\} \equiv \left\{ \begin{array}{l} \hat{p}_{-2} - \hat{q}_{-1} \\ \hat{p}_{-1} - \hat{p}_1 - \hat{q}_{-2} + \hat{q}_2 \\ \hat{p}_0 - \hat{q}_{-1} - \hat{q}_1 \\ \hat{p}_1 - \hat{q}_0 - \hat{q}_2 \\ \hat{p}_2 - \hat{q}_1 \end{array} \right\} \xrightarrow{\text{measure}} \left\{ \begin{array}{l} \hat{p}_{-2} - \hat{q}_{-1} \\ \hat{p}_{-1} - \hat{p}_1 - \hat{q}_{-2} + \hat{q}_2 \\ m_0 - \hat{q}_{-1} - \hat{q}_1 \\ \hat{p}_0 - m_0 \\ \hat{p}_2 - \hat{q}_1 \end{array} \right\}$$

$$\xrightarrow[\substack{\hat{p}_2 \to -\hat{p}_2 \\ \hat{q}_2 \to -\hat{q}_2}]{\substack{\hat{p}_1 \to \hat{q}_1 \\ \hat{q}_1 \to -\hat{p}_1}} \left\{ \begin{array}{l} \hat{p}_{-2} - \hat{q}_{-1} \\ \hat{p}_{-1} - \hat{q}_1 - \hat{q}_{-2} - \hat{q}_2 \\ m_0 - \hat{q}_{-1} + \hat{p}_1 \\ \hat{p}_0 - m_0 \\ -\hat{p}_2 + \hat{p}_1 \end{array} \right\} \equiv \left\{ \begin{array}{l} \hat{p}_{-2} - \hat{q}_{-1} \\ \hat{p}_{-1} - \hat{q}_1 - \hat{q}_{-2} - \hat{q}_2 \\ m_0 - \hat{q}_{-1} + \hat{p}_1 \\ \hat{p}_0 - m_0 \\ -(\hat{p}_2 - \hat{q}_{-1} + m_0) \end{array} \right\}$$

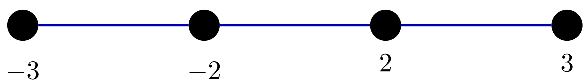


The state was symmetric, the measurement was also symmetric and the final result is not symmetric. This is confusing. We could have done very similar manipulations for the step where we rotated the first phase space and reflected the second one. We would get the final state that would have 1 connected to -2, -1, and 2, so the same state just reflected with respect to zero.



$$
\textit{nullifying set} = \left\{ \begin{array}{l} \hat{p}_{-3} - \hat{q}_{-2} \\ \hat{p}_{-2} - \hat{q}_{-3} - \hat{q}_{-1} \\ \hat{p}_{-1} - \hat{q}_{-2} - \hat{q}_{1} \\ \hat{p}_{1} - \hat{q}_{-1} - \hat{q}_{2} \\ \hat{p}_{2} - \hat{q}_{1} - \hat{q}_{3} \\ \hat{p}_{3} - \hat{q}_{2} \end{array} \right\} \equiv \left\{ \begin{array}{l} \hat{p}_{-3} - \hat{q}_{-2} \\ \hat{p}_{-2} - \hat{q}_{-3} - \hat{p}_{1} + \hat{q}_{2} \\ \hat{p}_{-1} - \hat{q}_{-2} - \hat{q}_{1} \\ \hat{p}_{1} - \hat{q}_{-1} - \hat{q}_{2} \\ \hat{p}_{2} - \hat{p}_{-1} + \hat{q}_{-2} - \hat{q}_{3} \\ \hat{p}_{3} - \hat{q}_{2} \end{array} \right\} \xrightarrow[\hat{p}_1, \hat{p}_{-1}]{\text{measure}} \left\{ \begin{array}{l} \hat{p}_{-3} - \hat{q}_{-2} \\ \hat{p}_{-2} - \hat{q}_{-3} - m_{1} + \hat{q}_{2} \\ \hat{p}_{-1} - m_{-1} \\ \hat{p}_{1} - m_{1} \\ \hat{p}_{2} - m_{-1} + \hat{q}_{-2} - \hat{q}_{3} \\ \hat{p}_{3} - \hat{q}_{2} \end{array} \right\}
$$

$$
\xrightarrow[\substack{\hat{p}_3 \to -\hat{p}_3 \\ \hat{q}_3 \to -\hat{q}_3}]{\substack{\hat{p}_2 \to -\hat{p}_2 \\ \hat{q}_2 \to -\hat{q}_2}} \left\{ \begin{array}{l} \hat{p}_{-3} - \hat{q}_{-2} \\ \hat{p}_{-2} - \hat{q}_{-3} - m_{1} - \hat{q}_{2} \\ \hat{p}_{-1} - m_{-1} \\ \hat{p}_{1} - m_{1} \\ -\hat{p}_{2} - m_{-1} + \hat{q}_{-2} + \hat{q}_{3} \\ -\hat{p}_{3} + \hat{q}_{2} \end{array} \right\}
$$



This way we achieve a "wire shortening". This can be generalized to more complicated cases.

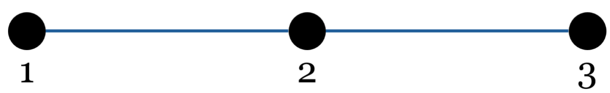


FIG. 1: 1 - 2 - 3

The state is:

$$
\begin{aligned}
|\psi\rangle &= C_Z^{12} C_Z^{23} \left|0\right\rangle_p^1 \left|0\right\rangle_p^2 \left|0\right\rangle_p^3 = e^{i\hat{q}_1\hat{q}_2} e^{i\hat{q}_2\hat{q}_3} \left|0\right\rangle_p^1 \left|0\right\rangle_p^2 \left|0\right\rangle_p^3 \\
&= \left(\frac{1}{\sqrt{2\pi}}\right)^3 \int d^3r \left[e^{i(\hat{q}_1\hat{q}_2 + \hat{q}_2\hat{q}_3)} \left|r_1\right\rangle_q^1 \left|r_2\right\rangle_q^2 \left|r_3\right\rangle_q^3 \right] \\
&= \left(\frac{1}{\sqrt{2\pi}}\right)^3 \int d^3r \left[e^{ir_2(r_1 + r_3)} \left|r_1\right\rangle_q^1 \left|r_2\right\rangle_q^2 \left|r_3\right\rangle_q^3 \right]
\end{aligned}
$$

A. q_i measurement (vertex removal)

Let's measure q_1. Ignoring prefactor, we get:

$$
\begin{aligned}
&|m_1\rangle_q^1 \langle m_1|_q^1 |\psi\rangle = \\
&= |m_1\rangle_q^1 \left(\frac{1}{\sqrt{2\pi}}\right)^3 \int \mathrm{d}^3 r \left[e^{ir_2(r_1+r_3)} \langle m_1|_q^1 |r_1\rangle_q^1 |r_2\rangle_q^2 |r_3\rangle_q^3 \right] \\
&= |m_1\rangle_q^1 \left(\frac{1}{\sqrt{2\pi}}\right)^3 \int \mathrm{d}^2 r_{23} \left[e^{ir_2(m_1+r_3)} |r_2\rangle_q^2 |r_3\rangle_q^3 \right] \\
&= \left(\frac{1}{\sqrt{2\pi}}\right)^3 \int \mathrm{d}^2 r_{23} \left[e^{i\hat{q}_1\hat{q}_2} e^{i\hat{q}_2\hat{q}_3} |m_1\rangle_q^1 |r_2\rangle_q^2 |r_3\rangle_q^3 \right] \\
&= \left(\frac{1}{\sqrt{2\pi}}\right) e^{i\hat{q}_1\hat{q}_2} e^{i\hat{q}_2\hat{q}_3} |m_1\rangle_q^1 |0\rangle_p^2 |0\rangle_p^3 \\
&= \left(\frac{1}{\sqrt{2\pi}}\right) e^{im_1\hat{q}_2} e^{i\hat{q}_2\hat{q}_3} |m_1\rangle_q^1 |0\rangle_p^2 |0\rangle_p^3 \\
&= \left(\frac{1}{\sqrt{2\pi}}\right) Z_2(m_1) e^{i\hat{q}_2\hat{q}_3} |m_1\rangle_q^1 |0\rangle_p^2 |0\rangle_p^3 \\
&= \left(\frac{1}{\sqrt{2\pi}}\right) e^{i\hat{q}_2\hat{q}_3} |m_1\rangle_q^1 |m_1\rangle_p^2 |0\rangle_p^3
\end{aligned}
$$

It means that the q measurement removes the vertex and displaces its neighbor. Let's analyze the same measurement with nullifiers. Nullifiers of the initial state are

$$\{H_1, H_2, H_3\} = \{\hat{p}_1 - \hat{q}_2,\, \hat{p}_2 - \hat{q}_1 - \hat{q}_3,\, \hat{p}_3 - \hat{q}_2\}. \tag{14}$$

q_1 commutes with all but first, so after measurement, we have:

$$\{\hat{q}_1 - m_1,\, \hat{p}_2 - \hat{q}_1 - \hat{q}_3,\, \hat{p}_3 - \hat{q}_2\} \to \{\hat{q}_1 - m_1,\, \hat{p}_2 - m_1 - \hat{q}_3,\, \hat{p}_3 - \hat{q}_2\} \tag{15}$$

Using this, we can generalize that q measurement removes the entanglement of a measured vertex with its neighbors and displaces them.

B. Measure p_1

Let's measure p_1. Ignoring prefactor we get:

$$
\begin{aligned}
&|m_1\rangle_p^1 \langle m_1|_p^1 |\psi\rangle = \\
&= |m_1\rangle_p^1 \left(\frac{1}{\sqrt{2\pi}}\right)^3 \int d^3r \left[e^{ir_2(r_1+r_3)} \langle m_1|_p^1 |r_1\rangle_q^1 |r_2\rangle_q^2 |r_3\rangle_q^3\right] \\
&= |m_1\rangle_p^1 \left(\frac{1}{\sqrt{2\pi}}\right)^3 \int d^3r \left[e^{ir_2(r_1+r_3)} \frac{e^{-im_1r_1}}{\sqrt{2\pi}} |r_2\rangle_q^2 |r_3\rangle_q^3\right] \\
&= |m_1\rangle_p^1 \left(\frac{1}{\sqrt{2\pi}}\right)^4 \int d^2r_{23} \left[2\pi\delta(r_2-m_1)e^{ir_2r_3} |r_2\rangle_q^2 |r_3\rangle_q^3\right] \\
&= |m_1\rangle_p^1 \left(\frac{1}{\sqrt{2\pi}}\right)^2 \int \mathrm{d}r_3 \left[e^{im_1r_3} |m_1\rangle_q^2 |r_3\rangle_q^3\right] \\
&= \frac{1}{2\pi} |m_1\rangle_p^1 |m_1\rangle_q^2 \int \mathrm{d}r_3 \left[e^{im_1r_3} |r_3\rangle_q^3\right] \\
&= \frac{1}{2\pi} e^{i\hat{q}_2\hat{q}_3} |m_1\rangle_p^1 |m_1\rangle_q^2 \int \mathrm{d}r_3 \left[|r_3\rangle_q^3\right] \\
&= \boxed{\frac{1}{\sqrt{2\pi}} e^{i\hat{q}_2\hat{q}_3} |m_1\rangle_p^1 |m_1\rangle_q^2 |0\rangle_p^3} \\
&= \frac{1}{\sqrt{2\pi}} |m_1\rangle_p^1 |m_1\rangle_q^2 |m_1\rangle_p^3
\end{aligned}
$$

Let's analyze the same measurement with nullifiers. Nullifiers of the initial state are

$$\{H_1, H_2, H_3\} = \{\hat{p}_1 - \hat{q}_2,\, \hat{p}_2 - \hat{q}_1 - \hat{q}_3,\, \hat{p}_3 - \hat{q}_2\}. \tag{16}$$

Since p_1 does not commute only with H_2 we have:

$$\{\hat{p}_1 - \hat{q}_2, \hat{p}_1 - m_1,\, \hat{p}_3 - \hat{q}_2\} \to \{m_1 - \hat{q}_2, \hat{p}_1 - m_1,\, \hat{p}_3 - m_1\} \to |m_1\rangle_p^1 |m_1\rangle_q^2 |m_1\rangle_p^3, \tag{17}$$

which is in agreement with our previous calculation. The answer is symmetric with respect to 1 and 3, I can not come up with a good reason for this. Also, if we wish to know what would happen if we measured p_3 instead, we can simply swap 1 and 3. Due to the symmetry, we will recover the same answer.

C. Measure p_2

Let's measure p_2. Ignoring prefactor we get:

$$
\begin{aligned}
&|m_2\rangle_p^2 \langle m_2|_p^2 |\psi\rangle = \\
&= |m_2\rangle_p^2 \left(\frac{1}{\sqrt{2\pi}}\right)^3 \int \mathrm{d}^3r \left[e^{ir_2(r_1+r_3)} |r_1\rangle_q^1 \langle m_2|_p^2 |r_2\rangle_q^2 |r_3\rangle_q^3\right] \\
&= |m_2\rangle_p^2 \left(\frac{1}{\sqrt{2\pi}}\right)^3 \int \mathrm{d}^3r \left[e^{ir_2(r_1+r_3)} |r_1\rangle_q^1 \frac{e^{-im_2r_2}}{\sqrt{2\pi}} |r_3\rangle_q^3\right] \\
&= |m_2\rangle_p^2 \left(\frac{1}{\sqrt{2\pi}}\right)^4 \int \mathrm{d}^2r_{13} \left[2\pi\delta(r_1+r_3-m_2) |r_1\rangle_q^1 |r_3\rangle_q^3\right] \\
&= \frac{1}{2\pi} |m_2\rangle_p^2 \int \mathrm{d}r_3 \left[|m_2-r_3\rangle_q^1 |r_3\rangle_q^3\right]
\end{aligned}
$$

Let's analyze the same measurement with nullifiers. Nullifiers of the initial state are

$$\{H_1, H_2, H_3\} = \{\hat{p}_1 - \hat{q}_2,\, \hat{p}_2 - \hat{q}_1 - \hat{q}_3,\, \hat{p}_3 - \hat{q}_2\}. \tag{18}$$

Since p_2 does not commute with both H_1 and H_3, we can replace one of them with the linear combination

$$\textit{nullifying set} = \{\hat{p}_1 - \hat{q}_2,\, \hat{p}_2 - \hat{q}_1 - \hat{q}_3,\, \hat{p}_3 - \hat{p}_1\}, \tag{19}$$

After the measurement, we get

$$\{\hat{p}_2 - m_2,\, m_2 - \hat{q}_1 - \hat{q}_3,\, \hat{p}_3 - \hat{p}_1\} \to |m_2\rangle_q^2\, |\textit{something}(m_2)\rangle_{13}\,. \tag{20}$$

$m_2 - \hat{q}_1 - \hat{q}_3$ agrees with our results with direct calculation as it is a nullifier of the final state,

$$(m_2 - \hat{q}_1 - \hat{q}_3)\, |m_2 - r_3\rangle_q^1\, |r_3\rangle_q^3 = 0. \tag{21}$$

To figure out the last one, we bring first and third qubits to p basis.

$$\begin{aligned}
&|m_2\rangle_p^2\, \langle m_2|_p^2\, |\psi\rangle \\
&= \frac{1}{2\pi}\, |m_2\rangle_p^2 \int \mathrm{d}r_3\, \left[|m_2 - r_3\rangle_q^1\, |r_3\rangle_q^3\right] \\
&= \frac{1}{2\pi}\, |m_2\rangle_p^2 \int \mathrm{d}r_3\, \left[\frac{1}{2\pi} \int \mathrm{d}s_1\, e^{-is_1(m_2 - r_3)}\, |s_1\rangle_p^1 \int \mathrm{d}s_3\, e^{-is_3 r_3}\, |s_3\rangle_p^3\right] \\
&= \frac{1}{(2\pi)^2}\, |m_2\rangle_p^2 \int \mathrm{d}r_3\, \mathrm{d}s_1\, \mathrm{d}s_3\, \left[e^{ir_3(s_1 - s_3)} e^{-is_1 m_2}\, |s_1\rangle_p^1\, |s_3\rangle_p^3\right] \\
&= \frac{1}{2\pi}\, |m_2\rangle_p^2 \int \mathrm{d}s_3\, \left[e^{-is_3 m_2}\, |s_3\rangle_p^1\, |s_3\rangle_p^3\right]
\end{aligned}$$

Now it's easy to see that this state is nullified by $\hat{p}_3 - \hat{p}_1$.

MS: The discussion in [1] about "wire shortening" is not general enough even for the linear graph states. I do not understand if they imply that their discussion is general or not.

III. TELEPORTATION

A. Ideal

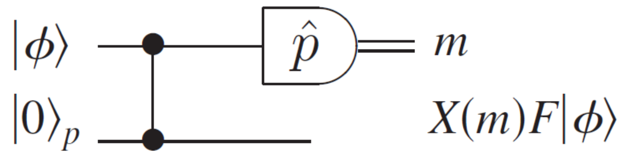


FIG. 2: Psi

The state is:

$$\begin{aligned}
|\psi\rangle &= C_Z^{12} |\phi\rangle^1 |0\rangle_p^2 \\
&= \frac{1}{\sqrt{2\pi}} \int \mathrm{d}^2 r_{12} \left[e^{i r_1 r_2} \phi_q(r_1) |r_1\rangle_q^1 |r_2\rangle_q^2 \right]
\end{aligned}$$

1. *Projective Measurement*

Let's measure p_1. Ignoring prefactor we get:

$$\begin{aligned}
& |m_1\rangle_p^1 \langle m_1|_p^1 |\psi\rangle \\
&= |m_1\rangle_p^1 \frac{1}{\sqrt{2\pi}} \int \mathrm{d}^2 r_{12} \left[e^{i r_1 r_2} \phi_q(r_1) \langle m_1|_p^1 |r_1\rangle_q^1 |r_2\rangle_q^2 \right] \\
&= |m_1\rangle_p^1 \left(\frac{1}{\sqrt{2\pi}} \right)^2 \int \mathrm{d}^2 r_{12} \left[e^{i r_1 r_2} \phi_q(r_1) e^{-i m_1 r_1} |r_2\rangle_q^2 \right] \\
&= \frac{1}{\sqrt{2\pi}} |m_1\rangle_p^1 \int \mathrm{d} r_1 \left[e^{-i m_1 r_1} \phi_q(r_1) |r_1\rangle_p^2 \right] \\
&= \frac{e^{-i m_1 \hat{p}_2}}{\sqrt{2\pi}} |m_1\rangle_p^1 \int \mathrm{d} r_1 \left[\phi_q(r_1) |r_1\rangle_p^2 \right] \\
&= \frac{e^{-i m_1 \hat{p}_2}}{\sqrt{2\pi}} |m_1\rangle_p^1 \int \mathrm{d} r_1 \left[\phi_q(r_1) \mathcal{F}_2 |r_1\rangle_q^2 \right] \\
&= X_2(m_1) \mathcal{F}_2 |m_1\rangle_p^1 |\phi\rangle^2
\end{aligned}$$

Measuring the first qumode caused it to "migrate" to the second one. I wrote migrate in quotes, as there is an extra operator needed to account for $X_2(m_1)\mathcal{F}_2$. Let's specify the input to be a Fock state $|\phi\rangle = |n\rangle$. The corresponding wavefunctions are:

$$\psi_n(x) = \frac{\pi^{-1/4}}{\sqrt{n!\, 2^n}} e^{-x^2/2} H_n(x). \tag{22}$$

In general, one can do a usual manipulation to represent a state for a given wavefunction as an operator acting on a zero-momentum eigenstate:

$$|\psi\rangle = \int dx\, \psi_q(x)\ |x\rangle_q = \int dx\, \psi_q(\hat{Q})\ |x\rangle_q = \sqrt{2\pi}\, \psi_q(\hat{Q})\ |0\rangle_p\ . \tag{23}$$

In our case, this leads to

$$|n\rangle = \frac{\pi^{1/4}}{\sqrt{n!\, 2^{n-1}}} e^{-(\hat{q})^2/2} H_n(\hat{q}) |0\rangle_p\,. \tag{24}$$

So, the state after the measurement is

$$|m_1\rangle_p^1 \langle m_1|_p^1 |\psi\rangle = \frac{\pi^{1/4}}{\sqrt{n!\,2^{n-1}}} |m_1\rangle_p^1 X_2(m_1)\mathcal{F}_2\, e^{-(\hat{q}_2)^2/2} H_n(\hat{q}_2)\, |0\rangle_p^2 \tag{25}$$

2. With Stabilizers

Let's analyze the same measurement with nullifiers. To find the stabilizers we remind ourselves that if K stabilizes $|\phi\rangle$, then $UKU^\dagger$ stabilizes $U|\phi\rangle$. Stabilizers of $|n\rangle^1 |0\rangle_p^2$ are $X(s)^2$, $e^{\alpha \hat{a}_1^{n+1}}$, and $\hat{N}_1 - (n-1)\hat{I}$.

$$\begin{aligned} C_z^{12} K (C_z^{12})^\dagger &= e^{i\hat{q}_1\hat{q}_2}[\hat{N}_1 - (n-1)\hat{I}](e^{i\hat{q}_1\hat{q}_2})^\dagger \\ &= e^{i\hat{q}_1\hat{q}_2}\hat{N}_1(e^{i\hat{q}_1\hat{q}_2})^\dagger - (n-1)e^{i\hat{q}_1\hat{q}_2}\hat{I}(e^{i\hat{q}_1\hat{q}_2})^\dagger \\ &= e^{i\hat{q}_1\hat{q}_2}\hat{N}_1(e^{i\hat{q}_1\hat{q}_2})^\dagger - (n-1)\hat{I} \\ &= \hat{N}_1 + \left[e^{i\hat{q}_1\hat{q}_2}, \hat{N}_1\right]\left(e^{i\hat{q}_1\hat{q}_2}\right)^\dagger - (n-1)\hat{I} \end{aligned} \tag{26}$$

We want to compute the commutator $\left[e^{i\hat{q}_2\hat{q}_1}, \hat{N}_1\right]$. We use the general identity:

$$\left[e^A, B\right] = \int_0^1 e^{sA}[A,B]e^{(1-s)A}\, ds, \tag{27}$$

with $A = i\hat{q}_2\hat{q}_1$, $B = \hat{N}_1$. First we evaluate

$$[i\hat{q}_2\hat{q}_1, \hat{N}_1] = i\hat{q}_2[\hat{q}_1, \hat{N}_1] = i\hat{q}_2(-i\hat{p}_1) = \hat{q}_2\hat{p}_1, \tag{28}$$

Where we used

$$[\hat{q}_1, \hat{N}_1] = \left[\frac{1}{\sqrt{2}}(a_1 + a_1^\dagger), a_1^\dagger a_1\right] = \frac{1}{\sqrt{2}}\left([a_1, a_1^\dagger a_1] + [a_1^\dagger, a_1^\dagger a_1]\right) = \frac{1}{\sqrt{2}}(a_1 - a_1^\dagger) = -i\hat{p}_1.$$

Now plug into the integral:

$$\left[e^{i\hat{q}_2\hat{q}_1}, \hat{N}_1\right] = \int_0^1 e^{si\hat{q}_2\hat{q}_1}\hat{q}_2\hat{p}_1 e^{(1-s)i\hat{q}_2\hat{q}_1} ds. \tag{29}$$

Since $\hat{q}_2$ commutes with $e^{i\hat{q}_2\hat{q}_1}$, we can factor it out:

$$= \hat{q}_2 \int_0^1 e^{si\hat{q}_2\hat{q}_1}\hat{p}_1 e^{(1-s)i\hat{q}_2\hat{q}_1} ds. \tag{30}$$

Now consider the conjugation of $\hat{p}_1$ under $e^{i\hat{q}_2\hat{q}_1}$:

$$e^{\alpha i\hat{q}_2\hat{q}_1}\hat{p}_1 e^{-\alpha i\hat{q}_2\hat{q}_1} = \hat{p}_1 + \alpha[i\hat{q}_2\hat{q}_1, \hat{p}_1] + \cdots = \hat{p}_1 + \alpha(-\hat{q}_2).$$

Here we used

$$e^X Y e^{-X} = Y + [X,Y] + \frac{1}{2!}[X,[X,Y]] + \frac{1}{3!}[X,[X,[X,Y]]] + \cdots \tag{31}$$

So:

$$e^{\alpha i\hat{q}_2\hat{q}_1}\hat{p}_1 e^{-\alpha i\hat{q}_2\hat{q}_1} = \hat{p}_1 + \alpha\hat{q}_2.$$

Applying this to the integral:

$$\begin{aligned}\int_0^1 e^{si\hat{q}_2\hat{q}_1}\hat{p}_1 e^{(1-s)i\hat{q}_2\hat{q}_1}ds &= \left[\int_0^1 e^{si\hat{q}_2\hat{q}_1}\hat{p}_1 e^{-si\hat{q}_2\hat{q}_1}ds\right]e^{i\hat{q}_2\hat{q}_1}\\ &= \left[\int_0^1 (\hat{p}_1+s\hat{q}_2)\,ds\right]e^{i\hat{q}_2\hat{q}_1}\\ &= \left[\hat{p}_1+\frac{1}{2}\hat{q}_2\right]e^{i\hat{q}_2\hat{q}_1}.\end{aligned}$$

The commutator is

$$\left[e^{i\hat{q}_2\hat{q}_1},\hat{N}_1\right] = \left(\hat{q}_2\hat{p}_1+\frac{1}{2}\hat{q}_2^2\right)e^{i\hat{q}_2\hat{q}_1}. \tag{32}$$

Using this we continue simplifying Eq. 26

$$\begin{aligned}C_z^{21}K(C_z^{21})^\dagger &= \hat{N}_1+\hat{q}_2\hat{p}_1+\frac{1}{2}\hat{q}_2^2-(n-1)\hat{I}\\ &= \boxed{\hat{N}_1-(n-1)\hat{I}+\hat{q}_2\hat{p}_1+\frac{1}{2}\hat{q}_2^2.}\end{aligned} \tag{33}$$

Let's check that this is actually a stabilizer of $C_Z^{12}\left|n\right\rangle^1\left|0\right\rangle_p^2$:

$$\begin{aligned}&\left(\hat{N}_1-(n-1)\hat{I}+\hat{q}_2\hat{p}_1+\tfrac{1}{2}\hat{q}_2^2\right)e^{i\hat{q}_1\hat{q}_2}\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}\\ &=\hat{N}_1e^{i\hat{q}_1\hat{q}_2}\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}-(n-1)e^{i\hat{q}_1\hat{q}_2}\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}+\hat{q}_2\hat{p}_1e^{i\hat{q}_1\hat{q}_2}\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}+\tfrac{1}{2}\hat{q}_2^2e^{i\hat{q}_1\hat{q}_2}\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}\\ &=\left(\left[\hat{N}_1,e^{i\hat{q}_1\hat{q}_2}\right]+e^{i\hat{q}_1\hat{q}_2}\hat{N}_1\right)\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}-(n-1)e^{i\hat{q}_1\hat{q}_2}\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}+\hat{q}_2\hat{p}_1e^{i\hat{q}_1\hat{q}_2}\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}+\tfrac{1}{2}\hat{q}_2^2e^{i\hat{q}_1\hat{q}_2}\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}\\ &=\left(-\hat{q}_2\hat{p}_1-\tfrac{1}{2}\hat{q}_2^2\right)e^{i\hat{q}_1\hat{q}_2}\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}+e^{i\hat{q}_1\hat{q}_2}\hat{N}_1\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}-(n-1)e^{i\hat{q}_1\hat{q}_2}\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}\\ &\quad+\hat{q}_2\hat{p}_1e^{i\hat{q}_1\hat{q}_2}\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}+\tfrac{1}{2}\hat{q}_2^2e^{i\hat{q}_1\hat{q}_2}\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}\\ &=e^{i\hat{q}_1\hat{q}_2}\left(\hat{N}_1-(n-1)\hat{I}\right)\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}=e^{i\hat{q}_1\hat{q}_2}\left|n\right\rangle^{(1)}\left|0\right\rangle_p^{(2)}\end{aligned} \tag{34}$$

It is very difficult (if not impossible) to extract nullifiers from this one, so let's try the second stabilizer $e^{\alpha\hat{a}_1^{n+1}}$: First we compute:

$$\begin{aligned}e^{i\hat{q}_1\hat{q}_2}\hat{a}_1^{n+1}\left(e^{i\hat{q}_1\hat{q}_2}\right)^\dagger &= \left(e^{i\hat{q}_1\hat{q}_2}\hat{a}_1e^{-i\hat{q}_1\hat{q}_2}\right)^{n+1} = \left(e^{i\hat{q}_1\hat{q}_2}\left(\frac{1}{\sqrt{2}}(\hat{q}_1+i\hat{p}_1)\right)e^{-i\hat{q}_1\hat{q}_2}\right)^{n+1}\\ &= \left(\frac{1}{\sqrt{2}}\left(\hat{q}_1+i(\hat{p}_1-\hat{q}_2)\right)\right)^{n+1} = \left(\frac{1}{\sqrt{2}}(\hat{q}_1+i\hat{p}_1-i\hat{q}_2)\right)^{n+1} = \left(\hat{a}_1-\frac{i}{\sqrt{2}}\hat{q}_2\right)^{n+1}.\end{aligned}$$

Hence, we have:

$$C_z^{12}K(C_z^{12})^\dagger = e^{i\hat{q}_1\hat{q}_2}e^{\alpha\hat{a}_1^{n+1}}(e^{i\hat{q}_1\hat{q}_2})^\dagger = \boxed{e^{\alpha\left(\hat{a}_1-\frac{i}{\sqrt{2}}\hat{q}_2\right)^{n+1}}.} \tag{35}$$

Let's see what happens with $X(s)^2$:

$$C_z^{12}X(s)^2(C_z^{12})^\dagger = C_z^{12}e^{-is\hat{p}_2}(C_z^{12})^\dagger = \boxed{e^{-is\hat{p}_2}e^{is\hat{q}_1}.} \tag{36}$$

Combined stabilizer needs to be simplified further to extract the nullifier.

$$e^{is(\hat{q}_1-\hat{p}_2)}e^{\alpha\left(\hat{a}_1-\frac{i}{\sqrt{2}}\hat{q}_2\right)^{n+1}} \tag{37}$$

Let's denote $\hat{q}_1 - \hat{p}_2 \equiv A$ and $\hat{a}_1 - \frac{i}{\sqrt{2}}\hat{q}_2 \equiv B$

$$[\hat{q}_1, B^{n+1}] = (n+1)[\hat{q}_1, B]B^n = -\frac{n+1}{\sqrt{2}}B^n \tag{38}$$

$$[\hat{p}_2, B^{n+1}] = (n+1)[\hat{p}_2, B]B^n = -\frac{n+1}{\sqrt{2}}B^n \tag{39}$$

$$[A, B] = \left(-\frac{n+1}{\sqrt{2}}B^n + \frac{n+1}{\sqrt{2}}B^n\right) = 0 \tag{40}$$

$$e^{is(\hat{q}_1-\hat{p}_2)}\, e^{\alpha\left(\hat{a}_1-\frac{i}{\sqrt{2}}\hat{q}_2\right)^{n+1}} = e^{is(\hat{q}_1-\hat{p}_2)+\alpha\left(\hat{a}_1-\frac{i}{\sqrt{2}}\hat{q}_2\right)^{n+1}} \tag{41}$$

So the nullifying set is:

$$\boxed{\mathcal{N} = \left\{\hat{q}_1 - \hat{p}_2, \left(\hat{a}_1 - \frac{i}{\sqrt{2}}\hat{q}_2\right)^{n+1}\right\}} \tag{42}$$

3. n = 0

Let's see how this looks for $n = 0$

$$\mathcal{N}\Big|_{n=0} = \left\{\hat{p}_2 - \hat{q}_1, \frac{1}{\sqrt{2}}(\hat{q}_1 + i\hat{p}_1 - i\hat{q}_2)\right\} \tag{43}$$

$\hat{p}_2$ commutes with the first one, so we have:

$$\mathcal{N}\Big|_{n=0} = \{m_2 - \hat{q}_1, \hat{p}_2 - m_2\} \tag{44}$$

The final state is

$$|m_2\rangle_q^1 \, |m_2\rangle_p^2 \tag{45}$$

We could have done this for any value of n. To measure p_1, we have to modify nullifiers first and then do the measurement

$$\mathcal{N}\Big|_{n=0} = \left\{\hat{p}_2 - \hat{q}_1, \frac{1}{\sqrt{2}}(\hat{p}_2 + i\hat{p}_1 - i\hat{q}_2)\right\} \xrightarrow{\text{measure}} \left\{\hat{p}_1 - m_1, \frac{1}{\sqrt{2}}(\hat{p}_2 + im_1 - i\hat{q}_2)\right\} \tag{46}$$

$$\equiv \left\{\hat{p}_1 - m_1, \frac{1}{\sqrt{2}}(\hat{q}_2 + i\hat{p}_2 - m_1)\right\} = \left\{\hat{p}_1 - m_1, \hat{a}_2 - \frac{1}{\sqrt{2}}m_1\right\} \tag{47}$$

We have successfully teleported a Fock state $|0\rangle$ to the second qumode. This is barely a success as $|0\rangle$ is really just a vacuum state.

4. n = 1

Let's see how it looks for $n = 1$

$$\mathcal{N}\Big|_{n=1} = \left\{\hat{p}_2 - \hat{q}_1, \frac{1}{2}(\hat{q}_1 + i\hat{p}_1 - i\hat{q}_2)^2\right\} \tag{48}$$

$$[\hat{p}_1, \hat{q}_1 + i\hat{p}_1 - i\hat{q}_2] = -i \implies [\hat{p}_1, (\hat{q}_1 + i\hat{p}_1 - i\hat{q}_2)^n] = -in(\hat{q}_1 + i\hat{p}_1 - i\hat{q}_2)^{n-1} \tag{49}$$

This means that for any $n > 0$, no linear combination of nullifiers will commute with $\hat{p}_1$. The nullifier formalism for quadrature measurements described in the appendix of [1] is not applicable for Fock state teleportation.

B. Finitely Squeezed

$$
\begin{aligned}
|\psi\rangle &= C_Z^{12}\,|\phi\rangle^1\,|0,\Omega\rangle_p^2 = C_Z^{12}\,|\phi\rangle^1\,|0,\Omega^{-1}\rangle_q^2 \\
&= (\pi\Omega^{-2})^{-1/4}\frac{1}{\sqrt{2\pi}}\int \mathrm{d}^2 r_{12}\;\left[e^{i\hat{q}_1\hat{q}_2}e^{-\frac{\Omega^2 r_2^2}{2}}\phi_q(r_1)\,|r_1\rangle_q^1\,|r_2\rangle_q^2\right] \\
&= \frac{\sqrt{\Omega}}{(2\pi)^{(3/4)}}\int \mathrm{d}^2 r_{12}\;\left[e^{ir_1r_2}e^{-\frac{\Omega^2 r_2^2}{2}}\phi_q(r_1)\,|r_1\rangle_q^1\,|r_2\rangle_q^2\right]
\end{aligned}
$$

Let's measure p_1. Ignoring prefactor we get:

$$
\begin{aligned}
&|m_1\rangle_p^1\,\langle m_1|_p^1\,|\psi\rangle \\
&= \frac{\sqrt{\Omega}}{(2\pi)^{(3/4)}}\,|m_1\rangle_p^1\int \mathrm{d}^2 r_{12}\;\left[e^{ir_1r_2}e^{-\frac{\Omega^2 r_2^2}{2}}\phi_q(r_1)\,\langle m_1|_p^1\,|r_1\rangle_q^1\,|r_2\rangle_q^2\right] \\
&= \frac{\sqrt{\Omega}\,e^{-\frac{\Omega^2\hat{q}_2^2}{2}}}{(2\pi)^{(5/4)}}\,|m_1\rangle_p^1\int \mathrm{d}^2 r_{12}\;\left[e^{ir_1r_2}\phi_q(r_1)e^{-im_1r_1}\,|r_2\rangle_q^2\right] \\
&= \frac{\sqrt{\Omega}\,e^{-\frac{\Omega^2\hat{q}_2^2}{2}}}{(2\pi)^{(3/4)}}\,|m_1\rangle_p^1\int \mathrm{d}r_1\;\left[\phi_q(r_1)e^{-im_1r_1}\,|r_1\rangle_p^2\right] \\
&= \frac{\sqrt{\Omega}}{(2\pi)^{(3/4)}}e^{-\frac{\Omega^2\hat{q}_2^2}{2}}e^{-im_1\hat{p}_2}\,|m_1\rangle_p^1\int \mathrm{d}r_1\;\left[\phi_q(r_1)\,|r_1\rangle_p^2\right] \\
&= \frac{\sqrt{\Omega}}{(2\pi)^{(3/4)}}e^{-\frac{\Omega^2\hat{q}_2^2}{2}}e^{-im_1\hat{p}_2}\,|m_1\rangle_p^1\int \mathrm{d}r_1\;\left[\phi_q(r_1)\mathcal{F}_2\,|r_1\rangle_q^2\right] \\
&= \frac{\sqrt{\Omega}}{(2\pi)^{(3/4)}}e^{-\frac{\Omega^2\hat{q}_2^2}{2}}X_2(m_1)\mathcal{F}_2\,|m_1\rangle_p^1\,|\phi\rangle^2
\end{aligned}
$$

$$
e^{-\frac{\Omega^2\hat{q}_2^2}{2}}X_2(m_1)\mathcal{F}_2\,|m_1\rangle_p^1\,|\phi\rangle^2 = e^{-\frac{\Omega^2\hat{q}_2^2}{2}}e^{-im_1\hat{p}_2}\mathcal{F}_2\,|m_1\rangle_p^1\,|\phi\rangle^2 \tag{50}
$$

We start with:

$$
e^{-\frac{\Omega^2}{2}\hat{q}_2^2}e^{-im_1\hat{p}_2} \tag{51}
$$

Let $A = -\frac{\Omega^2}{2}\hat{q}_2^2$, $B = -im_1\hat{p}_2$. Using the Baker–Campbell–Hausdorff identity:

$$
e^Ae^B = e^{A+B+\frac{1}{2}[A,B]+\frac{1}{12}[A,[A,B]]-\frac{1}{12}[B,[A,B]]+\cdots} \tag{52}
$$

We compute the relevant commutators. First:

$$
[A,B] = im_1\frac{\Omega^2}{2}[\hat{q}_2^2,\hat{p}_2] = im_1\frac{\Omega^2}{2}\cdot 2i\hat{q}_2 = -m_1\Omega^2\hat{q}_2 \tag{53}
$$

Next:

$$
[B,[A,B]] = [-im_1\hat{p}_2, -m_1\Omega^2\hat{q}_2] = im_1^2\Omega^2[\hat{p}_2,\hat{q}_2] = im_1^2\Omega^2(-i) = m_1^2\Omega^2 \tag{54}
$$

Therefore, the BCH expansion gives:

$$
e^Ae^B = e^Be^Ae^{[A,B]}e^{-\frac{1}{12}[B,[A,B]]} = e^Be^Ae^{[A,B]}\cdot e^{-\frac{1}{12}m_1^2\Omega^2} \tag{55}
$$

Thus, we have:

$$
e^{-\frac{\Omega^2}{2}\hat{q}_2^2}e^{-im_1\hat{p}_2} = e^{-im_1\hat{p}_2}e^{-\frac{\Omega^2}{2}\hat{q}_2^2}e^{-m_1\Omega^2\hat{q}_2}\cdot e^{-\frac{1}{12}m_1^2\Omega^2} \tag{56}
$$

Finally, the last two exponentials can be combined since both depend only on $\hat{q}_2$, and they commute:

$$e^{-\frac{\Omega^2}{2}\hat{q}_2^2}e^{-im_1\hat{p}_2} = e^{-\frac{1}{12}m_1^2\Omega^2}e^{-im_1\hat{p}_2}e^{-\Omega^2\left(\frac{1}{2}+m_1\right)\hat{q}_2} \tag{57}$$

$$\begin{aligned} |m_1\rangle_p^1 e^{-\frac{\Omega^2\hat{q}_2^2}{2}}X_2(m_1)\mathcal{F}_2\,|\phi\rangle^2 &= |m_1\rangle_p^1 e^{-\frac{1}{12}m_1^2\Omega^2}X_2(m_1)e^{-\Omega^2\left(\frac{1}{2}+m_1\right)\hat{q}_2}\mathcal{F}_2\,|\phi\rangle^2 \\ &= |m_1\rangle_p^1 e^{-\frac{1}{12}m_1^2\Omega^2}X_2(m_1)e^{-\Omega^2\left(\frac{1}{2}+m_1\right)\hat{q}_2}\frac{1}{\sqrt{2\pi}}\int \mathrm{d}s\,\mathrm{d}r\,\phi_q(s)e^{irs}\,|r\rangle_q^2 \end{aligned} \tag{58}$$

IV. SPIN ENTANGLED STATES

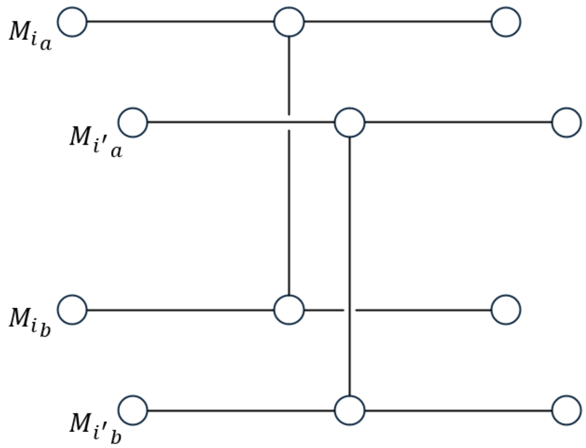


FIG. 3: asd

$$|\mathrm{EPR}^2\rangle = \sum_{n_1=0}^{\infty} \frac{(\tanh r_1)^{n_1}}{\cosh r_1} |n_1\rangle_{A_1} |n_1\rangle_{B_1} \otimes \sum_{n_2=0}^{\infty} \frac{(\tanh r_2)^{n_2}}{\cosh r_2} |n_2\rangle_{A_2} |n_2\rangle_{B_2} \tag{59}$$

$$|s_A, m_A\rangle_A = \left| \frac{n_{A1}+n_{A2}}{2}, \frac{n_{A1}-n_{A2}}{2} \right\rangle_A \tag{60}$$

$$|s_B, m_B\rangle_B = \left| \frac{n_{B1}+n_{B2}}{2}, \frac{n_{B1}-n_{B2}}{2} \right\rangle_B \tag{61}$$

$$|\mathrm{EPR}^2\rangle = \sum_{s=0}^{\infty} \sum_{m=-s}^{s} \frac{(\tanh r_1)^{s+m}}{\cosh r_1} \frac{(\tanh r_2)^{s-m}}{\cosh r_2} |s, m\rangle_A |s, -m\rangle_B \tag{62}$$

Setting $r_1 = r_2 \equiv r$ gives

$$|\mathrm{EPR}^2\rangle = \sum_{s=0}^{\infty} \left[\frac{(\tanh r)^s}{\cosh r} \right]^2 \sum_{m=-s}^{s} |s, m\rangle_A |s, -m\rangle_B \tag{63}$$

In addition, we can obtain the exact zero-total-spin eigenstate by phase shifting either the $A2$ or the $B2$ optical path by π:

$$e^{-i\pi N_{A_2}} |n_2\rangle_{A_2} = e^{-i\pi n_2} |n_2\rangle_{A_2} = (-1)^{n_2} |n_2\rangle_{A_2}, \tag{64}$$

which yields

$$|\mathrm{EPR}^2\rangle = \sum_{s=0}^{\infty} \sqrt{2s+1} \left[\frac{(\tanh r)^s}{\cosh r} \right]^2 |(ss)00\rangle. \tag{65}$$

[1] M. Gu, C. Weedbrook, N. C. Menicucci, T. C. Ralph, and P. van Loock, "Quantum computing with continuous-variable clusters," *Phys. Rev. A* **79** (Jun, 2009) 062318. `https://link.aps.org/doi/10.1103/PhysRevA.79.062318`.

[2] D. Gottesman, "The Heisenberg representation of quantum computers," in *22nd International Colloquium on Group Theoretical Methods in Physics*, pp. 32–43. 7, 1998. [quant-ph/9807006].

[3] N. C. Menicucci, P. van Loock, M. Gu, C. Weedbrook, T. C. Ralph, and M. A. Nielsen, "Universal quantum computation with continuous-variable cluster states," *Phys. Rev. Lett.* **97** (Sep, 2006) 110501. https://link.aps.org/doi/10.1103/PhysRevLett.97.110501.