

Integral MPC

mbedillion

November 2020

1 Introduction

$$x[k+1] = \begin{bmatrix} A & 0 \\ C & I \end{bmatrix} x[k] + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ I \end{bmatrix} r$$
$$y[k] = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} x[k]$$

With these we have

$$y = \underbrace{\begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^N \end{bmatrix}}_T x_0 + \underbrace{\begin{bmatrix} CB & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ CA^{N-1}B & CA^{N-1}B & \cdots & CB \end{bmatrix}}_S z + \underbrace{\begin{bmatrix} W \\ W + AW \\ \vdots \\ W + \sum_{i=1}^{N-1} A^i W \end{bmatrix}}_{WW} R$$

with R the position reference and $W = \begin{bmatrix} 0 \\ I \end{bmatrix}$. Now we have

$$J = \sum (RR - y)^T Q (RR - y) + z^T R z$$

with $RR = \begin{bmatrix} R \\ 0 \end{bmatrix}$.

From here I think the development is as normal. You have the additional term $2WW \cdot SRz$ (\pm some transposes. I don't think this quite matches your development.