Data Analytics with R

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Linear Regression

23 December 2020

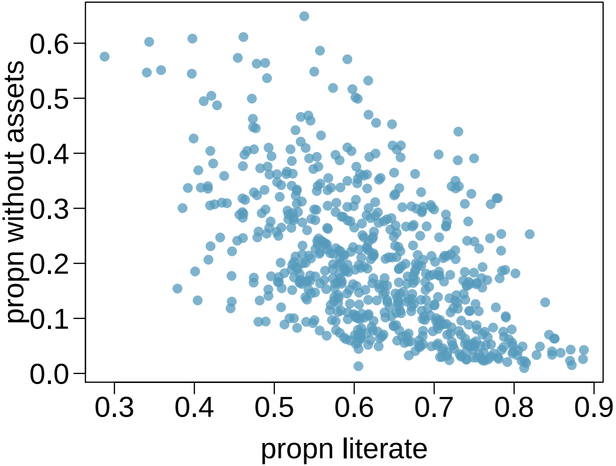
Line fitting, residuals, and correlation

Modeling numerical variables

In this unit we will learn to quantify the relationship between two numerical variables, as well as modeling numerical response variables using a numerical or categorical explanatory variable.

Asset Poverty vs. Literacy Rate

The *scatterplot* below shows the relationship between literacy rate in all 640 Indian districts and the % of households who don't own any assets.



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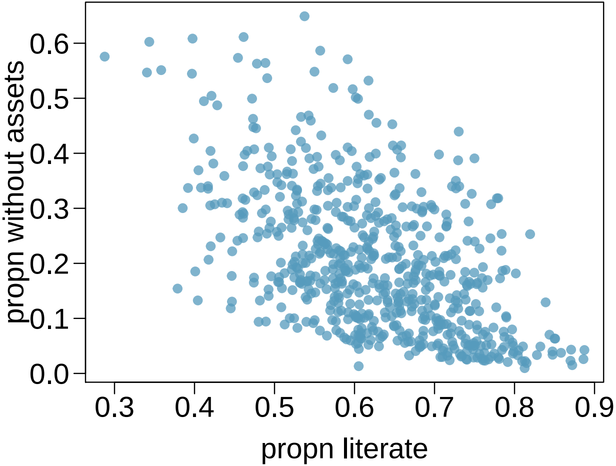

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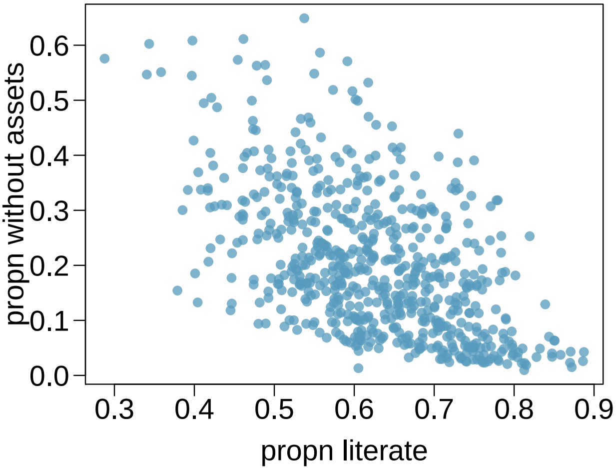

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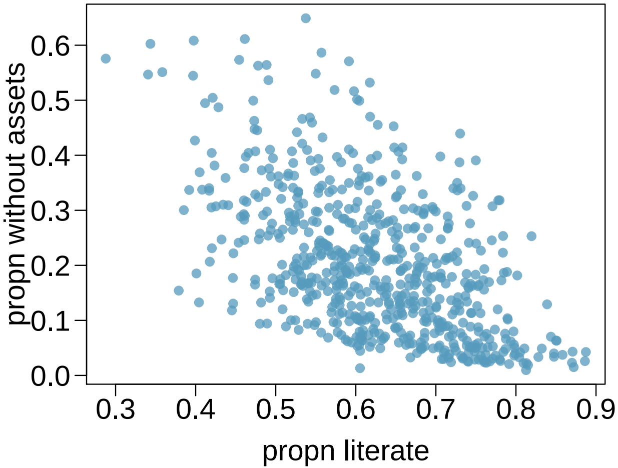

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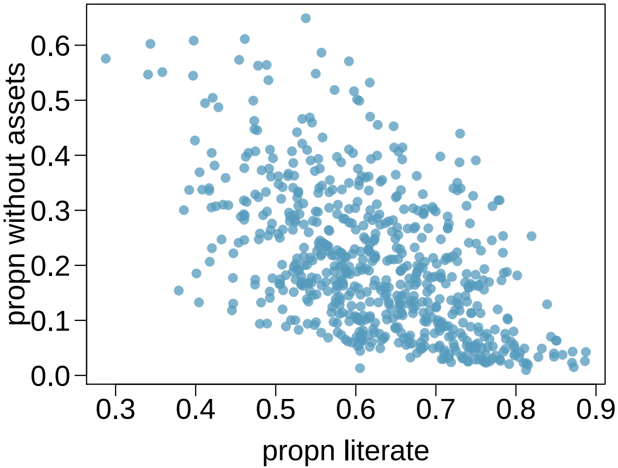

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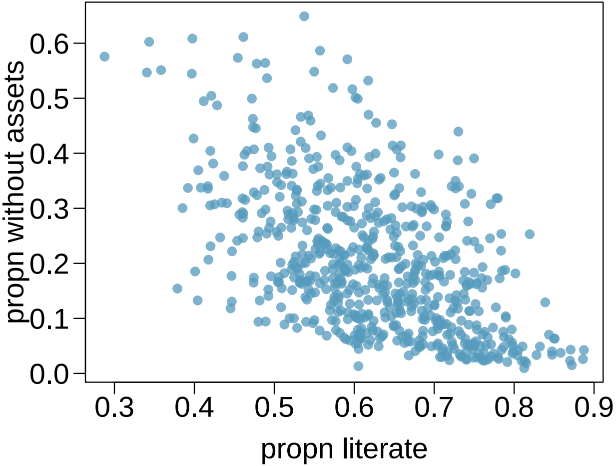

Outcome variable?

proportion of households without any assets

Predictor variable?

literacy proportion

Relationship?

linear, negative, moderately strong

The linear model for predicting asset-poverty from literacy rate in India is

$$\widehat{\text{poverty}} = 0.618 - 0.66 * \mathit{prop}_{Lit}$$

The "hat" is used to signify that this is an estimate.

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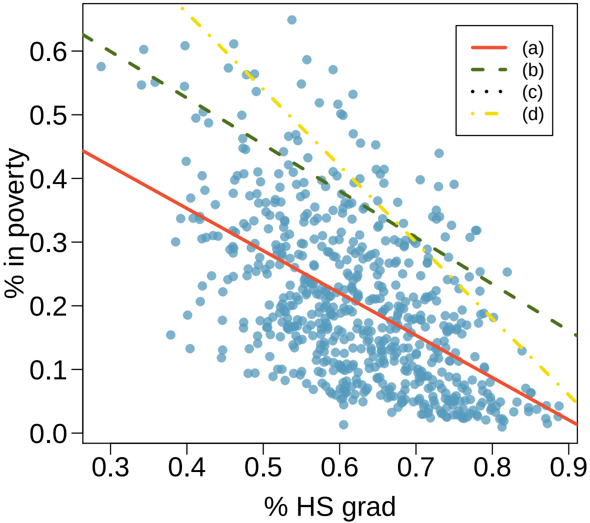
$$0.618 - 0.66 * 0.778 = 0.102$$

Eyeballing the line

Which of the following appears to be the line that best fits the linear relationship between % in poverty and % literate? Choose one.



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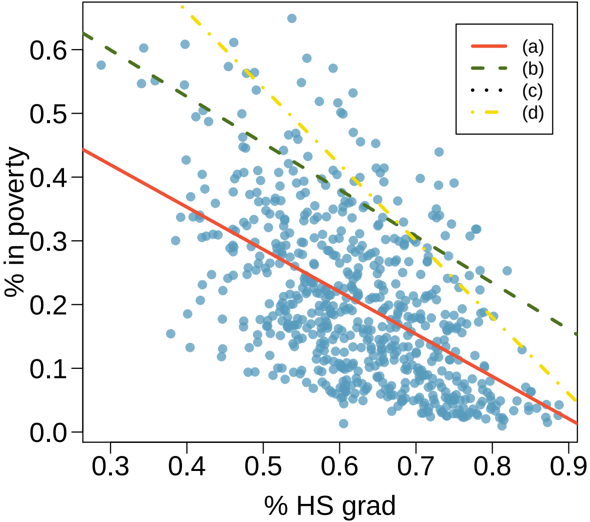
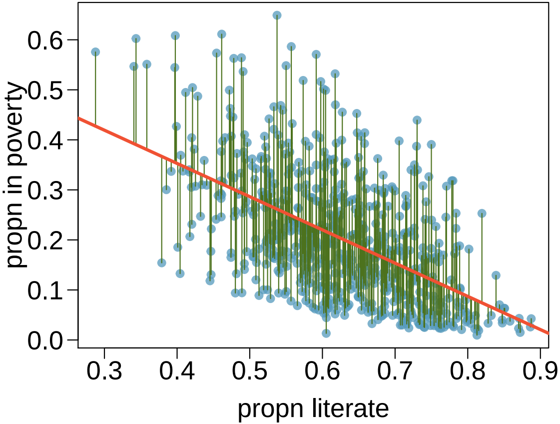


Residuals

Residuals are the leftovers from the model fit: Data = Fit + Residual

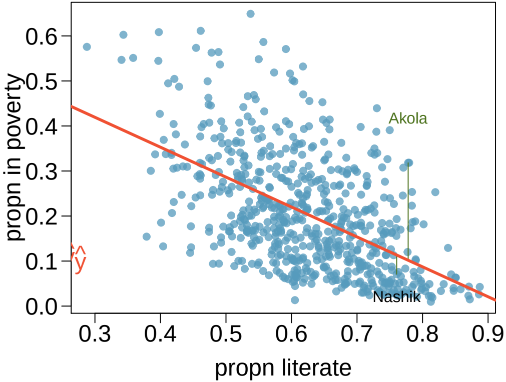


Residuals (cont.)

Residual

Residual is the difference between the observed (y_i) and predicted $\hat{y}_i$.

$$e_i = y_i - \hat{y}_i$$



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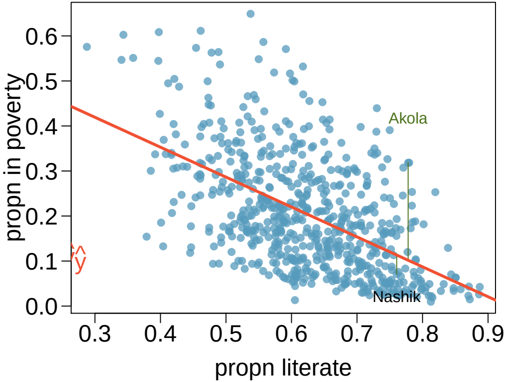

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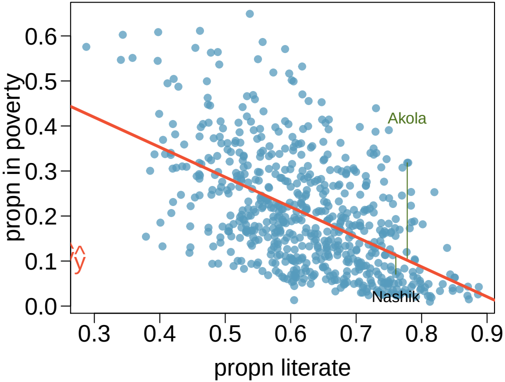

- % living in poverty in Akola is 21% more than predicted.
- % living in poverty in Nashik is 4% less than predicted.

Quantifying the relationship

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- A value of 0 indicates no linear association.

Guessing the correlation

Which of the following is the best guess for the correlation between propn in asset poverty and propn literate?

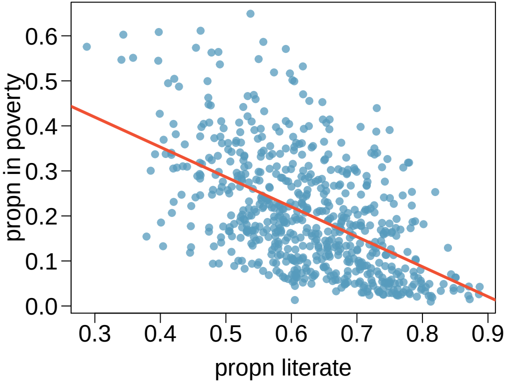
(a) 0.6

(b) -0.54

(c) -0.1

(d) 0.02

(e) -1.5



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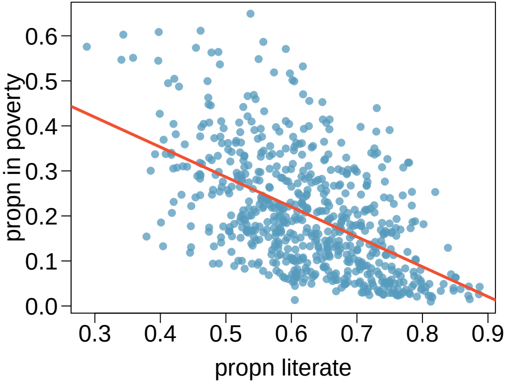

Guessing the correlation

Which of the following is the best guess for the correlation between propn in asset poverty and propn of SC-ST in a district?

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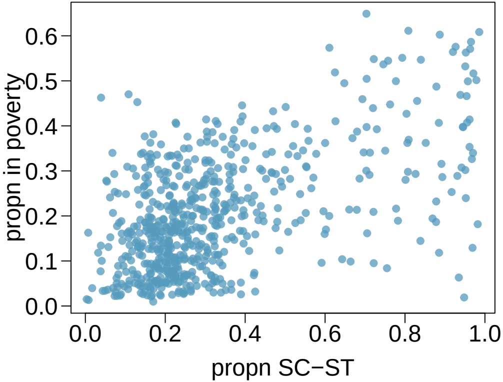
(b) -0.6
(c) -0.4
(d) 0.9
(e) 0.55



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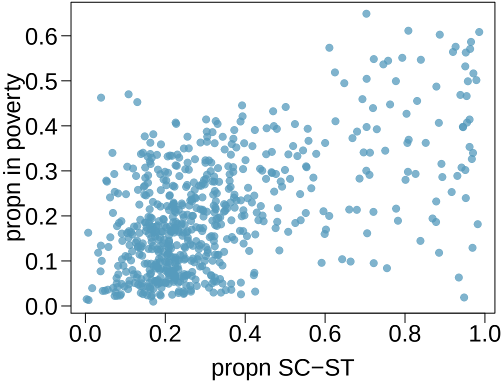

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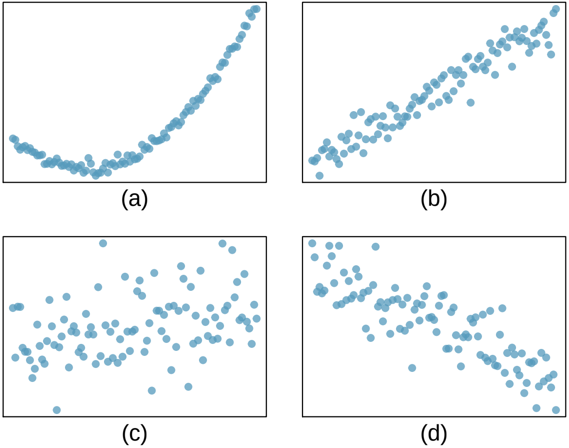

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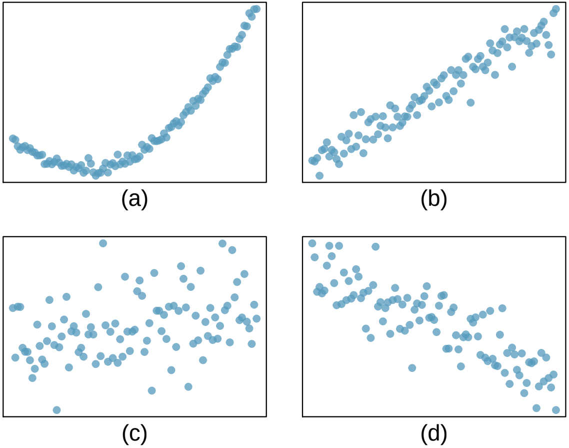

(b) → correlation means linear association

Fitting a line by least squares regression

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- Why least squares?
  1. Most commonly used
  2. Easier to compute by hand and using software
  3. In many applications, a residual twice as large as another is usually more than twice as bad

The least squares line

$$\hat{y} = \beta_0 + \beta_1 x$$

- $\hat{y}$: Predicted value of the outcome variable, y
- β_0: Intercept, parameter
  - b_0: Intercept, point estimate
- β_1: Slope, parameter
  - b_1: Slope, point estimate
- x: Predictor variable

Conditions for the least squares line

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3. Constant variability

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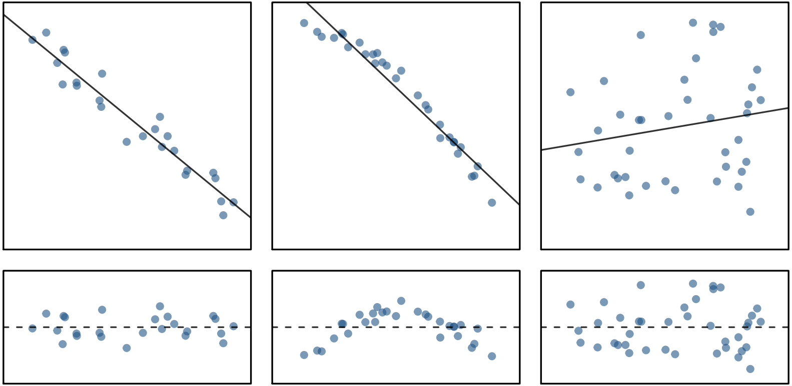
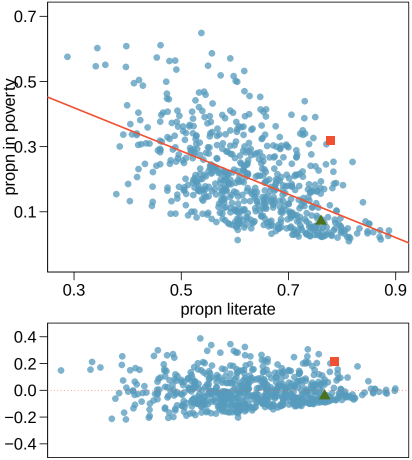
- Check using a scatterplot of the data, or a *residuals plot*.



Anatomy of a residuals plot



Akola:

$$pLit = 0.778 \qquad pNoAssets = 0.318$$

$$\widehat{pNoAssets} = 0.619 - 0.664 * 0.778 = 0.102$$

$$e = pNoAssets - \widehat{pNoAssets}$$

$$= 0.318 - 0.108 = {\color{red}0.217}$$

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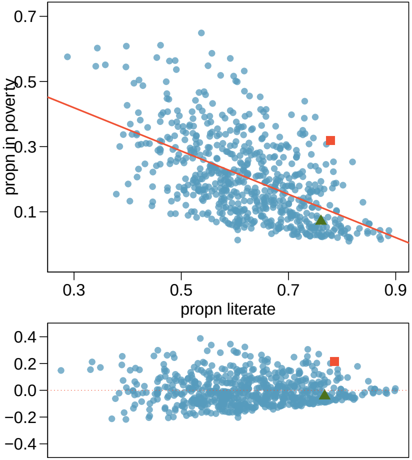

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▲ *Nashik:*

$$pLit = 0.761 \qquad pNoAssets = 0.071$$

$$\widehat{pNoAssets} = 0.619 - 0.664 * 0.761 = 0.113$$

$$e = pNoAssets - \widehat{pNoAssets}$$

$$= 0.071 - 0.113 = -0.042$$

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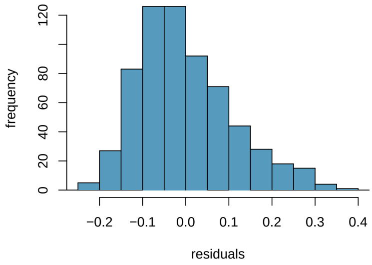
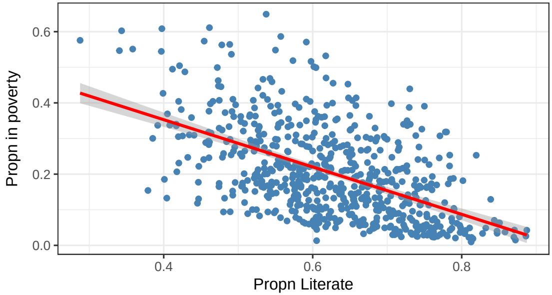
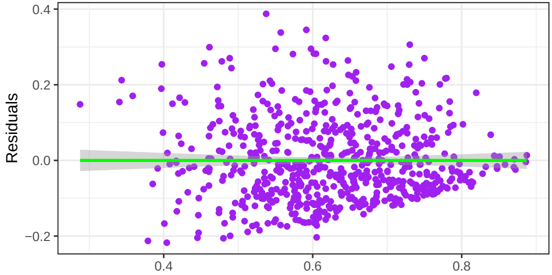
- Check using a histogram.



Conditions: (3) Constant variability





- The variability of points around the least squares line should be roughly constant.

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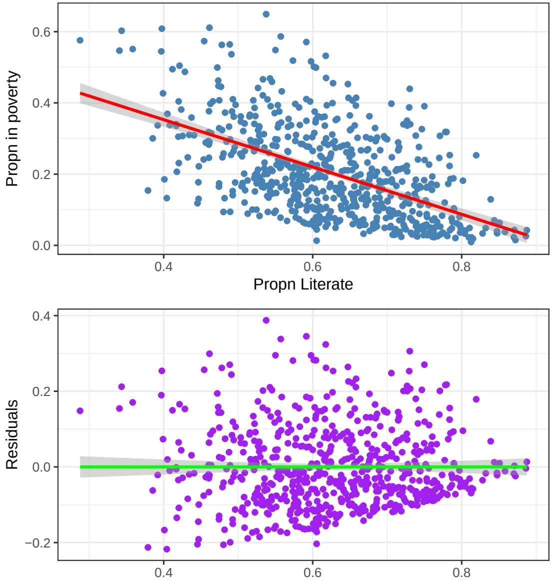

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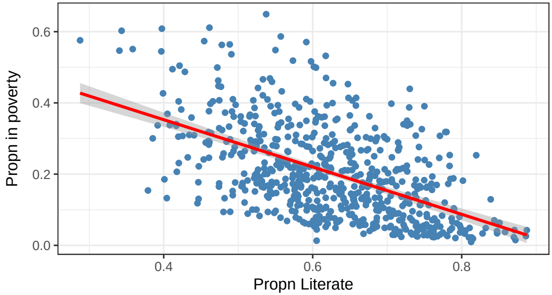

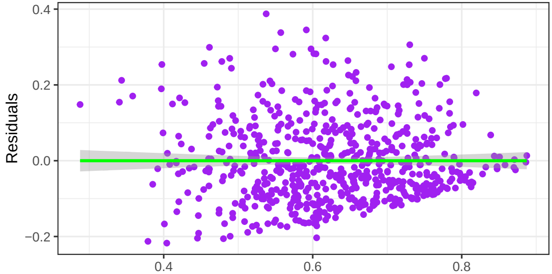

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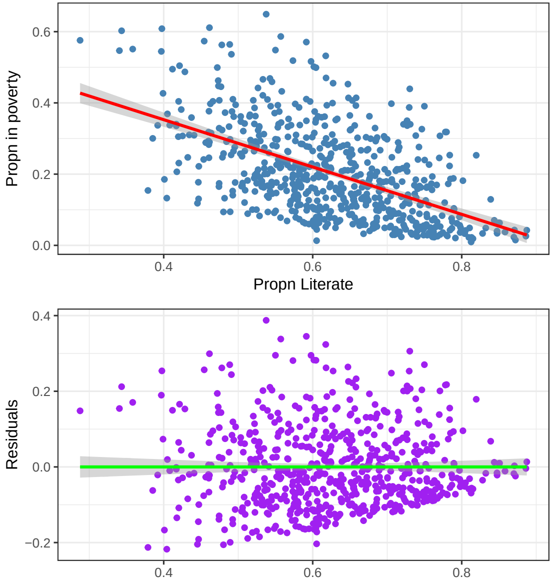

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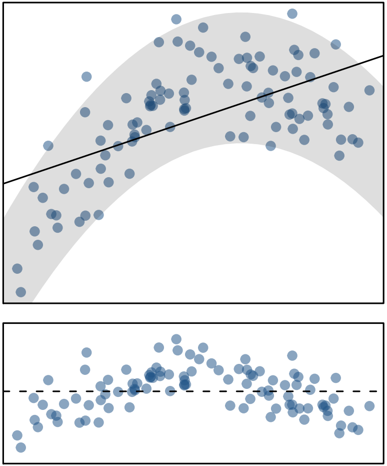
- Also called *homoscedasticity*.
- Check using a residuals plot.

Checking conditions

What condition is this linear model obviously violating?

(a) Constant variability
(b) Linear relationship
(c) Normal residuals
(d) No extreme outliers



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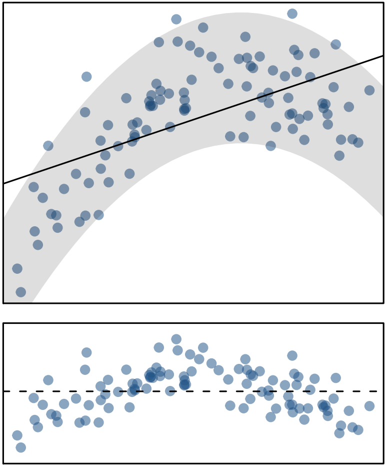

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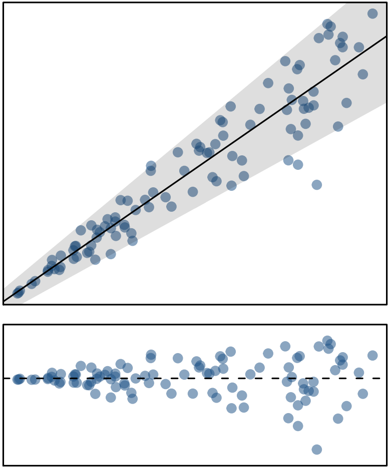

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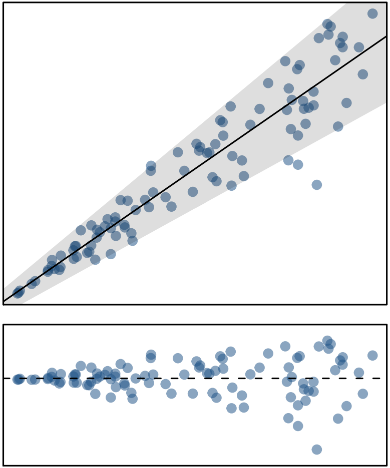
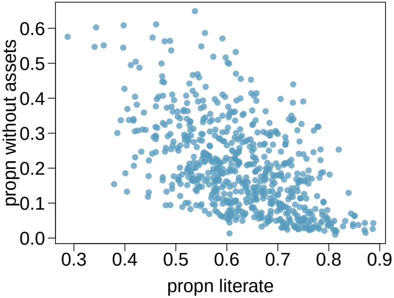


Given...



	propn literate (x)	propn in poverty (y)
mean	$\bar{x} = 0.6248$	$\bar{y} = 0.2036$
sd	$s_x = 0.105$	$s_y = 0.129$
	correlation	$R = -0.54$

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In context...

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Interpretation

For each additional percentage point in literacy rate, the asset poor rate would be lower on average by 0.66% points.

Intercept

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The intercept is where the regression line intersects the y-axis. The calculation of the intercept uses the fact the a regression line always passes through $(\bar{x}, \bar{y})$.

$$b_0 = \bar{y} - b_1\bar{x}$$

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$$\begin{aligned} b_0 &= 0.2036 - (-0.664) \times 0.6248 \\ &= 0.619 \end{aligned}$$

Which of the following is the correct interpretation of the intercept?

(a) For each % point increase in literacy rate, % living in poverty is expected to increase on average by 61.9%.

(b) For each % point decrease in literacy rate, % living in poverty is expected to increase on average by 61.9%.

(c) Having no literate person leads to 61.9% of households living without any assets.

(d) Districts with no literate population are expected on average to have 61.9% of households living in asset-poverty.

(e) In districts with no literate population % living in asset-poverty is expected to increase on average by 61.9%.

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(d) *Districts with no literate population are expected on average to have 61.9% of households living in asset-poverty.*

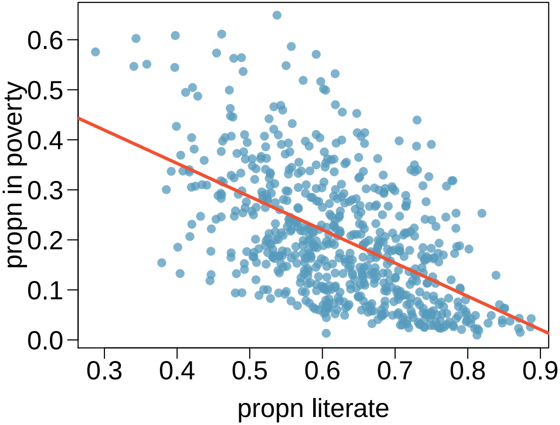
(e) In districts with no literate population % living in asset-poverty is expected to increase on average by 61.9%.

More on the intercept

Since there are no districts in the dataset with no literate population, the intercept is of no interest, not very useful, and also not reliable since the predicted value of the intercept is so far from the bulk of the data.

Regression line

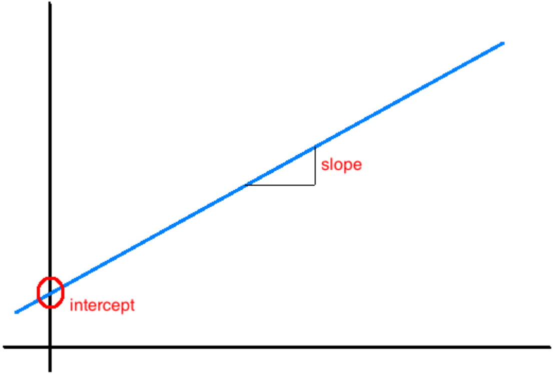
$$\widehat{pNoAssets} = 0.619 - 0.664 \times pLit$$



Interpretation of slope and intercept

- *Intercept:* When $x = 0$, y is expected to equal the intercept.

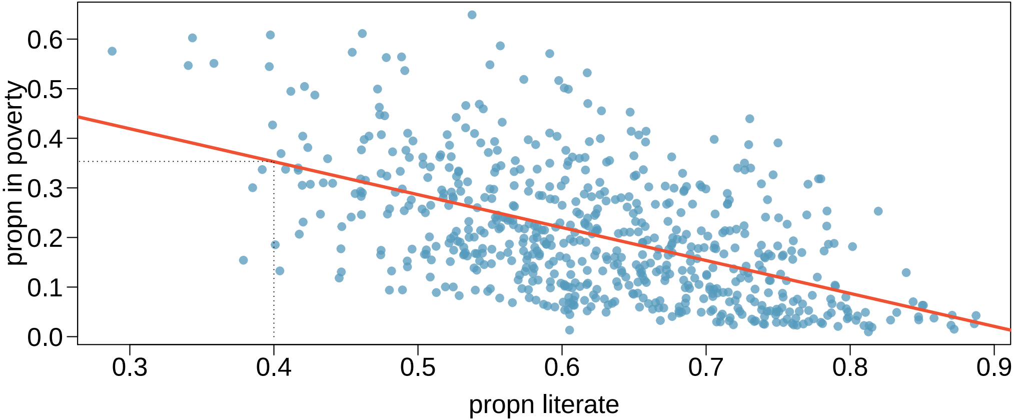
- *Slope:* For each unit in x, y is expected to increase / decrease on average by the slope.



Note: These statements are not causal, unless the study is a randomized controlled experiment.

Prediction

- Using the linear model to predict the value of the response variable for a given value of the explanatory variable is called *prediction*, simply by plugging in the value of x in the linear model equation.
- There will be some uncertainty associated with the predicted value.



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- The remainder of the variability is explained by variables not included in the model or by inherent randomness in the data.
- For the model we've been working with, $R^2 = -0.54^2 = 0.29$.

Interpretation of R^2

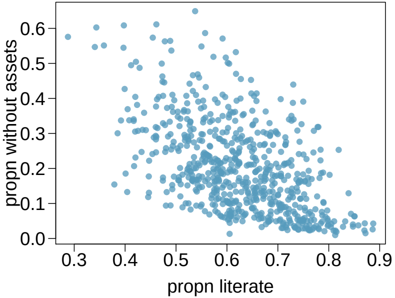
Which of the below is the correct interpretation of $R = -0.54$, $R^2 = 0.29$?

(a) 29% of the variability in the % of literates among the 640 districts is explained by the model.

(b) 29% of the variability in the % of households living in assets poverty among the 640 districts is explained by the model.

(c) 29% of the time % literates predict % living in asset poverty correctly.

(d) 71% of the variability in the % of households living in poverty among the 640 districts is explained by the model.



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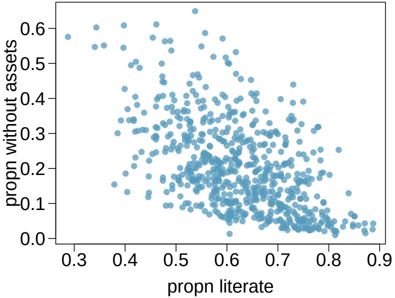
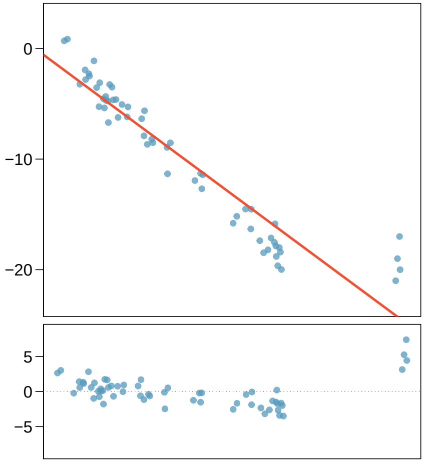


Types of outliers in linear regression

Types of outliers

How do outliers influence the least squares line in this plot?

To answer this question think of where the regression line would be with and without the outlier(s). Without the outliers the regression line would be steeper, and lie closer to the larger group of observations. With the outliers the line is pulled up and away from some of the observations in the larger group.



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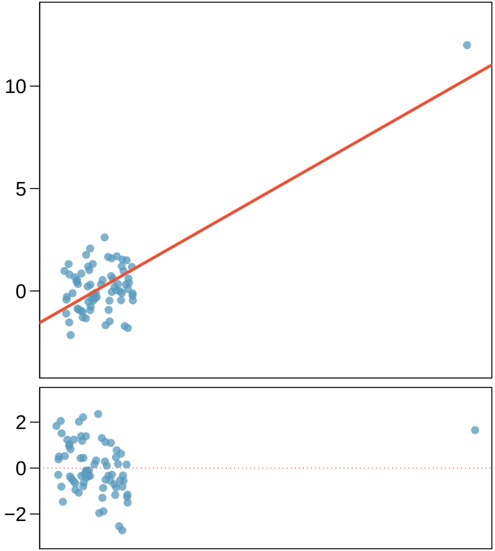

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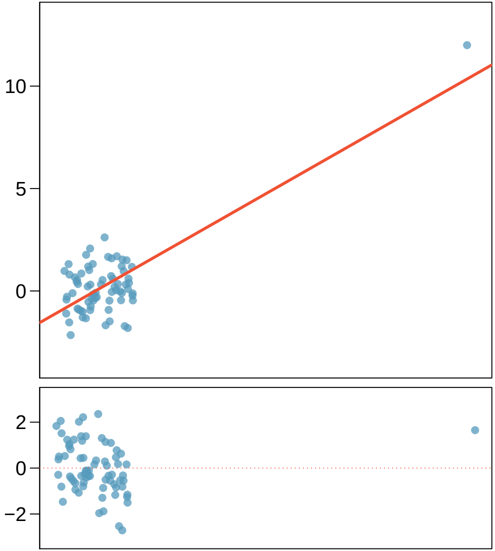
Without the outlier there is no evident relationship between x and y.



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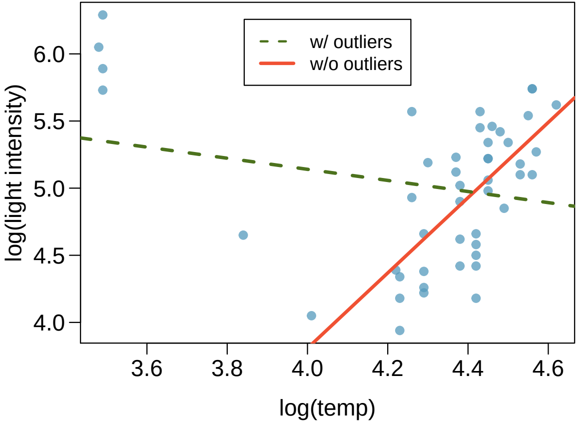
- High leverage points that actually influence the slope of the regression line are called *influential* points.
- In order to determine if a point is influential, visualize the regression line with and without the point. Does the slope of the line change considerably? If so, then the point is influential. If not, then it's not an influential point.

Influential points

Data are available on the log of the surface temperature and the log of the light intensity of 47 stars in the star cluster CYG OB1.



Types of outliers

Which of the below best describes the outlier?

(a) influential

(b) high leverage

(c) none of the above

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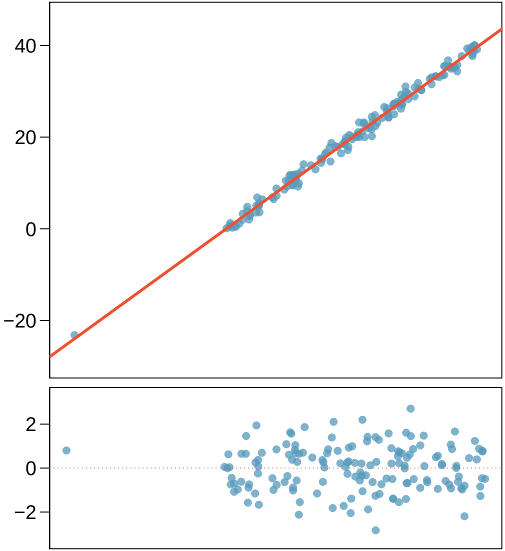

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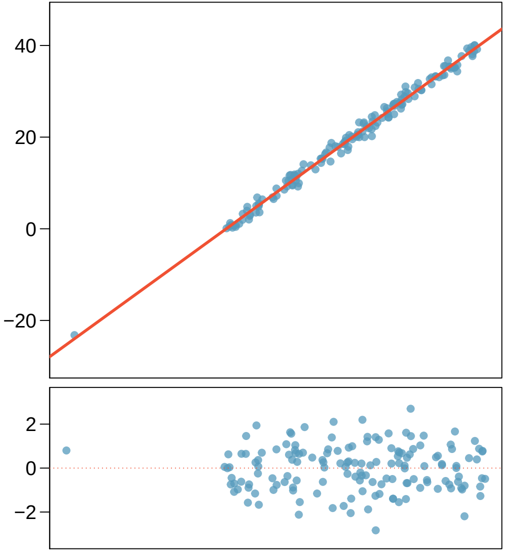

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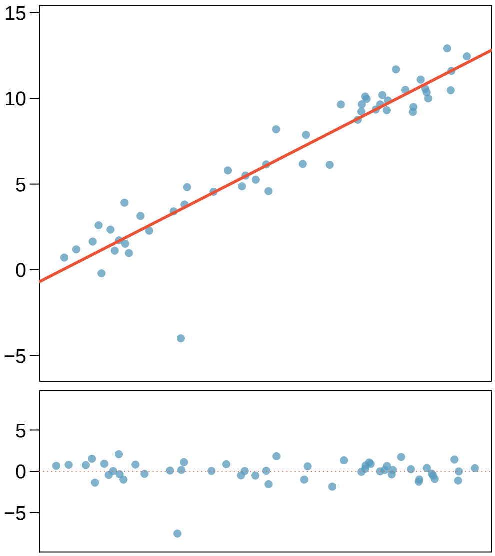
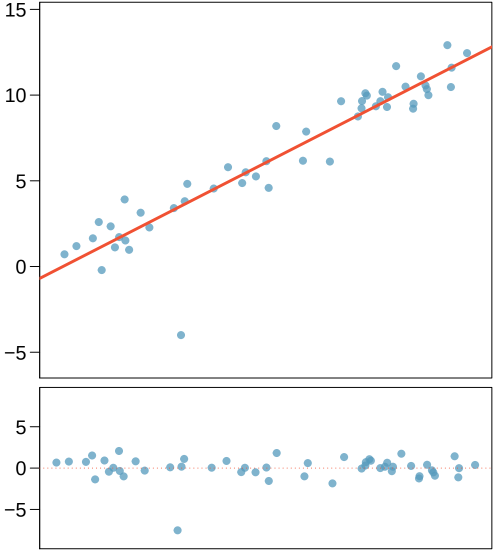


Types of outliers

Does this outlier influence the slope of the regression line?

Not much...



Recap

Which of following is true?

(a) Influential points always change the intercept of the regression line.

(b) Influential points always reduce R^2.

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(e) None of the above.

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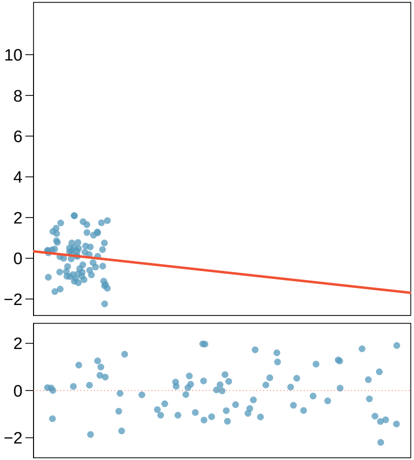
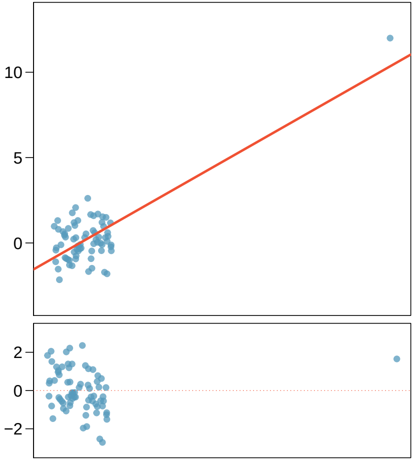
Recap (cont.)

$R = 0.08$, $R^2 = 0.0064$



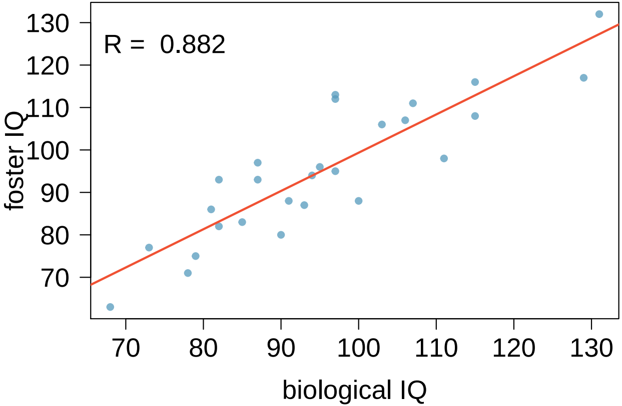
$R = 0.79$, $R^2 = 0.6241$



Inference for linear regression

Nature or nurture?

In 1966 Cyril Burt published a paper called "The genetic determination of differences in intelligence: A study of monozygotic twins reared apart?" The data consist of IQ scores for [an assumed random sample of] 27 identical twins, one raised by foster parents, the other by the biological parents.



Which of the following is false?

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Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)       9.20760    9.29990   0.990    0.332
bioIQ             0.90144    0.09633   9.358  1.2e-09

Residual standard error: 7.729 on 25 degrees of freedom
Multiple R-squared: 0.7779,Adjusted R-squared: 0.769
F-statistic: 87.56 on 1 and 25 DF,  p-value: 1.204e-09
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(a) Additional 10 points in the biological twin's IQ is associated with additional 9 points in the foster twin's IQ, on average.

(b) Roughly 78% of the foster twins' IQs can be accurately predicted by the model.

(c) The linear model is $\widehat{fosterIQ} = 9.2 + 0.9 \times bioIQ$.

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Testing for the slope

Assuming that these 27 twins comprise a representative sample of all twins separated at birth, we would like to test if these data provide convincing evidence that the IQ of the biological twin is a significant predictor of IQ of the foster twin. What are the appropriate hypotheses?

(a) $H_0 : b_0 = 0$; $H_A : b_0 \neq 0$

(b) $H_0 : \beta_0 = 0$; $H_A : \beta_0 \neq 0$

(c) $H_0 : b_1 = 0$; $H_A : b_1 \neq 0$

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  Remember: We lose 1 degree of freedom for each parameter we estimate, and in simple linear regression we estimate 2 parameters, β_0 and β_1.

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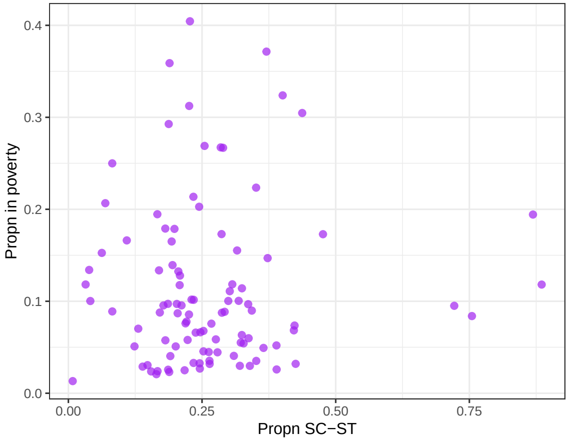
df &= 27 - 2 = 25 \\
p - value &= P(|T| > 9.36) < 0.01
\end{aligned}
$$

% in Poverty vs. % SC-ST

What can you say about the relationship between % in poverty and % SC-ST in a sample of 100 districts in India?



% in Poverty vs % SC-ST - linear model

Which of the below is the best interpretation of the slope?

	term	estimate	std.error	statistic	p.value
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2	pSCST	0.03	0.06	0.50	0.62

(a) A 1% increase in SC-ST population in a district is associated with a 3% increase in % of asset poor.

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(c) An additional 1% of SC-ST population increases the % of asset poor in a district by 10%.

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Do these data provide convincing evidence that there is a statistically significant relationship between % SC-ST and % asset poor in randomly chosen Indian districts?

	Estimate	Std. Error	t value	Pr($>$|t|)
(Intercept)	0.1049	0.0182	5.75	0.0000
pSCST	0.0301	0.0602	0.50	0.6179

How reliable is this p-value if these zip code areas are not randomly selected?

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Not very...

Confidence interval for the slope

Remember that a confidence interval is calculated as $point\ estimate \pm ME$ and the degrees of freedom associated with the slope in a simple linear regression is $n-2$. Which of the below is the correct 95% confidence interval for the slope parameter? Note that the model is based on observations from 27 twins.

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(0.7 &,\ 1.1)
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- The regression output gives b_1, SE_{b_1}, and *two-tailed* p-value for the t-test for the slope where the null value is 0.
- We rarely do inference on the intercept, so we'll be focusing on the estimates and inference for the slope.

Caution

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