Lecture Notes
Economic Growth

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Draft Notes for Chapter 10, 11 (Blanchard, 2013)

What do we learn in this set of lectures?

- The facts of growth.
  - There has been a large increase in output per person in advanced countries.
  - There has been a convergence of output person across countries.
- There was almost zero economic growth until 1500 AD.
- What are the sources of growth?
  1 Capital accumulation
  2 technological progress.

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1. Production Function & Economic Growth

We assume that there are two ingredients to produce goods in economy: **labour** (N), and **capital** (K). Now, we will define a function which uses these two ingredients to produce Y units of output.

$$Y = F(K, N) \tag{1}$$

1.1 Returns to Scale

- Double the labour and double the use of machines for the production of goods. The question is what happens to output? There are three scenarios.

  i Output is also doubled. We term this **constant returns to scale**.

  $$2 \cdot Y = F(2 \cdot K, 2 \cdot N)$$

  ii Output is more than doubled. We call this **increasing returns to scale**.

  iii Output rises by less than 100%. This scenario is known as **decreasing returns to scale**.

- The other question is what happens when you keep adding more and more labour hours or machines.

  – Each additional unit of capital thrown in leads to smaller change in output. Whatchamacallit? **Decreasing returns to capital**.

- Each additional unit of labour hour added in production process will yield smaller gain in output. This is **decreasing returns to labour**.

1.2 Output Per Worker

It makes more sense to look at output per worker rather than the aggregate output if we wish to understand productivity. Let's sketch a function using equation 1.

$$\frac{Y}{N} = F(\frac{K}{N}, \frac{N}{N})$$

$$\frac{Y}{N} = F(\frac{K}{N}, 1) \tag{2}$$

2. Growth Model (Solow)

Assumptions:

1 The population size, the participation rate, and the unemployment rate are all constant.

2 The labour force is equal to population times the participation rate.

3 Employment is the labour force times one minus the unemployment rate.

Recipe

- We simplify equation 2 as follows:

$$\frac{Y}{N} = F(\frac{K}{N}) \tag{3}$$

- We derive a relation between investment and savings assuming $T - G = 0$, and that people save a fixed fraction s of output every year.

$$I_t = s \cdot Y_t \tag{4}$$

- Capital and Investment: Let's assume that you have one machine today, and you know that you will need another one tomorrow. Will tomorrow's machine count be 2? The answer is no. The one that you have today will become older tomorrow, and you need to factor in its **depreciation**. Let's assume that machine depreciates by 10%. So, you will actually have 1.9 machines tomorrow.

$$\begin{aligned} Capital_{\text{Tomorrow}} = Capital_{\text{Today}} - \\ \text{Depreciation Rate} \cdot Capital_{\text{Today}} + \\ Investment_{\text{Today}} \end{aligned} \tag{5}$$

Now, we are in a position to formally establish a nice relationship between investment today and capital tomorrow. We will mathematize equation 5 by using standard notations.

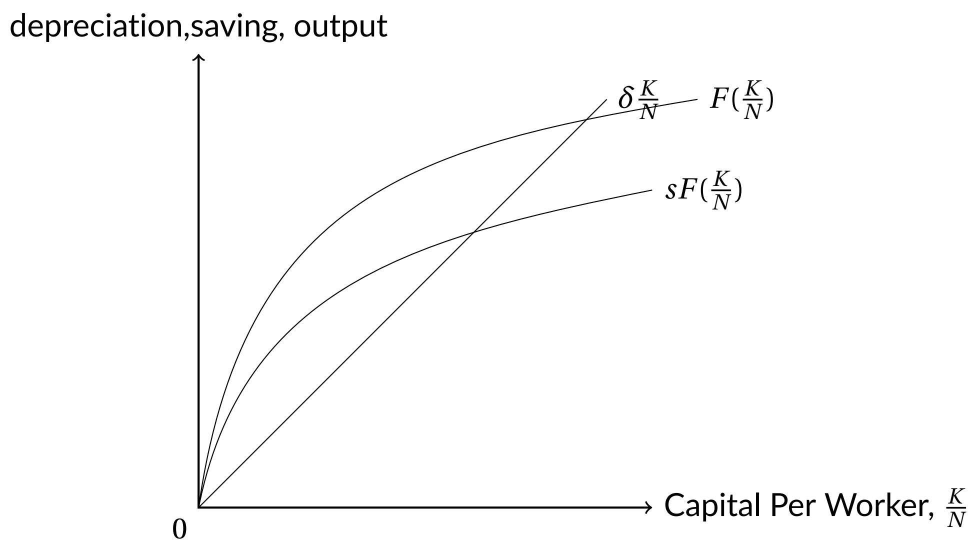
$$\underbrace{K_{t+1}}_{\text{Capital Tomorrow}} = \underbrace{K_t}_{\text{Capital Today}} - \underbrace{\delta \cdot K_t}_{\text{Depreciated Capital}} + \underbrace{I_t}_{\text{Investment Today}} \tag{6}$$

Since we are interested in productivity, we make some more changes to equation 6 (also using equation 3), and we finally have.

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = s \cdot \frac{Y_t}{N} - \delta \cdot \frac{K_t}{N} \tag{7}$$

- Take a look at figure 2. Let's characterize the steady state, the state at which output per worker and capital per worker are no longer changing. Set equation 7 to zero, and you get.

$$s \cdot \frac{K^*}{N} = \delta \cdot \frac{K^*}{N} \tag{8}$$



- Example: Assume that the production function is given by: $Y = \sqrt{K} \cdot \sqrt{N}$, savings rate is 10%, and depreciation is 10%. Find out the equilibrium output per worker.

Solution: First derive a relationship between output per worker and capital per worker.

$$
\begin{aligned}
Y &= \sqrt{K} \cdot \sqrt{N} \\
\frac{Y}{N} &= \sqrt{\frac{K}{N}}
\end{aligned}
$$

We also know the equilibrium condition from equation 7.

$$
\begin{aligned}
s \cdot \sqrt{\frac{K^*}{N}} &= \delta \cdot \frac{K^*}{N} \\
\frac{K^*}{N} &= \left(\frac{s}{\delta}\right)^2
\end{aligned}
$$

From the last piece of this equation, and the previous equation, we can compute steady-state output per worker.

$$
\frac{Y^*}{N} = \left(\frac{s}{\delta}\right)
$$

Plugging values of s and δ in the equation above, we get output per worker = 1.