

Consequences of Fat-Tailed Distributions

Understanding Risk

Overview of Fat-Tailed Distributions

- larger skewness and kurtosis than normal distributions
- potentially infinite variance
- commonly observed in nature and markets
- difficult to work with
- impossible to rule-out

TailRiskAnalyzer

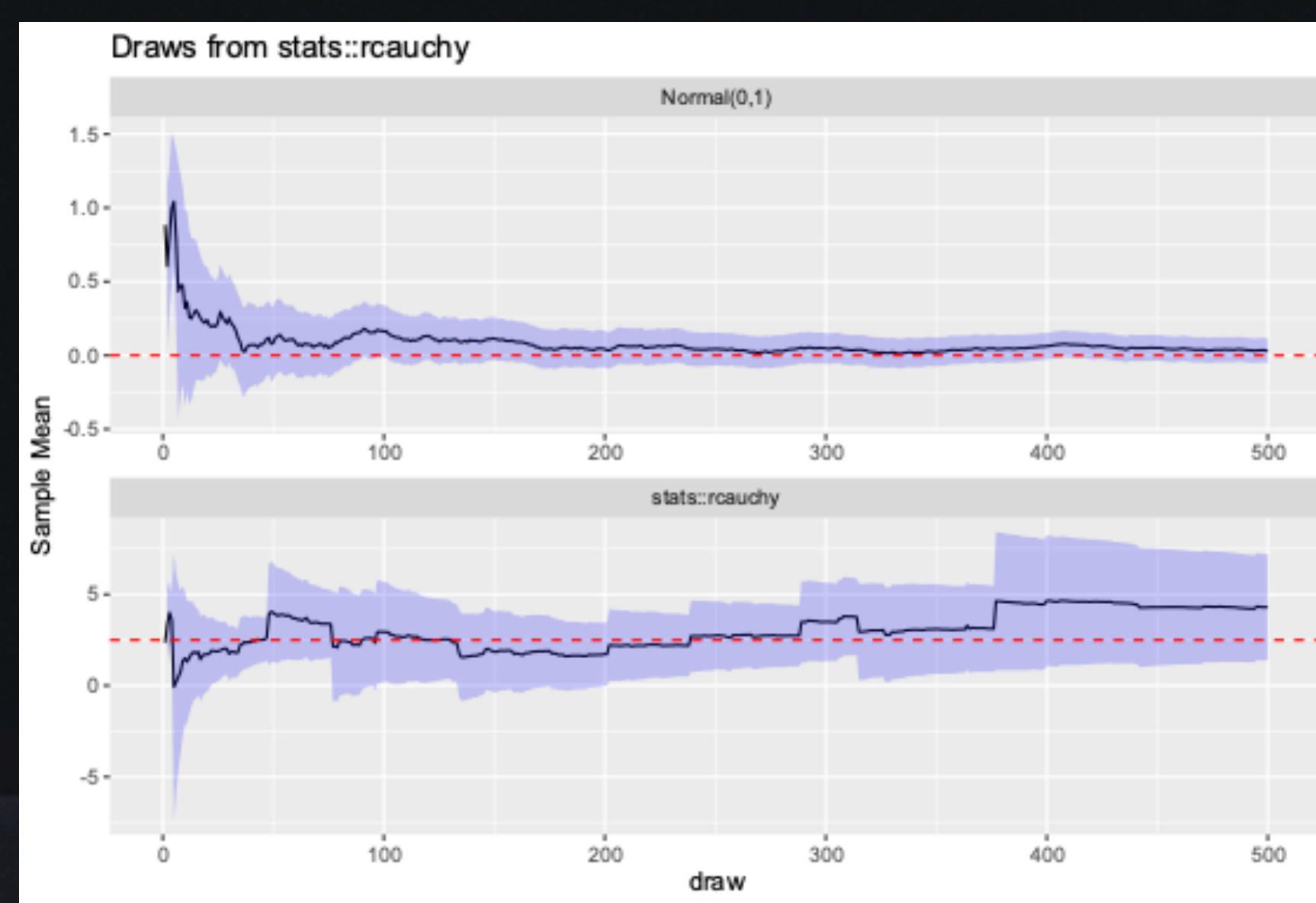
R package available at misikoff.github.io/TailRiskAnalyzer

- analyze distributions for "fat-tailedness", and understand the implications of "fat-tailedness"
- calculate the relative odds of data coming from a normal distribution compared to a distribution with heavier tails, e.g., Student's t -distribution, Pareto distribution, etc.
- determine optimal betting strategies using the Kelly Criterion
- plot outcome distributions for different betting strategies, highlighting the difference between median outcome and expected value, and ensemble averages compared to time averages

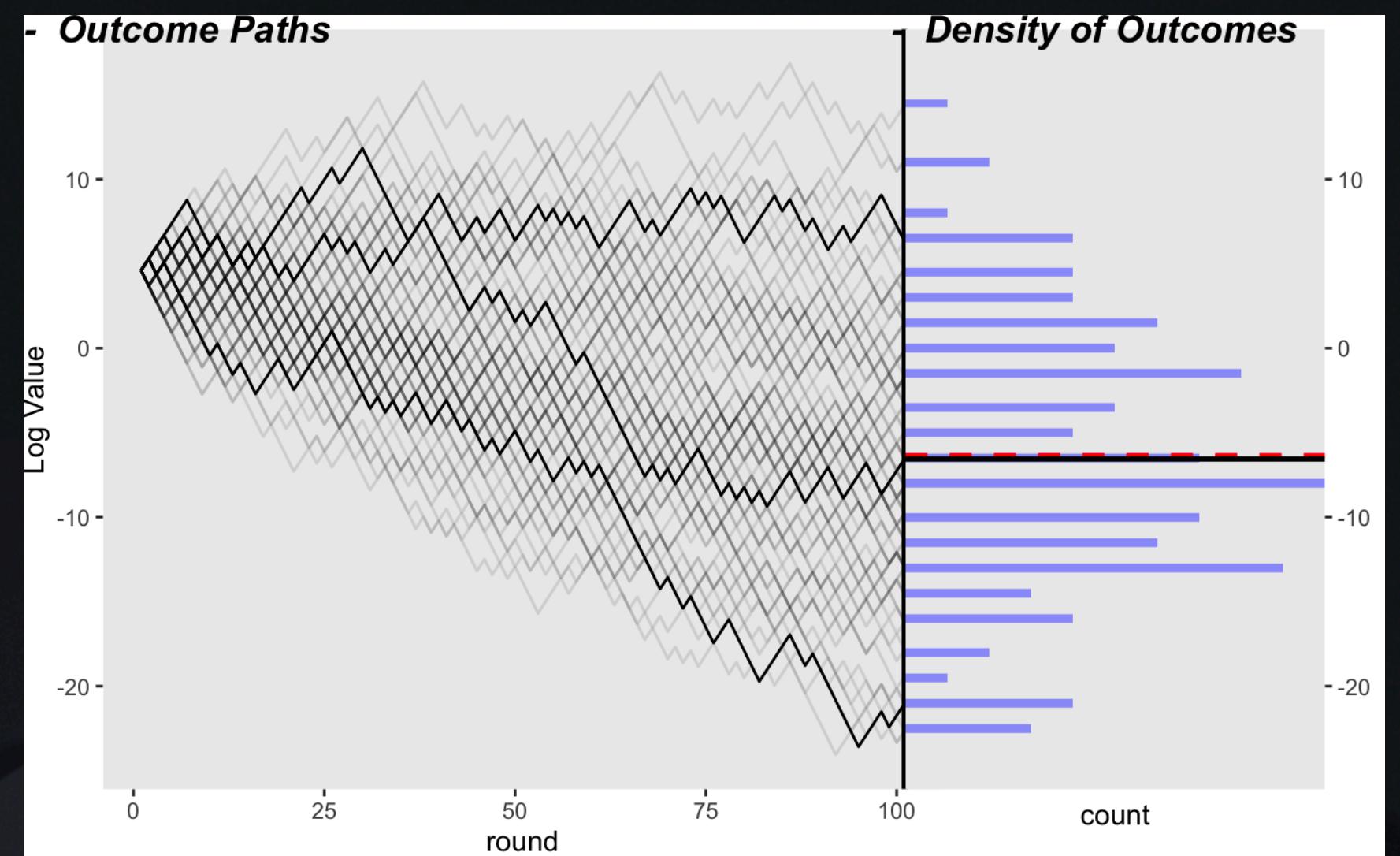
Examples

TailRiskAnalyzer Vignettes

Law of Large Numbers



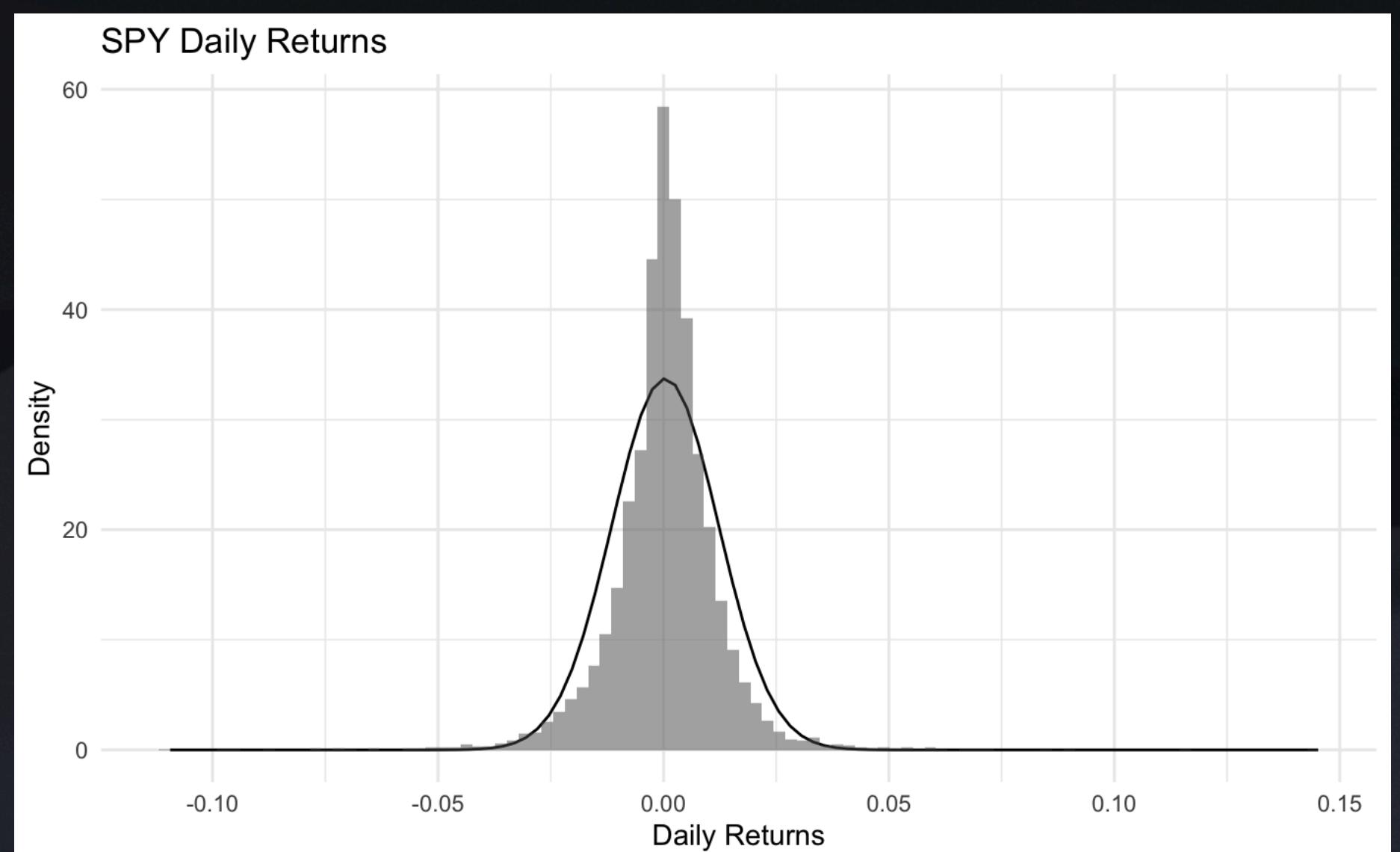
Patterns in Geometric Growth



Applications in Finance

The S & P 500

- Stock market index tracking the 500 largest US companies listed on US stock exchanges in the US, by market capitalization
- Can be traded via index funds, like SPDR S&P 500 ETF Trust (SPY)
- Returns generally fat tailed
 - Kurtosis of S&P 500 daily returns: 14.25316



From Yahoo Finance

Beating the Benchmark

The S&P 500

- A classic "Buy & Hold" strategy
- How can this be improved?
- Insurance focused on raising the lower quantile and median outcomes → lower risk and higher median returns

Characteristics of Insurance

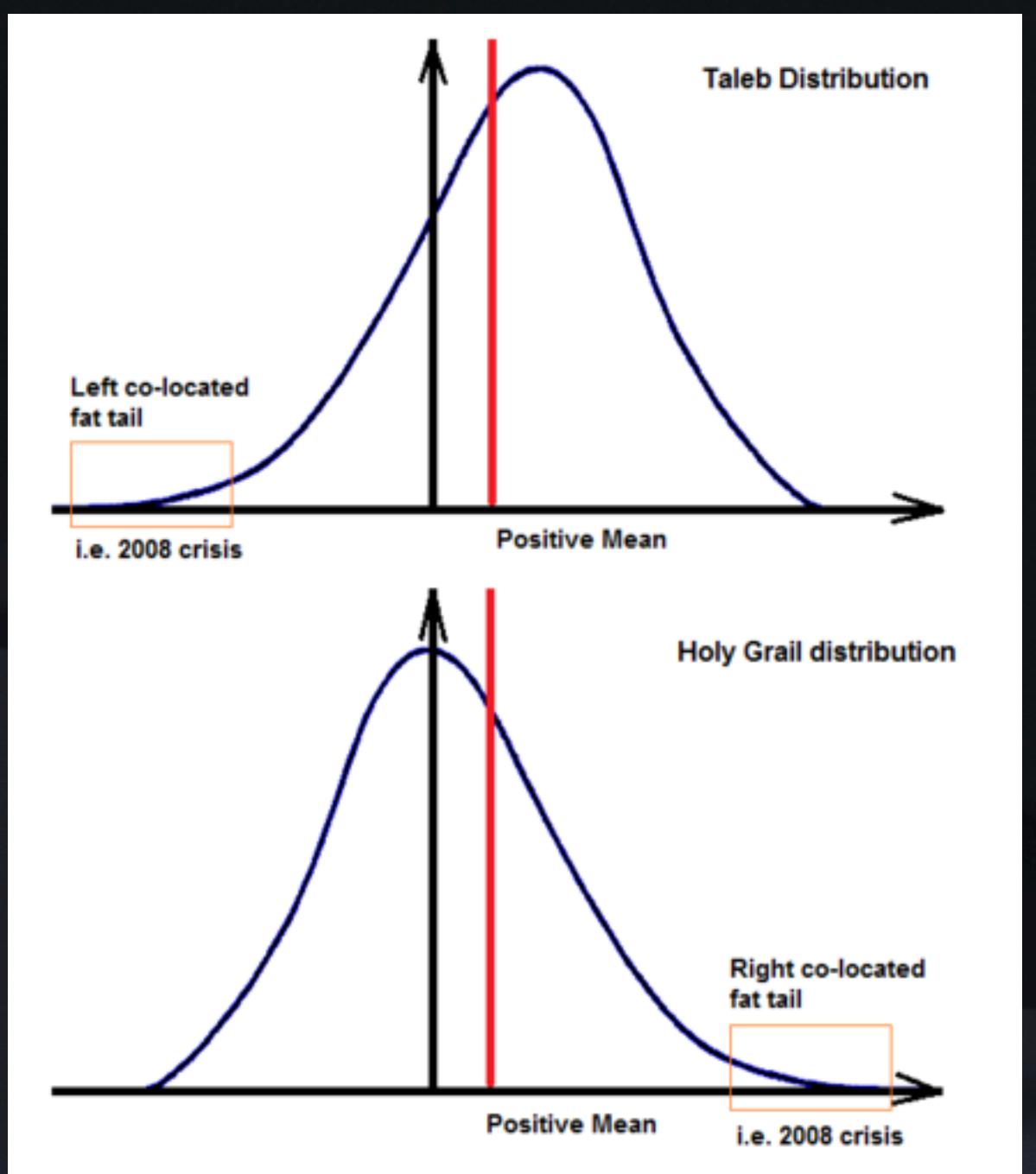
Predictable cost

Offset losses: partially, completely, or more than completely (insurance pays out more than the losses plus cost of contract)

Negative expected value

Optimizing an Investing Strategy

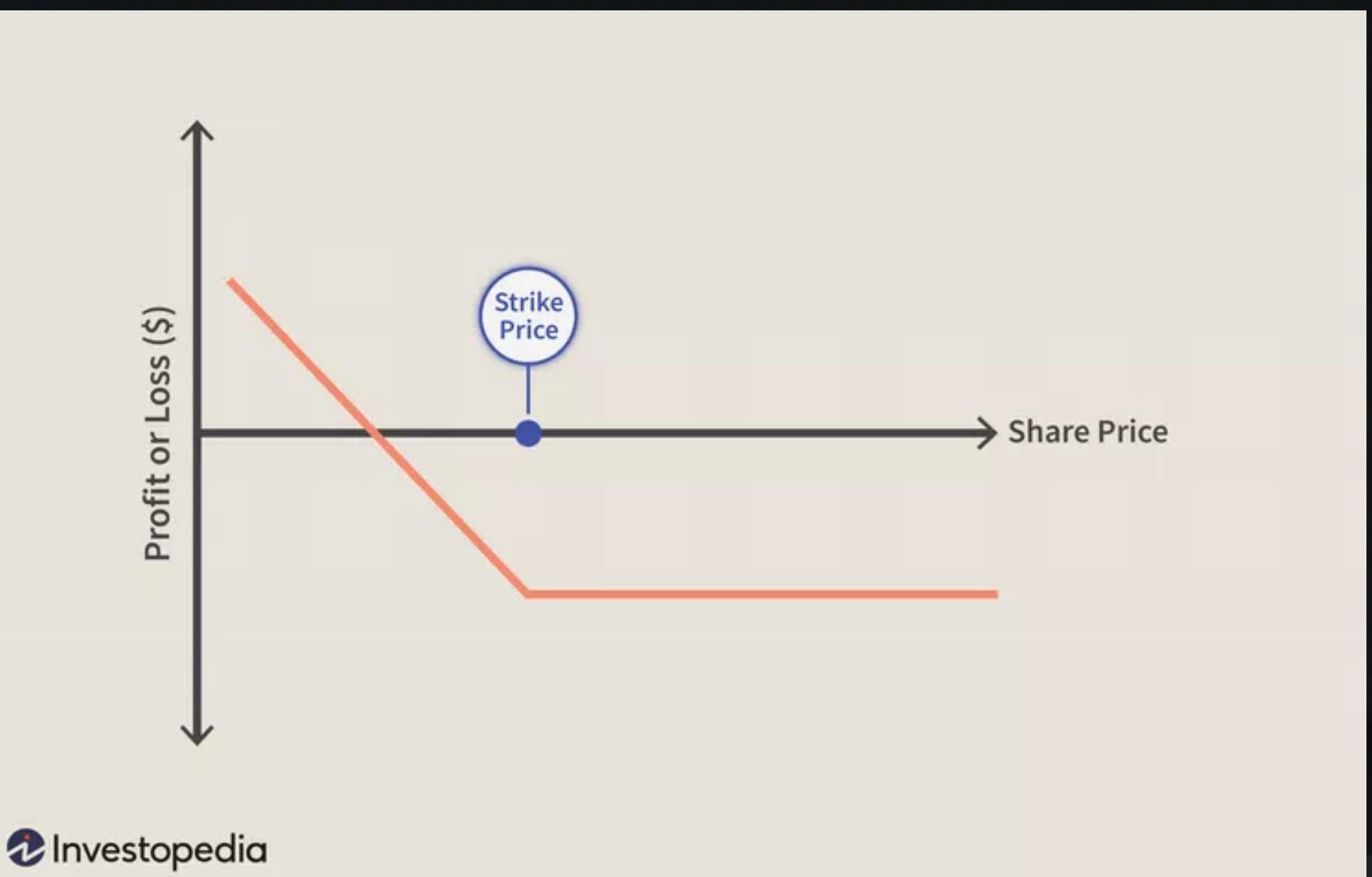
- the distribution of outcomes controlled by geometric growth can be affected by 'insurance', especially through impacts on the median outcome and lower quantiles.
- pair the S & P 500 with catastrophe insurance for a better payoff profile
- OOM options w/ a "minus-Taleb" distribution



Insurance Through Options

Put Options

- Put options give the owner the right (not obligation) to sell the underlying asset at a predetermined price
- Limited downside (the cost of the option)
- Becomes profitable as stock decreases past strike price*
*after accounting for transaction fees
- Since options have expiration dates, they must be bought and sold in cycles to maintain coverage
- Payoff profile
 - variable upside
 - fixed downside (\leq initial purchase price)



Metrics

- Beta
 - measures volatility of a portfolio relative to volatility of the market
- Treynor Ratio
 - measures return of the portfolio compared to "risk free" rate of return relative to volatility of portfolio

$$\beta_p = \frac{\text{Cov}(r_p, r_m)}{\text{Var}(r_m)}$$

$$T_p = \frac{r_p - r_f}{\beta_p}$$

Strategy

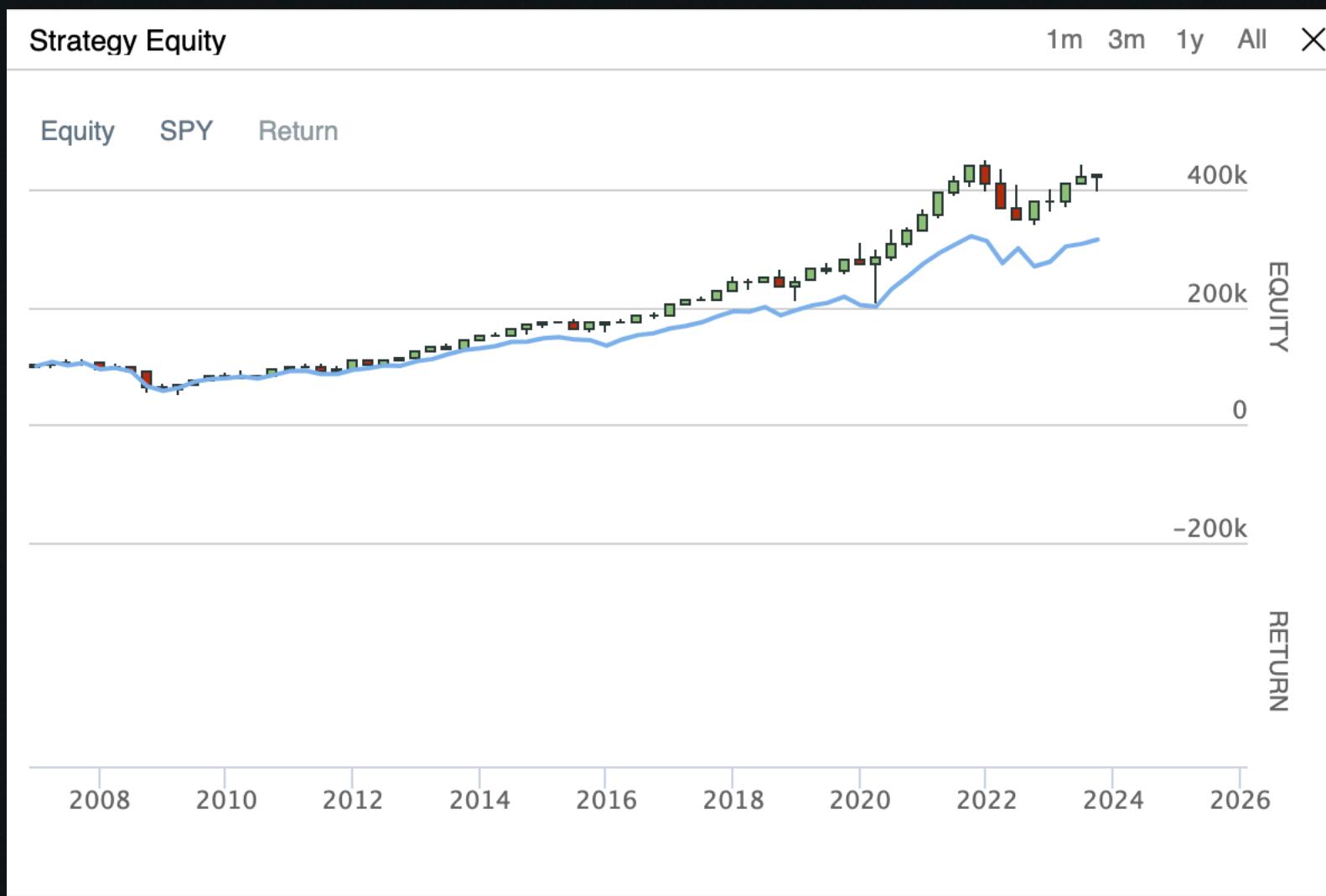
- Each month:
 - sell any held options
 - rebalance portfolio
 - 0.5%: options that will gain value when SPY drops significantly
 - 30% OOM puts, 60-day expiration*
 - 95.5%: SPY

*based on writings from Mark Spitznagel

Comparing the Baseline to the "Insured Strategy"

Implemented via [QuantConnect](#)

SPY



Return: 360.60%

Beta: 0.996

Treynor Ratio: 0.062

SPY + Puts



Return: 388.68%

Beta: 0.597

Treynor Ratio: 0.113

82%

Improvement in Treynor Score Achieved Through Insurance

Conclusions

- better understanding of fat-tailed distributions can be built through software 'playgrounds' like TailRiskAnalyzer
- wide-ranging applications including adjusting payoff profiles in financial markets

References

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