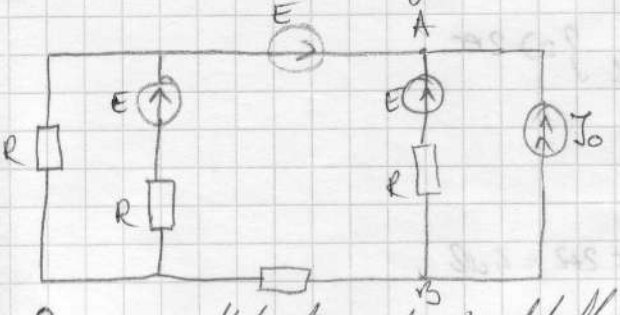
1. Se dă circuitul din figură



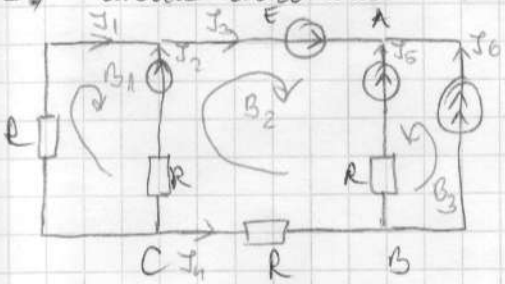
$E = 12\,V$

$J_0 = 1\,A$

$R = 2\,\Omega$

Se cere: 1) Met. teoremelor Kirchhoff
2) Met. curenților ciclici
3.) Met. potențialelor la noduri
4.) T. lui Thevenin
5.) U_{AB} pe mai multe trasee
6.) Teorema conservării puterilor

1) Metoda teoremelor lui Kirchhoff



Analiza topologică: $N = 4$, $L = 6$ $\Rightarrow B = L - N + 1 = 3$

$N = 4 \Rightarrow$ 3 ec. cu T.k.I
$B = 3 \Rightarrow$ 3 ec. cu T.k.II
$\Rightarrow$ 6 ec. ; $J_6 = J_0$ $\Rightarrow$ 5 ec.

$$\begin{cases} (A): J_3 + J_5 + J_6 = 0 \\ (B): J_5 + J_6 - J_4 = 0 \\ (C): J_1 + J_2 + J_4 = 0 \\ (B_1): R J_1 - R J_2 = -E \\ (B_2): R J_2 - R J_5 - R J_4 = E - E + E \\ (B_3): J_6 = J_0 = 1A \end{cases}$$

$$\begin{cases} J_3 + J_5 = -1 \\ -J_4 + J_5 = -1 \Rightarrow J_5 = J_4 - 1 \\ J_1 + J_2 + J_4 = 0 \Rightarrow 2J_3 + J_4 = 6 \\ 2J_1 - 2J_2 = -12 \Rightarrow J_1 = J_2 - 6 \\ 2J_2 - 2J_4 - 2J_5 = 12 \Rightarrow 2J_2 - 2J_4 + 2 = 12 \\ J_6 = J_0 = 1 \end{cases}$$

$$\begin{cases} 2J_2 + J_4 = 6 \\ 2J_2 - 4J_4 = 10 \end{cases} \Rightarrow -5J_4 = 4 \Rightarrow J_4 = -\frac{4}{5}\,A$$

$$\Rightarrow \cancel{J_4 = \frac{4}{5}\,A}$$

$$2J_2 = 6 - J_4 \Rightarrow J_2 = \frac{6 + \frac{4}{5}}{2} = \frac{17}{6}\,A$$

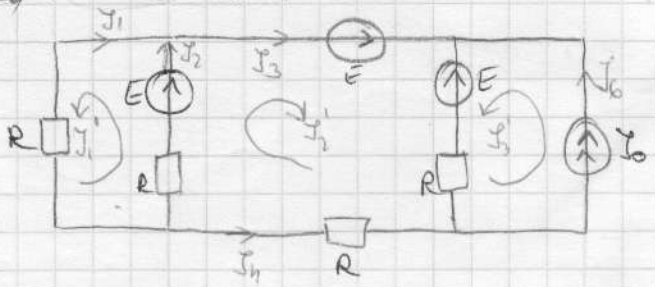
$$J_1 = J_2 - 6 = \frac{17}{5} - 6 = -\frac{13}{5}\,A$$

$$J_3 = -J_5 - 1 = \frac{4}{5}\,A$$

$$J_5 = J_4 - 1 = -\frac{9}{5}\,A$$

$$\begin{cases} J_1 = -\frac{13}{5}\,A \\ J_2 = \frac{17}{5}\,A \\ J_3 = \frac{4}{5}\,A \\ J_4 = -\frac{4}{5}\,A \\ J_5 = -\frac{9}{5}\,A \\ J_6 = 1A \end{cases}$$

2.) Metoda curenților ciclici



$N=4$
$L=3$
$B=3$
$J_3' = J_0 = 1$ } $\Rightarrow$ 2 ec.

$$\begin{cases} R_{11}J_1' + R_{12}J_2' + R_{13}J_3' = E \\ R_{21}J_1' + R_{22}J_2' + R_{23}J_3' = E + E - E \\ J_3' = J_0 = 1 \end{cases}$$

$R_{11} = R + R = 2 + 2 = 4\,\Omega$

$R_{12} = R_{21} = R = 2$

$R_{13} = R_{31} = 0$

$R_{22} = R + R + R = 3 \cdot 2 = 6\,\Omega$

$R_{23} = R = 2\,\Omega$

$$\begin{cases} 4J_1' + 2J_2' + 0 = 12 \\ 2J_1' + 6J_2' + 2 \cdot 1 = 12 \\ J_3' = 1 \end{cases}$$

$$(\Leftrightarrow) \begin{cases} 4J_1' + 2J_2' = 12 \,/:2 \\ 2J_1' + 6J_2' = 10 \,/:2 \end{cases} \Leftrightarrow \begin{cases} 2J_1' + J_2' = 6 \\ J_1' + 3J_2' = 5 \,/\cdot(-2) \end{cases} \Leftrightarrow \begin{cases} 2J_1' + J_2' = 6 \\ -2J_1' - 6J_2' = -10 \end{cases} \Rightarrow J_2' = \frac{4}{5}\,A$$

$J_1' = 5 - 3 \cdot J_2' = 5 - 3\,\frac{4}{5} = \frac{13}{5}\,A$

$$\begin{cases} J_1' = \frac{13}{5}\,A \\ J_2' = \frac{4}{5}\,A \\ J_3' = 1A \end{cases}$$

$J_1 = -\frac{13}{5}\,A = -J_1'$

$J_2 = \frac{17}{5}\,A = J_1' + J_2'$

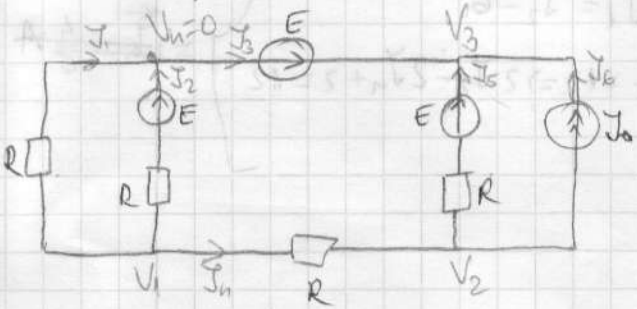
$J_3 = \frac{4}{5}\,A = J_2'$

$J_4 = -J_2 = -\frac{4}{5}\,A$

$J_5 = -J_2' - J_3' = -\frac{4}{5} - 1 = -\frac{9}{5}\,A$

$J_6 = J_2' = 1A$

3.) Metoda potențialelor la noduri



$N = 4 \Rightarrow$ 3 ec
$V_3 = E = 12V$ } $\Rightarrow$ 2 ec.

$$\begin{cases} G_{11}V_1 + G_{12}V_2 + G_{13}V_3 = J_{sc1} \\ G_{21}V_1 + G_{22}V_2 + G_{23}V_3 = J_{sc2} \\ V_3 = 12V \end{cases}$$

$G_{11} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{2}\,\Omega^{-1}$

$G_{12} = G_{21} = -\frac{1}{R} = -\frac{1}{2}\,\Omega^{-1}$

$G_{13} = 0$

$G_{22} = \frac{1}{R} + \frac{1}{R} = 1\,\Omega^{-1}$

$J_{sc1} = -\frac{E}{R} = -\frac{12}{2} = -6\,A$

$J_{sc2} = -\frac{E}{R} - J_0 = -6 - 1 = -7\,A$

$$\begin{cases} \frac{3}{2}V_1 - \frac{1}{2}V_2 + 0 = -6 \,/\cdot 2 \\ -\frac{1}{2}V_1 + V_2 - \frac{1}{2} \cdot 12 = -7 \,/\cdot 2 \end{cases} \qquad \begin{cases} 3V_1 - V_2 = -12 \,/\cdot 2 \\ -V_1 + 2V_2 = -2 \end{cases}$$

$6V_1 - 2V_2 = -24$

$-V_1 + 2V_2 = -2$

$5V_1 / = -26 \Rightarrow V_1 = -\frac{26}{5} V$

$V_2 = 3V_1 + 12 = -3 \cdot \frac{26}{5} + \frac{5}{12} = -\frac{18}{5} V$; $V_3 = 12V$

$$\begin{cases} V_1 = -\frac{26}{5} V \\ V_2 = -\frac{18}{5} V \\ V_3 = 12V \end{cases}$$

$I_1 = \frac{V_1 - V_4}{R} = -\frac{26}{5} \cdot \frac{1}{2} = -\frac{13}{5} A$

$I_5 = \frac{V_2 - V_3 + E}{R} = -\frac{9}{5} A$

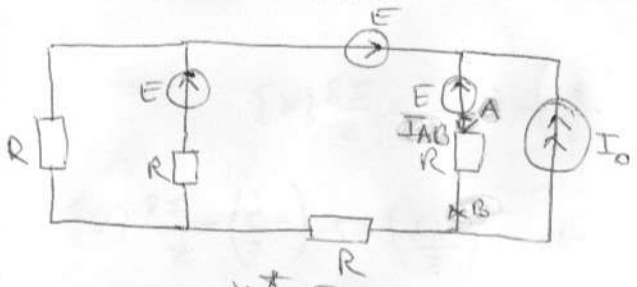
$I_2 = \frac{V_1 - V_4 + E}{R} = \frac{-\frac{26}{5} - 0 + 12}{2} = \frac{17}{5} A$

$I_6 = I_0 = 1A$

$I_3 = I_1 + I_2 = \frac{4}{5} A$

$I_4 = \frac{V_1 - V_2}{R} = \frac{-\frac{26}{5} + \frac{18}{5}}{2} = -\frac{4}{5} A$

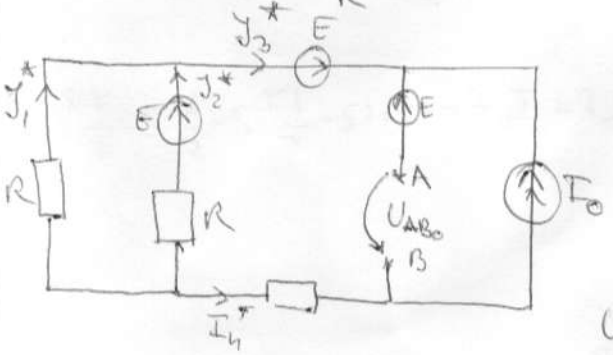
h) Teorema lui Thèvenin

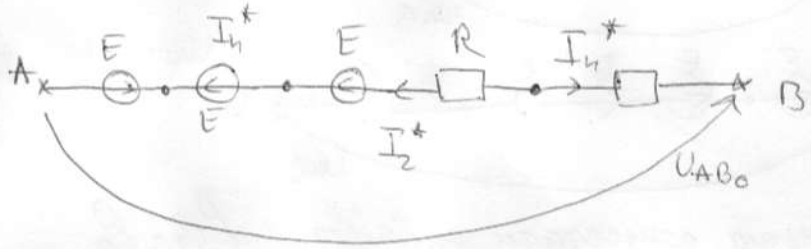


→ determinăm U_{AB_0}: facem mers în gol între A și B; refacem circuitul

Teorema lui Thèvenin: $I_{AB} = \frac{U_{AB_0}}{R_{AB} + R_{ABP}}$

$R_{AB} = R = 2\Omega$

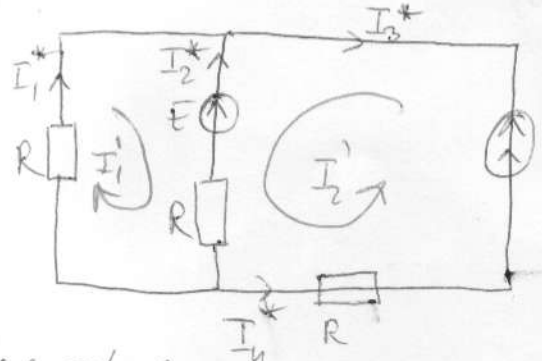




$U_{AB_0} = -E + E + E - R \cdot I_2^* + R I_4^* = E - R I_2^* + R \cdot I_4^*$

Obs: am pus steluțe la acești curenți pt. a fi foarte clar că aceștia sunt curenții din circuitul în care am făcut mers în gol NU curenții circuitului inițial.

!! nu cunoaștem I_2^*, I_4^*. Refacem circuitul pentru a vedea mai clar cum putem să îi calculăm.



$N = 2$, $L = 3$ } $B = 2$: avem un circuit cu 2 bucle și o sursă ideală de curent

⇒ cea mai rapidă metodă ar fi metoda curenților ciclici

$R_{11} = R + R = 4\Omega$

$R_{12} = R = 2\Omega$

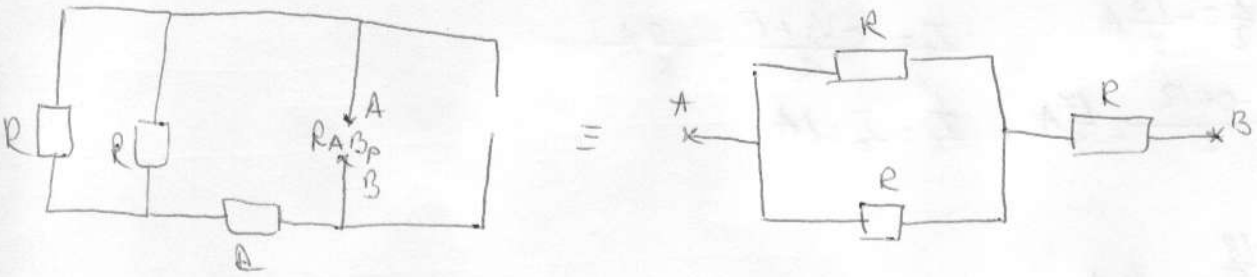
$\Rightarrow 4I_1' + 2 = -12 \Rightarrow I_1' = -\frac{7}{2} A$

$$\begin{cases} R_{11} I_1' + R_{12} I_2' = -E \\ I_2' = I_0 = 1A \end{cases}$$

$$\Rightarrow \begin{cases} I_1' = -\frac{7}{2} A \\ I_2' = I_0 = 1 A \end{cases} \Rightarrow$$

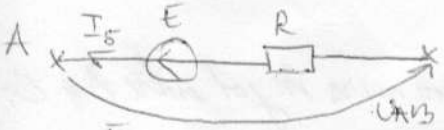
$$\Rightarrow \begin{cases} I_1^* = I_1' = -\frac{7}{2} [A] \\ I_2^* = -I_1' - I_2' = \frac{7}{2} - 1 = \frac{5}{2} [A] \\ I_3^* = -I_2' = -1 [A] \\ I_4^* = I_2' = 1 [A] \end{cases} \Rightarrow U_{AB_0} = E - R \cdot I_2^* + R \cdot I_4^* = 12 - 2 \cdot \frac{5}{2} + 2 \cdot 1 = 9 [V]$$

→ <u>Determinăm R_{AB_p}</u>: rezistența echivalentă văzută între A și B după pasivizarea circuitului:

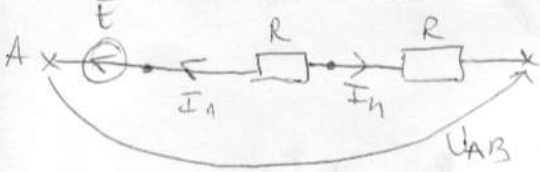


$$R_{AB_p} = \frac{R \cdot R}{R+R} + R = \frac{4}{4} + 2 = 3 [\Omega] \Rightarrow I_{AB} = \frac{9}{2+3} = \frac{9}{5} [A]$$

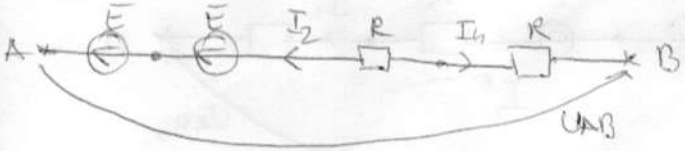
5.) U_{AB} pe mai multe trasee



$$U_{AB} = E - R \cdot I_5 = 12 - 2 \cdot \left(-\frac{8}{5}\right) = \frac{60+18}{5} = \frac{78}{5} [V]$$



$$U_{AB} = E - R \cdot I_1 + R \cdot I_4 = 12 - 2 \cdot \left(-\frac{13}{5}\right) + 2 \cdot \left(-\frac{4}{5}\right) = \frac{78}{5} [V]$$



$$U_{AB} = E + E - I_2 R + I_4 R = 12 + 12 - \frac{17}{5} \cdot 2 - \frac{4}{5} \cdot 2 = \frac{78}{5} [V]$$

6.) Teorema conservării puterilor $P_G = P_R$

$$P_G = \sum_{k=1}^{4} (E_k \cdot I_k + U_k \cdot I_0) \qquad P_R = \sum_{k=1}^{4} R_k \cdot I_k^2$$

$$P_G = E \cdot I_2 + E \cdot I_3 + E \cdot I_5 + U_{AB} \cdot I_0 =$$
$$= 12 \cdot \frac{17}{5} + 12 \cdot \frac{4}{5} + 12 \cdot \left(-\frac{9}{5}\right) + \frac{78}{5} =$$
$$= 12 \cdot \left(\frac{17+4-9}{5}\right) + \frac{78}{5} = \frac{222}{5} [W]$$

$$P_R = R \cdot I_1^2 + R \cdot I_2^2 + R \cdot I_4^2 + R \cdot I_5^2$$
$$= 2 \cdot \left(-\frac{13}{5}\right)^2 + 2 \cdot \left(\frac{17}{5}\right)^2 + 2 \cdot \left(-\frac{4}{5}\right)^2 + 2 \cdot \left(-\frac{9}{5}\right)^2$$
$$= 2 \cdot \frac{111}{5} = \frac{222}{5} [W]$$

$$\Rightarrow P_G = P_R$$