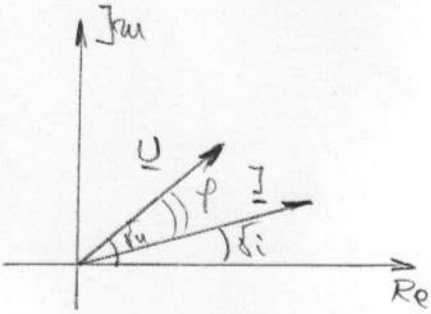
11.05.2010

Curs nr 10:

$\boxed{\text{fără pag. 229-238}}$

3. Impedanțe echivalente.

$$\left.\begin{aligned} u &= U\sqrt{2}\sin(\omega t+\gamma_u) \\ i &= I\sqrt{2}\sin(\omega t+\gamma_i) \end{aligned}\right\} \xrightarrow{\quad} \underline{U} = U\cdot e^{j\gamma_u}, \quad \gamma_u - \gamma_i = \varphi, \quad \xrightarrow{\quad} \underline{I} = I\cdot e^{j\gamma_i}$$



$$\boxed{\begin{cases} j = \sqrt{-1} \\ \frac{1}{j} = -j \\ j^2 = -1 \end{cases}}$$

$$\underline{Z} = \frac{U}{I}\cdot e^{j(\gamma_u-\gamma_i)} = \underbrace{\frac{U}{I}\cos\varphi}_{R} + j\underbrace{\left(\frac{U}{I}\right)\sin\varphi}_{X}$$

$$\underline{S} = \underline{U}\cdot\underline{I}^* = \underbrace{U\cdot I\cdot\cos\varphi}_{P} + j\underbrace{UI\cdot\sin\varphi}_{Q}$$

$$\underline{S} = \frac{\underline{U}\,\underline{I}^*\cdot\underline{I}}{\underline{I}} = \frac{\underline{U}}{\underline{I}}\cdot I^2 = \underline{Z}\cdot I^2$$

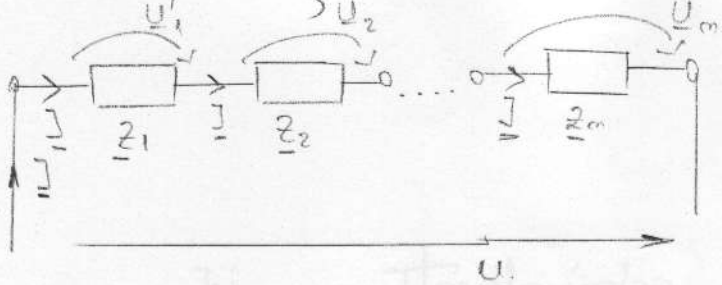
$$\underline{I}^*\cdot\underline{I} = I^2$$

$$\underline{Z}_e = \frac{\underline{U}}{\underline{I}}$$

3.1. Impedanțe echivalente în circuite necuplate.

$M>0 \quad K = \frac{|M|}{\sqrt{L_1 L_2}} \leq 1$

a) Impedanțe conectate în serie



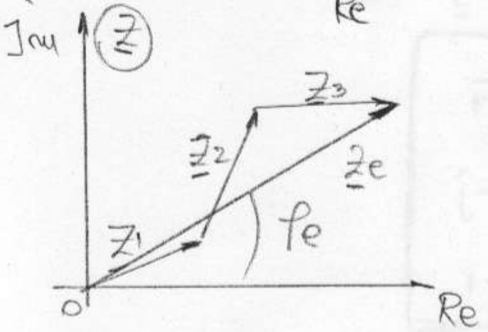
$U_1 = \underline{I} \cdot \underline{Z}_1$

$U_2 = \underline{I} \cdot \underline{Z}_2$

$U_n = \underline{I} \cdot \underline{Z}_n$

$$\underline{U} = \sum_1^n \underline{U}_k = \underline{I} \sum_1^n \underline{Z}_k$$

$$\underline{Z}_e = \sum_1^n \underline{Z}_k = \underbrace{(R_1 + R_2 + \dots + R_n)}_{R_e} + j\underbrace{(X_1 + X_2 + \dots + X_n)}_{X_e}$$

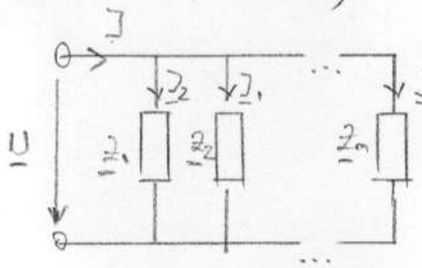


$R_k \geq 0 \Rightarrow R_e > R_k$

$X_k \gtrless 0 \qquad X_e \not> X_k$

$\boxed{X_e = 0}$ - rezonanță

b) Impedanțe conectate în paralel.



$\underline{Z}_e = \dfrac{\underline{U}}{\underline{I}}$

$\underline{Y}_e = \dfrac{1}{\underline{Z}_e} = \dfrac{\underline{I}}{\underline{U}}$

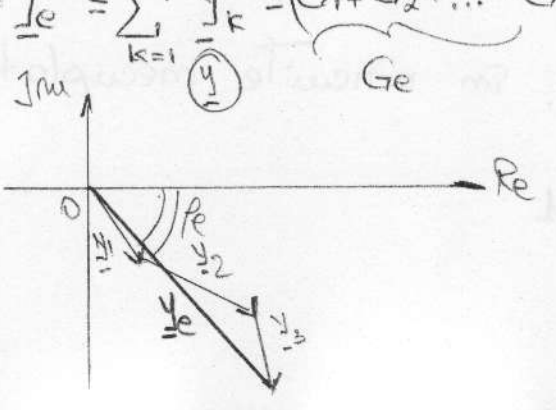
$\underline{I}_k = \dfrac{\underline{U}}{\underline{Z}_k} = \underline{U} \cdot \underline{Y}_k$

$$\underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{R + jX} = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2} = \frac{R}{Z^2} - j\frac{X}{Z^2}$$

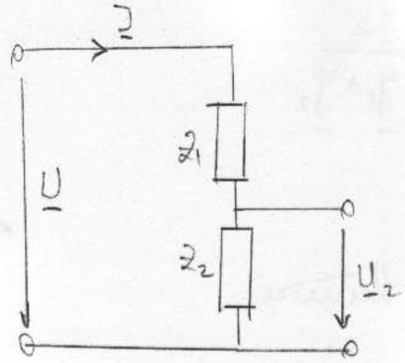
$= G - jB$

$$\underline{I} = \sum_1 \underline{I}_k = \underline{U} \sum_{k=1}^n \underline{Y}_k$$

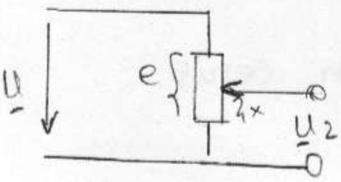
$$\frac{\underline{I}}{\underline{U}} = \underline{Y}_e = \sum_{k=1}^n \underline{Y}_k = \underbrace{(G_1 + G_2 + \dots + G_n)}_{G_e} + j\underbrace{(B_1 + B_2 + \dots + B_n)}$$



$$|\underline{Y}_e| < \sum_1 |\underline{Y}_k|$$



$$\underline{U}_2 = \underline{I} \cdot \underline{Z}_2 = \underline{U}\,\frac{\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2}$$

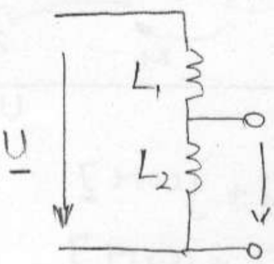


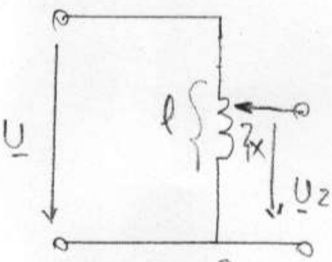
$$\underline{U}_2 = \underline{U} \cdot \frac{x}{e}$$

$$\underline{Z}_1 = j\omega L_1$$

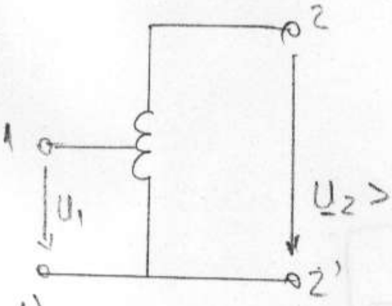
$$\underline{Z}_2 = j\omega L_2 .$$

$$\underline{U}_2 = \underline{U}\,\frac{L_2}{L_1+L_2}$$

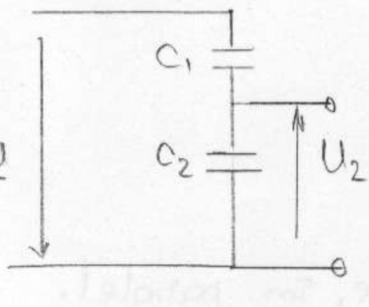




$$\underline{U}_2 = \underline{U}\,\frac{x}{l}$$



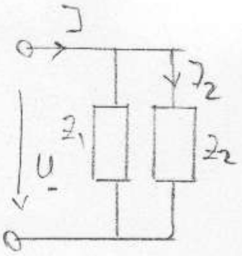
$U_1 = 110\,kV \rightarrow U_2 = 220\,kV$



$$\underline{Z}_1 = \frac{1}{j\omega C_1}$$

$$\underline{Z}_2 = \frac{1}{j\omega C_2}$$

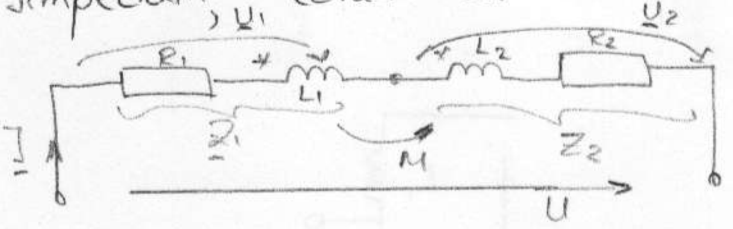
$$U_2 = \underline{U}\,\frac{C_1}{C_1+C_2}$$



$$\underline{I}_2 = \underline{I}\cdot\frac{\underline{Z}_1}{\underline{Z}_1+\underline{Z}_2} = \underline{I}\,\frac{\underline{Y}_2}{\underline{Y}_1+\underline{Y}_2}$$

3.2. Impedanțe echivalente cu laturi cuplate.

a) Impedanțe echiv. cu laturi cuplate în serie



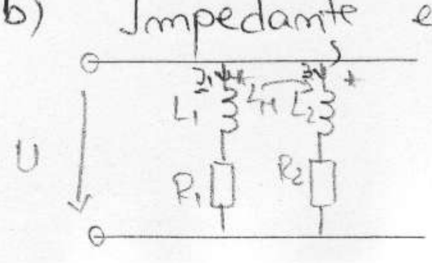
$$\underline{U}_1 = \underline{Z}_1\cdot\underline{I} + j\omega M\,\underline{I}$$
$$\underline{U}_2 = \underline{Z}_2\cdot\underline{I} + j\omega M\,\underline{I}$$

$$\underline{U} = \underline{U}_1 + \underline{U}_2 = (\underline{Z}_1 + \underline{Z}_2 + 2j\omega M)\,\underline{I}$$
$$\underline{Z}_e = \underline{Z}_1 + \underline{Z}_2 + 2j\omega M.$$
$$\underline{Z}_1 = R_1 + j\omega L_1$$
$$\underline{Z}_2 = R_2 + j\omega L_2$$
$$\underline{Z}_e = (R_1+R_2) + j\omega\underbrace{(L_1+L_2+2M)}_{L_e}$$

$$\begin{cases} L'_e = L_1 + L_2 + 2M \\ L''_e = L_1 + L_2 - 2M. \end{cases} \to \oplus \Rightarrow \boxed{M = \frac{L'_e - L''_e}{4}}$$

$$(\sqrt{L_1} - \sqrt{L_2})^2 \geq 0$$
$$L_1 + L_2 - 2\sqrt{L_1L_2} \geq 0$$
$$k = \frac{|M|}{\sqrt{L_1L_2}} = 1$$

b) Impedanțe echiv. cu laturi cuplate, în paralel.



$$\underline{I} = \underline{I}_1 + \underline{I}_2$$
$$\underline{U} = \underline{Z}_1\cdot\underline{I} + \underline{Z}_M\cdot\underline{I}_2$$

$$\begin{cases} \underline{Z}_1 = R_1 + j\omega L_1 \\ \underline{Z}_2 = R_2 + j\omega L_2 \\ \underline{Z}_M = j\omega M \end{cases}$$

$$\underline{U} = \underline{Z}_M \underline{I}_1 + \underline{Z}_2 \underline{I}_2$$

$$\underline{I}_1 = \frac{\underline{U} \begin{vmatrix} 1 & \underline{Z}_M \\ 1 & \underline{Z}_2 \end{vmatrix}}{\underline{Z}_1 \cdot \underline{Z}_2 - \underline{Z}_M^2} = \underline{U} \frac{\underline{Z}_2 - \underline{Z}_M}{\underline{Z}_1 \cdot \underline{Z}_2 - \underline{Z}_M^2}$$

$$\underline{I}_2 = \frac{\underline{Z}_1 - \underline{Z}_M}{\underline{Z}_1 \cdot \underline{Z}_2 - \underline{Z}_M^2}$$

$$\underline{I} = \underline{U} \frac{\underline{Z}_1 + \underline{Z}_2 - 2\underline{Z}_M}{\underline{Z}_1 \cdot \underline{Z}_2 - \underline{Z}_M^2}$$

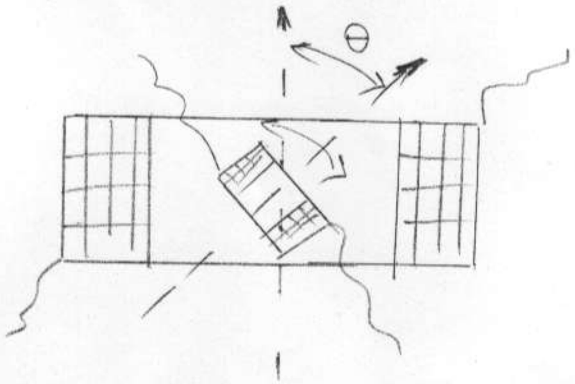
$$\frac{\underline{U}}{\underline{I}} = \underline{Z}_e = \frac{\underline{Z}_1 \cdot \underline{Z}_2 - \underline{Z}_M^2}{\underline{Z}_1 + \underline{Z}_2 - 2\underline{Z}_M}$$

$$R_1 = R_2 = 0$$

$$\Rightarrow \underline{Z}_e = \frac{-\omega^2 L_1 L_2 + \omega^2 M^2}{j\omega(L_1 + L_2 - 2M)}$$

$$\underline{Z}_e = \frac{j\omega(L_1 L_2 - M^2)}{L_1 + L_2 - 2M}$$

$$\boxed{L_e = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}}$$



- când cele 2 axe coincid, inductivitatea este maximă și pozitivă.

$$\begin{cases} L_e' = L_1 + L_2 + 2M \\ L_e'' = L_1 + L_2 - 2M \end{cases}$$

c. Transferul de putere în circuitele cuplate.

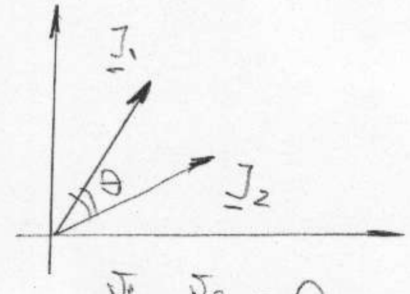
$$\underline{U} = \underline{Z}_1 \cdot \underline{I}_1 + \underline{Z}_M \cdot \underline{I}_2$$

$$\underline{U} = \underline{Z}_2 \cdot \underline{I}_2 + \underline{Z}_M \cdot \underline{I}_1$$

$$\underline{S} = \underline{U} \cdot \underline{I}^*$$

$$\underline{S}_1 = \underline{U} \cdot \underline{I}_1^*; \quad \underline{S}_2 = \underline{U} \cdot \underline{I}_2^*$$

$$\underline{S}_1 = \underline{Z}_1 \cdot I_1^2 + \underline{Z}_M \cdot \underline{I}_2 \cdot \underline{I}_1^*$$



$$\gamma_{i1} - \gamma_{i2} = \theta$$

$$\underline{Z}_M \cdot \underline{I}_2 \cdot \underline{I}_1^* = \underline{Z}_M \cdot I_2 \cdot I_1 \cdot e^{j(\gamma_{i2} - \gamma_{i1})} =$$

$$= \underline{Z}_M (I_1 \cdot I_2 \cdot \cos\theta - j I_1 I_2 \sin\theta) = j\omega M \cdot I_1 I_2 (\cos\theta - j \sin\theta) =$$

$$= \underbrace{\omega M I_1 I_2 \cdot \sin\theta}_{Re} + j\underbrace{\omega M \cdot I_1 I_2 \cos\theta}_{Im}$$

$$\begin{cases} P_1 = Re[\underline{S}_1] = R_1 \cdot I_1^2 + \omega M I_1 I_2 \sin\theta \\ P_2 = Re[\underline{S}_2] = R_2 I_2^2 - \omega M I_1 I_2 \sin\theta \end{cases}$$