27.04.2010

Curs nr 9:

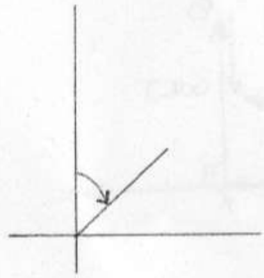
2.4. Reprezentarea simbolică a mărimilor sinusoidale.

$$\left.\begin{aligned} u_R &= R\cdot i \\ u_L &= L\cdot\frac{di}{dt} \\ u_C &= \frac{1}{C}\int i\cdot dt \end{aligned}\right\} \Rightarrow u = u_R + u_L + u_C$$

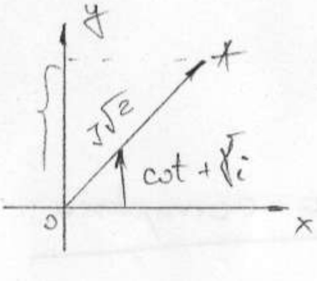
$$\begin{cases} i = I\sqrt{2}\sin(\omega t + \gamma_i) \\ u = U\sqrt{2}\sin(\omega t + \gamma_u) \end{cases}$$

val. efectivă → faza inițială

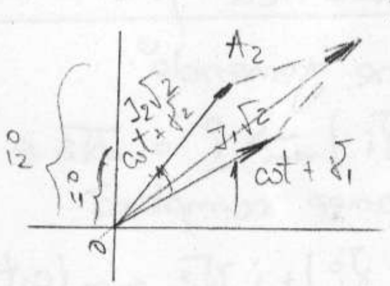
→ Reprezentarea geometrică prin fazori



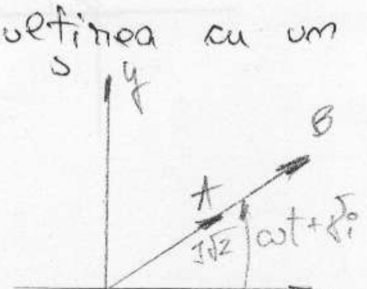
→ Reprezentarea simbolică - biuniformă
- intuitivă.



• Adunarea mărimilor:



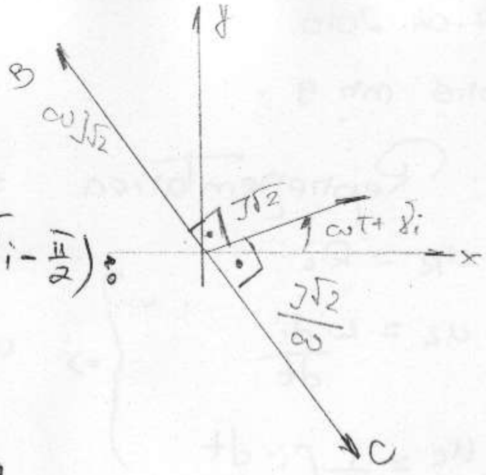
• Înmulțirea cu un scalar: $R i = R I\sqrt{2}\sin(\omega t + \gamma_i)$



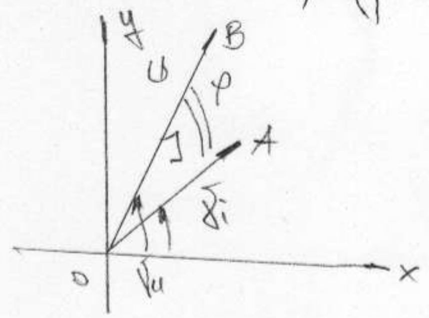
• Derivare în raport cu timpul:

$$\frac{di}{dt} = \omega I\sqrt{2}\,\sin\left(\omega t + \gamma_1 + \frac{\pi}{2}\right)$$

• Integrare: $\int i\,dt = \frac{I\sqrt{2}}{\omega}\sin\left(\omega t + \gamma_i - \frac{\pi}{2}\right)$:

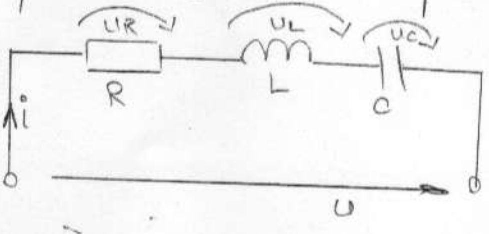


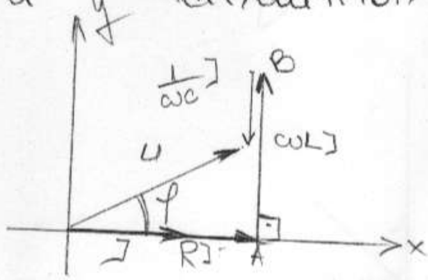
→ Reprezentare simplificată:



$$\boxed{\gamma_u - \gamma_i = \varphi}$$

Exemplu: Rezolvarea prin fazori a circuitelor RLC serie.





$$U^2 = (RI)^2 + I^2\left(\omega L - \frac{1}{\omega C}\right)^2$$

$$I = \frac{U}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\varphi = \operatorname{arctg}\frac{\omega L - \frac{1}{\omega C}}{R}$$

→ Reprezentare analitică prin numere complexe:

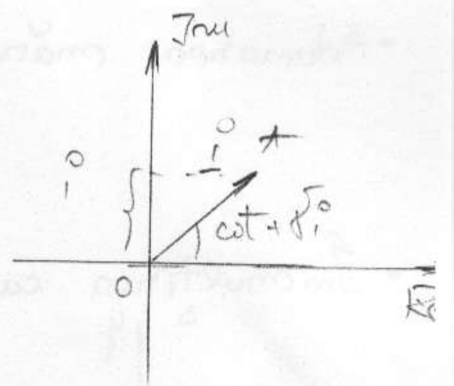
- reprezentare generală:

$$i = I\sqrt{2}\sin(\omega t + \gamma_i) \rightleftarrows \underline{i} = I\sqrt{2}\,e^{j(\omega t + \gamma_i)}$$

$\underline{i}$ - val. instantanee complexă

$$\underline{i} = I\sqrt{2}\cos(\omega t + \gamma_i) + jI\sqrt{2}\sin(\omega t + \gamma_i)$$

$$i = \operatorname{Im}[\underline{i}] = I\sqrt{2}\sin(\omega t + \gamma_i)$$

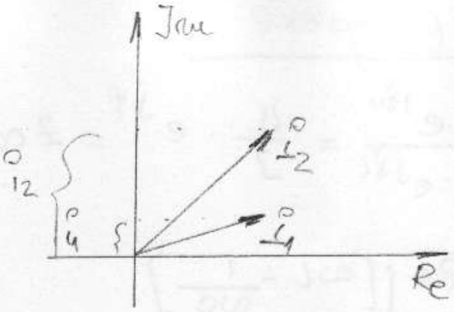


Adunare:

$$\underline{a} = a_1 + ja_2$$

$$\underline{b} = b_1 + jb_2$$

$$\underline{a} + \underline{b} = (a_1 + b_1) + j(a_2 + b_2)$$



- Înmulțirea cu un scalar:

$$R\cdot i \rightleftarrows \underline{i}\cdot R = R I\sqrt{2}\cdot e^{j(\omega t+\gamma_i)}$$

- Derivare:

$$L\frac{di}{dt} = \omega L\cdot I\sqrt{2}\sin\left(\omega t+\gamma_i+\frac{\pi}{2}\right)$$

$$L\frac{di}{dt} \Rightarrow \omega L I\sqrt{2}\cdot e^{j(\omega t+\gamma_i)}\cdot e^{j\frac{\pi}{2}}$$

$$e^{j\frac{\pi}{2}} = \cancel{\cos\frac{\pi}{2}} + j\sin\frac{\pi}{2} = j \Rightarrow \boxed{e^{j\frac{\pi}{2}} = j}$$

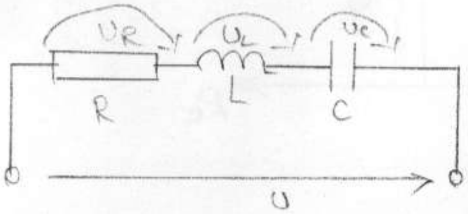
$$\boxed{e^{-j\frac{\pi}{2}} = -j}$$

$$\frac{di}{dt} \rightleftarrows j\omega\underline{i}$$

- Integrare: $\int i\,dt \rightleftarrows \frac{\underline{i}}{j\omega}$

$$-j = \frac{1}{j}$$

$$\underline{I} = I\cdot e^{j\gamma_i}$$



$$\underline{U} = R\cdot\underline{I} + j\omega L\cdot\underline{I} + \frac{1}{j\omega C}\underline{I}$$

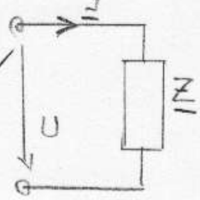
$$\underline{U} = \left[R + j\cdot\left(\omega L - \frac{1}{\omega C}\right)\right]\cdot\underline{I}$$

$$\underline{I} = \frac{\underline{U}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\begin{cases} I = \dfrac{U}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}} = |\underline{I}| \\ \varphi = \arctg\dfrac{\omega L - \frac{1}{\omega C}}{R} \end{cases}$$

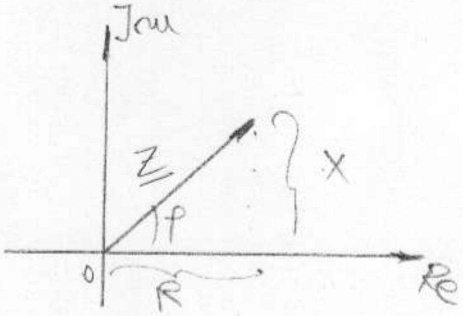
Impedanța și puterea complexe

impedanța complexă



$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{U \cdot e^{j\gamma_u}}{I \cdot e^{j\gamma_i}} = \frac{U}{I} \cdot e^{j\varphi} = Z\cos\varphi + jZ\sin\varphi$$

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = R + j\left[\omega L - \frac{1}{\omega C}\right]$$



$$\underline{Y} = \frac{1}{\underline{Z}}$$

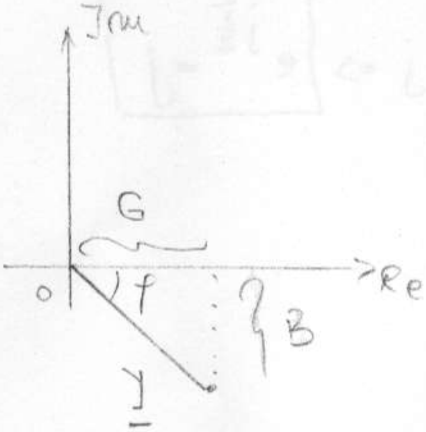
$$\underline{Y} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = \frac{R}{R^2+X^2} - j\frac{X}{R^2+X^2}$$

$$\boxed{R^2 + X^2 = Z^2}$$

$$\underline{Z} = -R + jX$$

$$\underline{Y} = G - jB$$

$$G = \frac{R}{R^2+X^2}\ ; \quad B = \frac{X}{R^2+X^2}$$



- Puterea complexă:

$$\begin{cases} P = U \cdot I \cos\varphi\ ; [W] \\ Q = U \cdot I \sin\varphi\ ; [VAR] \\ S = U \cdot I\ ; [VA] \end{cases}$$

$$\underline{S} = \underline{U} \cdot \underline{I}^* = U \cdot e^{j\gamma_u} \cdot I \cdot e^{-j\gamma_i}$$

$$\underline{S} = U \cdot I \cdot e^{j\varphi} = U I \cos\varphi + j U I \sin\varphi\ ;$$