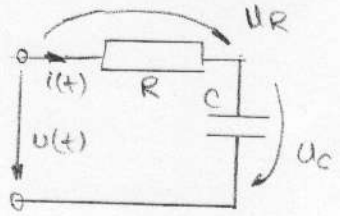
20.04.2010

Curs nr 2.

b. Circuitul R, C



$$\begin{cases} u_R = R \cdot i, & i_R = G \cdot u \\ u_L = L \cdot \frac{di}{dt}, & i_L = \frac{1}{L}\int u \cdot dt \\ u_C = \frac{1}{C}\int i\,dt, & i_C = C\frac{du}{dt} \end{cases}$$

$$i = \frac{dq}{dt}$$

$$u = u_R + u_C$$

$$R \cdot i + \frac{1}{C}\int i\,dt = u(t)$$

$$R\frac{dq}{dt} + \frac{1}{C}q = u(t)$$

$$\begin{cases} i = I\sqrt{2}\sin\omega t \\ u = U\sqrt{2}\sin(\omega t + \varphi) \end{cases}$$

$$RI\sqrt{2}\sin\omega t - \frac{I\sqrt{2}}{\omega C}\cos\omega t = U\sqrt{2}\sin\omega t\cos\varphi + U\sqrt{2}\cos\omega t\cdot\sin\varphi$$

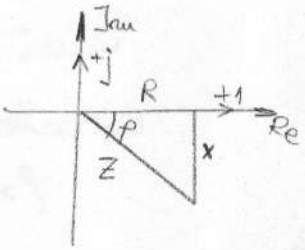
$$\begin{cases} RI = U\cos\varphi \\ \frac{1}{\omega C}I = U\sin\varphi \end{cases}$$

$$I = \frac{U}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

$$\varphi = \operatorname{arctg}\frac{-\frac{1}{\omega C}}{R}$$

$$X_C = \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + X^2}$$

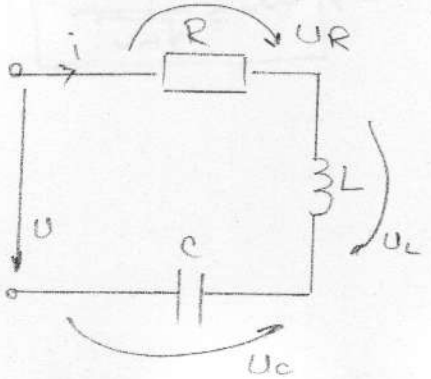


$$\begin{cases} R = Z\cos\varphi \\ X = Z\sin\varphi \end{cases}$$

$$i = \sqrt{-1}$$

$$j = \sqrt{-1} \quad ; \quad j^2 = -1 \quad ; \quad \frac{1}{j} = -j$$

c. Circuitul serie R, L, C



$$u_R + u_L + u_C = u$$

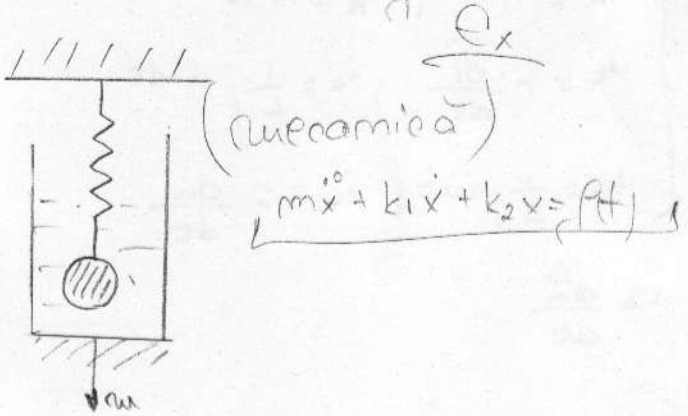
$$\begin{cases} i = I\sqrt{2}\sin\omega t \\ u = U\sqrt{2}\sin(\omega t + \varphi) \end{cases}$$

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = u(t)$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}\cdot q = u(t)$$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}\cdot q = \frac{1}{L}\cdot u(t)$$

$$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = f(t) \Rightarrow$$



$$m\ddot{x} + k_1\dot{x} + k_2 x = f(t)$$

-> continuare: $\omega_0 = \frac{1}{\sqrt{LC}}$

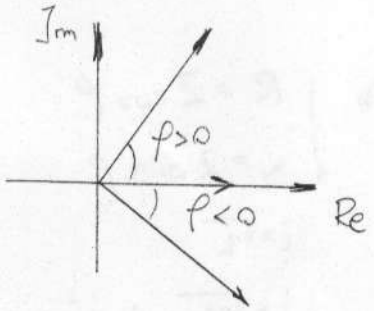
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$RI\sqrt{2}\sin\omega t + \omega L\, I\sqrt{2}\cos\omega t - \frac{1}{\omega C}\, I\sqrt{2}\cos\omega t \equiv U\sqrt{2}\sin\omega t\cos\varphi + U\sqrt{2}\cos\omega t\sin\varphi$$

$$\begin{cases} RI = U\cos\varphi \\ \left(\omega L - \frac{1}{\omega C}\right) = U\cdot\sin\varphi \end{cases}$$

$$\begin{cases} I = \dfrac{U}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \\ \varphi = \operatorname{arctg}\dfrac{\omega L - \frac{1}{\omega C}}{R} \end{cases}$$

$$\begin{cases} X = X_L - X_C \\ Z = \sqrt{R^2 + X^2} \end{cases} \quad ; \; X \gtrless 0$$



Dacă $X > 0 \Rightarrow X_L > X_C$ și circuitul are caracter inductiv

$\varphi > 0$ - defazată înaintea circuitul

Dacă $\omega L = \frac{1}{\omega C}$; $\omega^2 = \frac{1}{LC} \Rightarrow$

$$\Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

2.3 Puteri în regim sinusoidal

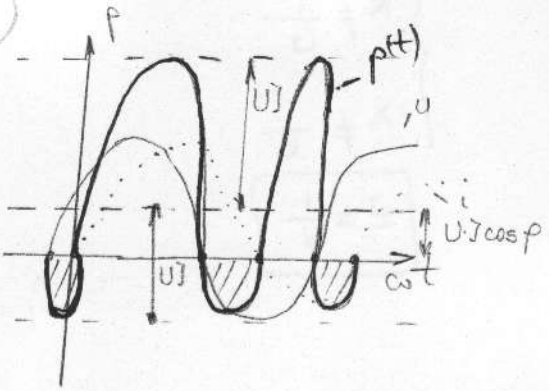
$$P = U\cdot I$$

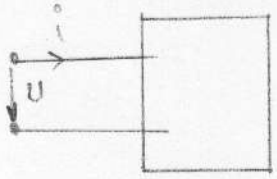
a. Puteri instantanee

$p = u \cdot i = U\sqrt{2}\sin(\omega t + \gamma_u) \cdot I\sqrt{2}\sin(\omega t + \gamma_i)$

$\gamma_u - \gamma_i = \varphi$

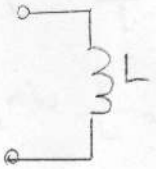
$p = U \cdot I\left[\cos\varphi - \cos(2\omega t + \gamma_u + \gamma_i)\right]$





$\Rightarrow p > 0$

$\Leftarrow p < 0$



↓ bobină

b. Putere activă

$$P = \tilde{p} = \frac{1}{T}\int_T p(t)\,dt = U \cdot I \cdot \cos\varphi$$

$$\boxed{P = U \cdot I \cos\varphi \ [W]}$$

c. Putere reactivă

$$\boxed{Q = U \cdot I \cdot \sin\varphi \ [VAR]}$$

d. Putere aparentă

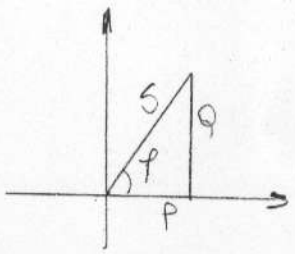
$$\boxed{S = U \cdot I \ [VA]}$$

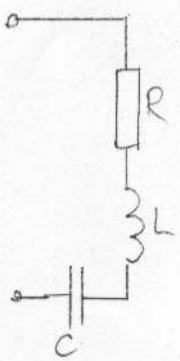
$$k_p = \frac{P}{S} = \cos\varphi$$

Motoare $\Rightarrow P$

Transformator $\Rightarrow S$

$$\begin{cases} S^2 = P^2 + Q^2 \\ Z^2 = R^2 + X^2 \end{cases}$$





$$\begin{cases} X = \frac{Q}{I^2}; & R = \frac{P}{I^2}; \\ B = \frac{Q}{U^2} & G = \frac{P}{U^2} \\ Z = \frac{S}{I^2}; & Y = \frac{S}{U^2}; \end{cases}$$

$$\begin{cases} R \neq \frac{1}{G} \\ X \neq \frac{1}{B} \end{cases}$$

$$\boxed{Z = \frac{1}{Y}}$$

$$R = \frac{U \cdot I \cos\varphi}{I^2} = \frac{U}{I}\cos\varphi;$$

$$G = \frac{U \cdot I \cos\varphi}{U^2} = \frac{I}{U}\cos\varphi$$

$$X = \frac{U \cdot I \sin\varphi}{I^2} = \frac{U}{I}\sin\varphi$$

$$B = \frac{U \cdot I \sin\varphi}{U^2} = \frac{I}{U}\sin\varphi$$

$$Z = \frac{U}{I^2} = \frac{U}{I}; \quad Y = \frac{U \cdot I}{U^2} = \frac{I}{U}$$