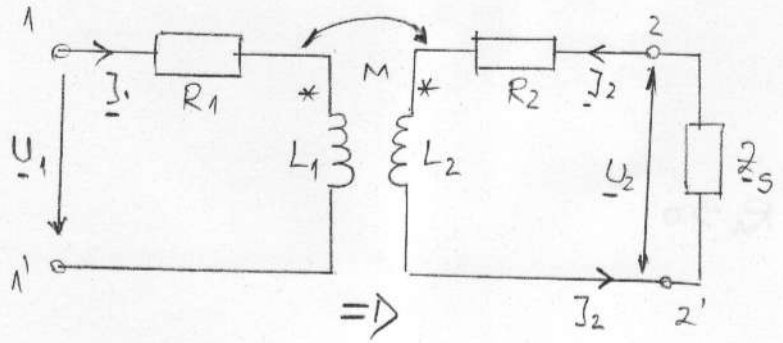
18.05.2010 TCE - curs.

Curs nr 11:

3.4. Transformatorul electric
- reprezintă un circuit cuplat.

a) Ecuațiile și schema echivalentă.



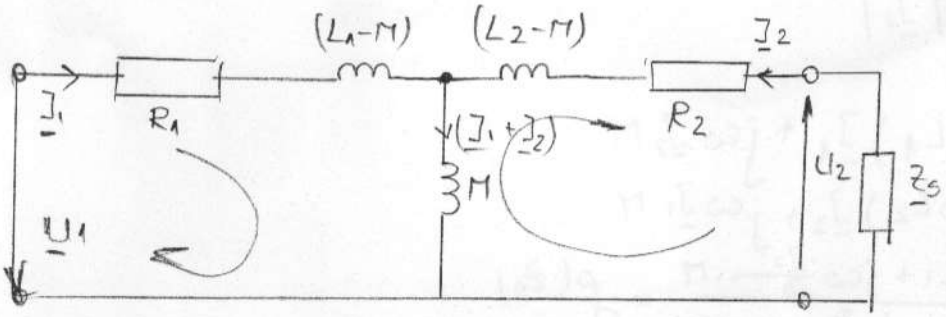

$$\Rightarrow$$

$$\begin{cases} \underline{U}_1 = R_1\underline{I}_1 + j\omega L_1\underline{I}_1 + j\omega M\underline{I}_2 \\ 0 = R_2\underline{I}_2 + j\omega L_2\underline{I}_2 + j\omega M\underline{I}_1 + \underbrace{\underline{Z}_S\cdot\underline{I}_2}_{\underline{U}_2} \pm j\omega M\underline{I}_2 \end{cases}$$

$$\begin{cases} \underline{U}_1 = \underline{Z}_1\underline{I}_1 + \underline{Z}_M\underline{I}_2 \\ -\underline{U}_2 = \underline{Z}_M\underline{I}_1 + \underline{Z}_2\cdot\underline{I}_2 \end{cases}$$

$$\begin{cases} \underline{Z}_1 = R_1 + j\omega L_1 \\ Z_2 = R_2 + j\omega L_2 \\ \underline{Z}_M = j\omega M \end{cases} \qquad \begin{aligned} \underline{Z}_S &= R_S + jX_S \\ \underline{U}_2 &= \underline{Z}_S\underline{I}_2 \end{aligned}$$

$$\Rightarrow \begin{cases} \underline{U}_1 = R_1\underline{I}_1 + j\omega(L_1 - M) + j\omega(\underline{I}_1 + \underline{I}_2) \\ -\underline{U}_2 = R_2\underline{I}_2 + j\omega(L_2 - M) + j\omega M(\underline{I}_1 + \underline{I}_2) \end{cases}$$




$$k = \frac{|M|}{\sqrt{L_1 \cdot L_2}} \leq 1$$

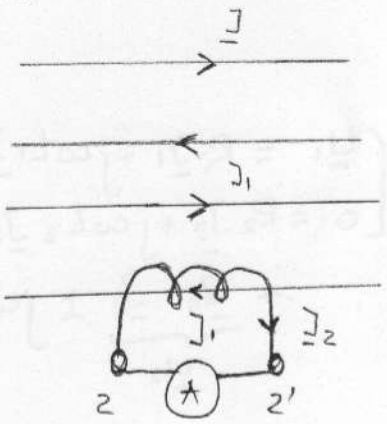
$$\begin{cases} L_1 = 4H \\ L_2 = 1H \\ \hline M = 1{,}8H \end{cases}$$

b) Raporturile de transformare și randamentul transf.

$$\underline{U}_1 = (R_1 + j\omega L_1)\underline{I}_1 + j\omega M \underline{I}_2$$

$$0 = (R_2 + j\omega L_2)\underline{I}_2 + j\omega \underline{I}_1 M + \underline{Z}_2 \underline{I}_2$$

$$-\frac{\underline{I}_2}{\underline{I}_1} = \frac{j\omega M}{R_2 + j\omega L_2 + Z_s} = f(Z_s)$$




$R_A \to 0$

$$-\frac{\underline{I}_2}{\underline{I}_1} = \frac{j\omega M}{R_2 + j\omega L_2 + Z_s} \approx \frac{M}{L_2}$$

$$\begin{cases} L_2 = k \cdot N_2^2 \\ M = k \cdot N_1 N_2 \\ L_1 = k \cdot N_1^2 \end{cases} \Rightarrow -\frac{\underline{I}_2}{\underline{I}_1} = \frac{N_1}{N_2}$$

$$\boxed{N_1 \cdot \underline{I}_1 + N_2 \cdot \underline{I}_2 = 0}$$ → legea solenațiilor

$$N \cdot I = \Theta$$

$$\boxed{N_1 \cdot \underline{I}_1 + N_2 \underline{I}_2 = N_1 \cdot \underline{I}_{10}}$$

$$|\underline{I}_{10}| \approx 5\% |\underline{I}_1|$$

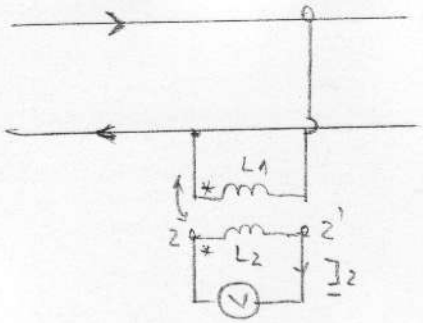
$$U_1 = (R_1 + j\omega L_1)\underline{I}_1 + j\omega \underline{I}_2 M$$

$$-U_2 = (R_2 + j\omega L_2)\underline{I}_2 + j\omega \underline{I}_1 M$$

$$-\frac{\underline{U}_1}{\underline{U}_2} = \frac{R_1 + j\omega L_1 + j\omega \frac{\underline{I}_2}{\underline{I}_1} \cdot M}{(R_2 + j\omega L_2)\frac{\underline{I}_2}{\underline{I}_1} \cdot j\omega M} = f(Z_s)$$

$$I_2 = 0 \Rightarrow \frac{-\underline{U}_1}{\underline{U}_2} = \frac{R_1 + j\omega L_1}{j\omega M} = \frac{L_1}{M}$$

$$\boxed{-\frac{\underline{U}_1}{\underline{U}_2} = \frac{N_1}{N_2}}$$



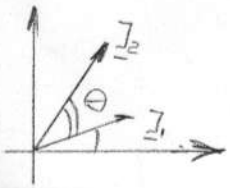

$$\begin{cases} R_v \to \infty \\ I_2 \to 0 \end{cases}$$

$$\begin{cases} \underline{U}_1 = (R_1 + j\omega L_1)\cdot \underline{I}_1 + j\omega M \underline{I}_2 \\ -\underline{U}_2 = (R_2 + j\omega L_2)\underline{I}_2 + j\omega M \underline{I}_1 \end{cases}$$

randamentul: $\eta = \frac{P_2}{P_1}$

$$S_1 = \underline{U}_1 \cdot \underline{I}_1^* = (R_1 + j\omega L_1) I_1^2 + j\omega M I_1 \cdot I_2 \cdot e^{j\theta}$$

$\underline{I}_1 \cdot \underline{I}_1^* = I_1^2$




$$P_1 = Re[S_1] = R_1 \cdot I_1^2 - \omega M I_1 \cdot I_2 \sin\theta$$

$$-P_2 = I_2 \cdot R_2^2 + \omega M I_1 \cdot I_2 \sin\theta$$

$$P_1 - P_2 = R_1 \cdot I_1^2 + R_2 \cdot I_2^2$$

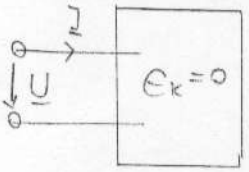
$$P_1 = P_2 + R_1 I_1^2 + R_2 I_2^2$$

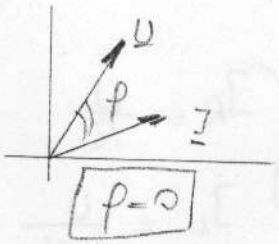
$\Rightarrow$ $\eta = \frac{P_2}{P_2 + R_1 \cdot I_1^2 + R_2 \cdot I_2^2} < 1$ → randamentul transformatorului

$R = R_0 (1 + \Delta T)$

## Cap 4: Rezonanta in circuite electrice

$$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = f(\omega t)$$

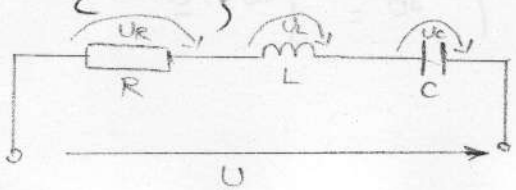



$$Q = U \cdot I \sin\varphi = 0$$

$$\begin{cases} Q = X \cdot I^2 = 0 \Rightarrow X = 0 \\ Q = B \cdot U^2 = 0 \Rightarrow B = 0 \end{cases}$$

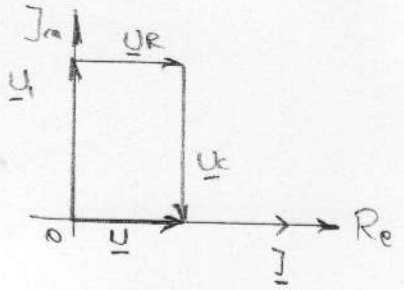
### 4.1. Rezonanta serie




$$\underline{U} = \underline{U}_R + \underline{U}_L + \underline{U}_C$$

$$\underline{U} = R \cdot \underline{I} + j\underbrace{\left(\omega L - \frac{1}{\omega C}\right)}_{X} \cdot \underline{I}$$

$\omega L - \frac{1}{\omega C} = 0 \Rightarrow \quad \omega_0 = \frac{1}{\sqrt{L \cdot C}}$




$U_L = U_C \Rightarrow \omega L = \frac{1}{\omega C}$

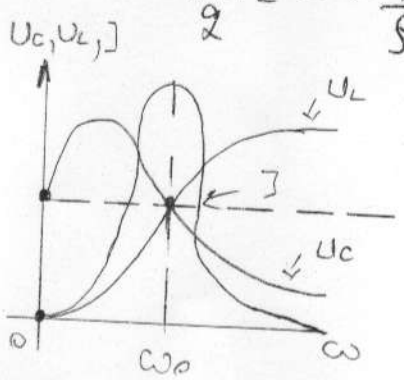
$\underline{U} = U_R = R \cdot \underline{I}$

- este posibil ca: $|U_L| > |\underline{U}|$

$\rho = \sqrt{\omega L \cdot \frac{1}{\omega C}} = \sqrt{\frac{L}{C}}$ - impedanta caract. a circuitul

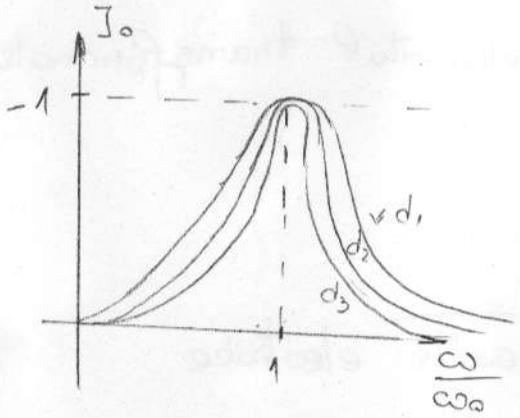
$Q = \frac{\rho}{R}$ - factor de calitate.

$\frac{1}{Q} = d = \frac{R}{\rho}$




$I = \frac{U}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$

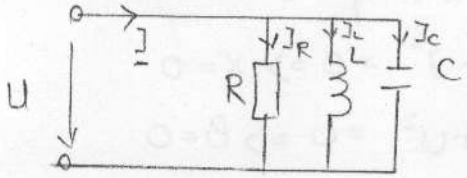
$\boxed{I_0 = \frac{U}{R}}$ - fm rezonanta




$X = \omega L$

$U_C = \frac{1}{\omega C} \cdot I$

## 4.2 Rezonanta paralel.




$\underline{I} = \underline{U}\left[\underbrace{\frac{1}{R}}_{G} + j\underbrace{\left(\omega C - \frac{1}{\omega L}\right)}_{B}\right]$

$\omega_0 = \frac{1}{\sqrt{L \cdot C}}$

$$\begin{cases} I_R = \frac{U}{R} \\ I_L = \frac{U}{j\omega L} \\ I_C = j\omega C \cdot \underline{U} \end{cases}$$

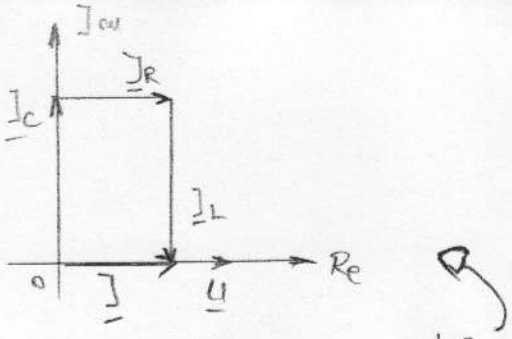
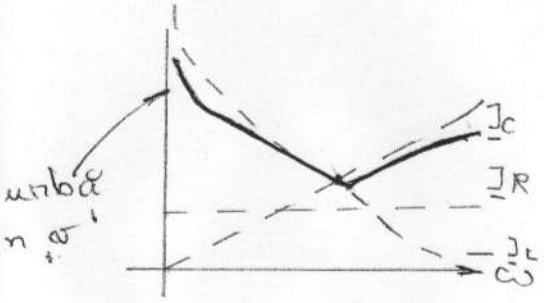
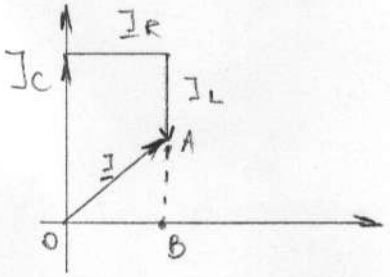



Diagrama pt. rezonanta.

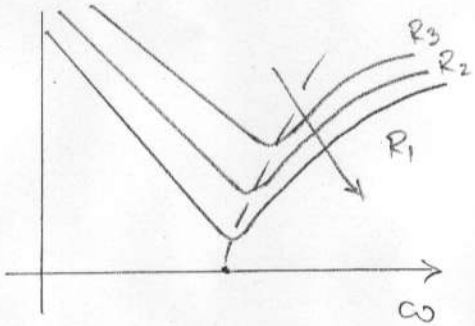
$$|\underline{I}_C| = |\underline{I}_L|$$
$$|\underline{I}| = |\underline{I}_R|$$




$\Rightarrow$ Curba de rezonanta.

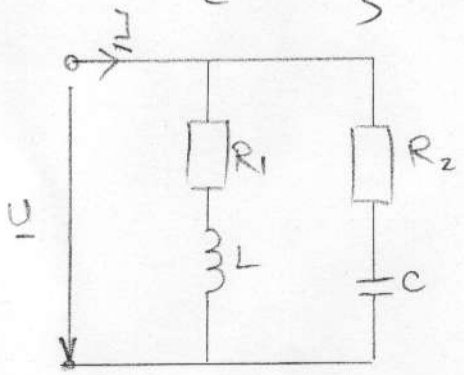



$$|\underline{I}| = \sqrt{|\underline{I}_R|^2 + (|\underline{I}_C| - |\underline{I}_L|)^2}$$




## 4.3. Rezonanta de tip serie paralel.

$X_e = 0$




$$Z_e = \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} = \frac{(R_1 + j\omega L)\left(R_2 + \frac{1}{j\omega C}\right)}{(R_1 + R_2) + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\mathrm{Im}[Z_e] = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \cdot \sqrt{\frac{\frac{L}{C} - R_1^2}{\frac{L}{C} - R_2^2}}$$

$R_1 > \sqrt{\frac{L}{C}}$ ; $R_2 < \sqrt{\frac{L}{C}}$

$\sqrt{\frac{L}{C}} = R_1 = R_2$ -> circuit complex aperiodic (Heavisido)