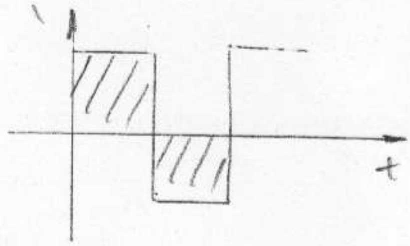
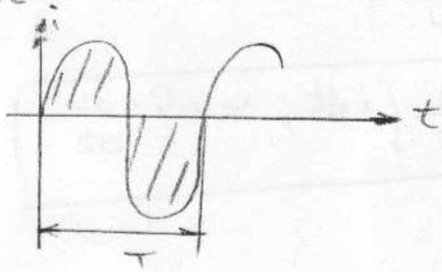
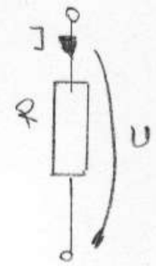
13.04.2010

Curs nr 7: C.2: Teoria circuitelor în curent alternativ

Mărimi, parametrii, legi și teoreme în circ. de curent alternativ.



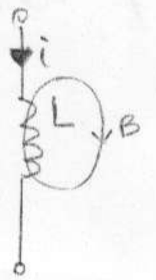




$R = \frac{U}{I}$

- litere mici - mărimile care sunt variabile în timp

$$\boxed{\begin{aligned} i_R &= G \cdot u \\ u_R &= R \cdot i \end{aligned}}$$



$\phi = \int_{S_\Gamma} \bar{B} \cdot d\bar{A}$

$\frac{\phi}{i} = L$; $\boxed{\phi = L \cdot i} = \int_{S_\Gamma} \bar{B} \cdot d\bar{A}$

$e_\Gamma = \frac{d}{dt} \int_{S_\Gamma} \bar{B} \cdot d\bar{A} = -\frac{d}{dt}\left[L \cdot i\right]$

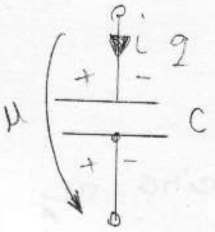
$e_\Gamma = -L \frac{di}{dt}$

t.e.m.s. electromotoare

$u + e = R \cdot i$

-> Legea lui Ohm: $u_L = L \cdot \frac{di}{dt}$

$\boxed{u_L = L \cdot \frac{di}{dt} ; \quad i_L = \frac{1}{L} \int u \cdot dt}$



$C = \frac{q}{u}$

$i = \frac{dq}{dt} \Rightarrow q = \int i \cdot dt$

$C \cdot u = \int i \cdot dt$

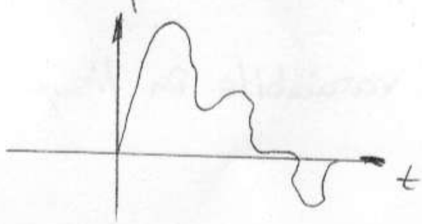
$$\boxed{u_c = \frac{1}{C}\int i\,dt;\ i_c = C \cdot \frac{du}{dt}}$$

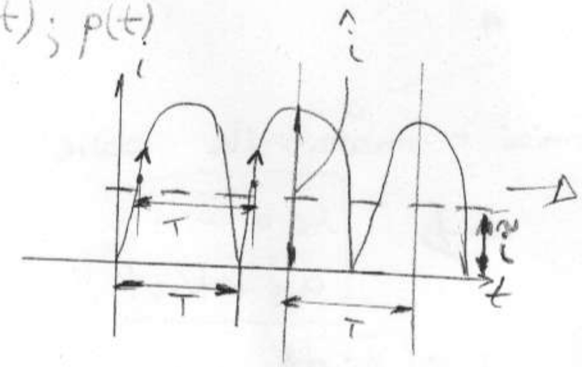
$\boxed{R, L, C}$

2.1. Mărimi variabile, mărimi sinusoidale.

$\Downarrow$

$e(t);\ i(t);\ u(t);\ p(t)$





→ mărime pulsatorie

$\tilde{i} = \frac{1}{T}\int_0^T i \cdot dt$

$$\boxed{I = \sqrt{\frac{1}{T}\int_0^T i^2 \cdot dt}}$$ → valoarea medie pătratică

$p = R \cdot i^2$

$\tilde{p} = R \cdot I^2$

bobina
$\begin{cases} W_m = \frac{1}{2} L \cdot i^2 \\ W_e = \frac{1}{2} C \cdot u^2 \end{cases}$

$p = \frac{\partial w}{\partial t}$

$p = L \cdot i \frac{di}{dt} = u_L \cdot i$

bobina

$p = C \cdot u \cdot \frac{\partial u}{\partial t} = i_c \cdot u$

condensator

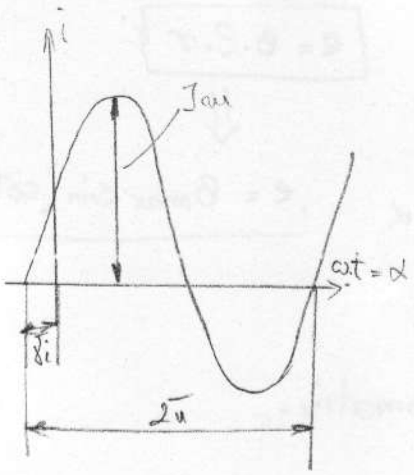
O mărime sinusoidală este o mărime periodică

$i = I_m \cdot \sin(\omega t + \gamma_i)$

$u = U_m \cdot \sin(\omega t + \gamma_u)$

$e = E_m \cdot \sin(\omega t + \gamma_e)$

$$\boxed{\sin^2 \overset{0}{=} \frac{1-\cos}{2}}$$

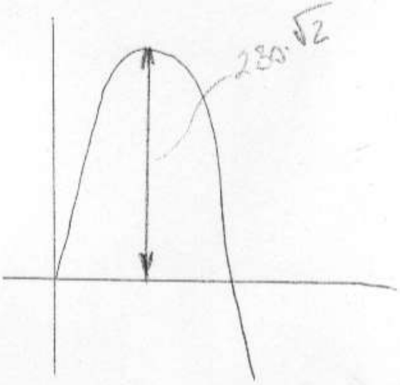


$$J = \sqrt{\frac{1}{T}\int_0^T i^2 \cdot dt}$$

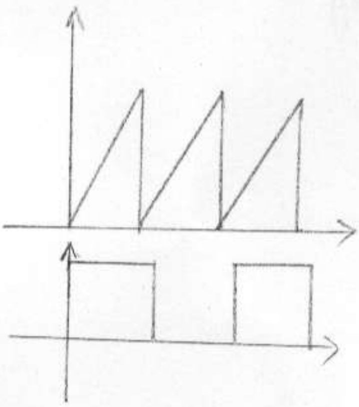
$$J = \sqrt{\frac{1}{T}\int_0^T J_m^2 \sin^2 \omega t \cdot dt}$$

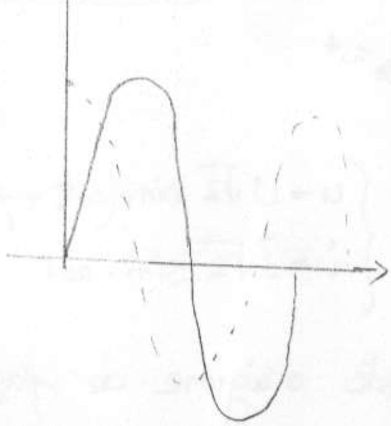
$$J = \sqrt{\frac{1}{T}\int_0^T J_m^2 \cdot \frac{1-\cos 2\omega t}{2}\, dt}$$

$$J^2 = \frac{J_m^2}{2} \Rightarrow \boxed{J = \frac{J_m}{\sqrt{2}}}$$

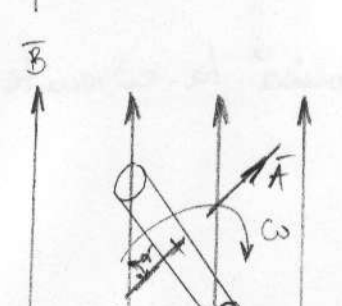


$$\begin{cases} i = J\sqrt{2}\sin(\omega t + \gamma_i) \\ u = U\sqrt{2}\sin(\omega t + \gamma_u) \end{cases}$$





Defazajul se modifică prin integrare sau derivare dar nu și prin formă.



$$\phi = \int_{S_\Gamma} \bar{B} \cdot d\bar{A} = \bar{B} \cdot \bar{A} = B \cdot A \cdot \cos\alpha$$

$$\omega = \frac{d\alpha}{dt} \qquad \boxed{\alpha = \omega t + \alpha_0}$$

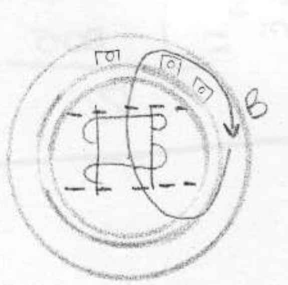
$$\omega = \frac{2\pi}{T} = \boxed{2\pi f}$$

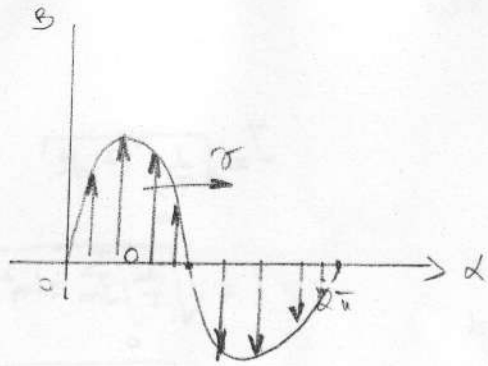
$$\frac{1}{T} = f$$

$$e = -\frac{d\phi}{dt} = B \cdot A \cdot \sin\alpha \cdot \frac{d\alpha}{dt}$$

$$e = \omega B \cdot A \cdot \sin(\omega t + \alpha_0)$$

$$= E\sqrt{2}$$

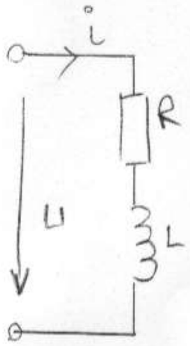




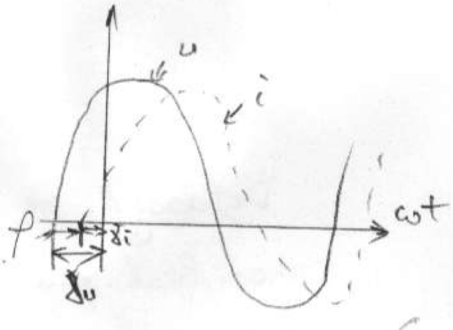
$$\boxed{e = B \cdot l \cdot v}$$
$$\Downarrow$$
$$e = B_{max} \cdot \sin(\omega t + \alpha_0) \cdot l$$

2.2. Circuite simple de curent alternativ. (circuite R, L, C)

a) R, L



a) $\begin{cases} u = U\sqrt{2}\sin(\omega t + \gamma_u) \\ i = I\sqrt{2}\sin(\omega t + \gamma_i) \end{cases}$

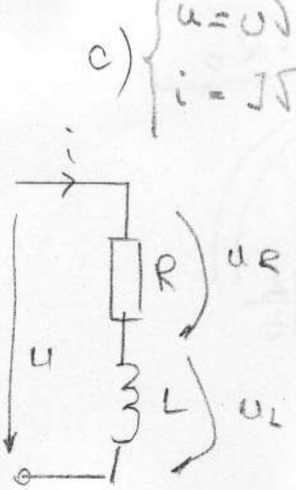


$$\boxed{\gamma_u - \gamma_i = \varphi}$$ defazaj

b) $\begin{cases} u = U\sqrt{2}\sin(\omega t + \varphi) \\ i = I\sqrt{2}\sin \omega t \end{cases}$

- curentul i este origine de fază; se aplică la circuite de tip serie.

c) $\begin{cases} u = U\sqrt{2}\sin \omega t \\ i = I\sqrt{2}\sin(\omega t - \varphi) \end{cases}$



$$u = u_R + u_L = R \cdot i + L\frac{di}{dt} = u$$

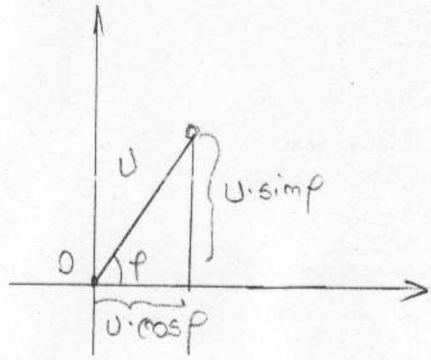
$i = i$

$i = i_l + i_p$

$I\sqrt{2}\sin\omega t + L\omega I\sqrt{2}\cos\omega t = U\sqrt{2}\sin\omega t\cdot\cos\varphi + U\sqrt{2}\cos\omega t\cdot\sin\varphi$

$$\left\{\begin{array}{l} R\cdot I = U\cos\varphi \\ \omega\cdot L\cdot I = U\cdot\sin\varphi \end{array}\right| ^2 \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} I = \dfrac{U}{\sqrt{R^2+(\omega L)^2}} \\ \varphi = \operatorname{arctg}\dfrac{\omega L}{R} \end{array}\right.$$



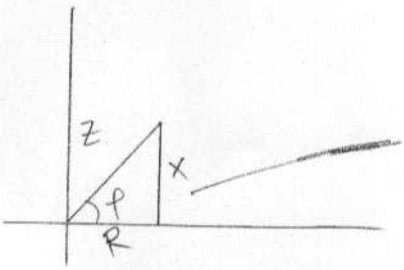
$U\cos\varphi = U_a$ – activ

$U\sin\varphi = U_r$ – reactiv

$\omega L = X_L$ – reactanța bobinei

$Z = \sqrt{R^2+X^2}$ – impedanță

R, X, Z



triunghiul impedanței