1 Introduction

Staff scheduling in healthcare continues to be a vexing problem that consumes valuable managerial resources and whose complexity can impact the quality of care delivered and the job satisfaction of its highly skilled labor force. []

Wide range of staff scheduling problems whose details depend on the specific healthcare delivery subsystem we are talking about. Examples: recovery room nurses, floor nurses, lab technicians, pharmacists and pharmacy technicians, surgical techs, ED nurses and techs, transporters.

Introduce motivation for tactical tour scheduling problems via real projects

For many systems, the kernel of the problem is a tour scheduling problem in which staffing levels needed vary both by time of day and day of week.

2 Tour scheduling problems

2.1 Scheduling environment

Features of tour scheduling problems including notions of planning cycle, periods, required staffing levels, shifts, tours, tour types, intra-tour start time flexibility,

To explicitly represent a tour we need of list of days worked, including the shift start times and lengths over the planning cycle. For example, consider the following very simple scenario for a two week planning cycle:

- only allowable tour type calls for five eight-hour shifts per week, M-F,
- within each tour, all shifts must start at 7a or they must all start at 3p,
- no restrictions on the number of weekend days worked over the planning cycle.

In this case, there are only two possible tours:

	Week 1							Week 2						
Tour Variable	Su	M	T	W	U	F	Sa	Su	M	T	W	U	F	Sa
x_1	x	7a	7a	7a	7a	7a	x	x	7a	7a	7a	7a	7a	x
x_2	x	3p	3p	3p	3p	3p	x	x	3p	3p	3p	3p	3p	x

In this case, the solution to the tour scheduling problem is fully specified by the number of people working Tour 1 and the number working Tour 2 and we only need two tour variables (x_1 and x_2) in our model. Of course, the number of tour variables can become enormous as we increase our scheduling options such as additional tour types, shift start times and shift lengths. We will explore this issue in the next section. Here, we want to show that there are other ways we could model this very simple scenario in terms of the variables we choose to use. For example, instead of explicit tour variables, we could use one variable to represent the number of people working a specific days worked pattern and another to represent the number of people working each shift. This is called an *implicitmodeling* approach [].

	Week 1							Week 2						
Days Worked Variable	Su	M	T	W	U	F	Sa	Su	M	T	W	U	F	Sa
y_1	x	On	On	On	On	On	x	x	On	On	On	On	On	x

Shift Variable	Shift Start Time
s_1	7a
s_2	3p

To specify a valid solution to our tour scheduling problem, we need values for s_1, s_2 and y_1. For this toy problem, it is easy to see the correspondence between variables in the two approaches. For given values of x_1 and x_2, $y_1 = x_1 + x_2$, $s_1 = x_1$, and $s_2 = x_2$.

2.2 Problem size explosion

2.3 Implicit modeling of tours

To

3 Related Work

Review one week implicit tour scheduling models including my AOR 2004 paper.

Recent multi-week implicit tour scheduling paper

4 Definitions and notation

Planning cycle related terms...

Planning cycle parameters

$$
\begin{aligned}
n_W &= \text{number of weeks in the planning cycle} \\
n_P &= \text{number of periods per day} \\
n_C &= 7 n_W n_P \text{ number of periods in planning cycle}
\end{aligned}
$$

Define a number of sets of indices.

$$
\begin{aligned}
\mathbb{P} &= \{1, 2, \ldots n_P\} \\
\mathbb{D} &= \{1, 2, \ldots 7\} \\
\mathbb{W} &= \{1, 2, \ldots n_W\}
\end{aligned}
$$

So, each period in the planning cycle is defined by a tuple $(i, d, w) \in B$ where $B = \mathbb{P} \times \mathbb{D} \times \mathbb{W}$.

Shift length and tour type parameters

$$n_H = \text{number of different shift lengths}$$
$$\mathbb{H} = \{1, 2, \ldots n_T\}$$
$$h_k = k\text{'th shift length in periods, for } k \in \mathbb{H}$$

$$n_T = \text{number of different tour types}$$
$$\mathbb{T} = \{1, 2, \ldots n_T\}$$
$$L(i) = \text{set of shift length indices allowed for tour type } i\text{, for } i \in \mathbb{T}$$

To smooth the postpartum occupancy across days of the week, we introduce two, non-negative, dummy variables, M^U and M^L. These are used to bound the mean occupancy across the week from above and below – see Equation (2). Then we minimize the difference between the bounds.

Model: OB-SMOOTH
Minimize

$$M^U - M^L \tag{1}$$

Subject to

$$0 \le M^L \le \mu_{i,j}^4 \le M^U \quad (i = 1,2,\dots P,\ j = 1,2,\dots 7) \tag{2}$$

$$L_{i,j}^a \le \Lambda_{i,j}^a \le U_{i,j}^a \quad (a = 5,6, i = 1,2,\dots P,\ j = 1,2,\dots 7) \tag{3}$$

$$L_{\cdot,j}^a \le \sum_{i=1}^{P} \Lambda_{i,j}^a \le U_{\cdot,j}^a \quad (a = 5,6,\ j = 1,2,\dots 7) \tag{4}$$

$$L^a \le \sum_{i=1}^{P} \sum_{j=1}^{7} \Lambda_{i,j}^a \le U^a \quad (a = 5,6) \tag{5}$$

Equations (**??**)-(**??**) for arrival rates (6)

Equation (**??**) for discharge rates (7)

Equation (12) for conservation of flow (8)

Equation (**??**) for mean occupancy (9)

Equations (**??**)-(**??**) for variance of occupancy (10)

$$\Lambda_{i,j}^a \ge 0 \text{ and integer}, \quad (a = 5,6, i = 1,2,\dots P,\ j = 1,2,\dots 7) \tag{11}$$

Equation (3) sets upper and lower bounds on the number of scheduled inductions and scheduled cesareans by time period of day and day of week. Equations (4) and (5) set upper and lower bounds on the number of scheduled inductions and scheduled cesareans by day of week and for the entire week, respectively. The remaining constraints constitute the patient flow portion of the model. Note that the only decision variables are $\Lambda_{i,j}^5$, the number of scheduled inductions, and $\Lambda_{i,j}^6$, the number of scheduled cesareans. The model OB-SMOOTH is a standard mixed integer linear program.

4.1 Computational experiments

Solve realistic problems. No competitive models to which to compare to other than reporting the size of comparable explicit tour scheduling models. I guess I could compare to the 5/7 model for the restricted environment for which it was designed.

5 Just some templates to use

$$\lambda_{i,j}^{h,s(h,m)} = D_{i,j}^{h,r(h,m)} \quad (i = 1,2,\dots P,\ j = 1,2,\dots 7, h \in \mathscr{H}, m = 2,3\dots n_h). \tag{12}$$

6 Conclusions and Future Work

BLAH, BLAH

Table 1: Patient care units

Unit #	Unit Name	Abbr.
1	Labor & Delivery	LD
2	Recovery room	R
3	Cesarean section procedure area	C
4	Postpartum unit	PP