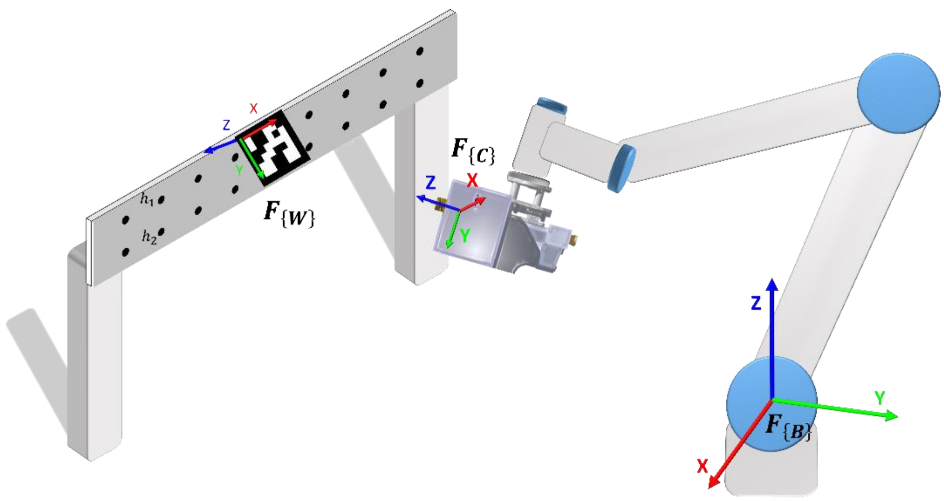
2. Question 2:

Kinematics:

- Consider the following notations:
  - The position of point A relative to point B in reference frame $F_{\{C\}}$ is given as: ${}^{C}_{B}\vec{p}_{A}$
  - The rotation matrix describing the rotation from reference frame $F_{\{A\}}$ to reference frame $F_{\{B\}}$ is given as: ${}_{A}R_{B}$
- Consider the following configuration:
  - The robot base frame $F_{\{B\}}$.
  - The camera frame $F_{\{C\}}$ where the camera is fixed to the end of the robot manipulator. $F_{\{C\}}$ is defined such that the z-axis extends out of the camera (known as the optical axis).
  - The workpiece Frame $F_{\{W\}}$, where a fiducial (ARUCO) is fixed to the workpiece.
  - The workpiece contains a set of 4 holes $h_i, i = 1, \dots, 4$, where the position of each hole relative to the fiducial is known in workpiece reference frame $F_{\{W\}}$. (i.e. ${}^{W}_{W}\vec{p}_{h_i}$ is known)
    ${}^{W}_{W}\vec{p}_{h_1} = [0.2, 0.2, 0]^T$, ${}^{W}_{W}\vec{p}_{h_2} = [0.2, -0.2, 0]^T$, ${}^{W}_{W}\vec{p}_{h_3} = [-0.2, 0.2, 0]^T$, ${}^{W}_{W}\vec{p}_{h_4} = [-0.2, -0.2, 0]^T$.



- Solve for the following:
  1. When the camera was in position: ${}^{B}_{B}\vec{p}_{C} = [0., -0.7, 0.7]^T$ and rotation: ${}_{B}R_{C} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$. It detected the ARUCO marker with the following readings: ${}^{C}_{C}\vec{p}_{W} = [0.2, 0.0, 0.5]^T$, ${}_{C}R_{W} = \begin{bmatrix} 0.93 & -0.353 & 0.0669 \\ 0.3535 & 0.8660 & -0.35355 \\ 0.669783 & 0.35355 & 0.9330 \end{bmatrix}$. Find the position of each hole in the robot base frame (i.e. ${}^{B}_{B}\vec{p}_{h_i}$).

2. Following 1., Find a target pose for the camera (both position and rotation) such that h_1 is 0.1 m in front of the camera. Do the same for all the holes.

3. Write a C++ program that solves for 1 and 2. The code should take in the pose of the camera, the pose of the ARUCO relative to the camera, and the pose of the holes relative to the ARUCO. The code should output the pose of the holes relative to robot base frame, and a set of camera poses that align the camera such that it is a target distance ahead of the holes.

Question 2 (1)

To find the position of each hole relative to the base frame ${}^{B}_{B}P_{h_i}$, we have to do a series of coordinate transformations from one frame to the next one to reach the base from the hole.

Now, we have the camera position relative to point the base in reference frame {B} is given as ${}^{B}_{B}P_{C} = [0, -0.7, 0.7]^T$ and the rotation matrix from base frame {B} to camera frame {C} is given as: ${}_{B}R_{C} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$ so we have to find the transformation rotation matrix from the camera frame {C} to the base frame {B} which is the inverse matrix of ${}_{B}R_{C}$ which is the same as follows ${}_{C}R_{B} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

Then, do the same for the workpiece and the camera frames.

the work-piece position relative to the camera in reference frame {C} is given as ${}^{C}_{C}P_{W} = [0.2, 0, 0.5\ ]^T$ and the rotation matrix from camera frame {C} to workpiece frame {W} is given as: ${}_{C}R_{W} = \begin{bmatrix} 0.93 & -0.353 & 0.0669 \\ 0.3535 & 0.866 & -0.35355 \\ 0.669783 & 0.35355 & 0.933 \end{bmatrix}$ so we have to find the transformation rotation matrix from the workpiece frame {W} to the camera frame {C} which is the inverse matrix of ${}_{C}R_{W}$which is the same as follows

$${}_{W}R_{C} = \begin{bmatrix} 0.899478 & 0.340327 & 0.0644669 \\ -0.5462739 & 0.793374 & 0.3397965 \\ -0.43871433 & -0.544941 & 0.89676901 \end{bmatrix}$$

The workpiece contains a set of 4 holes *h* , = 1, … ,4 , where the position of each hole relative to the fiducial is known in workpiece reference frame { }.

$${}^{W}_{W}P_{h_1} = [0.2, 0.2, 0]^{\mathrm{T}} , {}^{W}_{W}P_{h_2} = [0.2, -0.2, 0]^{\mathrm{T}} , {}^{W}_{W}P_{h_3} = [-0.2, 0.2, 0]^{\mathrm{T}} , {}^{W}_{W}P_{h_4} = [-0.2, -0.2, 0]^{\mathrm{T}}$$

$${}^{B}_{B}P_{h_i} = {}_{C}R_{B} * \left({}_{W}R_{C} * {}_{W}P_{h_i} + {}_{C}P_{W} \right) + {}_{B}P_{C}$$

So, for the 1st hole h_1

$$
{}^{B}_{B}P_{h_1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} * \left(\begin{bmatrix} 0.899478 & 0.340327 & 0.0644669 \\ -0.5462739 & 0.793374 & 0.3397965 \\ -0.43871433 & -0.544941 & 0.89676901 \end{bmatrix} * \begin{bmatrix} 0.2 \\ 0.2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.2 \\ 0 \\ 0.5 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ -0.7 \\ 0.7 \end{bmatrix} = \begin{bmatrix} -0.447961 \\ -1.00327 \\ 0.650587 \end{bmatrix}
$$

For the 2nd hole h_2

$$
{}^{B}_{B}P_{h_2} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} * \left(\begin{bmatrix} 0.899478 & 0.340327 & 0.0644669 \\ -0.5462739 & 0.793374 & 0.3397965 \\ -0.43871433 & -0.544941 & 0.89676901 \end{bmatrix} * \begin{bmatrix} 0.2 \\ -0.2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.2 \\ 0 \\ 0.5 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ -0.7 \\ 0.7 \end{bmatrix} = \begin{bmatrix} -0.31183 \\ -1.22125 \\ 0.967922 \end{bmatrix}
$$

For the 3rd hole h_3

$$
{}^{B}_{B}P_{h_3} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} * \left(\begin{bmatrix} 0.899478 & 0.340327 & 0.0644669 \\ -0.5462739 & 0.793374 & 0.3397965 \\ -0.43871433 & -0.544941 & 0.89676901 \end{bmatrix} * \begin{bmatrix} -0.2 \\ 0.2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.2 \\ 0 \\ 0.5 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ -0.7 \\ 0.7 \end{bmatrix} = \begin{bmatrix} -0.0881698 \\ -1.17875 \\ 0.432078 \end{bmatrix}
$$

For the 4th hole h_4

$$
{}^{B}_{B}P_{h_4} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} * \left(\begin{bmatrix} 0.899478 & 0.340327 & 0.0644669 \\ -0.5462739 & 0.793374 & 0.3397965 \\ -0.43871433 & -0.544941 & 0.89676901 \end{bmatrix} * \begin{bmatrix} -0.2 \\ -0.2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.2 \\ 0 \\ 0.5 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ -0.7 \\ 0.7 \end{bmatrix} = \begin{bmatrix} 0.0479613 \\ -1.39673 \\ 0.749413 \end{bmatrix}
$$

Question 2 (2)

Firstly, Transform Hole Position to the Camera Frame

For the 1st Hole

the work-piece position relative to the camera in reference frame {C} is given as ${}^{C}_{C}P_W = [0.2, 0, 0.5\ ]^T$ and the rotation matrix from camera frame {C} to work-piece frame {W} is given as: ${}_{C}R_W = \begin{bmatrix} 0.93 & -0.353 & 0.0669 \\ 0.3535 & 0.866 & -0.35355 \\ 0.669783 & 0.35355 & 0.933 \end{bmatrix}$ so we have to find the transformation rotation matrix from the workpiece frame {W} to the camera frame {C} which is the inverse matrix of ${}_{C}R_W$ which is the same as follows

$$
{}_{W}R_C = \begin{bmatrix} 0.899478 & 0.340327 & 0.0644669 \\ -0.5462739 & 0.793374 & 0.3397965 \\ -0.43871433 & -0.544941 & 0.89676901 \end{bmatrix}
$$

the position of first hole relative to the fiducial ${}^{W}_{W}P_{h1} = [0.2, 0.2, 0\ ]^T$

the position of h1 in the camera frame: ${}^{C}_{C}P_{h_1} = {}_{W}R_C * {}_{W}P_{h_1} + {}_{C}P_W$

$$
{}^{C}_{C}P_{h_1} = \left(\begin{bmatrix} 0.899478 & 0.340327 & 0.0644669 \\ -0.5462739 & 0.793374 & 0.3397965 \\ -0.43871433 & -0.544941 & 0.89676901 \end{bmatrix} * \begin{bmatrix} 0.2 \\ 0.2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.2 \\ 0 \\ 0.5 \end{bmatrix} \right) = \begin{bmatrix} 0.447961 \\ 0.0494127 \\ 0.303269 \end{bmatrix}
$$

Then computing the desired camera position relative to camera frame

$$
{}^{C}_{C}\acute{P}_{h_1} = {}^{C}_{C}P_{h_1} - \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.447961 \\ 0.0494127 \\ 0.203269 \end{bmatrix}
$$

Transform the desired camera position to the robot base frame by the using the given rotation matrix and translation position of camera to base

$$
{}^{B}_{B}\acute{P}_{C\_h1} = {}_{C}R_B * {}^{C}_{C}\acute{P}_{h1} + {}_{B}P_C
$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} * \begin{bmatrix} 0.447961 \\ 0.0494127 \\ 0.203269 \end{bmatrix} + \begin{bmatrix} 0 \\ -0.7 \\ 0.7 \end{bmatrix} = \begin{bmatrix} \mathbf{-0.447961} \\ \mathbf{-0.903269} \\ \mathbf{0.650587} \end{bmatrix}$$

Now, let's calculate these normalized vectors and construct the rotation matrix.

The orientation matrix is constructed using these normalized vectors to define the rotation of the camera. Normalized vectors simplify the interpretation of the matrix. Each column of the matrix represents a unit vector in a particular direction, making it easier to understand the transformation. In summary, normalization is applied to ensure that the orientation matrix accurately represents the rotation without introducing scaling effects. It simplifies subsequent calculations and facilitates a more intuitive interpretation of the resulting matrix.

Then moving to the computing the desired camera orientation of target pose for the fist hole too

where $\text{Direction Vector} = {}^{B}_{B}\acute{P}_{C\_h1} - {}^{B}_{B}P_{C} = \begin{bmatrix} -0.447961 \\ -0.903269 \\ 0.650587 \end{bmatrix} - \begin{bmatrix} 0 \\ -0.7 \\ 0.7 \end{bmatrix} = \begin{bmatrix} \text{-0.447961} \\ \text{-0.203269} \\ \text{-0.0494127} \end{bmatrix}$

$$\text{Normalized Direction Vector} = \frac{\text{Direction Vector}}{\|\text{Direction Vector}\|} = \begin{bmatrix} \text{- 0.906075} \\ \text{-0.411144} \\ \text{-0.0999452} \end{bmatrix}$$

The Perpendicular Vector is a vector perpendicular to the direction vector. We can choose any vector that is not collinear with the direction vector. Let Perpendicular Vector $= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$\text{Cross Product Vector} = \text{Normalized Direction Vector} \times \text{Perpendicular Vector}$$
$$= \begin{bmatrix} -0.906075 \\ -0.411144 \\ -0.0999452 \end{bmatrix} \text{X} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \text{-0.0999452} \\ 0.411144 \end{bmatrix}$$

Then the new rotaion matrix target pose of the first hole ${}_{C}\acute{R}_{B}$

$${}_{C}\acute{R}_{B} = \begin{bmatrix} \text{X - Axis} = \text{Perpendicular Vector} \\ \text{Y - Axis} = \text{Cross Product(Z - Axis, X - Axis)} \\ Z\text{ - Axis} = \text{Normalized Direction Vector} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \text{- 0.906075} \\ \text{-0.0999452} & \mathbf{0} & \text{- 0.411144} \\ 0.411144 & \mathbf{0} & \text{-0.0999452} \end{bmatrix}$$

Now, Repeating the same procedures for h2, h3 and h4

For the 2nd Hole

the position of 2nd hole relative to the fiducial ${}^{W}_{W}P_{h1} = [0.2, -0.2, 0]^T$

the position of h2 in the camera frame: ${}^{C}_{C}P_{h_2} = {}_{W}R_{C} * {}_{W}P_{h_2} + {}_{C}P_{W}$

$$ {}_{C}^{C}P_{h_2} = \left(\begin{bmatrix} 0.899478 & 0.340327 & 0.0644669 \\ -0.5462739 & 0.793374 & 0.3397965 \\ -0.43871433 & -0.544941 & 0.89676901 \end{bmatrix} * \begin{bmatrix} 0.2 \\ -0.2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.2 \\ 0 \\ 0.5 \end{bmatrix} \right) = \begin{bmatrix} 0.31183 \\ \text{-}0.267922 \\ 0.521245 \end{bmatrix} $$

Then computing the desired camera position relative to camera frame

$$ {}_{C}^{C}\acute{P}_{h_2} = {}_{C}^{C}P_{h_2} - \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.31183 \\ \text{-}0.267922 \\ 0.421245 \end{bmatrix} $$

Transform the desired camera position to the robot base frame by the using the given rotation matrix and translation position of camera to base

$$ {}_{B}^{B}\acute{P}_{C\_h2} = {}_{C}R_{B} * {}_{C}^{C}\acute{P}_{h2} + {}_{B}P_{C} $$

$$ = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} * \begin{bmatrix} 0.31183 \\ -0.267922 \\ 0.421245 \end{bmatrix} + \begin{bmatrix} 0 \\ -0.7 \\ 0.7 \end{bmatrix} = \begin{bmatrix} \mathbf{-0.31183} \\ \mathbf{-1.12125} \\ \mathbf{0.967922} \end{bmatrix} $$

Now, let's calculate these normalized vectors and construct the rotation matrix. The orientation matrix is constructed using these normalized vectors to define the rotation of the camera. Normalized vectors simplify the interpretation of the matrix. Each column of the matrix represents a unit vector in a particular direction, making it easier to understand the transformation. In summary, normalization is applied to ensure that the orientation matrix accurately represents the rotation without introducing scaling effects. It simplifies subsequent calculations and facilitates a more intuitive interpretation of the resulting matrix.

Then moving to the computing the desired camera orientation of target pose for the fist hole too

where $\text{Direction Vector} = {}_{B}^{B}\acute{P}_{C\_h2} - {}_{B}^{B}P_{C} = \begin{bmatrix} -0.31183 \\ -1.12125 \\ 0.967922 \end{bmatrix} - \begin{bmatrix} 0 \\ -0.7 \\ 0.7 \end{bmatrix} = \begin{bmatrix} \text{-}0.31183 \\ \text{-}0.421245 \\ 0.267922 \end{bmatrix}$

$$ \text{Normalized Direction Vector} = \frac{\text{Direction Vector}}{\|\text{Direction Vector}\|} = \begin{bmatrix} \text{- }0.529769 \\ \text{-}0.715655 \\ 0.455174 \end{bmatrix} $$

The Perpendicular Vector is a vector perpendicular to the direction vector. We can choose any vector that is not collinear with the direction vector. Let $\text{Perpendicular Vector} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$ \text{Cross Product Vector} = \text{Normalized Direction Vector} \times \text{Perpendicular Vector} $$

$$ = \begin{bmatrix} -0.529769 \\ -0.715655 \\ 0.455174 \end{bmatrix} \text{X} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.455174 \\ 0.715655 \end{bmatrix} $$

Then the new rotation matrix target pose of the first hole ${}_{C}\acute{R}_{B}$

$$ {}_{C}\acute{R}_{B} = \begin{bmatrix} X - Axis = \text{Perpendicular Vector} \\ Y - Axis = \ Cross\ Product(Z - Axis, X - Axis) \\ Z - Axis \ = \text{Normalized Direction Vector} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{-0.529769} \\ \mathbf{0.455174} & \mathbf{0} & \mathbf{-0.715655} \\ \mathbf{0.715655} & \mathbf{0} & \mathbf{0.455174} \end{bmatrix} $$

For the 3rd Hole

The position of 3rd hole relative to the fiducial ${}^{W}_{W}P_{h3} = [-0.2, 0.2, 0\,]^{T}$

The position of h3 in the camera frame: ${}^{C}_{C}P_{h_3} = {}_{W}R_{C} * {}_{W}P_{h_3} + {}_{C}P_{W}$

$$ {}^{C}_{C}P_{h_3} = \left(\begin{bmatrix} 0.899478 & 0.340327 & 0.0644669 \\ -0.5462739 & 0.793374 & 0.3397965 \\ -0.43871433 & -0.544941 & 0.89676901 \end{bmatrix} * \begin{bmatrix} -0.2 \\ 0.2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.2 \\ 0 \\ 0.5 \end{bmatrix} \right) $$
$$ = \begin{bmatrix} 0.0881698 \\ 0.267922 \\ 0.478755 \end{bmatrix} $$

Then computing the desired camera position relative to camera frame

$$ {}^{C}_{C}\acute{P}_{h_3} = {}^{C}_{C}P_{h_3} - \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.0881698 \\ 0.267922 \\ 0.378755 \end{bmatrix} $$

Transform the desired camera position to the robot base frame by the using the given rotation matrix and translation position of camera to base

$$ {}^{B}_{B}\acute{P}_{C\_h3} = {}_{C}R_{B} * {}^{C}_{C}\acute{P}_{h3} + {}_{B}P_{C} $$

$$ = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} * \begin{bmatrix} 0.0881698 \\ 0.267922 \\ 0.378755 \end{bmatrix} + \begin{bmatrix} 0 \\ -0.7 \\ 0.7 \end{bmatrix} = \begin{bmatrix} \mathbf{-0.0881698} \\ \mathbf{-1.07875} \\ \mathbf{0.432078} \end{bmatrix} $$

Now, let's calculate these normalized vectors and construct the rotation matrix.

The orientation matrix is constructed using these normalized vectors to define the rotation of the camera. Normalized vectors simplify the interpretation of the matrix. Each column of the matrix represents a unit vector in a particular direction, making it easier to understand the transformation. In summary, normalization is applied to ensure that the orientation matrix accurately represents the rotation without introducing scaling effects. It simplifies subsequent calculations and facilitates a more intuitive interpretation of the resulting matrix.

Then moving to the computing the desired camera orientation of target pose for the fist hole too

where Direction Vector $= {}^{B}_{B}\acute{P}_{C\_h3} - {}^{B}_{B}P_{C} = \begin{bmatrix} -0.0881698 \\ -1.07875 \\ 0.432078 \end{bmatrix} - \begin{bmatrix} 0 \\ -0.7 \\ 0.7 \end{bmatrix}$

$$= \begin{bmatrix} -0.0881698 \\ -0.378755 \\ -0.267922 \end{bmatrix}$$

Normalized Direction Vector $= \frac{\text{Direction Vector}}{\|\text{Direction Vector}\|} = \begin{bmatrix} -0.186705 \\ -0.802037 \\ -0.567343 \end{bmatrix}$

The Perpendicular Vector is a vector perpendicular to the direction vector. We can choose any vector that is not collinear with the direction vector. Let Perpendicular Vector $= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$\text{Cross Product Vector} = \text{Normalized Direction Vector} \times \text{Perpendicular Vector}$$
$$= \begin{bmatrix} -0.186705 \\ -0.802037 \\ -0.567343 \end{bmatrix} \text{X} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.567343 \\ 0.802037 \end{bmatrix}$$

Then the new rotaion matrix target pose of the first hole ${}_{C}\acute{R}_{B}$

$${}_{C}\acute{R}_{B} = \begin{bmatrix} X-Axis = \text{Perpendicular Vector} \\ Y-Axis = Cross\ Product(Z-Axis, X-Axis) \\ Z-Axis = \text{Normalized Direction Vector} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{-0.186705} \\ \mathbf{-0.567343} & \mathbf{0} & \mathbf{-0.802037} \\ \mathbf{0.802037} & \mathbf{0} & \mathbf{-0.567343} \end{bmatrix}$$

For the 4th Hole

the position of 4th hole relative to the fiducial ${}^{w}_{W}P_{h4} = [-0.2, -0.2, 0]^{T}$

the position of h4 in the camera frame: ${}^{C}_{C}P_{h_4} = {}_{W}R_{C} * {}_{W}P_{h_4} + {}_{C}P_{W}$

$${}^{C}_{C}P_{h_4} = \left(\begin{bmatrix} 0.899478 & 0.340327 & 0.0644669 \\ -0.5462739 & 0.793374 & 0.3397965 \\ -0.43871433 & -0.544941 & 0.89676901 \end{bmatrix} * \begin{bmatrix} -0.2 \\ -0.2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.2 \\ 0 \\ 0.5 \end{bmatrix} \right)$$
$$= \begin{bmatrix} -0.0479613 \\ -0.0494127 \\ 0.696731 \end{bmatrix}$$

Then computing the desired camera position relative to camera frame

$$ {}^{C}_{C}\acute{P}_{h_4} = {}^{C}_{C}P_4 - \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix} = \begin{bmatrix} -0.0479613 \\ -0.0494127 \\ 0.596731 \end{bmatrix} $$

Transform the desired camera position to the robot base frame by the using the given rotation matrix and translation position of camera to base

$$ {}^{B}_{B}\acute{P}_{C\_h4} = {}_{C}R_{B} * {}^{C}_{C}\acute{P}_{h4} + {}_{B}P_{C} $$

$$ = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} * \begin{bmatrix} -0.0479613 \\ -0.0494127 \\ 0.596731 \end{bmatrix} + \begin{bmatrix} 0 \\ -0.7 \\ 0.7 \end{bmatrix} = \begin{bmatrix} \mathbf{0.0479613} \\ \mathbf{-1.29673} \\ \mathbf{0.749413} \end{bmatrix} $$

Now, let's calculate these normalized vectors and construct the rotation matrix. The orientation matrix is constructed using these normalized vectors to define the rotation of the camera. Normalized vectors simplify the interpretation of the matrix. Each column of the matrix represents a unit vector in a particular direction, making it easier to understand the transformation. In summary, normalization is applied to ensure that the orientation matrix accurately represents the rotation without introducing scaling effects. It simplifies subsequent calculations and facilitates a more intuitive interpretation of the resulting matrix.

Then moving to the computing the desired camera orientation of target pose for the fist hole too where $\text{Direction Vector} = {}^{B}_{B}\acute{P}_{C\_h4} - {}^{B}_{B}P_{C} = \begin{bmatrix} 0.0479613 \\ -1.29673 \\ 0.749413 \end{bmatrix} - \begin{bmatrix} 0 \\ -0.7 \\ 0.7 \end{bmatrix} = \begin{bmatrix} 0.0479613 \\ -0.596731 \\ 0.0494127 \end{bmatrix}$

$$ \text{Normalized Direction Vector} = \frac{\text{Direction Vector}}{\|\text{Direction Vector}\|} = \begin{bmatrix} 0.0798435 \\ -0.993407 \\ 0.0822598 \end{bmatrix} $$

The Perpendicular Vector is a vector perpendicular to the direction vector. We can choose any vector that is not collinear with the direction vector. Let $\text{Perpendicular Vector} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$ \text{Cross Product Vector} = \text{Normalized Direction Vector} \times \text{Perpendicular Vector} $$
$$ = \begin{bmatrix} 0.0798435 \\ -0.993407 \\ 0.0822598 \end{bmatrix} \text{X} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.0822598 \\ 0.993407 \end{bmatrix} $$

Then the new rotation matrix target pose of the first hole ${}_{C}\acute{R}_{B}$

$$ {}_{C}\acute{R}_{B} = \begin{bmatrix} X-Axis = \text{Perpendicular Vector} \\ Y-Axis = Cross\ Product(Z-Axis, X-Axis) \\ Z-Axis = \text{Normalized Direction Vector} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0.0798435} \\ \mathbf{0.0822598} & \mathbf{0} & \mathbf{-0.993407} \\ \mathbf{0.993407} & \mathbf{0} & \mathbf{0.0822598} \end{bmatrix} $$