Simulating a flock of birds

STEFFEN RANDRUP, MICHEL SMOLA, ANDREI TIMUS

University of Copenhagen

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Abstract

The motion of a flock of birds appears complex and difficult to classify due to the large number of paticipants each with their own idea of where to go. Yet such flocks display clear collective motion. In this paper we describe such a flock similar to a system of self-propelled particles interacting with nearby neighbours. We extend this model by reducing the interaction of any given bird by only interacting with other birds within the bird's vision. Any bird will act as an average of any other birds it can see. We show that this simple model of limited interaction may cause the flock to align over distances larger than the extend of a single bird's interaction. Including how the noise in each bird's direction and density of birds affect the alignment. We compare the transition from unordered to ordered motion to a phase transition and comment on the effects of a vision based approach to this transition.

I. COLLECTIVE MOTION OF BIRDS

One may consider the collective movement of birds to be described by a simple set of rules. In this paper we describe the movement of such a flock as a system of self-propelled particles. Our model is an extension of the model presented by Tamás Vicsek and his group in their paper "Novel Type of Phase Transition in a System of Self-Driven Particles" [1]. The motion of each bird or particle is governed by the motion of other birds or particles nearby. In the following we may use birds and particles interchangably.

In our setup the particles where distributed randomly in a square lattice of size L. Each particle had assigned a random direction θ and a constant interaction radius r. One update consisted in modifying the position of all particles according to the following equation:

$$x_i(t+1) = x_i(t) + v_i(t)\Delta t \tag{1}$$

where Δt represents the time between 2 updates. The velocity of a particle had a constant absolute value and a direction given by:

$$\theta(t+1) = \langle\theta(t)\rangle_r + \Delta\theta \tag{2}$$

where

$$\langle\theta(t)\rangle_r = \arctan\frac{\langle\sin(\theta(t))\rangle_r}{\langle\cos(\theta(t))\rangle_r} \tag{3}$$

is the average direction of the velocities of nearby particles wich are inside the radius of influence, r, of the given particle. $\Delta\theta$ represents the noise in the system and takes values with equal probability within the interval $[-\eta/2, \eta/2]$. For a constant number of particles there are 3 parameters which can be varied: η,ρ,and v, where v is the distance a particle travels between 2 updates. η describes the interval in which $\Delta\theta$ is picked. ρ is the density of the particles. This is effectively a way of controlling the size of the lattice if the number of particles is fixed.

We further extend the model by implementing a cone-like vision. This means, that rather than averaging the velocities over all birds within a certian radius, we only look at a slice of this circle. For a bird to be within the vision of another bird, it must be within the interaction radius, r, and the angle between the bird's velocity and direction to the other bird must be within $\pm\theta_{\text{cone}}$. The limit $\theta_{\text{cone}} = \pi$ equals the case without limiting the cone vision and reproduces the results from [1].

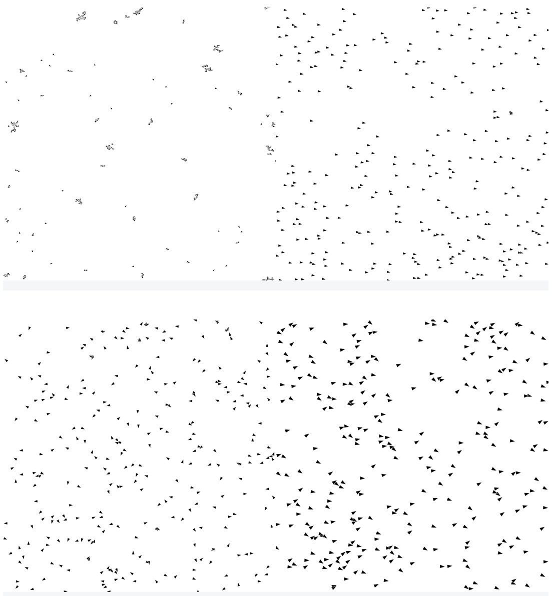

Figure 1: Different configurations of η and ρ
Top left: low η low ρ
Top right: Low η, high ρ
Bottm left: High η, low ρ
Bottom right: High η, high ρ

Figure 1 shows the configuration of birds for different values of ρ and η. For low values of both ρ and η the birds tend to form smaller groups in which the birds move in the same direction. The groups themselves move randomly. When the randomness is high and the density low, the overall motion of the birds is just random. In the case of high randomness and high density, the birds appear to move randomly but with an overall trend in directions shared among the all the birds. If the density is high enough, while the noise has low values, the birds move orderly in one direction. This overall trend in motion is similar to the system going through a kinetic phase transition where all the particles share the same direction. In order to characterize the behaviour of the flock we have determined the average normalized velocity of the system and investigated how it changes when the input parameters are modified:

$$v_a = \frac{1}{Nv}\left|\sum_{i=1}^{N} v_i\right| \tag{4}$$

For disordered motion v_a is aproximatively 0, while for ordered v_a has the value near 1.

The behaviour of v_a is found to be similar to that of the order parameter of some equilibrium systems close to their critical point. For large system sizes the data shows scaling which is an indication of a phase transition. Equations (6) and (7) may be used to describe such a system near the critical value, at which the phase transition occours.

i. Determining the critical noise level

To determine the critical noise parameter η_c we used a quadratic fit model to to fit the falling curve for $\eta < \eta_c$ and a linear fit for $\eta > \eta_c$, both visible in figure 2. η_c is then set to the intersection of the fit curves. Hereby we determine

$$\eta_c = 4.56 * ** \tag{5}$$

put in final number

$$v_a \approx [\eta_c(\rho) - \eta]^{\beta} \tag{6}$$

$$v_a \approx [\rho - \rho_c(\eta)]^{\gamma} \tag{7}$$

II. From disorder to order

The alignment, v_a, depends on the amount of randomness η in the system and the density ρ, as seen in equations (6) and (7). In figure 2 we see how the alignment of the birds decreases as the randomness of their motion increases. The decrease is to be expected, as a higher degree of randomness should disrupt alignment. This reflects to v_a, which approaches 0 in a totally unordered state and 1 in a totally ordered state. What might be more interesting is the general shape of the curves. We may then determine β from equation (6) to be get value of β

This is also shown in figure 3. The results presented here are all computed without the conevision (equivalent to $\theta_{\text{cone}} = \pi$). Figure 2 shows how v_a varies with the noise η and density ρ for a constant value of density and noise respectively. As seen in the graph the overall alignment of the particles depends on the noise and for a high number of particles N the system becomes less ordered. For high η the flock reaches a coherent moving phase, while for low η the particles move randomly in the lattice.

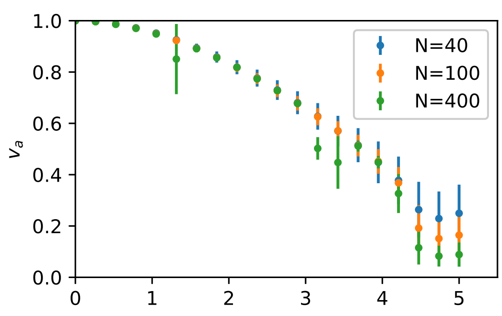

Figure 2: Dependence of alignment on randomness

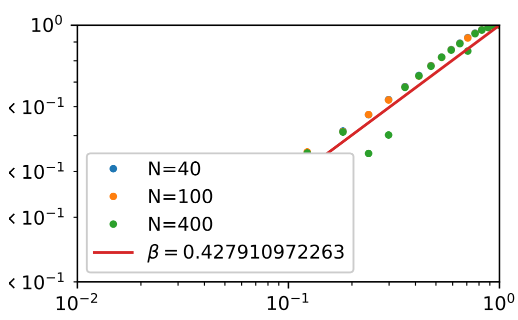

Figure 3: Determining β

Similarly we may see the effects of the density on the alignment of birds. This also fits the prediction from equation (7). This is shown in figure 4.

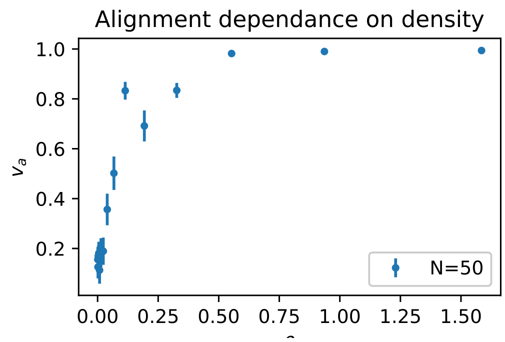

Figure 4: Dependance of alignment on density

i. Cone-like vision

The effects of our implementation of an angle based vision are only detectable on both low randomness and low density. This is expected as a higher density causes an effective long range interaction, which may span the entire flock. This negates the effect of an angle based vision as the flock moves in a coherent fashion. As for randomness of motion it decreases the overall alignment of the flock as seen in figure 2. In figure 5 we present the overall alignment's dependance of the angle describing this cone-like vision for different densities. The randomness is kept at a low value of $\eta = 0.1$. The results correspond quite well with our expectation of increasing the alignment with larger angle. For an angle of π we see, the alignment looks like it did when we considered every bird within the interaction radius. We also notice the effects of the density of birds. As described above we expect a long range interaction between the birds to arise when the density increases. This effect is visible in the figure as we see the alignment increase faster, when the density is higher. Again we see, how the system goes between an unordered state and an ordered state. For low densities we don't reach full alignment, but that might be due to the lack of interaction between far-away birds. The trend however is similar to the phase transition presented earlier.

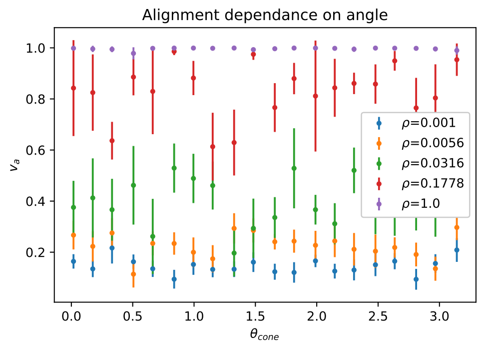

Figure 5: Conevision

III. Discussion

We have compared the collective motion of birds to a phase transition. The connection between these is not obvious. However we do see simlilarities in how increasing the noise in the system will distrupt alignment and how denser systems will display longer effective interaction range. Causing the entire system to behave coherently.

The addition of a cone-like vision is, as the name might suggests, inspired from birds rather than the study of phase transitions. The effects of the vision based approach were suttle. They were only detectable for low densities and noise, and for sufficiently large angles of vision the system would again display behavior to that without the vision based approach. As discussed this behavior is similar to that of a phase transition, however it is only detectable for low denisities. This also means, that the system doesn't align completely in the range where it is detectable, unlike for the other transitions presented in this paper. We have treated the angle of vision more like a property of the system than a variable. This is a naive approach as it assumes that any single bird only ever looks in one direction.

The values of v_a have been taken as a time average after some transient time after which the system has settled into a state, which doesn't change significantly.

IV. Conclusion

In this paper we have presented a model for studying the motion of a flock of birds. The model contained contained few variables; density of birds, noise in their direction and an angle controlling which other birds influence a given bird's motion. Each of these variables gave rise to the movement of the flock going from disordered to ordered. For the case of the angle of vision the transition was only detectable for low densities and low noise as higher densities made more birds be within sight and high noise reduced alignment enough for the effect of the vision to be hard to detect.

References

[1] Tamás Vicsek, András Czirók, Eshel Ben-Jacob, Inon Cohen and Ofer Shochet
Novel Type of Phase Transition in a System of Self-Driven Particles
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