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MASTER THESIS

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Height map compression techniques

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Study programme:	Informatics
Specialization:	Software Systems

Prague 2016

Dedication.

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Název práce: Komprese výškových map

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Katedra: Kabinet software a výuky informatiky

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Abstrakt:

Klíčová slova:

Title: Height map compression techniques

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Abstract: The goal of this thesis is to design a suitable method for lossy compression of heightmap terrain data. This method should accept blocks of float samples of dimensions $2^n x 2^n$ at the input, for which it should be able to perform progressive mip-maps (progressive lower-resolution representations) decompression. For every mip-map, it should keep the reconstructed data within a certain maximum absolute per-sample error bound in the unit of meters adjustable by the user. Given these constraints, it should be as efficient as possible. Our method is inspired by the second generation of progressive wavelet-based compression scheme modified to satisfy the maximum-error constraint. We simplified this scheme by factoring out unnecessary computations in order to improve the efficiency. Our method can compress a 256x256 block in about 30 ms and decompress it in about 1 ms. Thanks to these attributes, the method can be used in a real-time planet renderer. It achieves the compression ratio of 37:1 on the whole Earth 90m/sample terrain dataset transformed and separated into square blocks, while respecting the maximum error of 5m.

Keywords: heightmap, lossy, compression, mip-map, guaranteed maximum error bound

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1. Introduction

A real-time whole-planet renderer must work with huge amounts of terrain data. In order to reach reasonable frame rates, its rendering pipeline has to use some kind of multiresolution (LOD-ing[1]) approach. There is a survey paper summarizing the best known multiresolution terrain rendering methods [7]. Some of them are designed to render just a flat area, others are able to render the whole planet. In the rest of this chapter, we will briefly describe those of them which also contain terrain data compression.

C-BDAM[2] [5] and P-BDAM[3] [2] perform the compression in the refinement of a node of their LOD hierarchy. Once the values of a certain node are known, they are used to predict the values of its children as accurately as possible. After that, the differences between these predictions and the real values are computed. These are called residuals. With the help of them, the real values can be restored with absolute accuracy. However, the residuals are then quantized to achieve better compression ratio which means that the compression is lossy. Then, they are losslessly compressed by an entropy codec. Both these methods are able to compute the residuals in the way which ensures that the error of the reconstructed data is kept within a maximum error bound adjustable by the user in every node of their LOD hierarchy. This can be achieved by a slight modification of the second-generation wavelet lifting scheme [9]. C-BDAM is designed to render just a flat area, whereas P-BDAM is able to render the whole planet.

Another paper [6] describes a method for rendering a flat portion of terrain. This method contains data compression based on the same principle - the residuals needed to reconstruct the children of a square node of the terrain LOD hierarchy are compressed. The computation of residuals is based on the wavelet-based JPEG2000 standard. This method is not able to reconstruct the data within a certain maximum-error bound which makes it less interesting to us. Besides, the visual artifacts between adjacent nodes of different LODs are not handled by its rendering pipeline.

The aim of our meth

In practice, many applications handle the real-time rendering well with LOD schemes tailored to their needs. In such cases, a compression method tied to a concrete LOD scheme (which is the case of the mentioned methods) is not feasible. This method handles only the compression, so it can be used as a plug & play component in an existing real-time renderer. Its only job is to compress a block of terrain height samples sized $2^n x 2^n$ and to provide fast progressive decompression of its mip-maps, while respecting the maximum error bound at every mip-map. The source code of the method is written modularly, so that any representation of the height samples can be compressed - doubles, floats or even arbitrary structures. It is inspired by C-BDAM - the compression method is extracted from the LOD scheme and simplified.

As a case study we have implemented this method as a plugin into an appli-

[1]LOD is the abbreviation of level of detail - degradation of quality of the displayed data with the growing distance in order to optimize the rendering

[2]Compressed Batched Dynamic Adaptive Meshes

[3]Planet Sized Batched Dynamic Adaptive Meshes

cation, which transforms the heights on the planet surface into 256x256 blocks of 32-bit float samples in the unit of meters, which are then stored separately and during the run fetched into a quadtree-based LOD hierarchy. The mip-maps of the blocks are used while looking at them from a side.

This approach introduces heavy redundancy of the data - a block corresponding to a certain quadtree node contains simplified blocks of its children and all these blocks are stored separately. To the contrary, in C-BDAM only the residuals needed to reconstruct the children from the parent node are stored.[4] However, the reason why this approach is used is that the user can navigate to any area almost immediately - only the data needed for the scene has to be fetched, without having to reconstruct it by traversing from the root. Moreover, this approach enables the user to flexibly extend the terrain data by high-resolution insets. The mentioned redundancy of the data emphasizes the need for as efficient compression method as possible, doing only what is required - providing the mip-maps while respecting the maximum-error bound of the samples inside each one of them.

In Chapter 2, we briefly describe the basic theory of wavelets and link C-BDAM and this method to it, in Section **??**, we briefly describe the basic outline of the method. In Chapter 4, we describe the details of the method. In Chapter 5, we compare the core algorithm of this method to the algorithm of C-BDAM. We present the results in Section 6 and then discuss them in Section 7.

[4]The LOD structure in C-BDAM is not a quadtree, though

2. The wavelets

This chapter consists of two sections. In the first one (2.1), we will briefly and formally describe the main principle and usage of second generation wavelet transformation methods which are relevant for this thesis. In the second one (2.2), we will compare C-BDAM and our method to these methods. Even though C-BDAM is based on the same principle, it differs from these methods a bit, so we will describe the basic differences. Then we will perform the same basic comparison with our proposed method. Our method differs from the described wavelet scheme and C-BDAM a bit more which will be clarified in that section.

2.1 The introduction to second-generation wavelets

Basically, there are two generations of wavelets. The first generation uses dilated and translated wavelet function [1] for computation. The second one uses filter banks to perform high-pass and low-pass filtering [3]. The computational equivalency of these two approaches has been proven [4].

For this work, the second generation of discrete wavelet transform methods is most relevant, so we will briefly describe it in this section in order to give the reader an idea of the wavelet concept which is referred to in many places of this thesis. The second wavelet generation is much easier to understand than the first one, so it is possible to describe its basic idea in a few pages.

Every method of this generation consists of just several subsequent applications of lifting onto the input. The lifting is the basic step of the method. It splits the set of its input signal samples into two parts - low-pass (the low frequency information) and high-pass (residuals, the high frequency information). The lifting is firstly applied on the input set of signal samples and then is recursively applied to the low-pass part produced in the previous iteration until the length of the latest low-pass part is 1. In order to make this recursion possible, the count of samples of the original input of the method must be a certain power of two. If the length of the input is 2^n, the method performs n iterations of lifting. The described successive application of lifting on smaller and smaller input is called the bottom-top pass. We can imagine this as building a pyramid of low-pass outputs the first tier of which is the input itself and every following higher tier is the low-pass output of lifting applied to the tier right below. Every tier is half the width of the previous one and after the bottom-top pass, the highest tier has the width of 1.

After this bottom-top pass, we can perform the inverse top-bottom pass. This pass does not know how the produced pyramid looks, it only knows its highest tier, sized 1. However, it is supposed to be able to progressively reconstruct the whole pyramid from the top to the bottom, only utilizing the knowledge of the high-pass information. Producing a certain tier from the previos one is called the reconstruction which is the exact opposite of lifting.

At this point, you might ask what all these decompositions and backward compositions are good for. What makes them interesting is the fact that the bottom-top pass just needs the high-pass information (residuals) to fully reconstruct the input. This information tends to be sparse and input-dependent - the smoother the input, the less high-pass information it contains. If we compress it well, we can save much storage space. Thus, if we want to store a set of samples the count of which is a power of two in as little space as possible, we will not store the samples directly, but we will store just the compressed residuals produced by the successive iterations of lifting applied to the input. If we are not required to accurately reconstruct the input, we can even decimate (quantize) the residuals. Because this information often contains just details, its careful decimation does not deform the reconstruction much and ensures better compression ratio. One more interesting fact is that the residuals bound to lower tiers of the pyramid carry finer details than those bound to the higher ones. Thanks to this, the more-detailed (larger) sets of residuals can be compressed more aggressively than the less-detailed (smaller) ones. This is called progressive compression and it is used for example in JPEG standard [8].

In the following lines, we will describe the lifting and reconstruction steps more formally. Let us say that the lifting is given the input samples x_k. It splits them into the even ones: $x_{2k} = x_e$ and the odd ones: $x_{2k+1} = x_o$. This splitting is not yet based on any frequency properties of the samples, it is based just on their order. However, these two sets of samples will subsequently be modified, so that the even ones will contain the low-pass information and the odd ones will become the residuals - the high-pass information. This will be performed with the help of two operators: the prediction operator P and the update operator U. P will be used to produce the residuals d from x_o and U will be used to produce the low-pass part s from x_e.

Up to this point, just the common properties of the second-generation methods have been described. Now will come the differences between them. The only thing they differ in is the way they perform lifting and reconstruction. The way the lifting step is performed clearly determines the way how the reconstruction is performed, as the reconstruction must be the exact inverse of lifting. The lifting step varies in the order in which the operators P and U are applied. According to this, the methods can be split into two main groups - the prediction-first ones and the update-first ones.

In the prediction-first methods, the prediction is applied first:

$$d = x_o - P(x_e)$$
$$s = x_e + U(d)$$

The reconstruction must be the exact inverse:

$$x_e = s - U(d)$$
$$x_o = d + P(x_e)$$

In the update-first methods, the update operator is applied first:

$$s = x_e + U(x_o)$$
$$d = x_o - P(s)$$

Here is how the reconstruction looks then:

$$x_o = d + P(s)$$

$$x_e = s - U(x_o)$$

2.2 Comparisons between the wavelets, C-BDAM and our method

In this section, we will describe how C-BDAM and our method differ from the basic second-generation wavelet scheme and from each other. The lifting inside C-BDAM is a slight variation of the update-first approach. The main difference is that the input to the first update is not only x_o, but the whole x. In addition, the computation of s is not the summation of the product of x_e and U anymore, because inside $U(x)$, x_e is multiplied:

$$s = U(x)$$
$$d = x_o - P(s)$$

The inverse reconstruction is then:

$$x_o = d + P(s)$$
$$x_e = U^{-1}(x)$$

Moreover, the samples x are regularly distributed in the plane, so the spliting into x_o and x_e no longer depends on the indices of the samples, but on their positions instead (Fig. 2.1). Nevertheless, this is just a formal difference which has no effect on the computation. The size of x_o and x_e is still half the size of x which is crucial to keep the original form of lifting. Note that if the residuals d were simply quantized after lifting and used in the reconstructions inside the second top-bottom pass, each step of the reconstruction would increase the maximum absolute deviation from the original low-pass values produced on the first bottom-top pass. To ensure that the reconstructed values are within the maximum-error bound from their corresponding values produced in the first pass at each tier, the residuals computed in the first pass are slightly corrected according to the actual values in another additional top-bottom pass which then turns out to be identical to the reconstruction (decompression), except for the fact that in the following decompression, just the corrected residuals are used to progressively reconstruct the data.

The method proposed in this thesis shares the same main lifting principle with C-BDAM - it is update-first and uses the whole x as the input to U, but has several differences: the size of x_e and thus s is not half the size of x, but one fourth of it instead, as each four neighboring pixels of x are collapsed into one inside s (Fig. 4.1). Additionally, the lifting is not complete, because the prediction operator is not applied there and the computation of residuals is not performed there either. In the lifting of C-BDAM, just temporary approximate residuals are computed and they are corrected in the subsequent top-bottom pass, whereas in our method, the correct residuals which already ensure the satisfaction of the maximum-error bound constraint are computed directly in the second top-bottom pass, also utilizing the prediction operator. Similarly to C-BDAM, the computations inside this pass are identical to the reconstruction of the data, except for the fact that during the reconstruction, the residuals are not computed

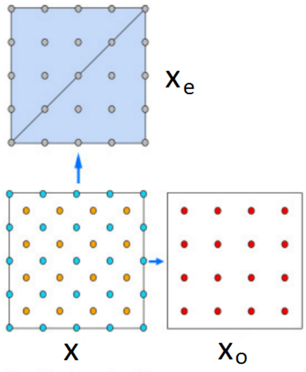


Figure 2.1: Lifting in C-BDAM - the samples x are split into the even ones (x_e) which will become low-pass (s) and the odd ones (x_o) which will become high-pass (d)
Source: C-BDAM [5] (edited)

anymore. Additionally, the prediction operator is applied multiple times in one step of the reconstruction which is explained in Chapter 4. The rationale behind all these differences is explained in Chapter. 5.

3. The outline of the method

In this chapter, we will briefly describe how the compression works. Basically, two consecutive passes are performed on the input heightmap. These passes are analogic to the passes of the described second-generation wavelet methods. The first bottom-top pass computes the target mip-maps - from the largest one to the smallest one. Those will be the mip-maps, against which the accuracy of reconstruction will be measured. The largest mip-map is the input itself. The second top-bottom pass constructs the compressed mip-maps from the smallest one to the largest one with respect to the target mip-maps in order to ensure that the maximum deviation of every compressed mip-map from its corresponding target mip-map is within the maximum error bound set by the user. The smallest of these mip-maps is just the suitably quantized sole value of the corresponding target mip-map. It is directly stored as first. The values of each following compressed mip-map are predicted from its previous compressed mip-map. For these mip-maps, we store just the residuals which are added to the predictions to satisfy the maximum deviation constraint.

More formally, the first pass is given the input square block of float height samples $\boldsymbol{L_n}$ sized $2^n x 2^n$ and produces n mip-maps $\boldsymbol{L_{n-1..0}}$ from it, one by one. The dimension of $\boldsymbol{L_i}$ is half the dimension of $\boldsymbol{L_{i+1}}$. Generally, $\boldsymbol{L_i}$ can be computed from $\boldsymbol{L_{i+1}}$ by any form of averaging of pixels - see the details in the following chapter.

The second top-bottom pass has already $\boldsymbol{L_{0..n}}$ available and computes $\boldsymbol{L^\bullet_{0..n}}$ - the compressed mip-maps. The dimension of $\boldsymbol{L_i}$ and $\boldsymbol{L^\bullet_i}$ is the same. The computation ensures that the maximum absolute deviation between their corresponding samples is not greater than $\boldsymbol{D}$ - the parameter set by the user. This will be denoted by:

$$maxdev(\boldsymbol{L_i}, \boldsymbol{L^\bullet_i}) \leq \boldsymbol{D},$$

where

$$maxdev(A, B) = \arg\max_{x,y} |A[x][y] - B[x][y]|$$

We will achieve this with the help of the uniform quantizer Q_D the quantization step of which is set to the maximum value which still respects this error bound:

$$maxdev(Q_D(x), x) \leq \boldsymbol{D},$$

where x is an arbitrary float sample or block of samples. The quantizing step of this quantizer is $2\boldsymbol{D} - 1$ in case $\boldsymbol{D} \geq 0.5$ and $2\boldsymbol{D}$ otherwise.

As we already mentioned, $\boldsymbol{L^\bullet_0}$ is just the quantized sole value of $\boldsymbol{L_0}$:

$$\boldsymbol{L^\bullet_0} = Q_D(\boldsymbol{L_0})$$

Thanks to the fact that the quantizer respects the maximum-error bound $\boldsymbol{D}$, $maxdev(\boldsymbol{L_0}, \boldsymbol{L^\bullet_0}) \leq \boldsymbol{D}$.

Then, the values of every following $\boldsymbol{L^\bullet_{i+1}}$ are predicted from the values of the previous $\boldsymbol{L^\bullet_i}$. The raw differences between the target values and the predicted values are denoted as $\boldsymbol{E_{i+1}}$ (the residuals). With the help of them and the predictions from $\boldsymbol{L^\bullet_i}$, we would be able to accurately reconstruct the target mip-map $\boldsymbol{L_{i+1}}$. However, these residuals are then quantized with the uniform quantizer Q_D to $\boldsymbol{E^{\bullet}_{i+1}}$. With the help of the quantized residuals, we are no longer able to accurately reconstruct $\boldsymbol{L_{i+1}}$, but thanks to the fact that the used quantizer keeps the maximum absolute error within the bound $\boldsymbol{D}$, we can guarantee that the reconstructed $\boldsymbol{L^{\bullet}_{i+1}}$ will satisfy the maximum-error constraint: $maxdev(\boldsymbol{L^{\bullet}_{i+1}}, \boldsymbol{L_{i+1}}) \leq \boldsymbol{D}$. Here is how we construct $\boldsymbol{L^{\bullet}_{i+1}}$:

$$\begin{aligned}
\boldsymbol{E_{i+1}} &= \boldsymbol{L_{i+1}} - P(\boldsymbol{L^{\bullet}_{i}}) \\
\boldsymbol{E^{\bullet}_{i+1}} &= Q_D(\boldsymbol{E_{i+1}}) \\
\boldsymbol{L^{\bullet}_{i+1}} &= P(\boldsymbol{L^{\bullet}_{i}}) + \boldsymbol{E^{\bullet}_{i+1}}
\end{aligned} \tag{3.1}$$

Thanks to the fact that the residuals $\boldsymbol{E_{i+1}}$ are computed with respect to the target mip-map $\boldsymbol{L_{i+1}}$, the maximum-error constraint is satisfied, no matter what values are in $\boldsymbol{L^{\bullet}_{i}}$ and what the prediction operator P looks like. At the end, the quantized residuals $\boldsymbol{E^{\bullet}_{0..n}}$ are compressed with the help of an entropy codec (Zlib) and stored ($\boldsymbol{E^{\bullet}_{0}} = \boldsymbol{L^{\bullet}_{0}}$). The order of their storage is from $\boldsymbol{E^{\bullet}_{0}}$ to $\boldsymbol{E^{\bullet}_{n}}$, so that progressive decompression is possible. The more accurate P is, the smaller the residuals are, thus the higher the compression ratio is. The details of the prediction operator used in this method are described in the following chapter. The higher $\boldsymbol{D}$ is, the less entropy there is among the residuals, thus the higher the compression ratio is, but the lower the reconstruction quality is.

The real-time decompression then just reads the stored quantized residuals and decompresses them with the help of the same entropy codec. Thanks to the fact that the residuals of a smaller mip-map are stored before the residuals of a larger mip-map, progressive decompression of mip-maps $\boldsymbol{L^{\bullet}_{0..n}}$ is possible, utilizing the same principle of producing predictions from the previous reconstructed mip-map and adding residuals to them (eq. 3.1). Of course, in order for this to work, the prediction operator must be identical to the one used in the compression.

4. Details of the method

In this chapter, the method is described in more detail. Unlike the previous outline, this description should be sufficient enough for the reader to implement this method by themselves. In Section 4.1, it will be said how exactly we construct the target mip-maps during the first bottom-top pass and what alternative constructions we also considered. In Section 4.2, we will explain what is the form of P - the prediction operator - and how exactly it is applied in the second top-bottom pass in order to compute the residuals needed to reconstruct a finer mip-map from the coarser one and also perform the reconstruction with the help of these residuals. Note that this method does not use an update operator, see Chapter 5 for the explanation and details.

4.1 Bottom-top pass

As we already said, in the first bottom-top pass, we just construct the target mip-maps one by one, from the largest one - the input itself - to the smallest one, sized 1. At each step, we construct a smaller mip-map from the last constructed one. The dimension of the new mip-map is half the dimension of the last one, in other words, it is half as detailed. Generally, we can build the new mip-map by any form of averaging of pixels of the larger mip-map. In the previous chapter, we explained that the maximum absolute error of the reconstruction is not dependent on how the mip-maps look, as long as they contain valid values (no infinities, NaNs). However, the appearance of the mip-maps affects the compression ratio. The closer the neighboring mip-maps are to each other, the lower the residuals of the transition from the smaller one to the larger one are, thus the higher the compression ratio is. Additionally, as described in Chapter **??**, inside the renderer in which the method has been applied, the mip-maps of a certain terrain square are carefully selected, so that aliasing is minimized. This decision is based on the area of the square projected to the screen. This means that while looking at a certain square from above, its finest mip-map is displayed and during a fixed-radius circular traversal around it up to the point when we look at it from a side, we will be gradually displaying coarser (less detailed) mip-maps of the square. This means that if the mip-maps are significantly different from each other, disturbing visual artifacts might occur during this traversal. The best way how to minimize these artifacts is to use the simplest form of averaging of heights when producing a lower-resolution mip-map where the height at every pixel of the smaller mip-map $\boldsymbol{L_i}$ will be the average of the heights of the four corresponding neighboring pixels inside the larger mip-map $\boldsymbol{L_{i+1}}$:

$$\boldsymbol{L_i}[m][n] = \frac{\sum_{om=0}^{1} \sum_{on=0}^{1} \boldsymbol{L_{i+1}}[2m+om][2n+on]}{4} \tag{4.1}$$

For a comparison, in transition to a coarser LOD in C-BDAM, a different form of heights averaging is utilized. It properly conforms to the standard lifting scheme - it uses the update operator to produce the coarser LOD. This averaging is even parametrized by one coeffiecient named subsampling weight the value of which can span from 0 to 1. To the contrary, our method does not use the update operator there which is explained in Chapter 5. However, we tried to use a similar averaging of pixels inspired by the one performed in the update operator in C-BDAM. With the subsampling weight well set, we achieved a slightly better compression ratio, but the mentioned visual artifacts were more disturbing. At last, we decided to minimize the visual artifacts, as they really affect the user experience, and stick to the described simple averaging (eq. 4.1).

4.2 Top-bottom pass

This pass is performed in the offline compression after the first bottom-top pass, in which case it computes the residuals $\boldsymbol{E^\bullet_{0..n}}$ needed to completely progressively reconstruct the compressed multiresolution representation of the input $\boldsymbol{L_n}$. During the following real-time decompression, this is the only pass which is performed, with the only difference that it no longer computes the residuals, but it just reads them and uses them to reconstruct the data. Let us describe it detailly, so that it becomes clear how this pass is implemented.

In the previous chapter describing the outline of the method, we claimed that we construct a larger compressed mip-map $\boldsymbol{L^\bullet_{i+1}}$ from the smaller $\boldsymbol{L^\bullet_i}$ in just one step (eq. 3.1). This is not exacly true, it is a simplification which we made for several reasons: to give the reader a simple high-level idea of the method, to make the fact that $maxdev(\boldsymbol{L^\bullet_{i+1}}, \boldsymbol{L_{i+1}}) < \boldsymbol{D}$ easier to see and to make it clear that the way the target mip-maps $\boldsymbol{L_{n-1..0}}$ look has no effect on this constraint. However, the truth is that to get $\boldsymbol{L^\bullet_{i+1}}$ from $\boldsymbol{L^\bullet_i}$, the prediction operator P is applied consecutively three times. Its form is different at each of these applications which reflects the fact that before each application, different height values are known - the later the application, the more values are known. After each of these applications, the residuals are computed during the compression and added to the already computed values during both the compression and decompression. Nevertheless, the two main principles which ensure that the maximum error bound between $\boldsymbol{L_{i+1}}$ and $\boldsymbol{L^\bullet_{i+1}}$ is kept remain unchanged - during the compression, the residuals are still computed against the target mip-map $\boldsymbol{L_{i+1}}$ after each application of P and all predictions are made from the compressed values which ensures that both the compression and the decompression add the residuals to the same values. Hence, let us explain how these three steps are performed.

When constructing the larger compressed mip-map $\boldsymbol{L^\bullet_{i+1}}$ from $\boldsymbol{L^\bullet_i}$, we can imagine it as every pixel p from $\boldsymbol{L^\bullet_i}$ being substituted by four pixels a, b, c, d in $\boldsymbol{L^\bullet_{i+1}}$ as depicted in Fig. 4.1. This substitution is the exact inverse of the one performed in the first bottom-top pass described in the previous section ???. We will apply the prediction operator and subsequent residuals computation and addition three times in order to compute the values of the four pixels and the residuals needed to reconstruct them from p during the decompression.

In the first of the three steps, we compute the pixels labeled a. To predict them from their corresponding p pixels inside $\boldsymbol{L^\bullet_i}$, we use a simple prediction operator $P_a(\boldsymbol{L^\bullet_i}) = p$. We compute the residuals $\boldsymbol{E_a}$ and $\boldsymbol{E^\bullet_a}$ with respect to the target value a_t in $\boldsymbol{L_{i+1}}$ and then assign a the final value $a\bullet$ (eq. 4.2, recall that Q_D is a uniform quantizer respecting the maximum deviation D: $maxdev(v, Q_D(v)) < D$. It is clear that $maxdev(a\bullet, a_t) \leq \boldsymbol{D}$. The explanation would be the same as in

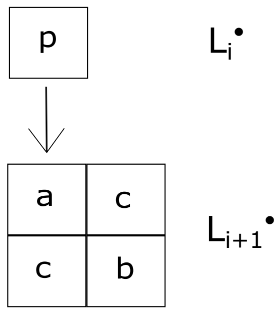


Figure 4.1: Substituting the pixel p in $\boldsymbol{L_i^\bullet}$ with four pixels in $\boldsymbol{L_{i+1}^\bullet}$

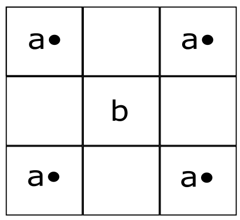


Figure 4.2: The prediction of b - $P_b(\boldsymbol{L_{i+1}^\bullet})$ - is the average of all the displayed $a\bullet$

Chapter 3.

$$\begin{aligned} \boldsymbol{E_a} &= a_t - p \\ \boldsymbol{E_a^\bullet} &= Q_D(\boldsymbol{E_a}) \\ a\bullet &= p + \boldsymbol{E_a^\bullet} \end{aligned} \tag{4.2}$$

In the second of the three steps, we compute the pixels labeled b. We do not predict them from $\boldsymbol{L_i^\bullet}$ anymore, but from the already available pixels $a\bullet$ inside $\boldsymbol{L_{i+1}^\bullet}$. The prediction operator P_b used for this has now the form of a straight-oriented Neville interpolating filter of order 2. All it does is that when it is requested to predict the height at some pixel, it just averages the heights of its certain four neighboring pixels as depicted in Fig. 4.2. It is easy to see that as long as it is requested to predict the height at pixels b, it always averages only the already known $a\bullet$ pixels. This is the same prediction operator as the one used in C-BDAM to predict the heights of the samples located at the border of a LOD hierarchy node. Once the predictions of b pixels are known, we perform an analogic computation of residuals $\boldsymbol{E_b}$ and their quantizations $\boldsymbol{E_b^\bullet}$, again with respect to the corresponding target values b_t in $\boldsymbol{L_{i+1}}$. Finally, we assign b its final value $b\bullet$ (eq. 4.3).

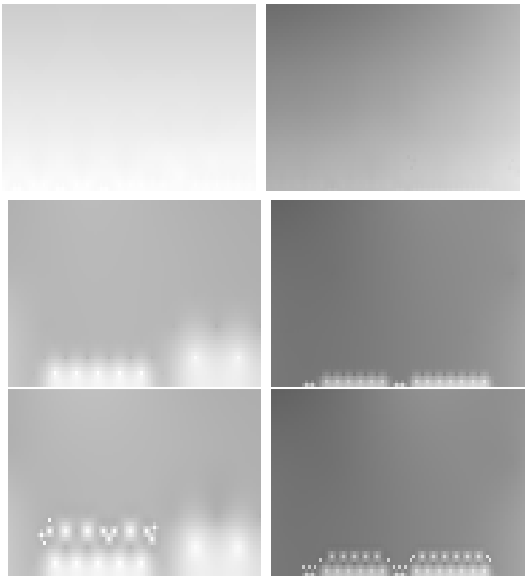
$$\begin{aligned} \boldsymbol{E_b} &= b_t - P_b(\boldsymbol{L_{i+1}^\bullet}) \\ \boldsymbol{E_b^\bullet} &= Q_D(\boldsymbol{E_b}) \\ b\bullet &= P_b(\boldsymbol{L_{i+1}^\bullet}) + \boldsymbol{E_b^\bullet} \end{aligned} \tag{4.3}$$

The reason why C-BDAM uses the order 2 Neville interpolating filter at the borders is that thanks to the way the samples are organized inside a node of its LOD hierarchy, the filter does not pick the samples behind the node's border. We can view the mip-map in our method as an analogy to the node in C-BDAM. However, the spatial organization of mip-map samples in our method differs from the organization of samples inside a LOD node in C-BDAM, so unlike in C-BDAM, in our method it might happen that this interpolating filter comes out of the underlying mip-map. We handle this by only including the valid interior values in the resulting average and completely ignoring the imaginary values behind the mip-map borders. Thus, when computing a certain prediction, we count how many times the filter has hit the interior of mip-map and divide the sum of the valid interior heights with this number of hits. Most of the times, the number of hits will be 4, but it will be 2 at the borders and just 1 at the corners. This way, it is always ensured that the filter does not make any data up, unlike the possible alternative of some mirror extension of data behind the borders. ??? Comparison with mirror extension ???

C-BDAM uses the larger order 4 Neville interpolating filter to predict the heights at the interior of a node of its LOD hierarchy. This filter covers larger area - it samples twelve points instead of four. In addition, it does not compute the average of these points, but their weighted sum. Just like in the case of simple averaging, the sum of the weights is 1. The difference is that the four closest points have a certain positive weight, whereas the remaining eight further points have a different negative weight, the absolute value of which is lower than the first weight (Fig. ???). The property with the lower absolute value indicates that the points which are further affect the result less. The fact that their weights are negative basically means that the valleys and hills are predicted better (Fig. ???).

Unlike C-BDAM, this method uses the smaller order 2 filter even for the interior samples. Let us explain the reasons why we decided to do so. The first reason is the increase of speed. The order 2 filter only averages four values, whereas the order 4 filter averages 12 values. Moreover, the subsequent averaging performed by the order 2 filter can easily be cached during the horizontal traversal which is an additional reduction of the computation overhead (Fig. ???). We also tried using the order 4 filter with various weights settings for the interior values, too. This slightly increased the compression ratio - probably because this filter is better at predicting hills and walleys - but worsened the quality of compression by producing more significant artifacts near smooth terrain's borders (Fig. 4.3) and sharp terrain transitions (Fig. 4.4). The most probable cause of this is that the predictions made by the order 4 filter tend to differ from the neighboring heights more. This emphasises the artifacts.

Generally, the reason why these artifacts occur is that as long as the predictions are close enough to the target mip-map and their quantized residuals are equal to zero, the compressed values might remain above/under the terrain for a long time, but only until one prediction gets a bit further from the target terrain. As soon as it happens, its associated residual will be quantized to a certain non-zero value which will result in the reconstructed value being flipped to the opposite side of the real terrain which produces a visual artifact. It is not a coincidence that this often occurs near a sharp change in the terrain. The predictions produced by the averaging filter get a bit different from the adjacent ones near this change, because at these places, the filter reaches out to the area behind the change (Fig. 4.5, 4.6). This difference might then cause the difference



Original

Order 2

Order 4

Figure 4.3: Two examples of the difference between artifacts caused by order 2 and order 4 filters near smooth terrain's border - in the first row there are the target heightmaps, in the second, there are the same heightmaps compressed using the order 2 filter, in the third row, the heightmaps compressed with the order 4 filter.

in residuals - the quantized residuals further from this change might be all zeroes, whereas the residual near this change not, causing a spike to occur. This spike will then get propagated to the following compressed mip-map levels. The only thing that is guaranteed is that the maximum error bound is still satisfied. ???(musim overit, clipping je nasadeny len v Bohemke, ale je to lepsi napad ako mirroring, tusim, ze to aj zlepsilo kompresny pomer, no nezistoval som pri nom, ake su artefakty)The clipping performed by the predicting filter near the mip-map borders creates the effect similar to a sharp terrain change, too, in a bit different way - by the sole fact that the terrain behind the border no longer follows its trend up to the border (rising, for example), but is practically mirrored behind the border (following the example, falling), because instead of reaching out to the non-existing values out of the mip-map, the existing ones are used.???

(potial iba)

The remaining pixels labeled c are predicted from the pixels $a\bullet$ and $b\bullet$ in $\boldsymbol{L^\bullet_{i+1}}$ by the diagonally-oriented order 2 Neville interpolating filter (fig. 4.7). The computation of residuals $\boldsymbol{E_c}$ and $\boldsymbol{E^\bullet_c}$ then follows according to the target value c_t from $\boldsymbol{L_{i+1}}$ and c is assigned the final value $c\bullet$ (eq. 4.3).

$$\begin{aligned} \boldsymbol{E_c} &= c_t - P_c(\boldsymbol{L^\bullet_{i+1}}) \\ \boldsymbol{E^\bullet_c} &= Q_D(\boldsymbol{E_c}) \\ c\bullet &= P_c(\boldsymbol{L^\bullet_{i+1}}) + \boldsymbol{E^\bullet_c} \end{aligned} \tag{4.4}$$

The cases when the filter comes out of the image are handled by a specific mirror extension (fig. 4.8). For the same reasons as in the prediction of b pixels, the order 2 filter is used for the prediction of all c pixels - both interior and

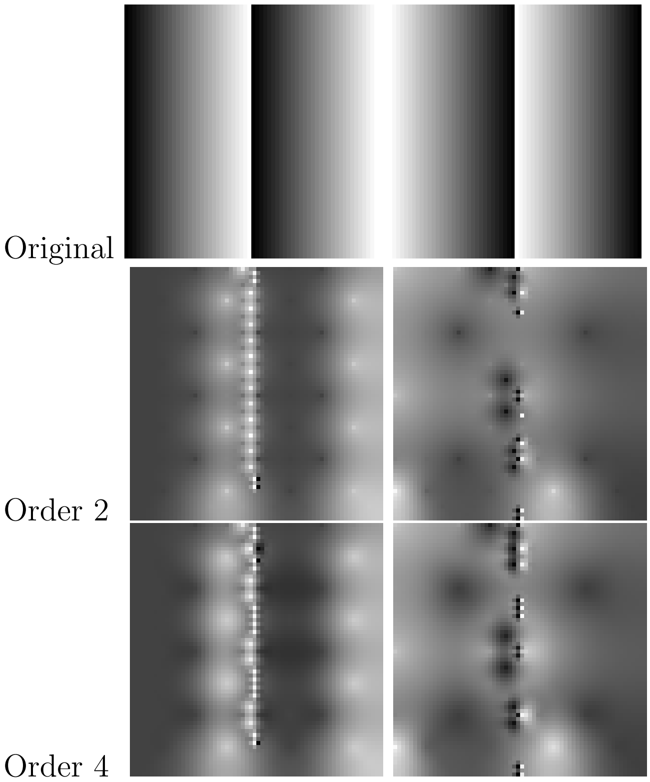


Figure 4.4: Two examples of the difference between artifacts caused by order 2 and order 4 filters near a sharp terrain change - in the first row there are the target heightmaps, in the second row, the same heightmaps compressed using the order 2 filter, in the third row, the heightmaps compressed with the order 4 filter. The span of the values in the original images is from 0 to 16 and the maximum absolute deviation (D) of compression is set to 9.

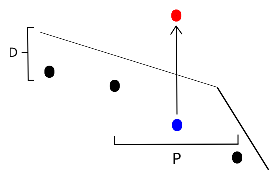


Figure 4.5: The illustration of how an artifact occurs - the black predictions are within the maximum-error bound D, so they are equal to the reconstructed values, but the blue one is not. Because a uniform quantizer is used, the blue prediction is shifted by $2D - 1$ to the top, creating an artifact.

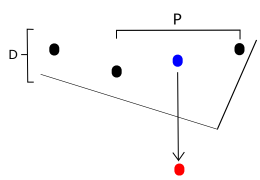


Figure 4.6: Another illustration of an artifact - the black predictions are within the maximum-error bound D, but the blue one is not. The blue prediction is shifted by $2D - 1$ to the bottom, creating an artifact.

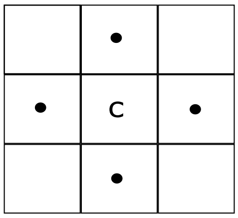


Figure 4.7: The prediction of c - $P_c(\boldsymbol{L^\bullet_{i+1}})$ - is the average of all the pixels marked with a dot - $\bullet$.

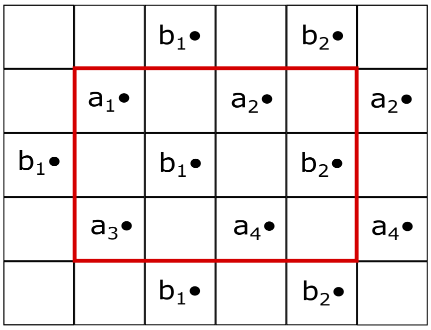


Figure 4.8: Handling of border cases in the computation of $P_c(\boldsymbol{L^\bullet_{i+1}})$ - the red line represents the border.

exterior ones. Similarly, the interpolation with such filter can be cached during diagonal traversal.

The residuals $\boldsymbol{E^\bullet_a}$, $\boldsymbol{E^\bullet_b}$ and $\boldsymbol{E^\bullet_c}$ are then encoded by an entropy codec and stored. The decompression is done in a similar manner with the only difference that the residuals are not computed anymore, but just decoded and read. So, we substitute every pixel from $\boldsymbol{L^\bullet_i}$ by four pixels in $\boldsymbol{L^\bullet_{i+1}}$, the value of which is computed in three passes of prediction followed by adding the read residual (the last lines of eq. 4.2, 4.3, 4.4).

5. Functional comparison to C-BDAM and wavelets

As we already mentioned, C-BDAM contains the whole rendering pipeline, whereas our method does not. However, it can be compared to C-BDAM in terms of how lifting is performed. As we already mentioned in the end of Section 2.1, C-BDAM omits a half of the samples while constructing a coarser LOD, whereas our method omits three fourths of the samples. This is spatially equivalent to two steps of lifting in C-BDAM (Fig. 2.1). The first step removes the pixels b and the second step removes the pixels c as seen in Fig. 4.1. Nevertheless, this equality is only spatial.

In our method, an analogy of the update operator of lifting is used to construct $\boldsymbol{L_i}$ from $\boldsymbol{L_{i+1}}$ (the averaging of four neighboring pixels - Sec. **??**). However, the lifting is not complete in our method as it does not contain the prediction operator - no residuals are computed there yet. In C-BDAM, also a prediction operator is used in the lifting to produce intermediate residuals. However, using just these residuals would not guarantee any maximum error bound, so C-BDAM makes another top-bottom pass to correct the residuals against the real values of samples produced in the first bottom-top pass. To make this correction fit into the original wavelet framework, several intricate computations need to be performed, including division, which is quite a large performance hit.

Our vision was that once it is needed to perform an extra top-bottom pass to correct the residuals so that the maximum error bound is guaranteed, it is not neccessary to compute any temporary values of the residuals during the lifting steps (the construction of the LOD pyramid). This is why we perform just an analogy of the update (the averaging of pixels) in the update-first scheme and let the following top-bottom pass compute suitable values of residuals. This is obviosly a major deviation from the wavelet scheme. In the top-bottom pass, we just predict the values in the finer LOD as accurately as possible, but these predictions have no linkage to the previous bottom-top pass, as they have not been used there at all. Then we directly compute the residuals with respect to the original values computed in the first bottom-top pass at the corresponding levels.

All in all, it can be said that the way the residuals are computed in this method is an extreme simplification of the way they are computed in C-BDAM. This way of computation does not even conform to the second-generation wavelet scheme - the lifting is not complete and the reconstruction is not the inverse of lifting. We think that without the residuals quantization or the per-level correction of residuals, respecting the wavelet scheme makes sense, as it ensures computational equivalency with the first-generation wavelets. However, in case the residuals need to be corrected at each level, we think that conforming to a wavelet scheme makes no sense, because this correction immediatelly destroys the mentioned equivalence - once a residual is cropped in order to get the resulting value closer to the actual data, it cannot be said that any of the following reconstruction is the inverse to the lifting performed before. Moreover, thanks to the mentioned deviation of C-BDAM from the classical update-first second-generation wavelet discussed in

Section 2.1, we question its computational equivalency with the first-generation wavelet even with no residuals quantization or cropping performed. Because of this, we think that the computations made in the second top-bottom pass can be optimized this way without any cost. Thus, this method would probably better be called wavelet-inspired than wavelet-based.

6. Results

This method has been applied in the real-time planet renderer mentioned in the introduction on height data of the whole Earth with the resolution of 90m (SRTM). Due to the redundancy of data in the applied LOD hierarchy, the size of the original data was 260GB. With the maximum error bound set to 5m, the size of the compressed data is 7GB, which yields the compression ratio of 37:1.

For a comparison, C-BDAM reached the compression ratio of 64:1 on the same dataset, but with the maximum error bound set to 16m. Thanks to the fact that the LOD hierarchy of C-BDAM contains no redundancy, the size of the original data was just 29GB and the size of the compressed data just 870MB. Under such circumstances, only the comparison in terms of the compression ratio is relevant.

In Fig. 6.1, you can see a part of a heightmap compressed by this method, together with the differences from the original.

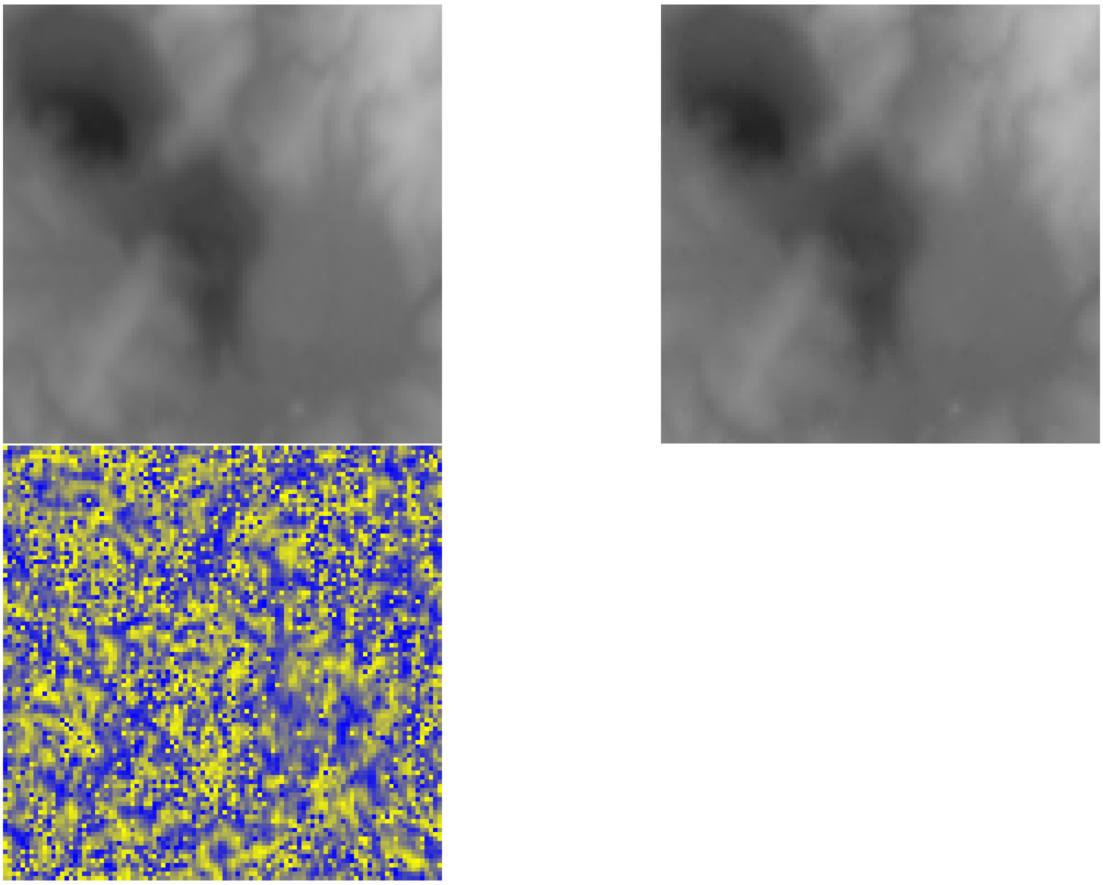


Figure 6.1: From the top to the bottom - the original terrain, the same terrain compressed with the maximum deviation of 5m, the difference between these two. The brighter the color, the greater the value. In the difference image, the yellow color means 4.5m, whereas the blue color means -4.5m.

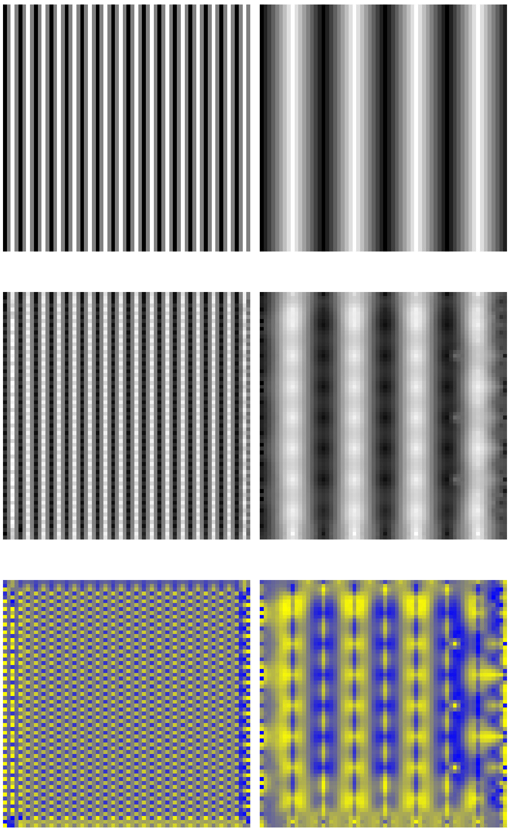


Figure 6.2: Two synthetic test images of size 64x64, each one containing spiky terrain with the heights ranging from -16 to 16. On the left, the longitude of spikes is 4, on the right, it is 16. From the top to the bottom - the original, compressed with the maximum deviation of 5, the difference between these two. The brighter the color, the greater the value. In the difference image, the yellow color means 4.5, whereas the blue color means -4.5.

7. Conclusion

In this paper, we described a heightmap compression method designed to be a plugin into an existing real-time planet renderer with its own rendering pipeline. The method proved to be convenient for the purpose, providing fast decompression (only about 1ms per block of data). Its compression ratio is comparable to C-BDAM, which is the method with the best compression ratio among the methods for the terrain compression, which guarantee a maximum error bound adjustable by the user.

Conclusion

[5]

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List of Tables

List of Abbreviations

Attachments