HW 3

2021 Q1 - 30 pts

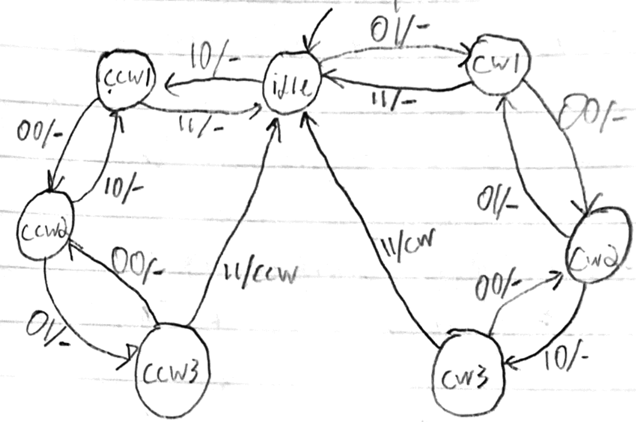
Original:

	In=0	In=1
a	b, 1	c, 0
b	f, 0	d, 0
c	a, 0	c, 0
d	c, 1	e, 0
e	a, 0	c, 0
f	b, 0	d, 0

Minimal:

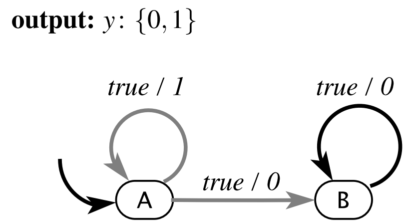
	In=0	In=1
a	b, 1	c, 0
b	b, 0	d, 0
c	a, 0	c, 0
d	c, 1	c, 0

2021 Q2 - 30 pts



This works for both a & b

3. Consider the following state machine:



Determine whether the following statement is true or false, and give a supporting argument:

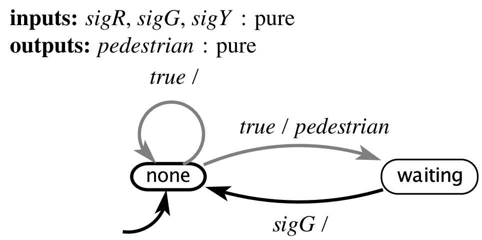
> The output will eventually be a constant 0, or it will eventually be a constant 1. That is, for some $n \in \mathbb{N}$, after the n-th reaction, either the output will be 0 in every subsequent reaction, or it will be 1 in every subsequent reaction.

Note that Chapter 13 gives mechanisms for making such statements precise and for reasoning about them.

Solution: TRUE. In an infinite execution, if the transition from A to B is ever taken, then after that point, the output will always be 0. If that transition is never taken, then the output will be a constant 1 for the entire execution. This too is allowed behavior for the state machine.

6. This problem considers variants of the FSM in Figure 3.11, which models arrivals of pedestrians at a crosswalk. We assume that the traffic light at the crosswalk is controlled by the FSM in Figure 3.10. In all cases, assume a time triggered model, where both the pedestrian model and the traffic light model react once per second. Assume further that in each reaction, each machine sees as inputs the output produced by the other machine *in the same reaction* (this form of composition, which is called synchronous composition, is studied further in Chapter 6).

   (a) Suppose that instead of Figure 3.11, we use the following FSM to model the arrival of pedestrians:

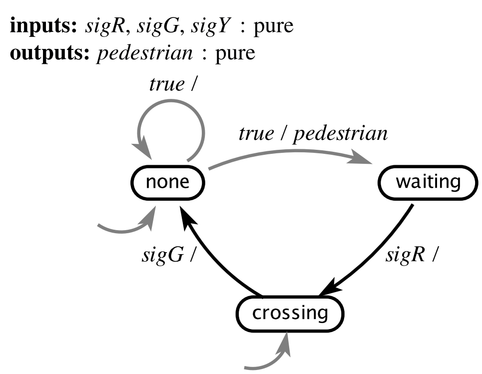
   

   Find a trace whereby a pedestrian arrives (the above machine transitions to waiting) but the pedestrian is never allowed to cross. That is, at no time after the pedestrian arrives is the traffic light in state red.

   Solution: The traffic light begins in state red while the pedestrian model begins in state none. Suppose that the pedestrian model transitions to waiting in exactly the same reaction where the traffic light transitions to state green. The system will now perpetually remain in the same state, where the pedestrian model is in waiting and the traffic light is in state green.

   Put another way, in the same reaction, we get red $\rightarrow$ green, which emits *sigG*, and none $\rightarrow$ waiting, which emits *pedestrian*. Once the composition is in state (green, none), all remaining reactions are stuttering transitions.

   (b) Suppose that instead of Figure 3.11, we use the following FSM to model the arrival of pedestrians:

   Here, the initial state is nondeterministically chosen to be one of none or crossing. Find a trace whereby a pedestrian arrives (the above machine transitions from none to waiting) but the pedestrian is never allowed to cross. That is, at no time after the pedestrian arrives is the traffic light in state red.

   Solution: Suppose the initial state is chosen to be (red, none) and sometime in the first 60 reactions transitions to (red, waiting). Then eventually the composite machine will transition to (green, waiting), after which all reactions will stutter.

7. Consider the state machine in Figure 3.2. State whether each of the following is a behavior for this machine. In each of the following, the ellipsis "$\cdots$" means that the last symbol is repeated forever. Also, for readability, *absent* is denoted by the shorthand a and *present* by the shorthand p.

   (a) $x = (p, p, p, p, p, \cdots), \quad y = (0, 1, 1, 0, 0, \cdots)$
   (b) $x = (p, p, p, p, p, \cdots), \quad y = (0, 1, 1, 0, a, \cdots)$
   (c) $x = (a, p, a, p, a, \cdots), \quad y = (a, 1, a, 0, a, \cdots)$
   (d) $x = (p, p, p, p, p, \cdots), \quad y = (0, 0, a, a, a, \cdots)$
   (e) $x = (p, p, p, p, p, \cdots), \quad y = (0, a, 0, a, a, \cdots)$

Solution:

(a) no
(b) yes
(c) no
(d) yes
(e) no

input: x: pure
output: y: $\{0, 1\}$

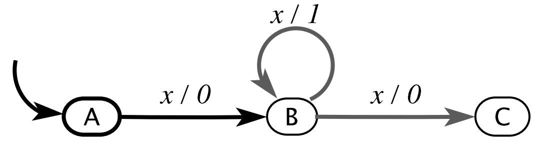


Figure 3.2: State machine for Exercise 7.