UCSB ECE 594BB – Winter 2023

Assignment 1: Software Simulation of Leaky Integrate-and-Fire Model

Total Points: 100

Due at 5pm, Friday February 3rd (no late submission)

Submit your solutions in PDF via Gauchospace

1 Background

In this assignment, you will build a model of LIF (Leaky Integrated-and-Fire) using software simulation based on Python, MATLAB, C, or any programing language of your choice.

The LIF neuron model is based on simplified modeling of extra/intracellular membrane behavior, and can be represented mathematically as below:

$$\tau \frac{dV}{dt} = RI(t) - (V(t) - V_{rest})$$

where τ is the time-constant (which is equal to RC), V is the membrane potential, V_{rest} is the resting membrane potential.

The input current *I(t)* is generated by the pre-synaptic neurons and can be defined in several ways, e.g. the 0th-order synaptic model, 1st-order synaptic model, etc:

$$\text{0th-order synaptic model:} \quad I_i(t) = \sum_j w_{ji} \cdot s_j(t) \qquad (1)$$

$$\text{1st-order synaptic model:} \quad I_i(t) = \sum_j w_{ji} \sum_f \alpha(t - t_j^{(f)}) \qquad (2)$$

In this assignment, we will only use the simple 0th-order synaptic model as represented above. In (1), w_{ji} is the synaptic weight between neurons *j* (pre-synaptic neuron) and *i* (post-synaptic neuron), $S_j(t)$ is the spike train of neuron *j*: $S_j(t) = \sum_f \delta(t - t_j^{(f)})$ where $t_j^{(f)}$ is a firing time of neuron *j*, and δ is the Dirac delta function. When $V(t) \geq V_\theta$(threshold), *V(t)* becomes V_{rest} and there will be an output spike.

To be more specific, in a multi-layered spiking neural network, e.g., a two-layer model in Figure 3, the current received by neurons after the first layer can be modeled by the 0th-order synaptic model specified in (1).

NOTE: For the rest of the homework, when modeling the output spike train $\delta(t - t^{(f)})$ for t he calculation of equation(1), the unit of $\delta(\cdot)$ is mV.

2 Problem and Credit Breakdown

2.1 Problem 1 (50 points)

Suppose that there is only a single neuron as shown in Fig. 1 with the following settings:

V_{rest} = -65mV, R=10^6 (Ohms), τ = 50ms;

time stepsize = 0.1ms, and total number of time steps = 20,000
(i.e. $t_n - t_{n-1}$ = 0.0001, and $0 \leq \mathrm{n} \leq 20{,}000$).

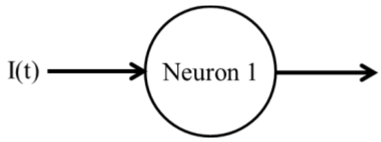

Fig. 1. A single LIF neuron with a current stimulus.

The total input current $I(t)$ is given directly as:

$$\mathrm{I(t)}\ (mA) = \begin{cases} 0 & (\text{if timestep } \leq 5000 \text{ or timestep } > 15000) \\ 0.00005 & (if\ 5000 < timestep \leq 15000) \end{cases}$$

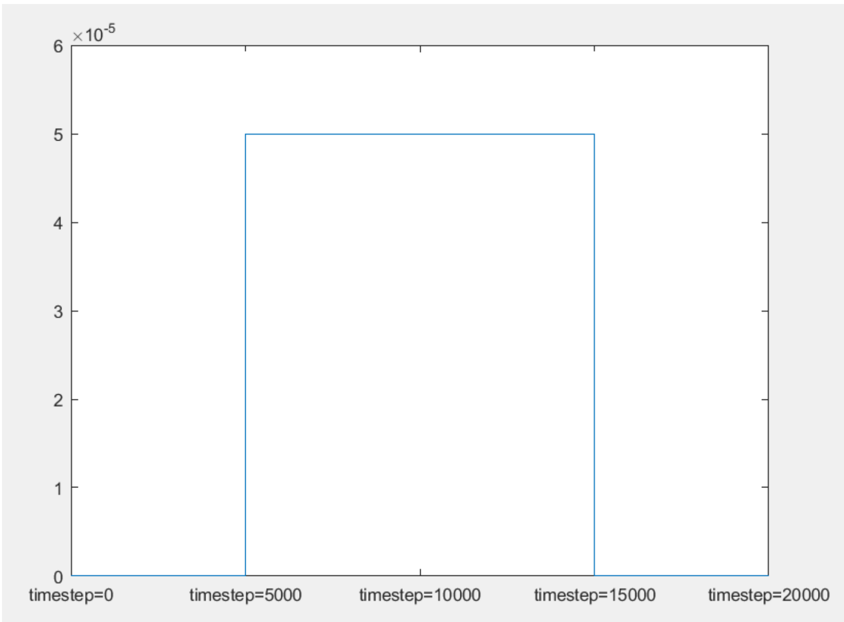

Fig. 2. The specified current stimulus *I(t)*.

Part 1) (25 points) with V_θ = -20mV:

(1) Simulate the LIF neuron and plot the waveform of *V(t)*.

(2) How many times does the neuron fires?

Part 2) (25 points) with V_θ = -30mV:

(1) Simulate the LIF neuron and plot the waveform of *V(t)*.

(2) How many times does the neuron fires?

2.2 Problem 2 (50 points)

Consider the spiking neural network of five interconnected LIF neurons as shown in Fig. 3.

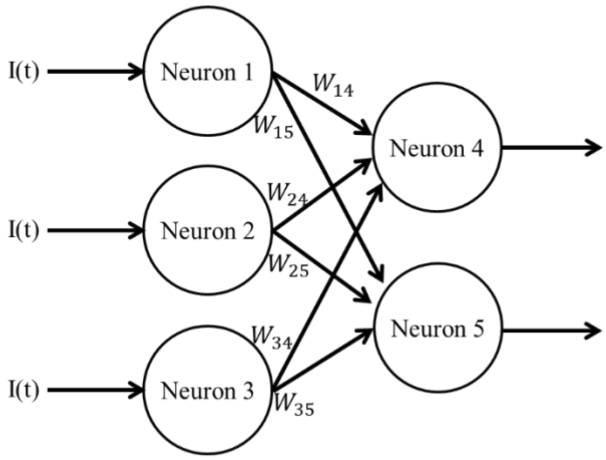

Fig. 3. A small spiking neural network.

Neurons 1 to 3 each receive an external current input is *I(t)*, which is same as we used for problem 1:

$$\mathrm{I(t)}\ (mA) = \begin{cases} 0 & (\text{if timestep } \leq 5000 \text{ or timestep } > 15000) \\ 0.00005 & (if\ 5000\ < timestep \leq 15000) \end{cases}$$

Again, we the following setting: time stepsize = 0.1ms, and total number of time steps = 20,000. Furthermore, we have:

Neurons 1~3, V_{rest} = -65mV, R=10^6 (Ohms), τ = 0.05(s), $V_\theta = -35mV$

Neurons 4 and 5, V_{rest} = -65mV, R=10^7 (Ohms), τ = 0.02(s), $V_\theta = -60mV$

and $w_{14} = -0.00001,\ w_{24} = 0.0001,\ w_{34} = 0.00001,\ w_{15} = 0.00003,\ w_{25} = -0.00004,\ w_{35} = 0.00005$, where the unit for the synaptic weights is mA/mV.

Part 1) (25 points)

(1) Simulate the SNN and generate the waveform of *V(t)* for Neuron 1, Neuron 4, and Neuron 5.

(2) Report the number of firing times for Neuron 1, 4 and 5.

Part 2) (25 points) Now change the synaptic weights to the following values and repeat part 1):

$w_{14} = 0.00001,\ w_{24} = 0.00001,\ w_{34} = 0.00001,\ w_{15} = 0.000005,\ w_{25} = 0.000005,\ w_{35} = -0.00001$, where the unit for the synaptic weights is mA/mV.

(1) Simulate the SNN and generate the waveform of *V(t)* for Neuron 1, Neuron 4, and Neuron 5.

(2) Report the number of firing times for Neuron 1, 4 and 5.