

Final Sheet

December 2021

1 Data

1.a Types of Variables

Qualitative/Categorical

- Outcomes fall into different categories
- Categories can be ordered

Quantitative

- Measured on a numeric scale

1.b Summarizing Data Visually

Qualitative/Categorical Data

- Frequency tables - displays all categories of a single categorical variable with associated frequencies
- Contingency tables - display two categorical variables simultaneously
- Marginal distributions - display distribution of one of the two variables only
- Conditional distributions - display distribution of one variable, satisfying a condition of the other variable
- Bar charts
- Pie charts

Quantitative Data

- **Graphically**
 - Histogram
 - Stem-and-leaf displays
 - Boxplots
- **Shape of the Distribution**
 - Modality (number of peaks):
 - * unimodal
 - * bimodal
 - * multimodal
 - Symmetry of distribution:
 - * unimodal

- * skewed to right (long right tail)
 - * skewed to left (long left tail)
- Presence of outliers
- **Numerically**
 - Measures of center:
 - * mean
 - * median
 - Measures of spread:
 - * variance: $s^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1}$
 - * standard deviation: $s = \sqrt{\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1}}$
 - * interquartile range $IQR = Q3 - Q1$
 - Percentiles (also called quantiles)
 - 5-number summary:
 - * minimum
 - * first quartile (Q1)
 - * second quartile (Q2)
 - * third quartile (Q3)
 - * maximum
- **Sensitivity to Outliers**
 - Sensitive to outliers:
 - * mean
 - * range, variance, standard deviation
 - Not sensitive to outliers
 - * median
 - * IQR

2 Normal Distribution

Characteristics of the Normal Model

- bell-shaped; unimodal
- perfectly symmetric about the mean
- spread of distribution determined by value of standard deviation
- mean μ and the standard deviation σ are parameters (numerical characteristics of a model)
- mean \bar{y} and standard deviation s are statistics (numerical characteristics of a sample)

The 68-95-99.7 Rule

- 68% of data falls within 1 σ of μ
- 95% of data falls within 2 σ of μ
- 99.7% of data falls within 3 σ of μ

Finding Areas Under the Normal Model

Algorithm

- Identify the:
 - μ - mean of the model
 - σ - standard deviation of the model
 - y - observed value
- Construct the normal model: $N(\mu, \sigma)$
- Calculate the z-score (z): $z = \frac{y - \mu}{\sigma}$
- Using R compute the p-value:
 - Area below y : `pnorm(z)`
 - Area above y : `pnorm(z, lower.tail = F)`
 - Area in between y_1 and y_2 (where $y_1 > y_2$): `pnorm(z_1) - pnorm(z_2)`
- **Finding Z-Score from the Area Under the Normal Model**
 - Area above unknown y : `qnorm(p, lower.tail = F)`
 - Area below unknown y : `qnorm(p)`

3 Binomial Distribution

4 Correlation and Association

Scatterplots

- Direction:
 - Positive (x and y) values tend to go in the same direction)
 - Negative (x and y values tend to go in the opposite direction)
- Form:
 - Linear
 - Non-linear
- Point relationship:
 - Strong relationship between points
 - weak or no relationship between points (randomly scattered)
- Outliers

Correlation (r)

- Positive correlation: large x values are linearly associated with large y values (r is positive)
- Negative correlation: large x values are linearly associated with small y values (r is negative)
- r has a value between 1 and -1, and has no units
- $r = \frac{\sum z_x * z_y}{n-1}$

Association vs Causality

- Association does not imply causation. There may be a lurking variable

5 Regression Analysis

The Regression Line

- Equation for regression line: $\hat{y} = intercept + (slope * x)$
- Equation for slope: $slope = r * \frac{s_y}{s_x}$
(where s_y and s_x are the standard deviations of y and x respectively)
- Equation for intercept: $intercept = \bar{y} - (slope * \bar{x})$
(where \bar{y} and \bar{x} are the mean y and x values respectively)

The Residuals

- The residual (e) is the difference between observed value y and the predicted value \hat{y} . Therefore:
 $e = y$ (from data) - \hat{y} (from model)
- The sum of residuals is equal to zero
- Linear model is obtained by minimizing the sum of the squared residuals. Therefore, also referred to as the least squares regression line
- To assess appropriateness of regression model, we use the residual plot (plots residuals against explanatory variable data). If plot shows no pattern, model is appropriate.

6 Experiments and Observational Studies

Types of Studies

- Observational Studies
 - Investigators have no control over either variable
 - No deliberate human intervention
 - Retrospective study: based on information from events that have taken place in the past
 - Prospective study: data and information is gathered in real time
- Experiments
 - Involves planned intervention on the exposure to a condition suspected of altering the response outcome
 - Most often control group(s) will be used

Randomized, Comparative Experiments

- Involves assessing the effect of an explanatory variable, called a factor, on a response variable
- Compares the response variable between different levels of the factor
- Experimenters control what type of treatment individuals receive, the treatment assignment is random
- Participants referred to as subjects or experimental units
- The treatment a subject receives will be a combination of the levels from different factors

Principles of Experimental Design

- Randomize
 - Treatments are randomly assigned to subjects

- Replicate
 - Comparison between different treatment groups will not be reliable unless more individuals receive each treatment
- Blocking
 - May be beneficial to control for variables that are not factors but are believed to have some influence on the response variable
 - Subjects are divided into blocks (ex. male and female groups). Treatment assignment and comparisons are done within each block separately

Blinding and Placebo

- Single Blind: either the subjects or the evaluators are blinded as to treatment assignment
- Double Blind: neither the subjects nor the evaluators knows the treatment assignments
- Blinding is usually done using a placebo which is designed to look like the treatment but has no real treatment value

7 Types of Sampling

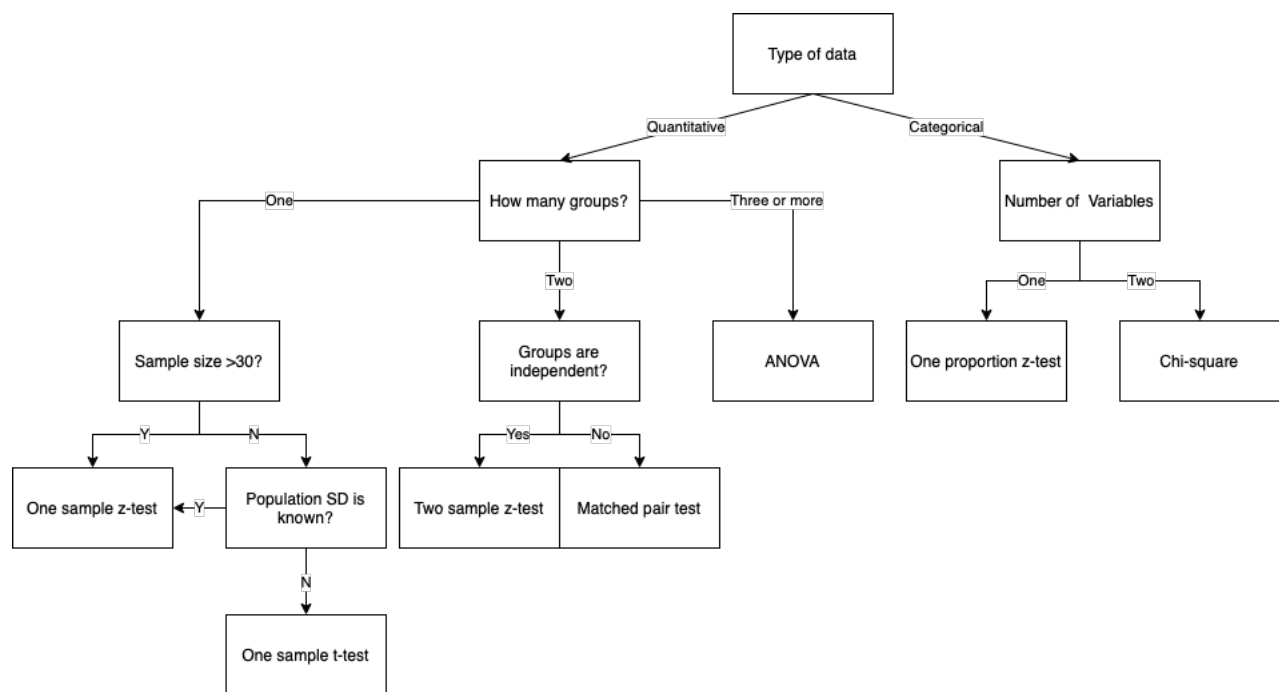
Sampling Methods

- Simple Random Sampling
 - Consists of n individuals sampled at random from the population
 - Each individual has an equal chance of being selected
 - Each possible sample size n is equally likely
- Stratified Sampling
 - Population is divided into strata (a stratum is a subset of the population that shares a particular characteristic)
 - Simple random sample is drawn from each stratum
 - Stratified sample has smaller variability across samples and hence give more reliable results
- Cluster Sampling
 - Can be used when natural groups in a population exist
 - Population is divided into those groups/clusters
 - Simple random sample from all clusters is obtained
 - If all individuals in a selected cluster are included, final sample is a one-stage cluster sample
 - If additional simple random sample is drawn from selected clusters, final sample is a two-stage cluster sample
 - This method is used for the sake of convenience, practicality, and cost-efficiency
- Multistage Sampling
 - Involves more than one stage or more than one sampling procedure in obtaining a sample
- Systematic Sampling
 - Obtained by selecting every k th individual from the sampling frame
 - Method can be used as long as list being sampled from does not contain a hidden order

Bad Sampling Procedures and Biases

- Undercoverage
 - When sampling frame or sampling procedure excludes or under-represents certain types of individuals from the population
- Convenience Sampling
 - Selecting individuals from a population based on availability and access
- Voluntary Response Bias
 - If responses are voluntary, those with strong opinions tend to be over-represented
- Non-response Bias
 - Individuals who do not respond in a survey might differ from the respondents in certain aspects
 - Including only the respondents in a sample will result in non-response bias
- Response Bias
 - Subject's response is influenced by how the question was phrased or asked, or due to misunderstanding of a question, or unwillingness to disclose the truth

8 Hypothesis Testing



8.a One sample z-test

Algorithm

- Identify parameter of interest. Find the null and alternative hypotheses.
- s - The standard deviation of the sample.
- n - The sample size.
- μ - Hypothesised population mean.
- $SE(\bar{y}) = \frac{s}{\sqrt{n}}$ - Standard error of the statistic.

- Construct the null-model: $\mathbf{N}(\mu, \frac{s}{\sqrt{n}})$
- Find the test-statistic(t): $\mathbf{Z} = \frac{x-\mu}{\mathbf{SE}(\bar{y})}$
- Using R compute the p-value:
 - One-sided hypothesis : `pnorm(t)`
 - Two-sided hypothesis : `2 * pnorm(t)`
- If the p-value is less than α - reject the null-hypothesis. Otherwise, you fail to reject the null-hypothesis.

8.b One proportion z-test

Algorithm

- Identify parameter of interest. Find the null and alternative hypotheses.
- n - The sample size.
- p_0 - Hypothesized proportion.
- $\mathbf{SD} = \sqrt{\frac{p_0(1-p_0)}{n}}$ - Standard error of the statistic.
- Construct the null-model: $\mathbf{N}(p_0, \sqrt{\frac{p_0(1-p_0)}{n}})$
- Find the test-statistic(t): $\mathbf{Z} = \frac{x-p_0}{\mathbf{SD}}$
- Using R compute the p-value:
 - One-sided hypothesis : `pnorm(t)`
 - Two-sided hypothesis : `2 * pnorm(t)`
- If the p-value is less than α - reject the null-hypothesis. Otherwise, you fail to reject the null-hypothesis.

8.c Two sample z-test

Algorithm

- Identify parameter of interest. Find the null and alternative hypotheses.
- s - The standard deviation of the sample.
- n - The sample size.
- μ - Hypothesized population mean.
- $\mathbf{SD}(\bar{y}_1 - \bar{y}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ - Standard error of the statistic.
- Construct the null-model: $\mathbf{N}(\mu, \frac{s}{\sqrt{n}})$
- Find the test-statistic(t): $\mathbf{Z} = \frac{x-p_0}{\mathbf{SE}(\bar{y})}$
- Using R compute the p-value:
 - One-sided hypothesis : `pnorm(t)`
 - Two-sided hypothesis : `2 * pnorm(t)`
- If the p-value is less than α - reject the null-hypothesis. Otherwise, you fail to reject the null-hypothesis.

8.d Matched pair

8.e One sample t-test

8.f ANOVA