COMENIUS UNIVERSITY IN BRATISLAVA
FACULTY OF MATHEMATICS, PHYSICS AND INFORMATICS
DEPARTMENT OF APPLIED INFORMATICS

Neural Networks

Project 2c – Hopfield network

April 15, 2025

OVERVIEW

Task: Implement a deterministic synchronous Hopfield network (wiki), then examine its ability to store and recall the following 5x7 patterns[1] (letters X, H, O and Z, drawn here with . and # instead of -1 and +1 for legibility)

```
.#...#.    .#...#.    ..###..    .#####.
..#.#..    .#...#.    .#...#.    ....#..
...#...    .#####.    .#...#.    ...#...
..#.#..    .#...#.    .#...#.    ..#....
.#...#.    .#...#.    ..###..    .#####.
```

Deadline: May 11, 23:59
Late submissions are penalized by -2 points each day. **It is not possible to submit a project more than 5 days after the deadline.**

REPORT

- test the noise-correcting recall ability of the network – for every input pattern:
  - corrupt the input with different amounts of discrete noise $k \in \{0, 7, 14, 21\}$
    - select k random pixel positions and flip them ($-1 \leftrightarrow +1$)
  - let the network relax until reaching a fixed point or a limit cycle
  - for each step, plot:
    - the amount of overlap[2] of the current configuration with each stored pattern
    - the energy[3] of the current configuration

[1] do not remove the empty columns to form a 5x5 shape – the recall will not work
[2] the percentage of neurons sharing its value (± 1) with the corresponding one in a pattern
[3] for some configuration $\boldsymbol{x}$ (activations of all the neurons at a specific time), the energy is: $E(\boldsymbol{x}) = -\frac{1}{2}\sum_i \sum_j (W_{i,j} \cdot x_i \cdot x_j)$

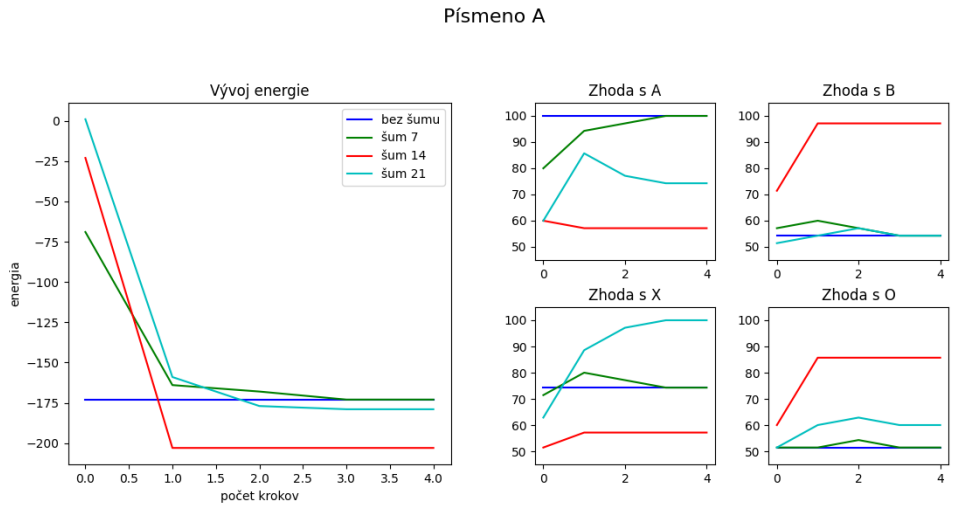
- investigate the dynamics of the network - for ≥ 10000 random input patterns[4]:
  - count the number of inputs leading to:
    - *true attractors* – stable states identical to input patterns (or negatives)
    - *false attractors* – stable states not matching any input pattern
    - *limit cycles* – periodic trajectories
  - plot the 10 most frequent final states or cycles
    - merge duplicates that differ only in the sign or in the phase of the cycle

EXAMPLES

These examples are from a network trained on a different dataset - letters A, B, X, O

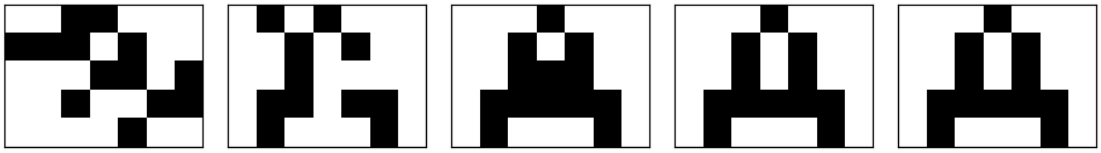
```
...#...    .####..    .#...#.    ..###..
..#.#..    .#...#.    ..#.#..    .#...#.
..#.#..    .####..    ...#...    .#...#.
.#####.    .#...#.    ..#.#..    .#...#.
.#...#.    .####..    .#...#.    ..###..
```

Noise-correcting recall ability:



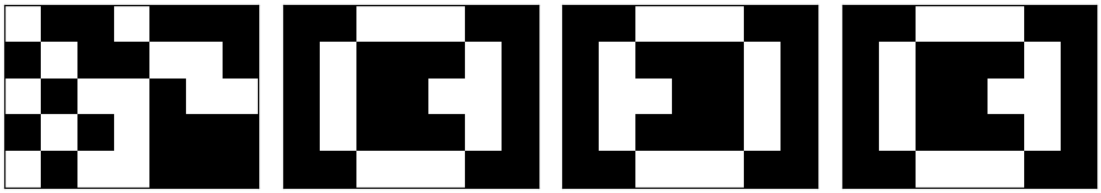
[4] this should not take very long, a few seconds at most

Correct recall - true attractor:



Incorrect recall - false attractor:

Incorrect recall - limit cycle:



BONUS

Examine the effect of storing more 5x7 patterns in the network (choose and draw some letters). Evaluate how the success rate of recall[5] depends on the number of stored patterns and for different amounts of added noise.

[5] the percentage of attempts, where the network converged to the original pattern (without noise)