Neural Networks

2nd Assignment, Hopfield Network, May 2025

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Results

The goal of this project was to implement a synchronous deterministic Hopfield network trained on four letters: *X, H, O, Z*.

Below are the results for all four letters. The graph shows the energy convergence and the overlap with each letter. Some curves terminate early because the states either reached a fixed point or entered a limit cycle. The network successfully copes with noise level $k = 7$, always converging to the correct state. At noise level $k = 14$, achieving 100% recall is not guaranteed, but it remains possible. The network fails to recall the correct pattern at noise level $k = 21$, which corresponds to more than half of the elements being corrupted.

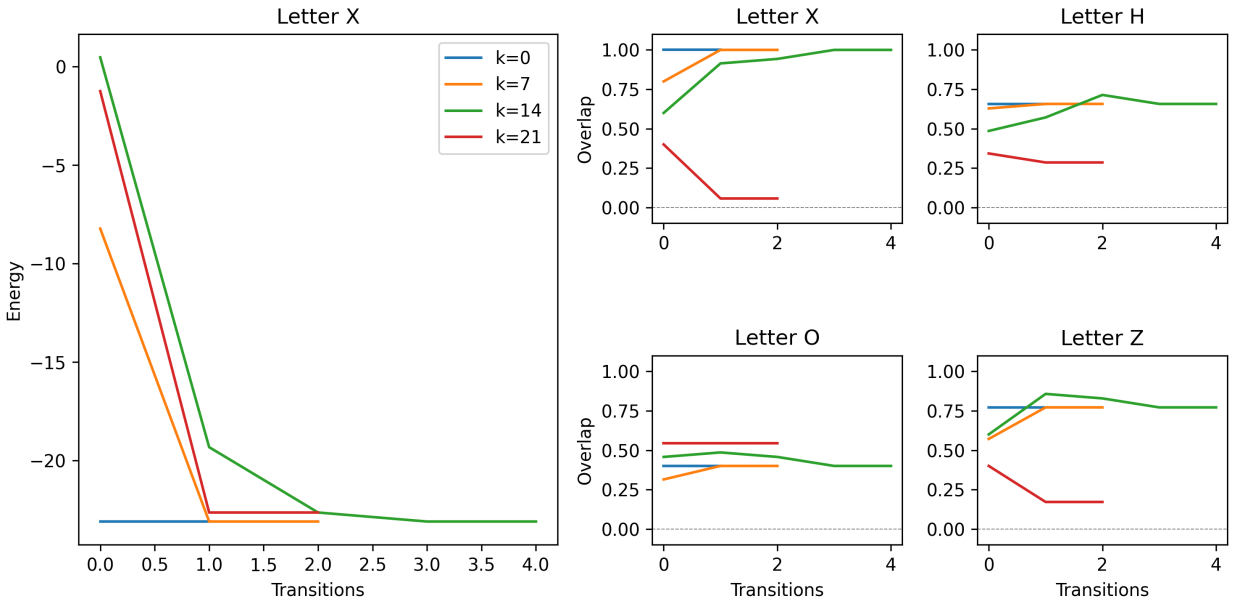

Figure 1: Results for letter X

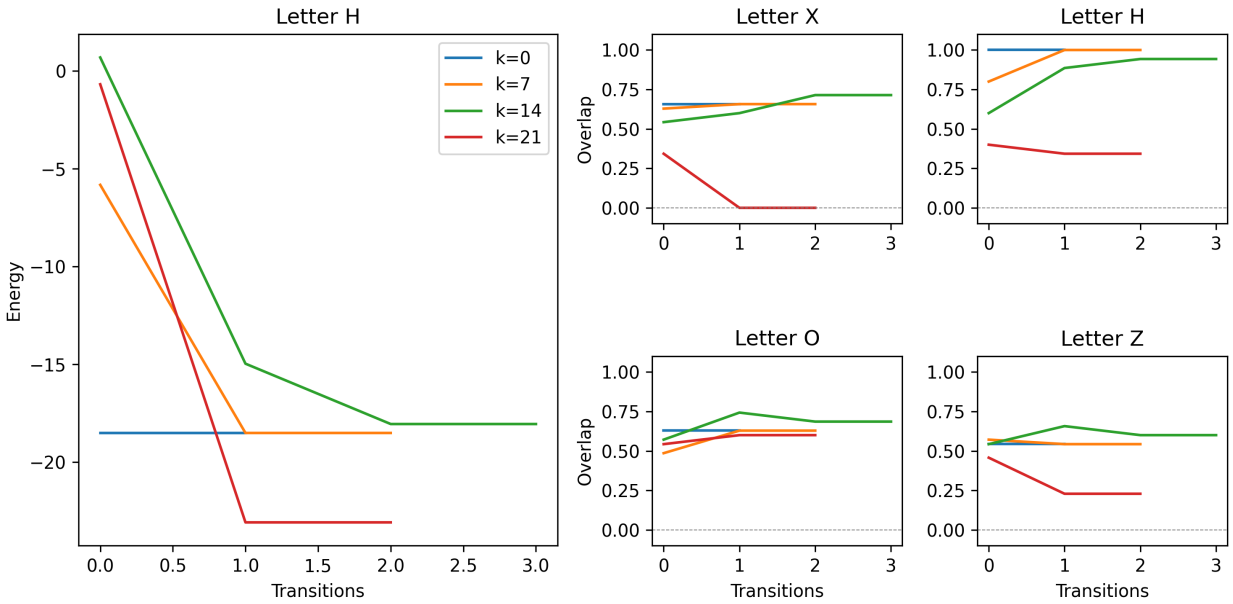

Figure 2: Results for letter H

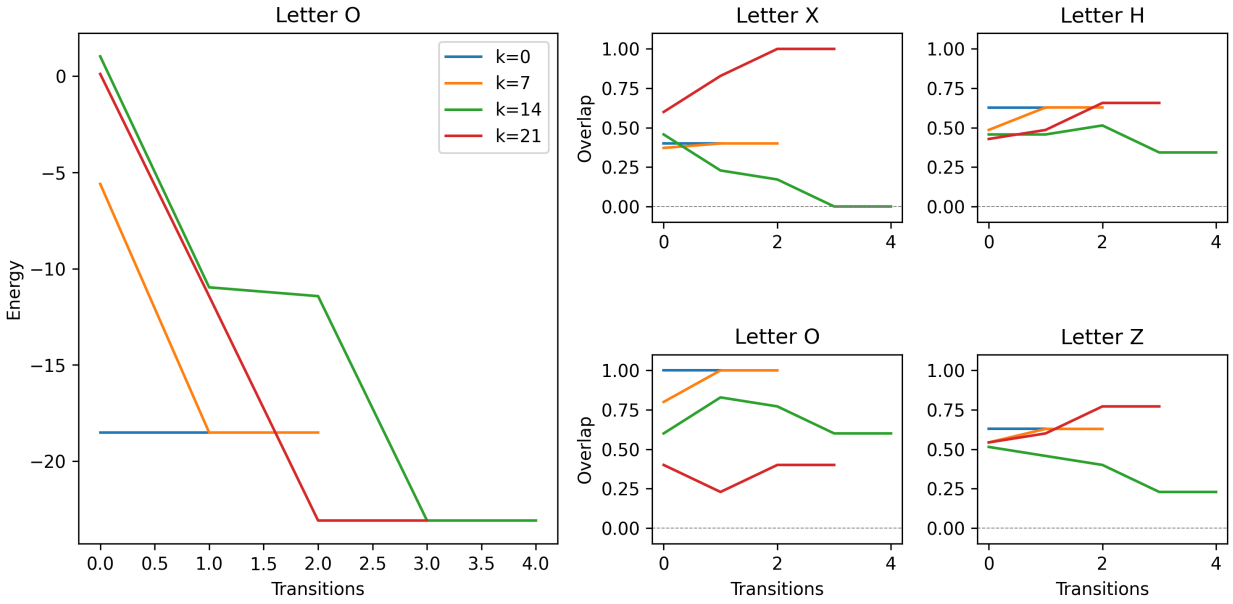

Figure 3: Results for letter O

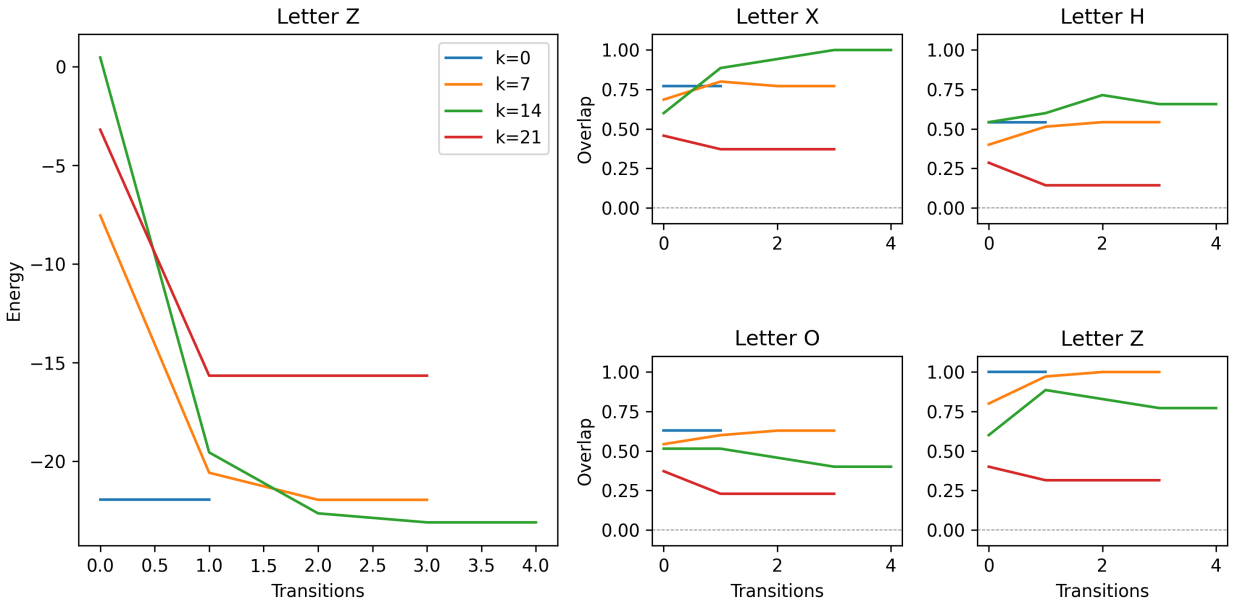

Figure 4: Results for letter Z

Recall rate

In this section, we provide results on the recall ability of the trained Hopfield network. We performed 10,000 runs with random input states. Below, we present the distributions of limit cycles, true and false attractors, as well as the 10 most frequently reached final states.

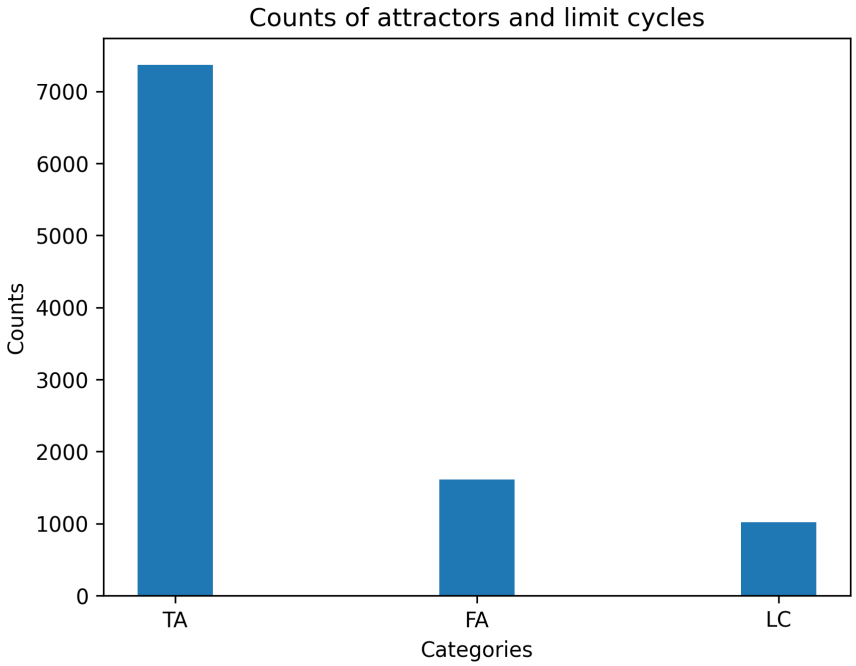

Figure 5: True attractors, false attractors and limit cycles distribution

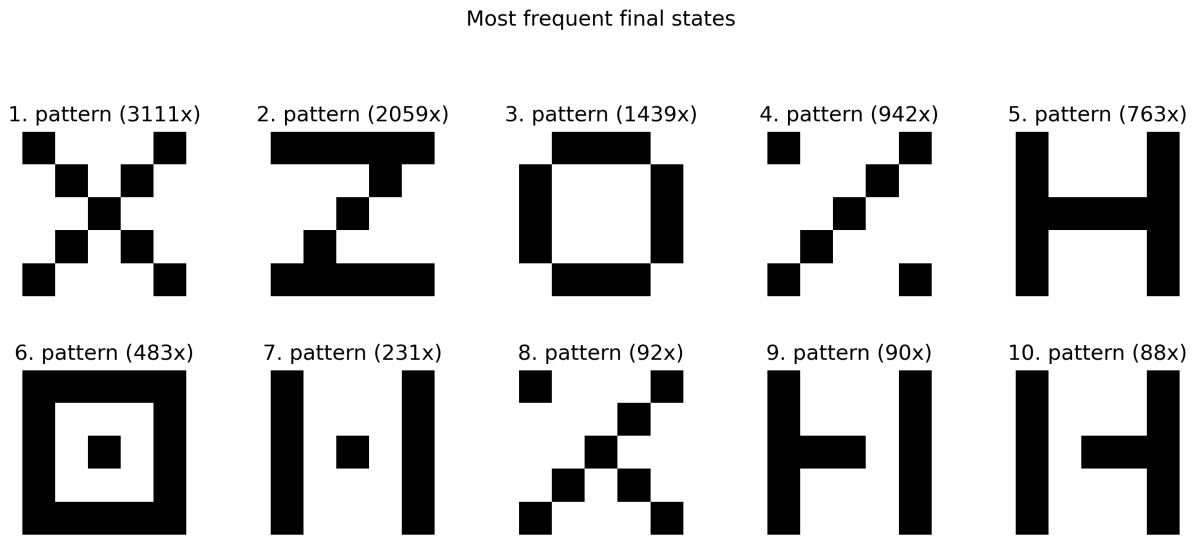

Figure 6: Recall rate for network trained on letters X, H, O, Z

Bonus

In addition to the existing dataset, I created two more datasets by augmenting the original one. Therefore, we now have three datasets with the following letters:

4-letter dataset: `X, H, O, Z`

6-letter dataset: `X, H, O, Z, J, K`

8-letter dataset: `X, H, O, Z, J, K, L, E`

Based on the plot below, we can see that the network trained on 4 patterns can successfully recall the correct pattern with 7 flipped bits in above 80% of cases. It achieves a recall rate slightly above 30% when 14 bits are flipped, and slightly above 20% when 21 bits are flipped.

Networks trained on 6 and 8 patterns essentially do not work at all. A very small yellow bar is visible only upon zooming in. This is due to the fact that a Hopfield network can store up to approximately $0.14N$ patterns, which in our case is approximately 4.9. Even a single additional pattern can lead to unpredictable behavior.

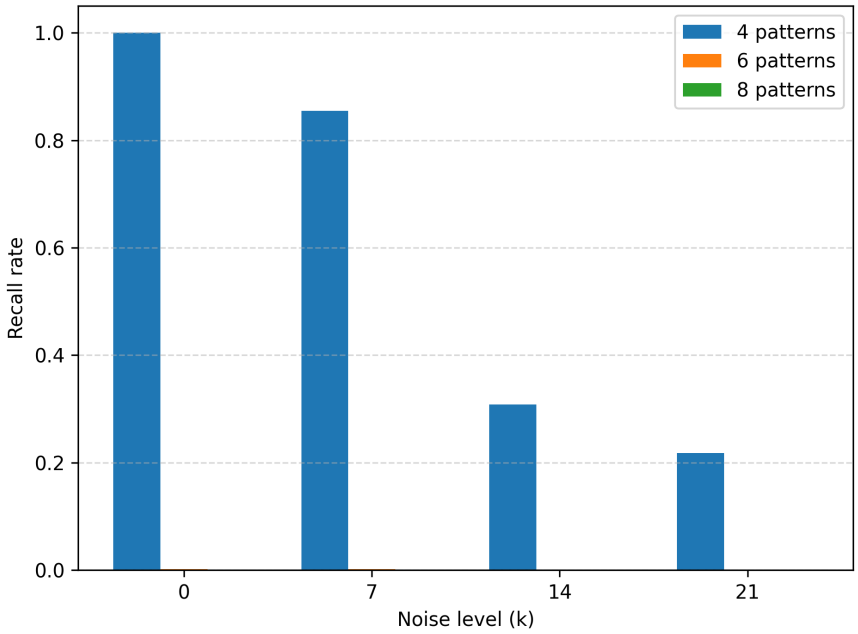

Figure 7: Recall rates for various datasets (1000 samples, patterns evenly distributed)