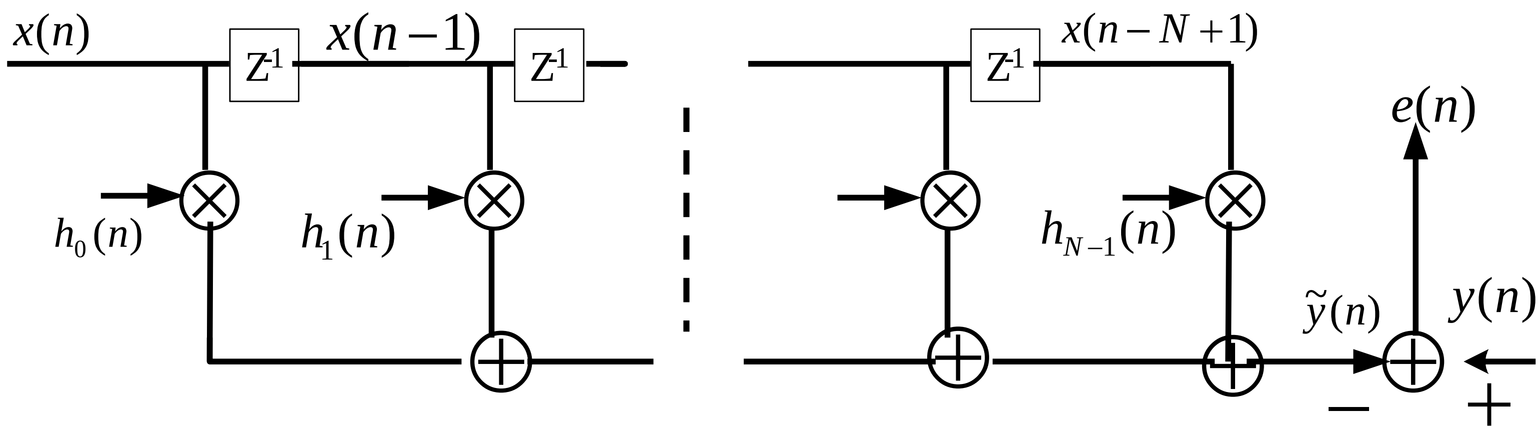
第七章　最小二乘自适应滤波

(Least-Squares　Adaptive Filters)

7.3　最小二乘自适应滤波器矢量空间分析

一 最小二乘滤波器的矢量空间分析



$$\xi[n]=\sum_{i=1}^{n}\lambda^{n-i}e^2(i|n)=\min \quad \lambda\rightarrow \text{遗忘因子, 加权因子}$$

$$e(i|n)=y(i)-\hat{y}(i), i=1,..,n$$

$$\underleftarrow{\mathbf{x}}(i)=[x(i),x(i-1),\ldots,x(i-N+1)]^T$$

$$\mathbf{H}(n)=[h_0(n),h_1(n),\ldots,h_{N-1}(n)]^T$$

$$\hat{y}(i)=\sum_{k=0}^{N-1}h_k(n)x(i-k)=\mathbf{H}^T(n)\underleftarrow{\mathbf{x}}(i)=\underleftarrow{\mathbf{x}}^T(i)\mathbf{H}(n)$$

$$\hat{y}(i) = \sum_{k=0}^{N-1} h_k(n)x(i-k) = \mathbf{H}^T(n)\underline{\mathbf{x}}(i) = \underline{\mathbf{x}}^T(i)\mathbf{H}(n)$$

$$J[\mathbf{H}(n)] = \xi[n] = \sum_{i=1}^{n} \lambda^{n-i} e^2(i|n)$$

$$\frac{\partial J[\mathbf{H}(n)]}{\partial \mathbf{H}(n)} = 0 \Rightarrow \sum_{i=1}^{n} \lambda^{n-i} e(i|n)\underline{\mathbf{x}}(i) = 0$$

$$\underbrace{\left[\sum_{i=1}^{n} \lambda^{n-i} \underline{\mathbf{x}}(i)\underline{\mathbf{x}}^T(i)\right]}_{\Phi(n)} \mathbf{H}(n) = \underbrace{\sum_{i=1}^{n} \lambda^{n-i} y(i)\underline{\mathbf{x}}(i)}_{\mathbf{z}(n)}$$

$$\mathbf{\Phi}(n)\mathbf{H}(n) = \mathbf{z}(n)$$

$$\mathbf{\Phi}(n) = \sum_{i=1}^{n} \lambda^{n-i} \underline{\mathbf{x}}(i)\underline{\mathbf{x}}^T(i)$$

$$\mathbf{z}(n) = \sum_{i=1}^{n} \lambda^{n-i} y(i)\underline{\mathbf{x}}(i)$$

定义：

$$\mathbf{e}(n|n)=[e(1|n),\ldots,e(i|n),\ldots,e(n|n)]^T$$

$$\mathbf{y}(n)=[y(1),\ldots,y(i),\ldots,y(n)]^T$$

$$\hat{\mathbf{y}}(n)=[\hat{y}(1),\ldots,\hat{y}(i),\ldots,\hat{y}(n)]^T$$

$$\mathbf{H}(n)=[h_0(n),h_1(n),\ldots,h_{N-1}(n)]^T$$

$$\mathbf{X}_{0,N-1}(n)=\begin{bmatrix} x(1) & 0 & 0 & 0 \\ x(2) & x(1) & & \\ \ldots\ldots & \ldots\ldots & & \ldots\ldots \\ x(n-1) & x(n-2) & & x(n-N) \\ x(n) & x(n-1) & & x(n-N+1) \end{bmatrix}_{n\times N}$$

$$\boxed{\hat{y}(i)=\sum_{k=0}^{N-1}h_k(n)x(i-k)=\mathbf{H}^T(n)\underleftarrow{\mathbf{x}}(i)=\underleftarrow{\mathbf{x}}^T(i)\mathbf{H}(n)}$$

$$\xi[n]=\sum_{i=1}^{n}\lambda^{n-i}e^2(i|n)\Rightarrow\xi[n]=\mathbf{e}^T(n|n)\mathbf{\Lambda}\mathbf{e}(n|n)$$

$$\mathbf{\Lambda}=diag[\lambda^{n-1},\lambda^{n-2},\ldots,\lambda,1]$$

$$\mathbf{\Phi}(n)=\sum_{i=1}^{n}\underleftarrow{\mathbf{x}}(i)\underleftarrow{\mathbf{x}}^T(i)=\mathbf{X}_{0,N-1}^T(n)\mathbf{X}_{0,N-1}(n),\lambda=1$$

$$\mathbf{z}(n)=\sum_{i=1}^{n}y(i)\underleftarrow{\mathbf{x}}(i)=\mathbf{X}_{0,N-1}^T(n)\mathbf{y}(n)$$

$$\mathbf{H}(n)=[\mathbf{X}_{0,N-1}^T\mathbf{X}_{0,N-1}]^{-1}\mathbf{X}_{0,N-1}^T\mathbf{y}(n) \qquad \boxed{\mathbf{\Phi}(n)\mathbf{H}(n)=\mathbf{z}(n)}$$

$$=\left\langle\mathbf{X}_{0,N-1},\mathbf{X}_{0,N-1}\right\rangle^{-1}\left\langle\mathbf{X}_{0,N-1},\mathbf{y}(n)\right\rangle$$

$\boxed{\text{定义}:<\mathbf{U},\mathbf{V}>=\mathbf{U}^\mathrm{T}\mathbf{V}}$

$$\hat{\mathbf{y}}(n)=\mathbf{X}_{0,N-1}(n)\mathbf{H}(n)$$

$$=\mathbf{X}_{0,N-1}(n)\left\langle\mathbf{X}_{0,N-1},\mathbf{X}_{0,N-1}\right\rangle^{-1}\mathbf{X}_{0,N-1}^\mathrm{T}\mathbf{y}(n)$$

$$\hat{\mathbf{y}}(n)=\mathbf{X}_{0,N-1}(n)\mathbf{H}(n)=\mathbf{X}_{0,N-1}(n)\left\langle \mathbf{X}_{0,N-1},\mathbf{X}_{0,N-1}\right\rangle^{-1}\mathbf{X}_{0,N-1}^{T}\mathbf{y}(n)$$

$$\mathbf{X}_{0,N-1}(n)=\begin{bmatrix} x(1) & 0 & 0 & 0 \\ x(2) & x(1) & & \\ & & & \\ x(n-1) & x(n-2) & & x(n-N) \\ x(n) & x(n-1) & & x(n-N+1) \end{bmatrix}$$

以$\mathbf{X}_{0,N-1}(n)$的N个列向量 $z^{0}\mathbf{x}(n), z^{-1}\mathbf{x}(n),\ldots, z^{-(N-1)}\mathbf{x}(n)$ 为基底，构成n维空间的N维子空间$\{\mathbf{X}_{0,N-1}(n)\}$

$$\mathbf{x}(n)=[x(1),x(2),\ldots,x(n)]^{T}$$

$$z^{-j}\mathbf{x}(n)=[0,\ldots,x(1),x(2),\ldots,x(n-j)]^{T}$$

$$\mathbf{X}_{0,N-1}(n)=[z^{0}\mathbf{x}(n),z^{-1}\mathbf{x}(n),\ldots,z^{-(N-1)}\mathbf{x}(n)]$$

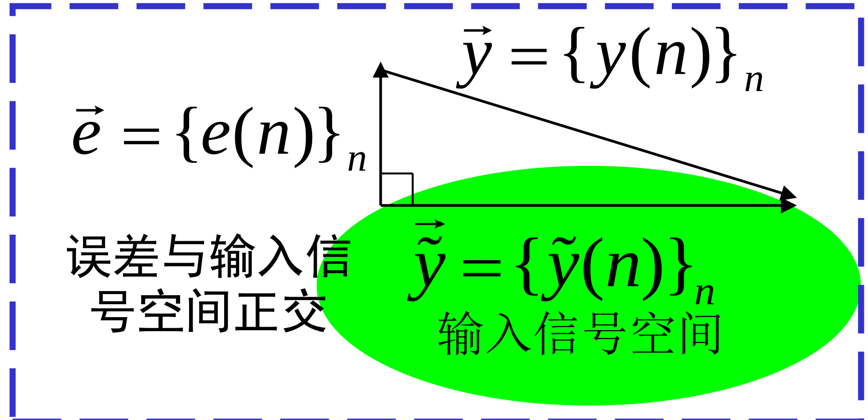
$$\hat{\mathbf{y}}(n)=\mathbf{X}_{0,N-1}(n)\mathbf{H}(n)=[z^{0}\mathbf{x}(n),z^{-1}\mathbf{x}(n),\ldots,z^{-(N-1)}\mathbf{x}(n)]\mathbf{H}(n)$$

$$=h_{0}(n)\mathbf{x}(n)+h_{1}(n)z^{-1}\mathbf{x}(n),\ldots,h_{N-1}(n)z^{-(N-1)}\mathbf{x}(n)$$

$\Rightarrow$ 线性组合，估计是数据矢量空间的一个矢量

$$\widehat{\mathbf{y}}(n)=\mathbf{X}_{0,N-1}(n)\mathbf{H}(n)=\mathbf{X}_{0,N-1}(n)\left\langle \mathbf{X}_{0,N-1},\mathbf{X}_{0,N-1}\right\rangle^{-1}\mathbf{X}_{0,N-1}^{T}\mathbf{y}(n)$$

正交原理:线性最优滤波的充要条件是滤波的输出(期望信号的估计)与误差(估计与期望信号的差)正交.



$$\mathbf{y}(n)=\tilde{\mathbf{y}}(n)+\mathbf{e}(n|n)$$

直和

$$\{\mathbf{Y}\}=\{\mathbf{X}_{0,N-1}(n)\}\oplus\{\mathbf{e}(n|n)\}$$

$$\mathbf{e}(n|n)=[e(1|n),\dots,e(i|n),\dots,e(n|n)]^T$$

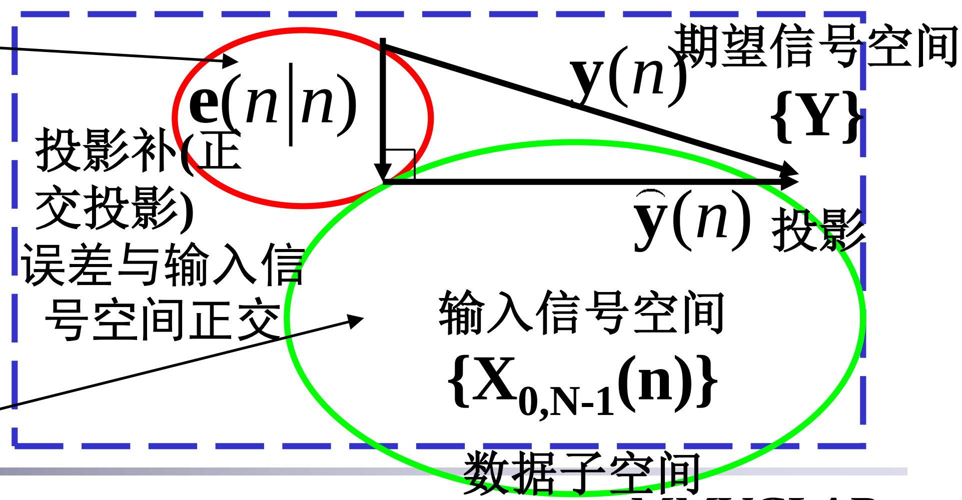
$$\mathbf{y}(n)=[y(1),\dots,y(i),\dots,y(n)]^T$$

$$\widehat{\mathbf{y}}(n)=[\widehat{y}(1),\dots,\widehat{y}(i),\dots,\widehat{y}(n)]^T$$

$\{\mathbf{e}(n|n)\}$
误差矢量空间
正交补空间

$$\widehat{\mathbf{y}}(n)=h_1(n)\mathbf{x}(n)+h_2(n)z^{-1}\mathbf{x}(n),\ \dots,h_N(n)z^{-(N-1)}\mathbf{x}(n)$$

$$z^0\mathbf{x}(n),z^{-1}\mathbf{x}(n),\dots,z^{-(N-1)}\mathbf{x}(n)$$



$$\boxed{\widehat{\mathbf{y}}(n)=\mathbf{X}_{0,N-1}(n)\left\langle\mathbf{X}_{0,N-1},\mathbf{X}_{0,N-1}\right\rangle^{-1}\mathbf{X}_{0,N-1}^{T}\mathbf{y}(n)}$$

二 投影矩阵和正交投影矩阵

$$\boxed{\mathbf{y}(n)=\tilde{\mathbf{y}}(n)+\mathbf{e}(n|n)}$$

输入信号空间{$\mathbf{X}_{0,N-1}(\mathbf{n})$}（数据子空间）的投影矩阵：

$\boxed{n\times n}$

$$\mathbf{P}_{0,N-1}(n)=\mathbf{X}_{0,N-1}(n)\left\langle\mathbf{X}_{0,N-1},\mathbf{X}_{0,N-1}\right\rangle^{-1}\mathbf{X}_{0,N-1}^{T}$$

U，{U}, $\mathbf{P}_U,\mathbf{P}_U^{\perp}$

$$\widehat{\mathbf{y}}(n)=\mathbf{P}_{0,N-1}(n)\mathbf{y}(n)$$

$$\mathbf{e}(n|n)=[\mathbf{I}-\mathbf{P}_{0,N-1}(n)]\mathbf{y}(n)=\mathbf{P}_{0,N-1}^{\perp}(n)\mathbf{y}(n)$$

$$\mathbf{P}_{0,N-1}^{\perp}(n)=[\mathbf{I}-\mathbf{P}_{0,N-1}(n)]$$

输入信号空间{$\mathbf{X}_{0,N-1}(\mathbf{n})$}（数据子空间）的正交投影矩阵

$$\mathbf{P}_U=\mathbf{U}\left\langle\mathbf{U},\mathbf{U}\right\rangle^{-1}\mathbf{U}^T,\quad\mathbf{P}_U^{\perp}=\mathbf{I}-\mathbf{U}\left\langle\mathbf{U},\mathbf{U}\right\rangle^{-1}\mathbf{U}^T$$

$$\mathbf{P}_U = \mathbf{U}\langle \mathbf{U},\mathbf{U}\rangle^{-1}\mathbf{U}^T, \mathbf{P}_U^{\perp} = \mathbf{I} - \mathbf{U}\langle \mathbf{U},\mathbf{U}\rangle^{-1}\mathbf{U}^T$$

投影矩阵$\mathbf{P}_U$ 及正交投影矩阵$\mathbf{P}_U^{\perp}$ 的性质：

1) $\mathbf{P}_U\mathbf{P}_U = \mathbf{P}_U, \mathbf{P}_U^{\perp}\mathbf{P}_U^{\perp} = \mathbf{P}_U^{\perp}$

2) $\mathbf{P}_U^T = \mathbf{P}_U, (\mathbf{P}_U^{\perp})^T = \mathbf{P}_U^{\perp}$

3) $\langle \mathbf{P}_U\mathbf{x}, \mathbf{P}_U\mathbf{y}\rangle = \langle \mathbf{P}_U\mathbf{x}, \mathbf{y}\rangle = \langle \mathbf{x}, \mathbf{P}_U\mathbf{y}\rangle$

$\langle \mathbf{P}_U^{\perp}\mathbf{x}, \mathbf{P}_U^{\perp}\mathbf{y}\rangle = \langle \mathbf{P}_U^{\perp}\mathbf{x}, \mathbf{y}\rangle = \langle \mathbf{x}, \mathbf{P}_U^{\perp}\mathbf{y}\rangle$

$$\boxed{\mathbf{P}_U = \mathbf{U}\left\langle \mathbf{U},\mathbf{U}\right\rangle^{-1}\mathbf{U}^T, \mathbf{P}_U^{\perp} = \mathbf{I} - \mathbf{U}\left\langle \mathbf{U},\mathbf{U}\right\rangle^{-1}\mathbf{U}^T}$$

$$4)\mathbf{P}_U^{\perp}\mathbf{P}_U = 0$$

$$5)\mathbf{P}_{Uv} = \mathbf{P}_{Uw} = \mathbf{P}_U + \mathbf{P}_w, \mathbf{P}_{Uv}^{\perp} = \mathbf{P}_{Uw}^{\perp} = \mathbf{I} - \mathbf{P}_{Uv} = \mathbf{P}_U^{\perp} - \mathbf{P}_w$$

$$w = \mathbf{P}_U^{\perp} v$$

$$6)\mathbf{P}_{Uv} = \mathbf{P}_U + \mathbf{P}_U^{\perp}\mathbf{v}\left\langle \mathbf{P}_U^{\perp}\mathbf{v}, \mathbf{P}_U^{\perp}\mathbf{v}\right\rangle^{-1}\mathbf{v}^T\mathbf{P}_U^{\perp}$$

$$\mathbf{P}_{Uv}^{\perp} = \mathbf{P}_U^{\perp} - \mathbf{P}_U^{\perp}\mathbf{v}\left\langle \mathbf{P}_U^{\perp}\mathbf{v}, \mathbf{P}_U^{\perp}\mathbf{v}\right\rangle^{-1}\mathbf{v}^T\mathbf{P}_U^{\perp}$$

$$w = \mathbf{P}_U^{\perp} v$$

$$\boxed{6)\mathbf{P}_{Uv} = \mathbf{P}_U + \mathbf{P}_U^{\perp}\mathbf{v}\left\langle \mathbf{P}_U^{\perp}\mathbf{v}, \mathbf{P}_U^{\perp}\mathbf{v} \right\rangle^{-1} \mathbf{v}^T \mathbf{P}_U^{\perp}}$$

$$7)\mathbf{P}_{Uv}\mathbf{y} = \mathbf{P}_U\mathbf{y} + \mathbf{P}_U^{\perp}\mathbf{v}\left\langle \mathbf{P}_U^{\perp}\mathbf{v}, \mathbf{P}_U^{\perp}\mathbf{v} \right\rangle^{-1} \left\langle \mathbf{P}_U^{\perp}\mathbf{v}, \mathbf{y} \right\rangle$$

$$\mathbf{P}_{Uv}^{\perp}\mathbf{y} = \mathbf{P}_U^{\perp}\mathbf{y} - \mathbf{P}_U^{\perp}\mathbf{v}\left\langle \mathbf{P}_U^{\perp}\mathbf{v}, \mathbf{P}_U^{\perp}\mathbf{v} \right\rangle^{-1} \left\langle \mathbf{P}_U^{\perp}\mathbf{v}, \mathbf{y} \right\rangle$$

8)对任意矢量**z,y**

$$\left\langle \mathbf{z}, \mathbf{P}_{Uv}\mathbf{y} \right\rangle = \left\langle \mathbf{z}, \mathbf{P}_U\mathbf{y} \right\rangle + \left\langle \mathbf{z}, \mathbf{P}_U^{\perp}\mathbf{v} \right\rangle \left\langle \mathbf{P}_U^{\perp}\mathbf{v}, \mathbf{P}_U^{\perp}\mathbf{v} \right\rangle^{-1} \left\langle \mathbf{P}_U^{\perp}\mathbf{v}, \mathbf{y} \right\rangle$$

$$\left\langle \mathbf{z}, \mathbf{P}_{Uv}^{\perp}\mathbf{y} \right\rangle = \left\langle \mathbf{z}, \mathbf{P}_U^{\perp}\mathbf{y} \right\rangle - \left\langle \mathbf{z}, \mathbf{P}_U^{\perp}\mathbf{v} \right\rangle \left\langle \mathbf{P}_U^{\perp}\mathbf{v}, \mathbf{P}_U^{\perp}\mathbf{v} \right\rangle^{-1} \left\langle \mathbf{P}_U^{\perp}\mathbf{v}, \mathbf{y} \right\rangle$$

$$\mathbf{P}_{0,N-1}(n)=\mathbf{X}_{0,N-1}(n)\left\langle \mathbf{X}_{0,N-1},\mathbf{X}_{0,N-1}\right\rangle^{-1}\mathbf{X}_{0,N-1}^{T}$$

三 时间更新

$$\hat{\mathbf{y}}(n)=\mathbf{P}_{0,N-1}(n)\mathbf{y}(n)$$

- 数据的更新的表示

单位现时矢量： $\boldsymbol{\pi}(n)=[0,\ldots,0,1]^T$

$\{\boldsymbol{\pi}(n)\}$的投影矩阵和正交投影矩阵

$$\mathbf{P}_\pi(n)=\boldsymbol{\pi}(n)\langle\boldsymbol{\pi}(n),\boldsymbol{\pi}(n)\rangle^{-1}\boldsymbol{\pi}^T(n)=diag[0,\ldots0,1]$$

$$\mathbf{P}_\pi^\perp(n)=\mathbf{I}-\mathbf{P}_\pi(n)=diag[1,\ldots,1,0]$$

例: $\mathbf{P}_\pi(n)\mathbf{x}(n)=[0,\ldots0,x(n)]$

$$\mathbf{P}_\pi^\perp(n)\mathbf{x}(n)=[x(1),\ldots,x(n-1),0]$$

$$5)\mathbf{P}_{Uv}=\mathbf{P}_{Uw}=\mathbf{P}_{U}+\mathbf{P}_{w},\ \mathbf{P}_{Uv}^{\perp}=\mathbf{P}_{Uw}^{\perp}=\mathbf{I}-\mathbf{P}_{Uv}=\mathbf{P}_{U}^{\perp}-\mathbf{P}_{w}$$

$$\mathbf{P}_{Uv}^{\perp}=\mathbf{P}_{U}^{\perp}-\mathbf{P}_{U}^{\perp}\mathbf{v}\left\langle \mathbf{P}_{U}^{\perp}\mathbf{v},\mathbf{P}_{U}^{\perp}\mathbf{v}\right\rangle^{-1}\mathbf{v}^{T}\mathbf{P}_{U}^{\perp}$$

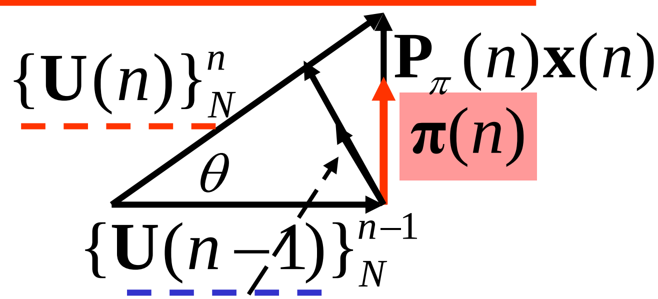
- 时间更新公式

$$\mathbf{v}=\boldsymbol{\pi}(n),\ \mathbf{v}\perp\{\mathbf{U}(n-1)\}\quad \{\mathbf{U}(n-1)\}_N^{n-1}=\{z^0\mathbf{x}(n-1),\dots,z^{-(N-1)}\mathbf{x}(n-1)$$

$$\{\mathbf{U}(n),\boldsymbol{\pi}\}=\{\mathbf{U}(n-1),\boldsymbol{\pi}\}\quad \{\mathbf{U}(n)\}_N^{n}=\{z^0\mathbf{x}(n),\dots,z^{-(N-1)}\mathbf{x}(n)\}$$

$$\mathbf{P}_{U\pi}(n)\Rightarrow\mathbf{P}_{U}(n-1)\oplus\mathbf{P}_{\pi}(n)=\mathbf{P}_{U}(n-1)\oplus diag[0,\dots,0,1]$$

$$\mathbf{P}_{U\pi}(n)=\begin{bmatrix}\mathbf{P}_{U}(n-1) & \mathbf{0}_{n-1}\\ \mathbf{0}_{n-1}^{T} & 1\end{bmatrix}$$



$$\mathbf{P}_{U\pi}^{\perp}(n)=\mathbf{P}_{U}^{\perp}(n)-\mathbf{P}_{U}^{\perp}(n)\boldsymbol{\pi}(n)\left\langle \mathbf{P}_{U}^{\perp}\boldsymbol{\pi}(n),\mathbf{P}_{U}^{\perp}\boldsymbol{\pi}(n)\right\rangle^{-1}\boldsymbol{\pi}^{T}(n)\mathbf{P}_{U}^{\perp}$$

$$\gamma_U(n)\triangleq\left\langle \mathbf{P}_{U}^{\perp}\boldsymbol{\pi}(n),\mathbf{P}_{U}^{\perp}\boldsymbol{\pi}(n)\right\rangle=\left\langle \boldsymbol{\pi}(n),\mathbf{P}_{U}^{\perp}\boldsymbol{\pi}(n)\right\rangle$$

角参量 $\gamma_U(n)=\cos^2\theta$ 新息的度量

$$\boxed{\mathbf{P}_{U\pi}^{\perp}(n)=\mathbf{P}_{U}^{\perp}(n)-\mathbf{P}_{U}^{\perp}(n)\boldsymbol{\pi}(n)\left\langle \mathbf{P}_{U}^{\perp}\boldsymbol{\pi}(n),\mathbf{P}_{U}^{\perp}\boldsymbol{\pi}(n)\right\rangle^{-1}\boldsymbol{\pi}^{T}(n)\mathbf{P}_{U}^{\perp}}$$

$$\mathbf{P}_{U\pi}^{\perp}(n)=\mathbf{P}_{U}^{\perp}(n)-\frac{\mathbf{P}_{U}^{\perp}(n)\boldsymbol{\pi}(n)\boldsymbol{\pi}^{T}(n)\mathbf{P}_{U}^{\perp}}{\gamma_{U}(n)}$$

$$\boldsymbol{\pi}(n)\boldsymbol{\pi}^{T}(n)=diag[0,\ldots,1]$$

$$=\mathbf{P}_{U}^{\perp}(n)-\frac{\mathbf{P}_{U}^{\perp}(n)\mathbf{P}_{\boldsymbol{\pi}}(n)\mathbf{P}_{U}^{\perp}(n)}{\gamma_{U}(n)}$$

$$\mathbf{P}_{U\pi}^{\perp}(n)=\mathbf{I}_{n\times n}-\mathbf{P}_{U\pi}(n)$$

$$=\begin{bmatrix}\mathbf{P}_{U}^{\perp}(n-1) & \mathbf{0}_{n-1}\\ \mathbf{0}_{n-1}^{T} & 0\end{bmatrix}$$

$$\mathbf{P}_{U\pi}(n)=\begin{bmatrix}\mathbf{P}_{U}(n-1) & \mathbf{0}_{n-1}\\ \mathbf{0}_{n-1}^{T} & 1\end{bmatrix}$$

$$\boxed{\begin{bmatrix}\mathbf{P}_{U}^{\perp}(n-1) & \mathbf{0}_{n-1}\\ \mathbf{0}_{n-1}^{T} & 0\end{bmatrix}=\mathbf{P}_{U}^{\perp}(n)-\frac{\mathbf{P}_{U}^{\perp}(n)\mathbf{P}_{\boldsymbol{\pi}}(n)\mathbf{P}_{U}^{\perp}(n)}{\gamma_{U}(n)}}$$

7.4　最小二乘格形（LSL）自适应算法

一 前向预测和后向预测误差滤波的矢量空间分析

$$e(i|n) = y(i) - \sum_{k=0}^{N-1} h_k(n) x(i-k), i = 1,..,n$$

正向预测误差:

$$e_N^f(i) = x(i) - \hat{x}_f(i) = x(i) - \sum_{k=1}^{N} a_{Nk} x(i-k), 1 \le i \le n$$

$$\varepsilon_N^f(n) = \sum_{i=1}^{n} [e_N^f(i)]^2 \overset{\{a_{Nk}\}_{k=1}^{N}}{\Longrightarrow} \min$$

定义:

$$\mathbf{e}_N^f(n) = [e_N^f(1), \ldots, e_N^f(i), \ldots, e_N^f(n)]^T$$

$$\mathbf{x}(n-1) = [x(1), x(2), \ldots, x(n-1)]^T \Leftarrow \mathbf{x}(n)$$ 输入信号矢量

$$\mathbf{x}(n) = [x(1), x(2), \ldots, x(n)]^T \Leftarrow \mathbf{y}(n)$$ 参考信号矢量

$$\widehat{\mathbf{x}}_f(n) = [\widehat{x}_f(1), \ldots, \widehat{x}_f(i), \ldots, \widehat{x}_f(n)]^T \Leftarrow \widehat{\mathbf{y}}(n)$$

$$\mathbf{A}_N(n) = [a_{N1}(n), a_{N2}(n), \ldots, a_{NN}(n)]^T \Leftarrow \mathbf{H}(n)$$

$$\mathbf{X}_{1,N}(n)=\begin{bmatrix} 0 & 0 & 0 & 0 \\ x(1) & 0 & & \\ & & & \\ x(n-2) & x(n-3) & & x(n-N-1) \\ x(n-1) & x(n-2) & & x(n-N) \end{bmatrix}$$

$$=[z^{-1}\mathbf{x}(n), z^{-2}\mathbf{x}(n), \ldots, z^{-N}\mathbf{x}(n)]^T$$

则：

$$\hat{\mathbf{x}}_f(n)=\mathbf{X}_{1,N}(n)\mathbf{A}_N(n)=\mathbf{P}_{1,N}(n)\mathbf{x}(n)$$

$$\mathbf{e}_N^f(n)=\mathbf{x}(n)-\mathbf{X}_{1,N}(n)\mathbf{A}_N(n)=\mathbf{P}_{1,N}^{\perp}\mathbf{x}(n)$$

$$\{\mathbf{X}_{1,N}(n)\}\begin{cases}\mathbf{P}_{1,N}(n)=\mathbf{X}_{1,N}(n)\left\langle \mathbf{X}_{1,N},\mathbf{X}_{1,N}\right\rangle^{-1}\mathbf{X}_{1,N}^T \\ \mathbf{P}_{1,N}^{\perp}(n)=\mathbf{I}-\mathbf{P}_{1,N}(n)\end{cases}$$

$$e_N^f(n)=\boldsymbol{\pi}^T(n)\mathbf{e}_N^f(n)=\left\langle \boldsymbol{\pi}(n),\mathbf{P}_{1,N}^{\perp}\mathbf{x}(n)\right\rangle$$

$$\varepsilon_N^f(n)=\sum_{i=1}^{n}[e_N^f(i)]^2=\left\langle \mathbf{e}_N^f(n),\mathbf{e}_N^f(n)\right\rangle$$

反向（后向）预测误差:

$$e_N^b(i) = x(i-N) - \hat{x}_b(i-N) = x(i-N) - \sum_{k=1}^{N} b_{Nk} x(i-N+k)$$

$$\varepsilon_N^f(n) = \sum_{i=1}^{n} [e_N^b(i)]^2 \overset{\{b_{Nk}\}_{k=1}^N}{\Longrightarrow} \min \qquad 1 \le i \le n$$

定义:

$$\mathbf{e}_N^b(n) = [e_N^b(1), \ldots, e_N^b(i), \ldots, e_N^b(n)]^T$$

$$\mathbf{x}(n) = [x(1), x(2), \ldots, x(n)]^T \quad \text{输入信号矢量}$$

$$\mathbf{x}_b(n-N) = [x_b(1-N), \ldots, x_b(i-N), \ldots, x_b(n-N)]^T \Leftarrow \mathbf{y}(n)$$

$$\widehat{\mathbf{x}}_b(n-N) = [\widehat{x}_b(1-N), \ldots, \widehat{x}_b(i-N), \ldots, \widehat{x}_b(n-N)]^T \Leftarrow \widehat{\mathbf{y}}(n)$$

$$\mathbf{B}_N(n) = [b_{NN}(n), b_{N(N-1)}(n), \ldots, b_{N1}(n)]^T \Leftarrow \mathbf{H}(n)$$

$$\mathbf{X}_{0,N-1}(n)=\begin{bmatrix} x(1) & 0 & 0 & 0 \\ x(2) & x(1) & & \\ & & & \\ x(n-1) & x(n-2) & & x(n-N) \\ x(n) & x(n-1) & & x(n-N+1) \end{bmatrix}$$

$$=[z^{0}\mathbf{x}(n),z^{-1}\mathbf{x}(n),\dots,z^{-(N-1)}\mathbf{x}(n)]^{T}$$

则：

$$\hat{\mathbf{x}}_b(n-N)=\mathbf{X}_{0,N-1}(n)\mathbf{B}_N(n)=\mathbf{P}_{0,N-1}(n)z^{-N}\mathbf{x}(n)$$

$$\mathbf{e}_N^b(n)=z^{-N}\mathbf{x}(n)-\hat{\mathbf{x}}_b(n-N)=\mathbf{P}_{0,N-1}^{\perp}z^{-N}\mathbf{x}(n)$$

$$\{\mathbf{X}_{0,N-1}(n)\}\begin{cases}\mathbf{P}_{0,N-1}(n)=\mathbf{X}_{0,N-1}(n)\left\langle \mathbf{X}_{0,N-1},\mathbf{X}_{0,N-1}\right\rangle^{-1}\mathbf{X}_{0,N-1}^{T} \\ \mathbf{P}_{0,N-1}^{\perp}(n)=\mathbf{I}-\mathbf{P}_{0,N-1}(n)\end{cases}$$

$$e_N^b(n)=\boldsymbol{\pi}^{T}(n)\mathbf{e}_N^b(n)=\left\langle \boldsymbol{\pi}(n),\mathbf{P}_{0,N-1}^{\perp}z^{-N}\mathbf{x}(n)\right\rangle$$

$$\varepsilon_N^b(n)=\sum_{i=1}^{n}[e_N^b(i)]^2=\left\langle \mathbf{e}_N^b(n),\mathbf{e}_N^b(n)\right\rangle$$

二 预测误差滤波器的格形结构

$$e_N^f(n) = \left\langle \boldsymbol{\pi}(n), \mathbf{P}_{1,N}^{\perp}\mathbf{x}(n) \right\rangle$$

$$e_N^f(n) \Rightarrow e_{N+1}^f(n); e_N^b(n) \Rightarrow e_{N+1}^b(n)$$

$$e_N^b(n) = \left\langle \boldsymbol{\pi}(n), \mathbf{P}_{0,N-1}^{\perp} z^{-N}\mathbf{x}(n) \right\rangle$$

$$e_{N+1}^f(n) = \left\langle \boldsymbol{\pi}(n), \mathbf{P}_{1,N+1}^{\perp}\mathbf{x}(n) \right\rangle \qquad \{\mathbf{X}_{1,N+1}(n)\}$$

$$\mathbf{X}_{1,N}(n) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ x(1) & 0 & 0 & 0 \\ x(2) & x(1) & & \\ & & & \\ x(n-2) & x(n-3) & & x(n-N+1) \\ x(n-1) & x(n-2) & & x(n-N) \end{bmatrix} \quad \mathbf{X}_{1,N+1}(n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ x(1) & 0 & 0 & 0 & 0 \\ x(2) & x(1) & & & \\ & & & & \\ x(n-2) & x(n-3) & & x(n-N+1) & x(n-N) \\ x(n-1) & x(n-2) & & x(n-N) & x(n-N-1) \end{bmatrix}$$

$$\mathbf{X}_{1,N+1}(n) = [\underbrace{\mathbf{X}_{1,N}(n)}_{\mathbf{U}} \quad \underbrace{z^{-(N+1)}\mathbf{x}(n)}_{\mathbf{v}}] = [\mathbf{U}, \mathbf{v}]$$

$$\mathbf{y} = \mathbf{x}(n) \qquad \mathbf{z} = \boldsymbol{\pi}(n)$$

$$\left\langle \mathbf{z}, \mathbf{P}_{Uv}^{\perp}\mathbf{y} \right\rangle = \left\langle \mathbf{z}, \mathbf{P}_{U}^{\perp}\mathbf{y} \right\rangle - \left\langle \mathbf{z}, \mathbf{P}_{U}^{\perp}\mathbf{v} \right\rangle \left\langle \mathbf{P}_{U}^{\perp}\mathbf{v}, \mathbf{P}_{U}^{\perp}\mathbf{v} \right\rangle^{-1} \left\langle \mathbf{P}_{U}^{\perp}\mathbf{v}, \mathbf{y} \right\rangle$$

$$\left\langle \mathbf{z}, \mathbf{P}_{Uv}^{\perp}\mathbf{y} \right\rangle = \left\langle \mathbf{z}, \mathbf{P}_{U}^{\perp}\mathbf{y} \right\rangle - \left\langle \mathbf{z}, \mathbf{P}_{U}^{\perp}\mathbf{v} \right\rangle \left\langle \mathbf{P}_{U}^{\perp}\mathbf{v}, \mathbf{P}_{U}^{\perp}\mathbf{v} \right\rangle^{-1} \left\langle \mathbf{P}_{U}^{\perp}\mathbf{v}, \mathbf{y} \right\rangle$$

$$\{\mathbf{X}_{1,N+1}(n)\}$$

$$\mathrm{U}=\{\mathbf{X}_{1,N}(n)\}$$

$$\mathbf{y} = \mathbf{x}(n)$$

$$\mathbf{z} = \boldsymbol{\pi}(n)$$

$$\mathbf{P}_{Uv}^{\perp} = \mathbf{P}_{1,N+1}^{\perp}(n)$$

$$\mathbf{P}_{U}^{\perp} = \mathbf{P}_{1,N}^{\perp}(n)$$

$$\mathbf{P}_{U}^{\perp}\mathbf{v} = \mathbf{P}_{1,N}^{\perp}(n) z^{-(N+1)}\mathbf{x}(n) = z^{-1}\mathbf{e}_{N}^{b}(n)$$

$$\left\langle \mathbf{P}_{U}^{\perp}\mathbf{v}, \mathbf{P}_{U}^{\perp}\mathbf{v} \right\rangle = \left\langle z^{-1}\mathbf{e}_{N}^{b}(n), z^{-1}\mathbf{e}_{N}^{b}(n) \right\rangle = \varepsilon_{N}^{b}(n-1)$$

$$\left\langle \mathbf{z}, \mathbf{P}_{U}^{\perp}\mathbf{y} \right\rangle = \left\langle \boldsymbol{\pi}(n), \mathbf{P}_{1,N}^{\perp}\mathbf{x}(n) \right\rangle = e_{N}^{f}(n)$$

$$\left\langle \mathbf{z}, \mathbf{P}_{Uv}^{\perp}\mathbf{y} \right\rangle = \left\langle \boldsymbol{\pi}(n), \mathbf{P}_{1,N+1}^{\perp}\mathbf{x}(n) \right\rangle = e_{N+1}^{f}(n)$$

$$e_{N+1}^{f}(n) = e_{N}^{f}(n) - \frac{1}{\varepsilon_{N}^{b}(n-1)} \left\langle \boldsymbol{\pi}(n), z^{-1}\mathbf{e}_{N}^{b}(n) \right\rangle \left\langle z^{-1}\mathbf{e}_{N}^{b}(n), \mathbf{x}(n) \right\rangle$$

$$k_{N+1}^{b} = \frac{\Delta_{N+1}(n)}{\varepsilon_{N}^{b}(n-1)}$$

$$e_{N}^{b}(n-1) \qquad \left\langle z^{-1}\mathbf{e}_{N}^{b}(n), \mathbf{e}_{N}^{f}(n) \right\rangle \triangleq \Delta_{N+1}(n)$$

$$\boxed{e_{N+1}^{f}(n) = e_{N}^{f}(n) - k_{N+1}^{b} e_{N}^{b}(n-1)}$$

$$\boxed{\left\langle \mathbf{P}_{U}^{\perp}\mathbf{x}, \mathbf{P}_{U}^{\perp}\mathbf{y} \right\rangle = \left\langle \mathbf{P}_{U}^{\perp}\mathbf{x}, \mathbf{y} \right\rangle = \left\langle \mathbf{x}, \mathbf{P}_{U}^{\perp}\mathbf{y} \right\rangle}$$

$$e_N^b(n) = \left\langle \boldsymbol{\pi}(n), \mathbf{P}_{0,N-1}^{\perp} z^{-N}\mathbf{x}(n) \right\rangle$$

$$e_{N+1}^b(n) = \left\langle \boldsymbol{\pi}(n), \mathbf{P}_{0,N}^{\perp} z^{-(N+1)}\mathbf{x}(n) \right\rangle \qquad \{\mathbf{X}_{0,N}(n)\}$$

$$\mathbf{X}_{0,N-1}(n) = \begin{bmatrix} x(1) & 0 & 0 & 0 \\ x(2) & x(1) & & \\ & & & \\ x(n-1) & x(n-2) & & x(n-N) \\ x(n) & x(n-1) & & x(n-N+1) \end{bmatrix}$$

$$\mathbf{X}_{0,N}(n) = \begin{bmatrix} x(1) & 0 & 0 & 0 & 0 \\ x(2) & x(1) & & & \\ & & & & \\ x(n-1) & x(n-2) & & x(n-N+2) & x(n-N+1) \\ x(n) & x(n-1) & & x(n-N+1) & x(n-N) \end{bmatrix}$$

$$[\mathbf{X}_{0,N}(n)] = [\mathbf{x}(n), \mathbf{X}_{1,N}(n)] = [\mathbf{v}, \mathbf{U}]$$

$$\mathbf{v} \qquad \mathbf{U} \qquad \mathbf{y} = z^{-(N+1)}\mathbf{x}(n) \qquad \mathbf{z} = \boldsymbol{\pi}(n)$$

$$\boxed{\left\langle \mathbf{z}, \mathbf{P}_{Uv}^{\perp}\mathbf{y} \right\rangle = \left\langle \mathbf{z}, \mathbf{P}_{U}^{\perp}\mathbf{y} \right\rangle - \left\langle \mathbf{z}, \mathbf{P}_{U}^{\perp}\mathbf{v} \right\rangle \left\langle \mathbf{P}_{U}^{\perp}\mathbf{v}, \mathbf{P}_{U}^{\perp}\mathbf{v} \right\rangle^{-1} \left\langle \mathbf{P}_{U}^{\perp}\mathbf{v}, \quad \mathbf{y} \right\rangle}$$

$$\{\mathbf{X}_{0,N}(n)\}$$

$$\text{U}=\{\mathbf{X}_{1,N}(n)\}$$

$$\mathbf{y} = z^{-(N+1)}\mathbf{x}(n)$$

$$\mathbf{z} = \boldsymbol{\pi}(n)$$

$$\mathbf{v} = \mathbf{x}(n)$$

$$\mathbf{P}_{Uv}^{\perp} = \mathbf{P}_{0,N}^{\perp}(n)$$

$$\mathbf{P}_{U}^{\perp} = \mathbf{P}_{1,N}^{\perp}(n)$$

$$\mathbf{P}_{U}^{\perp}\mathbf{v} = \mathbf{P}_{1,N}^{\perp}(n)\mathbf{x}(n) = \mathbf{e}_{N}^{f}(n)$$

$$\left\langle \mathbf{P}_{U}^{\perp}\mathbf{v}, \mathbf{P}_{U}^{\perp}\mathbf{v} \right\rangle = \left\langle \mathbf{e}_{N}^{f}(n), \mathbf{e}_{N}^{f}(n) \right\rangle = \varepsilon_{N}^{f}(n)$$

$$\left\langle \mathbf{z}, \mathbf{P}_{U}^{\perp}\mathbf{y} \right\rangle = \left\langle \boldsymbol{\pi}(n), \mathbf{P}_{1,N}^{\perp} z^{-(N+1)}\mathbf{x}(n) \right\rangle = e_{N}^{b}(n-1)$$

$$\left\langle \mathbf{z}, \mathbf{P}_{Uv}^{\perp}\mathbf{y} \right\rangle = \left\langle \boldsymbol{\pi}(n), \mathbf{P}_{0,N}^{\perp} z^{-(N+1)}\mathbf{x}(n) \right\rangle = e_{N+1}^{b}(n)$$

$$e_{N+1}^{b}(n) = e_{N}^{b}(n-1) - \frac{1}{\varepsilon_{N}^{f}(n)} \left\langle \boldsymbol{\pi}(n), \mathbf{e}_{N}^{f}(n) \right\rangle \left\langle \mathbf{e}_{N}^{f}(n), z^{-(N+1)}\mathbf{x}(n) \right\rangle$$

$$k_{N+1}^{f} = \frac{\Delta_{N+1}(n)}{\varepsilon_{N}^{f}(n)}$$

$$e_{N}^{f}(n) \qquad \left\langle \mathbf{e}_{N}^{f}(n), z^{-1}\mathbf{e}_{N}^{b}(n) \right\rangle \triangleq \Delta_{N+1}(n)$$

$$\boxed{e_{N+1}^{b}(n) = e_{N}^{b}(n-1) - k_{N+1}^{f} e_{N}^{f}(n)}$$

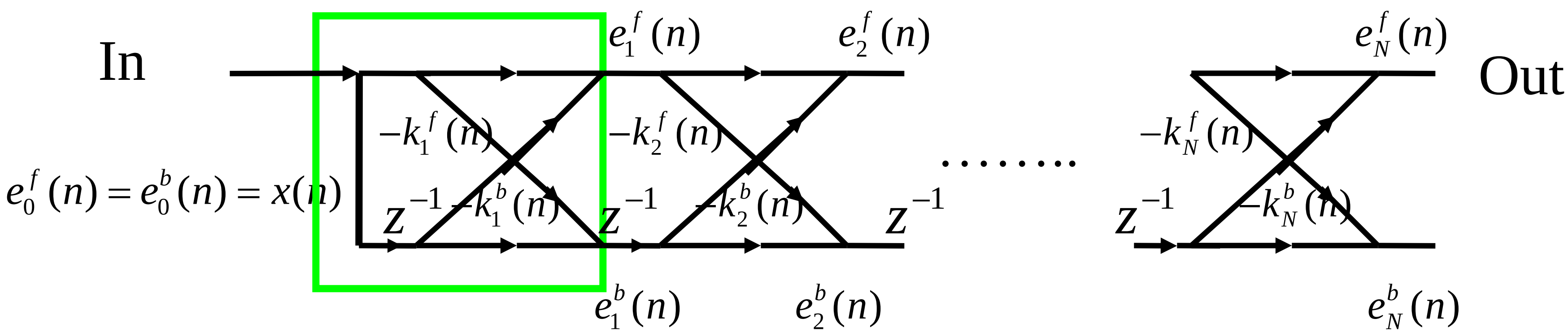
$$\boxed{\left\langle \mathbf{P}_{U}^{\perp}\mathbf{x}, \mathbf{P}_{U}^{\perp}\mathbf{y} \right\rangle = \left\langle \mathbf{P}_{U}^{\perp}\mathbf{x}, \mathbf{y} \right\rangle = \left\langle \mathbf{x}, \mathbf{P}_{U}^{\perp}\mathbf{y} \right\rangle}$$

$$\mathbf{X}_{1,N+1}(n) = [\mathbf{X}_{1,N}(n) \quad z^{-(N+1)}\mathbf{x}(n)] = [\mathbf{U}, \mathbf{v}]$$

$$[\mathbf{X}_{0,N}(n)] = [\mathbf{x}(n), \mathbf{X}_{1,N}(n)] = [\mathbf{v}, \mathbf{U}]$$

$$e_{N+1}^{f}(n) = e_{N}^{f}(n) - k_{N+1}^{b} e_{N}^{b}(n-1)$$

$$e_{N+1}^{b}(n) = e_{N}^{b}(n-1) - k_{N+1}^{f} e_{N}^{f}(n)$$



$$\left\langle \mathbf{e}_N^f(n), z^{-1}\mathbf{e}_N^b(n) \right\rangle \triangleq \Delta_{N+1}(n)$$

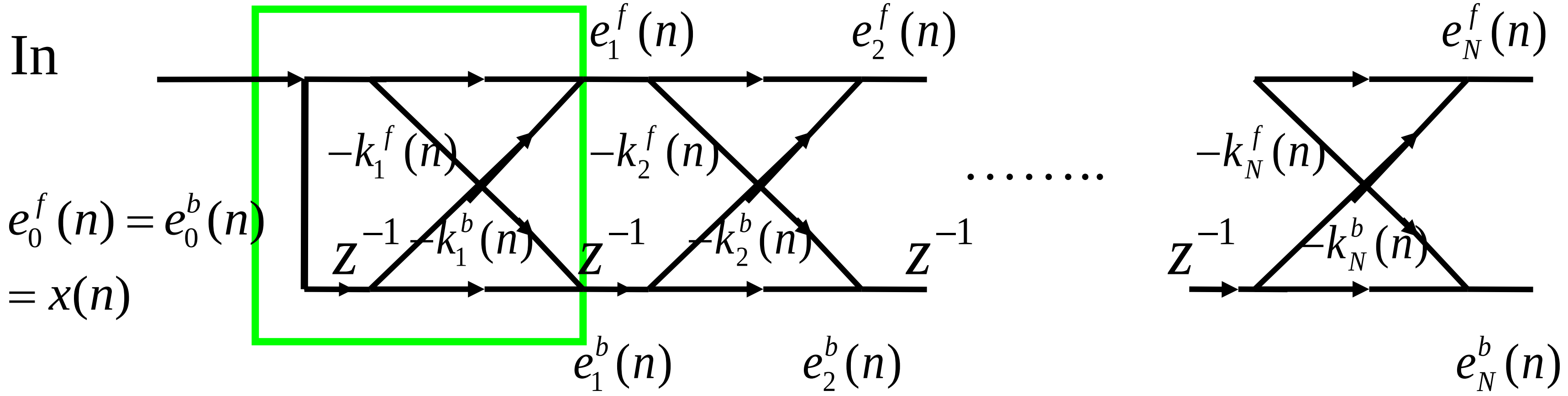
$$k_{N+1}^{b}(n) = \frac{\Delta_{N+1}(n)}{\varepsilon_N^b(n-1)} \qquad k_{N+1}^{f}(n) = \frac{\Delta_{N+1}(n)}{\varepsilon_N^f(n)}$$

$$\left\langle \mathbf{e}_N^f(n), \mathbf{e}_N^f(n) \right\rangle = \varepsilon_N^f(n)$$

$$\left\langle z^{-1}\mathbf{e}_N^b(n), z^{-1}\mathbf{e}_N^b(n) \right\rangle = \varepsilon_N^b(n-1)$$

三 LSL(Least Square Lattice)自适应算法

问题的提出：
LS准则下的预测误差滤波器的自适应计算?



$$k_{N+1}^b(n) = \frac{\Delta_{N+1}(n)}{\varepsilon_N^b(n-1)} \qquad k_{N+1}^f(n) = \frac{\Delta_{N+1}(n)}{\varepsilon_N^f(n)}$$

$$\left\langle \mathbf{e}_N^f(n), z^{-1}\mathbf{e}_N^b(n) \right\rangle \triangleq \Delta_{N+1}(n)$$

$$\left\langle \mathbf{e}_N^f(n), \mathbf{e}_N^f(n) \right\rangle = \varepsilon_N^f(n)$$

$$\left\langle z^{-1}\mathbf{e}_N^b(n), z^{-1}\mathbf{e}_N^b(n) \right\rangle = \varepsilon_N^b(n-1)$$

需解决的问题:以下量的递推计算(阶次叠代)

$$\boxed{e_N^f(n) \Rightarrow e_{N+1}^f(n); e_N^b(n) \Rightarrow e_{N+1}^b(n)}$$

格形结构

$$k_{N+1}^b(n) = \frac{\Delta_{N+1}(n)}{\varepsilon_N^b(n-1)} \quad k_{N+1}^f(n) = \frac{\Delta_{N+1}(n)}{\varepsilon_N^f(n)}$$

$$\varepsilon_N^f(n) \Rightarrow \varepsilon_{N+1}^f(n); \varepsilon_N^b(n) \Rightarrow \varepsilon_{N+1}^b(n)$$

$$\Delta_{N+1}(n-1) \Rightarrow \Delta_{N+1}(n) \quad \gamma_N(n) = \left\langle \boldsymbol{\pi}(n), \mathbf{P}_{0,N-1}^{\perp}(n)\boldsymbol{\pi}(n) \right\rangle$$

$$\gamma_N(n) \Rightarrow \gamma_{N+1}(n)$$

?

基本方法:

1)N维数据子空间(矩阵)和N+1维数据子空间(矩阵)之间的关系

$$\mathbf{X}_{1,N+1}(n)=[\underset{\mathbf{U}}{\underset{\Downarrow}{\mathbf{X}_{1,N}(n)}}\quad \underset{\mathbf{v}}{\underset{\Downarrow}{z^{-(N+1)}\mathbf{x}(n)}}]=[\mathbf{U},\mathbf{v}]$$

$$\mathbf{y}=\mathbf{x}(n)=\mathbf{z}$$

2)有关量的阶次迭代关系看作是N维数据子空间和N+1维数据子空间的投影之间的关系

$$\varepsilon_N^f(n)=\left\langle \mathbf{P}_{1,N}^{\perp}\mathbf{x}(n),\mathbf{P}_{1,N}^{\perp}\mathbf{x}(n)\right\rangle=\left\langle \mathbf{x}(n),\mathbf{P}_{1,N}^{\perp}\mathbf{x}(n)\right\rangle$$

3)运用投影的有关性质,如

$$\left\langle \mathbf{z},\mathbf{P}_{Uv}^{\perp}\mathbf{y}\right\rangle=\left\langle \mathbf{z},\mathbf{P}_{U}^{\perp}\mathbf{y}\right\rangle-\left\langle \mathbf{z},\mathbf{P}_{U}^{\perp}\mathbf{v}\right\rangle\left\langle \mathbf{P}_{U}^{\perp}\mathbf{v},\mathbf{P}_{U}^{\perp}\mathbf{v}\right\rangle^{-1}\left\langle \mathbf{P}_{U}^{\perp}\mathbf{v},\ \mathbf{y}\right\rangle$$

$$\varepsilon_N^f(n) = \left\langle \mathbf{e}_N^f(n), \mathbf{e}_N^f(n) \right\rangle$$

$$\varepsilon_N^b(n) = \left\langle \mathbf{e}_N^b(n), \mathbf{e}_N^b(n) \right\rangle$$

$$\varepsilon_N^f(n) \Rightarrow \varepsilon_{N+1}^f(n); \varepsilon_N^b(n) \Rightarrow \varepsilon_{N+1}^b(n)$$

$$\varepsilon_N^f(n) = \left\langle \mathbf{P}_{1,N}^{\perp}\mathbf{x}(n), \mathbf{P}_{1,N}^{\perp}\mathbf{x}(n) \right\rangle = \left\langle \mathbf{x}(n), \mathbf{P}_{1,N}^{\perp}\mathbf{x}(n) \right\rangle$$

$$\varepsilon_N^b(n) = \left\langle \mathbf{P}_{0,N-1}^{\perp} z^{-N}\mathbf{x}(n), \mathbf{P}_{0,N-1}^{\perp} z^{-N}\mathbf{x}(n) \right\rangle = \left\langle z^{-N}\mathbf{x}(n), \mathbf{P}_{0,N-1}^{\perp} z^{-N}\mathbf{x}(n) \right\rangle$$

$$\varepsilon_{N+1}^f(n) = \left\langle \mathbf{x}(n), \mathbf{P}_{1,N+1}^{\perp}\mathbf{x}(n) \right\rangle \qquad \{\mathbf{X}_{1,N+1}(n)\}$$

$$\mathbf{X}_{1,N+1}(n) = [\underbrace{\mathbf{X}_{1,N}(n)}_{\mathbf{U}} \quad \underbrace{z^{-(N+1)}\mathbf{x}(n)}_{\mathbf{v}}] = [\mathbf{U}, \mathbf{v}]$$

$$\mathbf{y} = \mathbf{x}(n) = \mathbf{z}$$

$$\varepsilon_{N+1}^b(n) = \left\langle z^{-(N+1)}\mathbf{x}(n), \mathbf{P}_{0,N}^{\perp} z^{-(N+1)}\mathbf{x}(n) \right\rangle \qquad \{\mathbf{X}_{0,N}(n)\}$$

$$[\mathbf{X}_{0,N}(n)] = [\underbrace{\mathbf{x}(n)}_{\mathbf{v}}, \underbrace{\mathbf{X}_{1,N}(n)}_{\mathbf{U}}] = [\mathbf{v}, \mathbf{U}]$$

$$\mathbf{y} = z^{-(N+1)}\mathbf{x}(n) = \mathbf{z}$$

$$\boxed{\left\langle \mathbf{z}, \mathbf{P}_{U\mathbf{v}}^{\perp}\mathbf{y}\right\rangle = \left\langle \mathbf{z}, \mathbf{P}_{U}^{\perp}\mathbf{y}\right\rangle - \left\langle \mathbf{z}, \mathbf{P}_{U}^{\perp}\mathbf{v}\right\rangle \left\langle \mathbf{P}_{U}^{\perp}\mathbf{v}, \mathbf{P}_{U}^{\perp}\mathbf{v}\right\rangle^{-1} \left\langle \mathbf{P}_{U}^{\perp}\mathbf{v}, \quad \mathbf{y}\right\rangle}$$

$$\boxed{k_{N+1}^{b}(n) = \frac{\Delta_{N+1}(n)}{\varepsilon_{N}^{b}(n-1)}}$$

$$\boxed{k_{N+1}^{f}(n) = \frac{\Delta_{N+1}(n)}{\varepsilon_{N}^{f}(n)}}$$

$$\boxed{\varepsilon_{N+1}^{f}(n) = \varepsilon_{N}^{f}(n) - \frac{\Delta_{N+1}^{2}(n)}{\varepsilon_{N}^{b}(n-1)}}$$

$$\varepsilon_{N+1}^{f}(n) = \varepsilon_{N}^{f}(n) - k_{N+1}^{b}(n)\Delta_{N+1}(n)$$

$$\boxed{\varepsilon_{N+1}^{b}(n) = \varepsilon_{N}^{b}(n-1) - \frac{\Delta_{N+1}^{2}(n)}{\varepsilon_{N}^{f}(n)}}$$

$$\varepsilon_{N+1}^{b}(n) = \varepsilon_{N}^{b}(n) - k_{N+1}^{f}(n)\Delta_{N+1}(n)$$

$$\mathbf{e}_N^f(n) = \mathbf{P}_{1,N}^{\perp}\mathbf{x}(n)$$

$$\Delta_{N+1}(n-1) \Rightarrow \Delta_{N+1}(n)$$

$$z^{-1}\mathbf{e}_N^b(n) = \mathbf{P}_{1,N}^{\perp}(n) z^{-(N+1)}\mathbf{x}(n)$$

$$\langle \mathbf{P}_U^{\perp}\mathbf{x}, \mathbf{P}_U^{\perp}\mathbf{y} \rangle = \langle \mathbf{P}_U^{\perp}\mathbf{x}, \mathbf{y} \rangle = \langle \mathbf{x}, \mathbf{P}_U^{\perp}\mathbf{y} \rangle$$

$$\langle z^{-1}\mathbf{e}_N^b(n), \mathbf{e}_N^f(n) \rangle \triangleq \Delta_{N+1}(n)$$

$$\Delta_{N+1}(n) = \Delta_{N+1}(n-1) + \frac{e_N^f(n)e_N^b(n-1)}{\gamma_N(n-1)}$$

$$\gamma_N(n) = \langle \boldsymbol{\pi}(n), \mathbf{P}_{0,N-1}^{\perp}(n)\boldsymbol{\pi}(n) \rangle$$ 角参量

推导过程参见P88—90

$$\boxed{\gamma_N(n) = \left\langle \boldsymbol{\pi}(n), \mathbf{P}^{\perp}_{0,N-1}(n)\boldsymbol{\pi}(n) \right\rangle}$$

$$\boxed{\gamma_N(n) \Rightarrow \gamma_{N+1}(n)}$$

$$\gamma_N(n) = \left\langle \boldsymbol{\pi}(n), \mathbf{P}^{\perp}_{0,N-1}(n)\boldsymbol{\pi}(n) \right\rangle$$

$$\boxed{\gamma_{N+1}(n) = \gamma_N(n) - \frac{[e^b_N(n)]^2}{\varepsilon^b_N(n)}}$$

算法总结：

1）初始化，N=1,2,,…P

$$e_N^b(0)=0,\Delta_N(0)=0,\gamma_N(0)=1,\varepsilon_N^b(0)=\varepsilon_N^f(0)=\delta$$

***For n*=1,2,3,… Repeat 2) and 3):**

2） *n*时刻初始化(零阶预测）（*n*=1,2,3,…)

$$e_0^f(n)=e_0^b(n)=x(n)$$

$$\varepsilon_0^b(n)=\varepsilon_0^f(n)=\varepsilon_0^f(n-1)+x^2(n)$$

$$\gamma_0(n)=1$$

3）n时刻的阶次迭代（N=0,1,2,,…P-1)

$$\Delta_{N+1}(n)=\Delta_{N+1}(n-1)+\frac{e_N^f(n)e_N^b(n-1)}{\gamma_N(n-1)}$$

$$k_{N+1}^b(n)=\frac{\Delta_{N+1}(n)}{\varepsilon_N^b(n-1)} \quad k_{N+1}^f(n)=\frac{\Delta_{N+1}(n)}{\varepsilon_N^f(n)}$$

$$e_{N+1}^f(n)=e_N^f(n)-k_{N+1}^b e_N^b(n-1)$$

$$e_{N+1}^b(n)=e_N^b(n-1)-k_{N+1}^f e_N^f(n)$$

$$\varepsilon_{N+1}^f(n)=\varepsilon_N^f(n)-k_{N+1}^b(n)\Delta_{N+1}(n)$$

$$\varepsilon_{N+1}^b(n)=\varepsilon_N^b(n)-k_{N+1}^f(n)\Delta_{N+1}(n)$$

$$\gamma_{N+1}(n-1)=\gamma_N(n-1)-\frac{[e_N^b(n-1)]^2}{\varepsilon_N^b(n-1)}$$

7.5　快速横向滤波（FTF）自适应算法

$$\boxed{\widehat{\mathbf{y}}(n)=\mathbf{X}_{0,N-1}(n)\left\langle\mathbf{X}_{0,N-1},\mathbf{X}_{0,N-1}\right\rangle^{-1}\mathbf{X}_{0,N-1}^{T}\mathbf{y}(n)}$$

$$\boxed{\widehat{\mathbf{y}}(n)=\mathbf{X}_{0,N-1}(n)\mathbf{H}(n)}$$

一 问题

输入信号空间{$\mathbf{X}_{0,N-1}$(n)}（数据子空间）的投影矩阵：

$$\mathbf{P}_{0,N-1}(n)=\mathbf{X}_{0,N-1}(n)\left\langle\mathbf{X}_{0,N-1},\mathbf{X}_{0,N-1}\right\rangle^{-1}\mathbf{X}_{0,N-1}^{T}$$

$$\widehat{\mathbf{y}}(n)=\mathbf{P}_{0,N-1}(n)\mathbf{y}(n)$$

U，{U}, $\mathbf{P}_U,\mathbf{P}_U^{\perp}$

$$\mathbf{e}(n|n)=\mathbf{P}_{0,N-1}^{\perp}(n)\mathbf{y}(n)$$

$$\mathbf{P}_{0,N-1}^{\perp}(n)=[\mathbf{I}-\mathbf{P}_{0,N-1}(n)]$$

输入信号空间{$\mathbf{X}_{0,N-1}$(n)}（数据子空间）的正交投影矩阵

$$\mathbf{H}(n)=[\mathbf{X}_{0,N-1}^{T}(n)\mathbf{X}_{0,N-1}(n)]^{-1}\mathbf{X}_{0,N-1}^{T}(n)\mathbf{y}(n)$$

$$\mathbf{K}_{0,N-1}(n)=[\mathbf{X}_{0,N-1}^{T}(n)\mathbf{X}_{0,N-1}(n)]^{-1}\mathbf{X}_{0,N-1}^{T}(n)$$ 横向滤波算子

$$\mathbf{H}(n)=\mathbf{K}_{0,N-1}(n)\mathbf{y}(n)\Rightarrow\boxed{\mathbf{H}(n-1)=\mathbf{K}_{0,N-1}(n-1)\mathbf{y}(n-1)\overset{?}{\Rightarrow}\mathbf{H}(n)}$$

$$\hat{\mathbf{y}}(n)=\mathbf{X}_{0,N-1}(n)\mathbf{H}(n)$$

H(n)是数据矩阵$\mathbf{X}_{0,N-1}$(n)的横向滤波算子对y(n)作用的结果

二 FTF涉及到的4个横向滤波器

1）最小二乘横向滤波器

$$\mathbf{H}(n)=\mathbf{K}_{0,N-1}(n)\mathbf{y}(n)$$

$$\mathbf{K}_{0,N-1}(n)=[\mathbf{X}_{0,N-1}^T(n)\mathbf{X}_{0,N-1}(n)]^{-1}\mathbf{X}_{0,N-1}^T(n)$$

$\mathbf{H}(n)$是数据矩阵$\mathbf{X}_{0,N-1}(n)$对$\mathbf{y}(n)$的最小二乘估计器

$$\mathbf{H}(n-1)=\mathbf{K}_{0,N-1}(n-1)\mathbf{y}(n-1)\Rightarrow\mathbf{H}(n)=\mathbf{K}_{0,N-1}(n)\mathbf{y}(n)$$

$$\mathbf{K}_{0,N-1}(n-1)\Rightarrow\mathbf{K}_{0,N-1}(n)$$

$$\mathbf{y}(n-1)\Rightarrow\mathbf{y}(n)$$

输入信号空间{$\mathbf{X}_{0,N-1}$(n)}或数据矩阵$\mathbf{X}_{0,N-1}$(n)横向滤波算子

$\boxed{\hat{\mathbf{x}}_f(n) = \mathbf{X}_{1,N}(n)\mathbf{A}_N(n)}$ $\boxed{\mathbf{A}_N(n) = [a_{N1}(n), a_{N2}(n), \ldots, a_{NN}(n)]^T}$

2）前向预测误差滤波器

$$\mathbf{A}(n) = [\mathbf{X}_{1,N}^T(n)\mathbf{X}_{1,N}(n)]^{-1}\mathbf{X}_{1,N}^T(n)\mathbf{x}(n)$$

$$\mathbf{A}(n) = \mathbf{K}_{1,N}(n)\mathbf{x}(n)$$

$$\mathbf{K}_{1,N}(n) = [\mathbf{X}_{1,N}^T(n)\mathbf{X}_{1,N}(n)]^{-1}\mathbf{X}_{1,N}^T(n)$$

$\mathbf{A}(n)$是数据矩阵$\mathbf{X}_{1,N}(n)$对$\mathbf{x}(n)$的最小二乘估计器

$$\mathbf{A}(n-1) = \mathbf{K}_{1,N}(n-1)\mathbf{x}(n-1) \Rightarrow \mathbf{A}(n) = \mathbf{K}_{1,N}(n)\mathbf{x}(n)$$

$$\mathbf{K}_{1,N}(n-1) \Rightarrow \mathbf{K}_{1,N}(n)$$

输入信号空间$\{\mathbf{X}_{1,N}(\mathbf{n})\}$或数据矩阵$\mathbf{X}_{1,N}(\mathbf{n})$横向滤波算子

$\hat{\mathbf{x}}_b(n-N)=\mathbf{X}_{0,N-1}(n)\mathbf{B}_N(n)$ $\mathbf{B}_N(n)=[b_{NN}(n),b_{N,N-1}(n-1),\ldots,b_{N,1}(n)]^T$

3）后向预测误差滤波器

$\mathbf{B}(n)$是数据矩阵$\mathbf{X}_{0,N-1}(n)$对$z^{-N}\mathbf{x}(n)$的最小二乘估计器

$$\mathbf{B}(n)=[\mathbf{X}_{0,N-1}^T(n)\mathbf{X}_{0,N-1}(n)]^{-1}\mathbf{X}_{0,N-1}^T(n)z^{-N}\mathbf{x}(n)$$

$$\mathbf{B}(n)=\mathbf{K}_{0,N-1}(n)z^{-N}\mathbf{x}(n)$$

$$\mathbf{K}_{0,N-1}(n)=[\mathbf{X}_{0,N-1}^T(n)\mathbf{X}_{0,N-1}(n)]^{-1}\mathbf{X}_{0,N-1}^T(n)$$

$$\mathbf{B}(n-1)=\mathbf{K}_{0,N-1}(n-1)z^{-N}\mathbf{x}(n-1)\Rightarrow\mathbf{B}(n)=\mathbf{K}_{0,N-1}(n)z^{-N}\mathbf{x}(n)$$

$$\mathbf{K}_{0,N-1}(n-1)\Rightarrow\mathbf{K}_{0,N-1}(n)$$

输入信号空间$\{\mathbf{X}_{0,N-1}(\mathbf{n})\}$或数据矩阵$\mathbf{X}_{0,N-1}(\mathbf{n})$横向滤波算子

4）增益滤波器

$\mathbf{G}_N(n)$是数据矩阵$\mathbf{X}_{0,N-1}(n)$对$\boldsymbol{\pi}(n)$的最小二乘估计器

$$\mathbf{G}_N(n)=\mathbf{K}_{0,N-1}(n)\boldsymbol{\pi}(n)$$

$$\mathbf{K}_{0,N-1}(n)=[\mathbf{X}_{0,N-1}^T(n)\mathbf{X}_{0,N-1}(n)]^{-1}\mathbf{X}_{0,N-1}^T(n)$$

$$\mathbf{G}_N(n-1)=\mathbf{K}_{0,N-1}(n-1)\boldsymbol{\pi}(n-1)\Rightarrow\mathbf{G}_N(n)=\mathbf{K}_{0,N-1}(n)\boldsymbol{\pi}(n)$$

$$\Downarrow$$

$$\mathbf{K}_{0,N-1}(n-1)\Rightarrow\mathbf{K}_{0,N-1}(n)$$

输入信号空间$\{\mathbf{X}_{0,N-1}(\mathbf{n})\}$或数据矩阵$\mathbf{X}_{0,N-1}(\mathbf{n})$横向滤波算子

$$\boxed{\mathbf{K}_{0,N-1}(n-1) \Rightarrow \mathbf{K}_{0,N-1}(n)}$$

$$\boxed{\mathbf{K}_{1,N}(n-1) \Rightarrow \mathbf{K}_{1,N}(n)}$$

三 横向滤波算子的时间更新

利用横向滤波算子的性质（**P.97~98**），得横向滤波算子的时间更新:

$$1)\mathbf{K}_{(0,N-1)\pi}(n) = \begin{bmatrix} \mathbf{K}_{0,N-1}(n-1) & \mathbf{0}_N \\ \mathbf{C}^T(n-1) & 1 \end{bmatrix}$$

$$2)\mathbf{K}_{(1,N)\pi}(n) = \begin{bmatrix} \mathbf{K}_{1,N}(n-1) & \mathbf{0}_N \\ \mathbf{B}^T(n-1) & 1 \end{bmatrix}$$

四 FTF自适应算法中的时间更新关系

1）最小二乘横向滤波器权矢量的时间更新

利用横向滤波算子的性质及横向滤波算子的时间更新关系：

$$\mathbf{K}_{0,N-1}(n-1)\mathbf{y}(n-1)=\mathbf{K}_{0,N-1}(n)\mathbf{y}(n)-\mathbf{G}_N(n)\frac{\left\langle \boldsymbol{\pi}(n),\mathbf{P}_{0,N-1}^{\perp}(n)\mathbf{y}(n)\right\rangle}{\gamma_N(n)}$$

$$\boxed{\mathbf{H}(n)=\mathbf{H}(n-1)+\frac{e(n|n)}{\gamma_N(n)}\mathbf{G}_N(n)}$$

$$\boxed{\mathbf{K}_{1,N}(n)=[\mathbf{X}_{1,N}^T(n)\mathbf{X}_{1,N}(n)]^{-1}\mathbf{X}_{1,N}^T(n)}$$

$$\boxed{\mathbf{G}_N(n)=\mathbf{K}_{0,N-1}(n)\boldsymbol{\pi}(n)}$$

2）增益滤波器权矢量的时间更新

利用横向滤波算子的性质及横向滤波算子的时间更新关系：

$$\mathbf{K}_{0,N}(n)\boldsymbol{\pi}(n)=\begin{bmatrix}\mathbf{0}_n^T\\ \mathbf{K}_{1,N}(n)\end{bmatrix}\boldsymbol{\pi}(n)+\begin{bmatrix}1\\ -\underbrace{\mathbf{K}_{1,N}(n)\mathbf{x}(n)}_{\mathbf{A}_N(n)}\end{bmatrix}\frac{e^f(n|n)}{\varepsilon^f(n)}$$

$$\mathbf{G}_{N+1}(n)\triangleq\begin{bmatrix}\mathbf{k}_N(n)\\ k(n)\end{bmatrix}$$

$$\mathbf{X}_{1,N}(n)=\begin{bmatrix}z^{-1}\mathbf{x}(n) & \ldots & z^{-N}\mathbf{x}(n)\end{bmatrix}=\begin{bmatrix}\mathbf{0}_N^T\\ \mathbf{X}_{0,N-1}(n-1)\end{bmatrix}$$

$$\mathbf{G}_N(n-1)=\mathbf{K}_{0,N-1}(n-1)\boldsymbol{\pi}(n-1)=\mathbf{K}_{1,N}(n)\boldsymbol{\pi}(n)$$

$$\mathbf{G}_{N+1}(n)=\begin{bmatrix}\mathbf{k}_N(n)\\ k(n)\end{bmatrix}=\begin{bmatrix}0\\ \mathbf{G}_N(n-1)\end{bmatrix}+\frac{e^f(n|n)}{\varepsilon^f(n)}\begin{bmatrix}1\\ -\mathbf{A}_N(n)\end{bmatrix}$$

$$\boxed{\mathbf{G}_N(n)=\mathbf{k}_N(n)+k(n)\mathbf{B}_N(n)}$$

3）前向预测误差滤波器参量的时间更新

利用横向滤波算子的性质及横向滤波算子的时间更新关系：

$$\boxed{\mathbf{A}_N(n)=\mathbf{A}_N(n-1)+\frac{e^f(n|n)}{\gamma_N(n-1)}\mathbf{G}_N(n-1)}$$

$$e^f(n|n)=\gamma_N(n-1)e^f(n|n-1)$$

$$e^f(n|n-1)=x(n)-\mathbf{x}_N^T(n-1)\mathbf{A}_N(n-1)$$

$$\varepsilon^f(n)=\varepsilon^f(n-1)+e^f(n|n)e^f(n|n-1)$$

4）后向预测误差滤波器参量的时间更新

利用横向滤波算子的性质及横向滤波算子的时间更新关系：

$$\boxed{\mathbf{B}_N(n)=\mathbf{B}_N(n-1)+\frac{e^b(n|n)}{\gamma_N(n)}\mathbf{G}_N(n)}$$

$$e^b(n|n)=\gamma_N(n)e^b(n|n-1)$$

$$e^b(n|n-1)=x(n-N)-\mathbf{x}_N^T(n)\mathbf{B}_N(n-1)$$

$$\varepsilon^b(n)=\varepsilon^b(n-1)+e^b(n|n)e^b(n|n-1)$$

5）角参量的时间更新

利用横向滤波算子的性质及横向滤波算子的时间更新关系：

$$\boxed{\gamma_{N+1}(n)=\gamma_N(n-1)\frac{\varepsilon^f(n)}{\varepsilon^f(n-1)}}$$

$$\boxed{\gamma_N(n)=\gamma_{N+1}(n)\frac{\varepsilon^b(n)}{\varepsilon^b(n-1)}}$$

$$\boxed{or \quad \gamma_N(n)=[1-k(n)e^b(n|n-1)]^{-1}\gamma_{N+1}(n)}$$

五 FTF自适应算法流程

$$\boxed{\mathbf{H}(n)=\mathbf{H}(n-1)+\frac{e(n|n)}{\gamma_N(n)}\mathbf{G}_N(n)}$$

$$e(n|n)=\gamma_N(n)e(n|n-1)$$

$$\mathbf{H}(n)=\mathbf{H}(n-1)+e(n|n-1)\mathbf{G}_N(n)$$

$$\boxed{\mathbf{G}_N(n)=\mathbf{k}_N(n)+k(n)\mathbf{B}_N(n)}$$

$$\mathbf{B}_N(n)=\mathbf{B}_N(n-1)+\frac{e^b(n|n)}{\gamma_N(n)}\mathbf{G}_N(n)$$

$$\mathbf{G}_N(n)=[\mathbf{k}_N(n)+k(n)\mathbf{B}_N(n-1)]\frac{\gamma_N(n)}{\gamma_N(n)-k(n)e^b(n|n)}$$

$$e^b(n|n)=\gamma_N(n)e^b(n|n-1)$$

$$\gamma_N(n)=[1-k(n)e^b(n|n-1)]^{-1}\gamma_{N+1}(n)$$

$$\mathbf{G}_N(n)=[\mathbf{k}_N(n)+k(n)\mathbf{B}_N(n-1)]\frac{\gamma_N(n)}{\gamma_{N+1}(n)}$$

$$\boxed{\mathbf{B}_N(n) = \mathbf{B}_N(n-1) + \frac{e^b(n|n)}{\gamma_N(n)}\mathbf{G}_N(n)}$$

$$\mathbf{G}_N(n) = [\mathbf{k}_N(n) + k(n)\mathbf{B}_N(n-1)]\frac{\gamma_N(n)}{\gamma_{N+1}(n)}$$

$$\mathbf{B}_N(n) = \mathbf{B}_N(n-1) + e^b(n|n-1)\mathbf{G}_N(n)$$

$$\boxed{\mathbf{A}_N(n) = \mathbf{A}_N(n-1) + \frac{e^f(n|n)}{\gamma_N(n-1)}\mathbf{G}_N(n-1)}$$

$$e^f(n|n) = \gamma_N(n-1)e^f(n|n-1)$$

$$\mathbf{A}_N(n) = \mathbf{A}_N(n-1) + e^f(n|n-1)\mathbf{G}_N(n-1)$$

FTF自适应算法流程：

1 初始化

$$\mathbf{A}_N(0)=\mathbf{0},\mathbf{B}_N(0)=\mathbf{0},\mathbf{H}_N(0)=\mathbf{0},\mathbf{G}_N(0)=0,\gamma_N(0)=1.0$$

$$\varepsilon^f(0)=\varepsilon^b(0)=\delta,0<\delta<1$$

2 按时间叠代计算（n=1,2,…)

(1)前向预测误差滤波器参量的时间更新

$$e^f(n|n-1)=x(n)-\mathbf{x}_N^T(n-1)\mathbf{A}_N(n-1)$$

$$e^f(n|n)=\gamma_N(n-1)e^f(n|n-1)$$

$$\varepsilon^f(n)=\varepsilon^f(n-1)+e^f(n|n)e^f(n|n-1)$$

$$\mathbf{A}_N(n)=\mathbf{A}_N(n-1)+e^f(n|n-1)\mathbf{G}_N(n-1)$$

(2) N+1阶角参量的时间更新和阶次更新

$$\gamma_{N+1}(n)=\frac{\varepsilon^{f}(n-1)}{\varepsilon^{f}(n)}\gamma_{N}(n-1)$$

(3)N+1阶增益滤波器权矢量的时间更新和阶次更新

$$\mathbf{G}_{N+1}(n)=\begin{bmatrix}\mathbf{k}_{N}(n)\\ k(n)\end{bmatrix}=\begin{bmatrix}0\\ \mathbf{G}_{N}(n-1)\end{bmatrix}+\frac{e^{f}(n|n)}{\varepsilon^{f}(n)}\begin{bmatrix}1\\ -\mathbf{A}_{N}(n)\end{bmatrix}$$

（4）后向预测误差滤波器参量， N阶角参量， N阶增益滤波器权矢量的时间更新

$$e^{b}(n|n-1)=x(n-N)-\mathbf{x}_{N}^{T}(n)\mathbf{B}_{N}(n-1)$$

$$\gamma_{N}(n)=[1-k(n)e^{b}(n|n-1)]^{-1}\gamma_{N+1}(n)$$

$$e^{b}(n|n)=\gamma_{N}(n)e^{b}(n|n-1)$$

$$\varepsilon^b(n)=\varepsilon^b(n-1)+e^b(n|n)e^b(n|n-1)$$

$$\mathbf{G}_N(n)=[\mathbf{k}_N(n)+k(n)\mathbf{B}_N(n-1)]\frac{\gamma_N(n)}{\gamma_{N+1}(n)}$$

$$\mathbf{B}_N(n)=\mathbf{B}_N(n-1)+e^b(n|n-1)\mathbf{G}_N(n)$$

（5）最小二乘横向滤波器权矢量的时间更新

$$e(n|n-1)=y(n)-\mathbf{x}_N^T(n)\mathbf{H}_N(n-1)$$

$$\mathbf{H}(n)=\mathbf{H}(n-1)+e(n|n-1)\mathbf{G}_N(n)$$

六 FTF自适应算法的特点

- **FTF比LMS算法收敛速度快**
- **运算量：8N；**