PHYS 203 Final Project: Greenhouse Gas Climate Modelling

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Abstract

A model is developed to explore the effects of CO_2 and water vapour as greenhouse gases on the equilibrium temperature of the Earth's climate system. This model relies on the selective absorption of the two molecules and their effect on the Earth's outgoing black-body spectrum. The model is then implemented in Java and compared to measured climate data. Reasons are then proposed for any discrepancies seen between the model and real world data.

1 Background

Without the atmosphere, Earth would come to an equilibrium temperature of 255K with the Sun, which is 32K below the average surface temperature of 287K currently observed on Earth. This difference is due to the presence of selective absorbers, objects which only absorb and emit specific wavelengths of light, in the atmosphere of Earth. The most significant selective absorbers in the Earth's atmosphere are water vapor and carbon dioxide (CO_2). Other important selective absorbers such as methane (CH_4) and nitrous oxide (N_2O) exist but are not as significant as water vapor and carbon dioxide on the atmospheric greenhouse effect[IPC13]. The selective absorbers present in our atmosphere affect incoming radiations from the Sun and outgoing radiation emitted from the surface of Earth differently due to the different blackbody spectrum of the Sun and the Earth. The surface of the Sun, being at 5800K has a blackbody spectrum that emits almost all of its energy at wavelengths below 3um. In contrast, the surface of Earth, being at 288K, emits almost all of its radiation at wavelengths between 4μm to 30μm. While water vapor has some absorption in the range of wavelength emitted from the Sun, water vapor and CO_2 have a much higher absorption over the wavelengths of longwave radiation (LWR) emitted from Earth [AH18]. The different absorptivity of our atmosphere to solar radiation and LWR emitted from Earth is shown in Figure 1.

When the atmosphere has high absorptivity at a specific wavelength, any radiation emitted by the surface of Earth at this wavelength would be absorbed by the atmosphere. Therefore, any radiation of that wavelength that eventually make it to space would instead come from emission higher up in the atmosphere as only at that height would the atmosphere above be thin enough to allow the radiation to escape to space without getting absorbed again. The higher the absorptivity of the atmosphere to a wavelength, the higher the fraction of the radiation of that wavelength that reach space come from emission higher up in the atmosphere, which are colder and therefore emit at a lower power [ZH13]. Figure 2 shows the emission spectrum of Earth at the top of atmosphere (TOA) with blackbody radiation at different temperatures as reference.

The temperature at which a specific wavelength is primarily being emitted at can be deduced by looking at the blackbody spectrum which best fits the intensity being emitted at that wavelength as seen in Figure 2. The TOA emission spectrum closely matches the blackbody radiation of 287K,

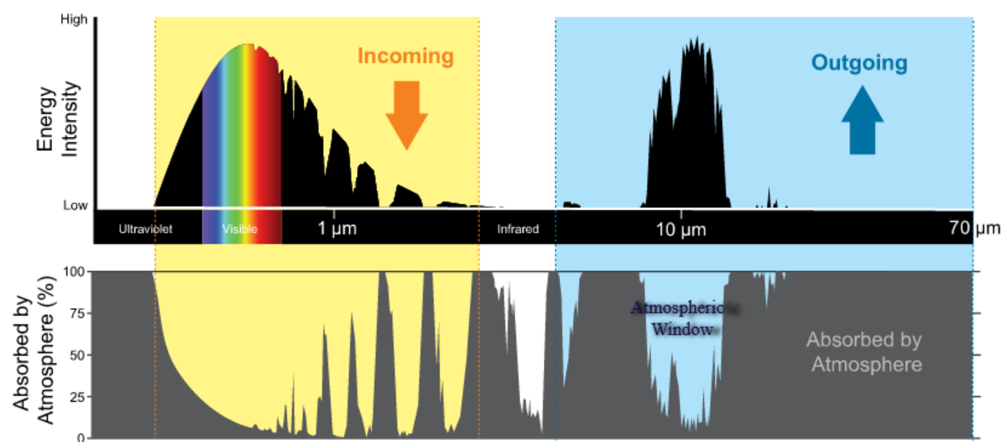

Figure 1: Intensity of outgoing and incoming radiation that directly passes through the atmosphere (above) Percent of outgoing and incoming radiation that is absorbed by the atmosphere (below). Figure taken from [US 19]

the temperature at the surface of Earth between 8-14 μm (715 - 1250 cm^{-1}). This means that the LWR emitted from the surface of Earth at those wavelengths can directly escape to space due to the low absorptivity of the atmosphere at those wavelengths. This region corresponds to the wavelengths where water vapor have very low absorptivity and are referred to as the atmospheric windows [Lin09].

Water vapor is also partially transparent from 14-19 μm (530 - 715 cm^{-1}). Almost all LWR emitted by the surface of Earth below 8μm and above 14μm are absorbed by water vapor in the atmosphere of Earth as seen in Figure 1. Figure 3 shows the absorption spectrum of CO_2 and water vapor on a logarithmic scale. The high absorptivity of the atmosphere to LWR below 8μm and above 14μm are primarily due to the high absorptivity of water vapor at these wavelengths. While water vapor has a broad absorption spectrum, CO_2 has strong absorption at peaks centered at 15μm and 4.3μm (670 cm^{-1} and 2330 cm^{-1}). These both correspond to points where water vapor shows low absorptivity and therefore the absorption at these wavelengths are not saturated by water vapor. The surface of Earth however, emits little radiation at 4.3μm, and therefore the most significant absorption peak of CO_2 occurs at 15μm, which is not only at wavelengths where water vapor has low absorptivity, but are also near the peak of the blackbody spectrum at 287K [ZH13]. The absorption by CO_2 at 15μm has a significant effect on the TOA emission in as seen in Figure 2.

2 Model

2.1 Method Overview

The basis of the model constructed is to do a time step simulation, and at each time step to find the total energy coming into the Earth as well as the energy radiating away from it. Based on this net energy, the estimated specific heat capacity of the Earth of 0.53 GJm^{-2}K^{-1} from [Sch07] for the Earth can be used to find the incremental change to the surface temperature. This is repeated many times to find the final equilibrium temperature and the rate at which the Earth achieves it.

To get the radiation coming into the Earth, we assume both the Earth and the sun are perfect blackbodies. Then the power coming out of the sun is $H_{sun} = \sigma T^4 A_{sun} = 5.67 \cdot 10^{-8} \cdot 5778^4 \cdot$

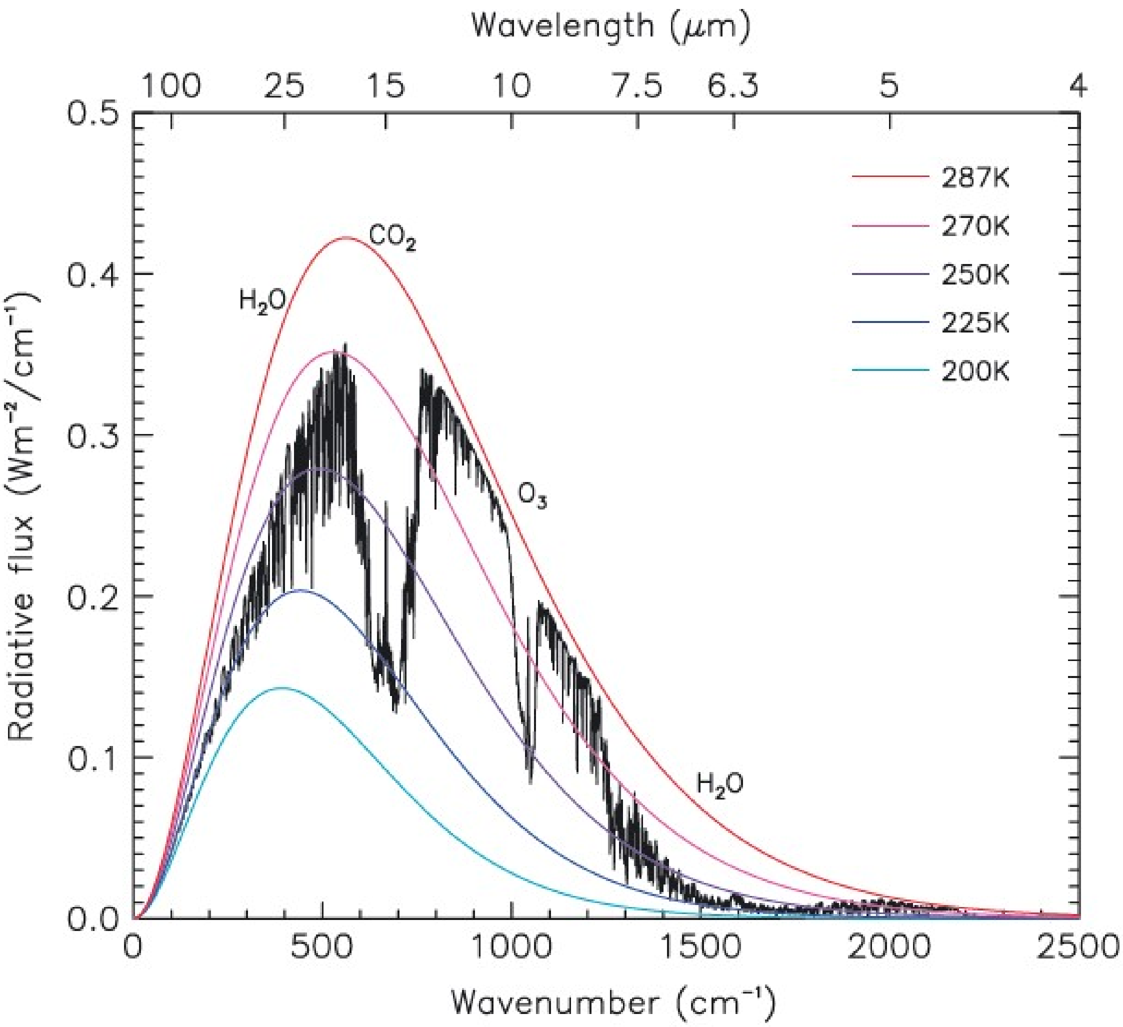

Figure 2: Spectrum of radiation emitted by Earth at the top of atmosphere (black curve). Colored curves are the blackbody spectrum at different temperature, with the red curve (287K) being the blackbody spectrum for the surface temperature of Earth. The points where TOA spectrum matches 287K blackbody are at wavelengths where the atmosphere have very little absorption in Figure 1 as it indicate that the radiation emitted at the surface are able to directly escape to space. Figure taken from [ZH13]

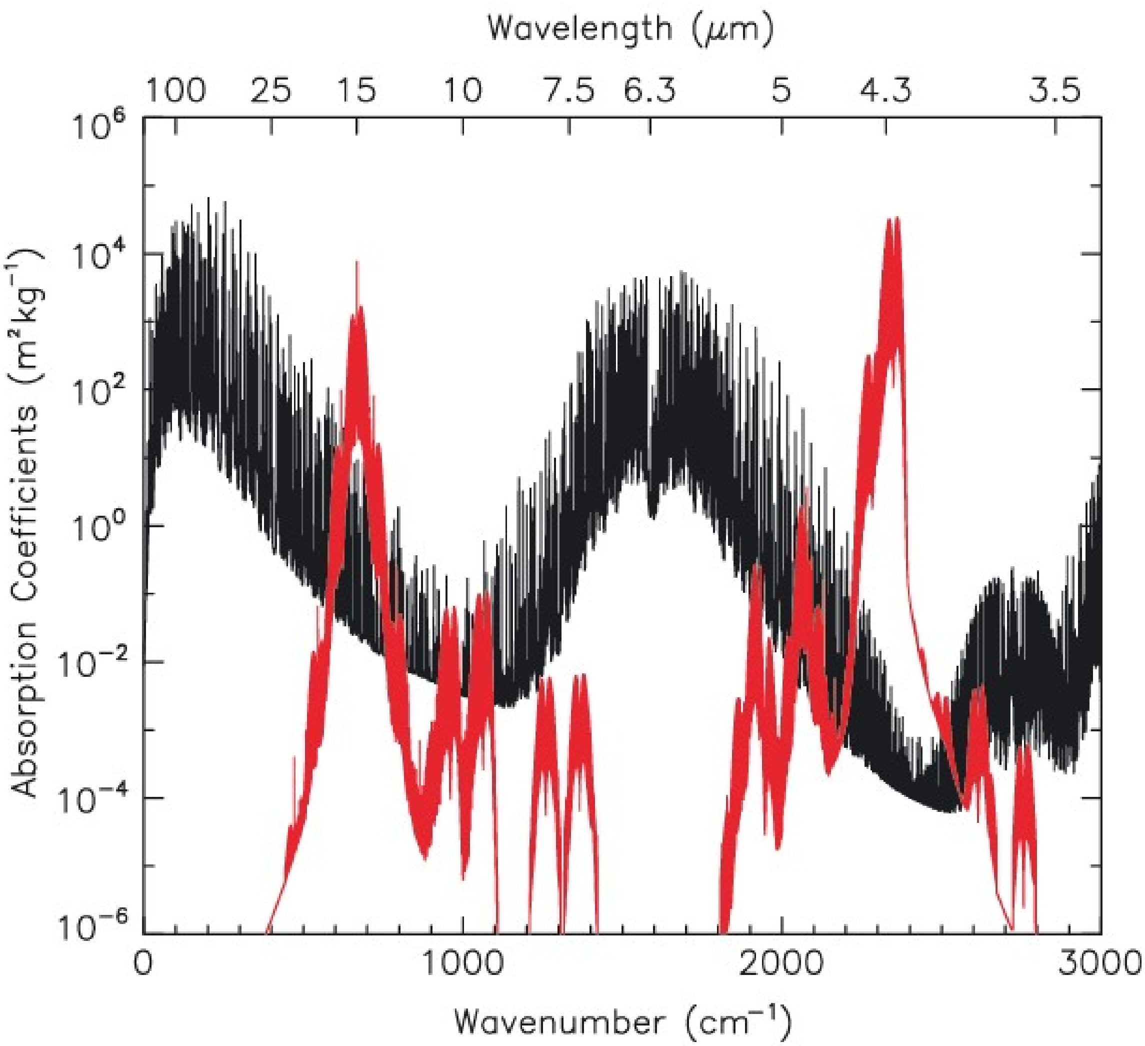

Figure 3: Absorption coefficient of CO2 and water vapor at different wavelengths emitted by the surface of Earth. Water vapor is Water vapor shows high absorptivity below 8μm and above 14μm. While the peaks in CO2 absorption coincide at 15μm and 4.3μm Figure taken from [ZH13]

$4\pi 696340000^2 = 3.85 \cdot 10^{26}$ W. Because the Earth is far away from the sun[Citation needed], only a fraction of this energy makes it to the Earth, which is proportional to the disk the Earth outlines on the sphere projected by the sun. Thus the power coming into the Earth is

$$H_{Earth} = H_{sun} \frac{A_{Earth-disk}}{A_{sun-sphere}} = 1.7 \cdot 10^{17} \text{ W} \tag{1}$$

To find the power radiating out of the Earth, wavelengths along the emitted spectrum are programmaticly looped over. Using the black-body emittance spectrum, it is known that

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)} \tag{2}$$

, where I is the spectral emittance, λ is the wavelength, and T is the temperature of the black-body. This can be used to find the infinitesimal power output per unit area of the Earth, which when repeated over the entire wavelength spectrum gives the total power output. However, the temperature in the formula depends on the wavelength in question, as depending on the absorbance of the atmosphere for the wavelength the wavelength could either be emitted from the high temperature surface or the low temperature upper troposphere. For example, if for a specific wavelength CO_2 absorbed 20% of the outgoing radiation and H_2O absorbed 30%, $(1 - 0.3)(1 - 0.2) = 56\%$ of the emission would be straight from the surface of the Earth, while the remaining 44% would be emitted from the top of the atmosphere. How exactly these percentages are determined is described in section 2.3. By summing all these small powers, we get the net power of the Earth and can thus find the temperature change as described above.

2.2 Greenhouse Gas Assumptions

The goal of the climate model constructed is primarily to demonstrate the effects of the two greenhouse gases H_2O and CO_2 on the equilibrium temperature of the Earth. The true climate system of the Earth is incredibly complex and includes many other factors, and for this reason several simplifying assumptions were made.

In the model it is assumed that the only greenhouse gases present on the Earth are water vapour and CO_2 gas. Importantly, it is also assumed that the absorption spectrum of both gases does not change with temperature nor concentration and is constant. The consequences of this assumption will be discussed later in this paper. To find what frequencies of light are absorbed by these two greenhouse gases, we can use publicly available data from [Lab]. Graphs of water vapour and CO_2 absorbance cross section can be seen in figure 4. In the model it is assumed that there are four regions in which the cross section of water behaves differently. For wavelengths less than 8 μm and greater than 19 μm, water absorbs all outgoing radiation (meaning all of it is emitted from the Earth at the lower atmospheric temperature). For wavelengths between 8 and 14 μm is is assumed that all radiation is let through by water vapour. Finally, for wavelengths between 14 and 19 μm it is assumed that water vapour has an absorbance cross section of $4.045 \cdot 10^{-22}$ cm^2/ molecule, the value of which was found by numerical integration of the absorbance cross section of water vapour in figure 4.

The cross section of carbon dioxide has a much simpler absorbance cross section estimate. Based on the graph of the absorbance cross section in figure 4, it was estimated that between wavelengths of 14.3 and 15.6 μm the average cross section was $0.613 \cdot 10^{-18}$ cm^2/ molecule and is approximately zero at all other relevant wavelength.

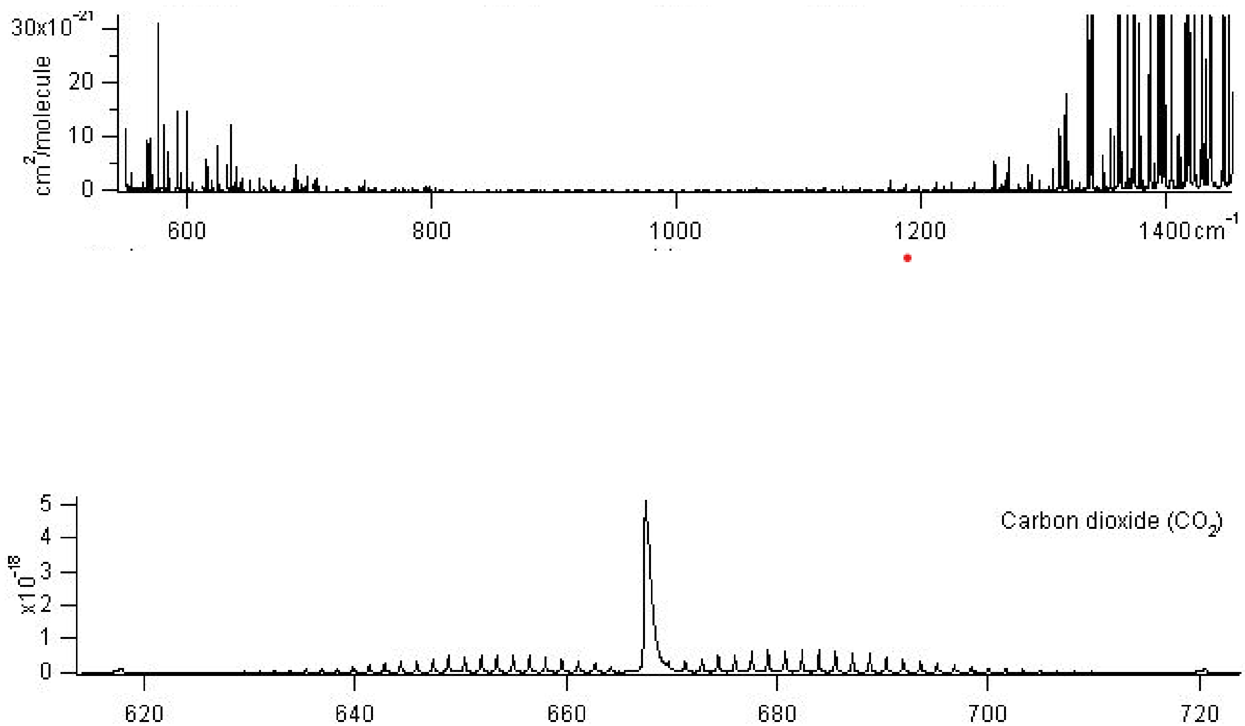

Figure 4: Absorbance cross section for H_2O (top) and CO_2 (bottom) molecules. Water has a high cross section outside of the shown range, while carbon dioxide absorbs no other relevant wavelengths. Graphs are taken from [Lab]

2.3 Calculating Absorbance

The percent of radiation absorbed by the atmosphere for a given wavelength and greenhouse gas was computed using Beer-Lambert Law for gas

$$A = \sigma_\lambda N l = -log(\frac{I_{Transmitted}}{I_{Emitted}}) \tag{3}$$

where σ_λ is the absorption cross-section at that wavelength, N is the number density, l is length traveled by the radiation and I is the intensity of the light.

Therefore $t = \frac{I_{Transmitted}}{I_{Emitted}} = e^{-\sigma_\lambda N l}$, and for a nonuniform medium, $t = e^{-\int_0^l \sigma_\lambda N dx}$. Since gas particles higher up in the atmosphere have a higher gravitational potential energy, it will have a higher chemical potential by mgz. Assume that the atmosphere is in diffusive equilibrium, we can derive that

$$N(z) = N(0)e^{-\frac{mgz}{kT}} \tag{4}$$

The transmission through the entire atmosphere will therefore be

$$t = e^{-\int_0^\infty \sigma_\lambda N(0) e^{-\frac{mgz}{kT}} dx} \tag{5}$$

Where $N(0)$ is the number density at the surface of Earth

This gives us

$$t = e^{-\sigma N(0)\frac{kT}{mg}} \tag{6}$$

Using the mean atmospheric temperature of 250K for T and mean molecular mass for m (in kg), we get $\frac{kT}{mg} = 7310$ m. Thus in each time step as the model loops over the many wavelengths possible, the percent that each wavelength is transmitted and calculated can be found using the estimations from section 2.2, and then factored into the appropriate infinitesimal power contribution to be summed.

2.4 Water Vapour Concentrations

As part of the absorbance model described in section 2.3, it is necessary to have the molar density of water vapour at every point of time. Unlike CO_2, which stays constant with temperature, H_2O changes molar density as the temperature changes and water evaporates as a result. Luckily, it is possible to estimate the vapour pressure of water as a function of temperature using a variety of formulas, the one chosen for this model is the August-Roche-Magnus equation as described in [AE96]. It states that the vapour pressure of water (in kPa) as a function of temperature (in celsius) is

$$P = 0.61094e^{17.625T/(T+243.04)} \tag{7}$$

Since water vapour acts approximately as an ideal gas, the molar density can then be estimated by using the ideal gas law:

$$\frac{N_{Water}}{V} = \frac{P_{Partial}}{RT} \tag{8}$$

This can then be used to find the absorbance of H_2O for a given wavelength of radiation.

2.5 Implementation

To implement this model, Java was used. The entire model as described above was put into code, and run with various parameters to explore the effect of the greenhouse gases on the Earth's temperature. The resulting code is available on Github to be seen[1].

3 Results

This section describes the usage of the model and the resulting graphs, while the following discussion section will talk about the meaning and implications behind each of the graphs.

3.1 Temperature Equilibrium

The first interesting thing to do with the model is to see what difference the addition of greenhouse gases has on the equilibrium temperature of the Earth. To show this, two versions of the Earth were modeled, one of which with all the greenhouse effects described above (with real Earth value for CO_2 concentration in the air), and the other with the assumption that all radiation was emitted directly from the Earth (and none from the top of the atmosphere). Both started at an arbitrary temperature of 270K, but as can be seen in figure 5, the resulting temperatures of the two versions of our world varied significantly. The non-greenhouse version came to the expected temperature of approximately 255K, but the greenhouse gas version came to approximately 283K. This is as expected, as the addition of greenhouse gases should lower the amount of energy emitted and thus raise the final equilibrium temperature to account for this.

[1] https://github.com/misprit7/PHYS-203-Final-Project

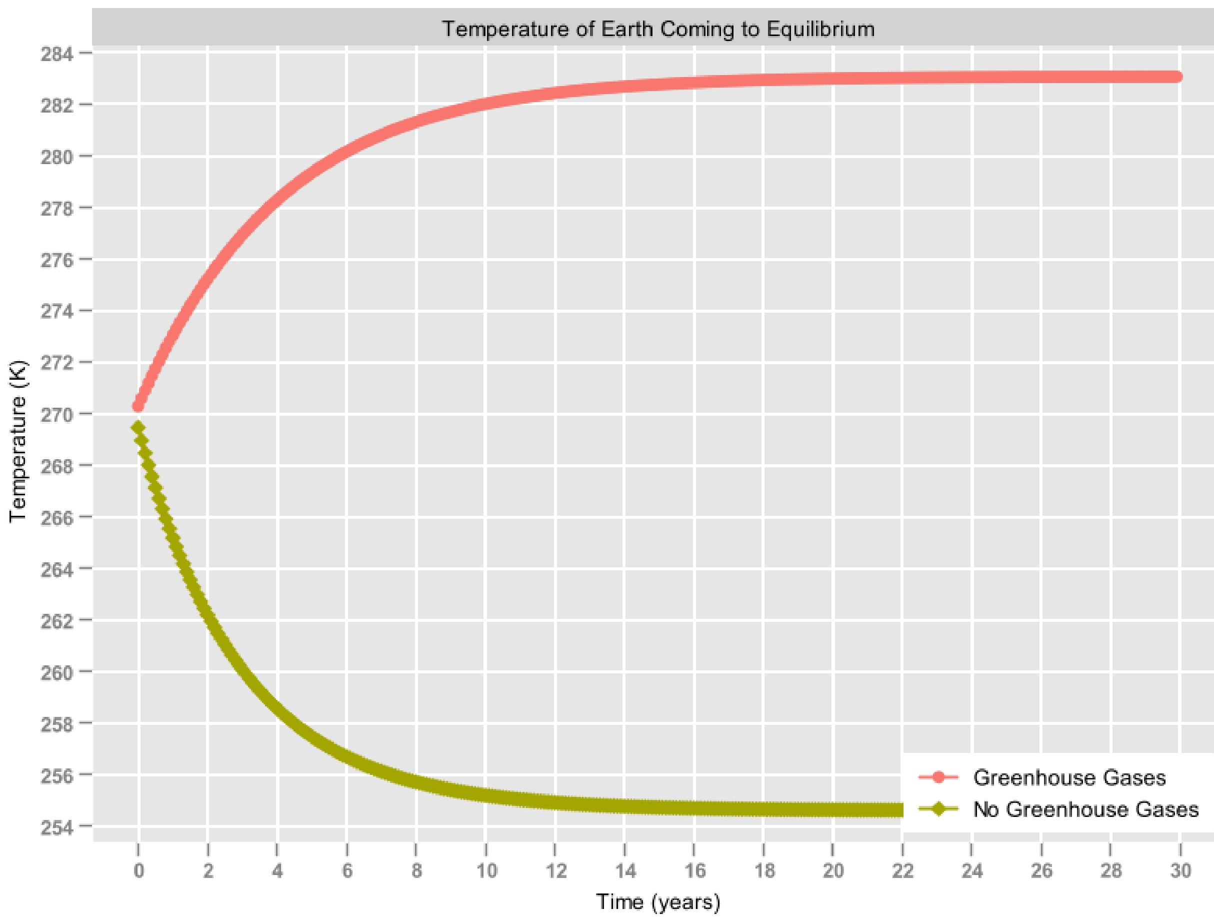

Figure 5: Earth coming to equilibrium with and without the effect of greenhouse gases at real Earth values. Both models started at the arbitrary temperature of 270K, but their final equilibrium temperatures varied significantly.

3.2 Emittance Spectrum

Another interesting usage of this model is to look at the resulting spectrum of intensities for the emitted wavelengths. For this emissions spectrum the real temperature and CO_2 concentrations for the Earth were used, and the resulting final spectrum accounting for all the effects described in the model section was graphed in figure 6. For reference, the spectrums for both the Earth and top of troposphere temperatures are also graphed, as these are what the spectrum would look like if regions had either complete or zero absorbance. It is clear that in the high and low wavelength regions the water vapour absorbs all outgoing radiation, hence it being emitted at a low temperature. In the atmospheric window more radiation is emitted at the higher surface temperature, but the region between 14.3 and 15.6 μm is blocked by the CO2 absorption as expected.

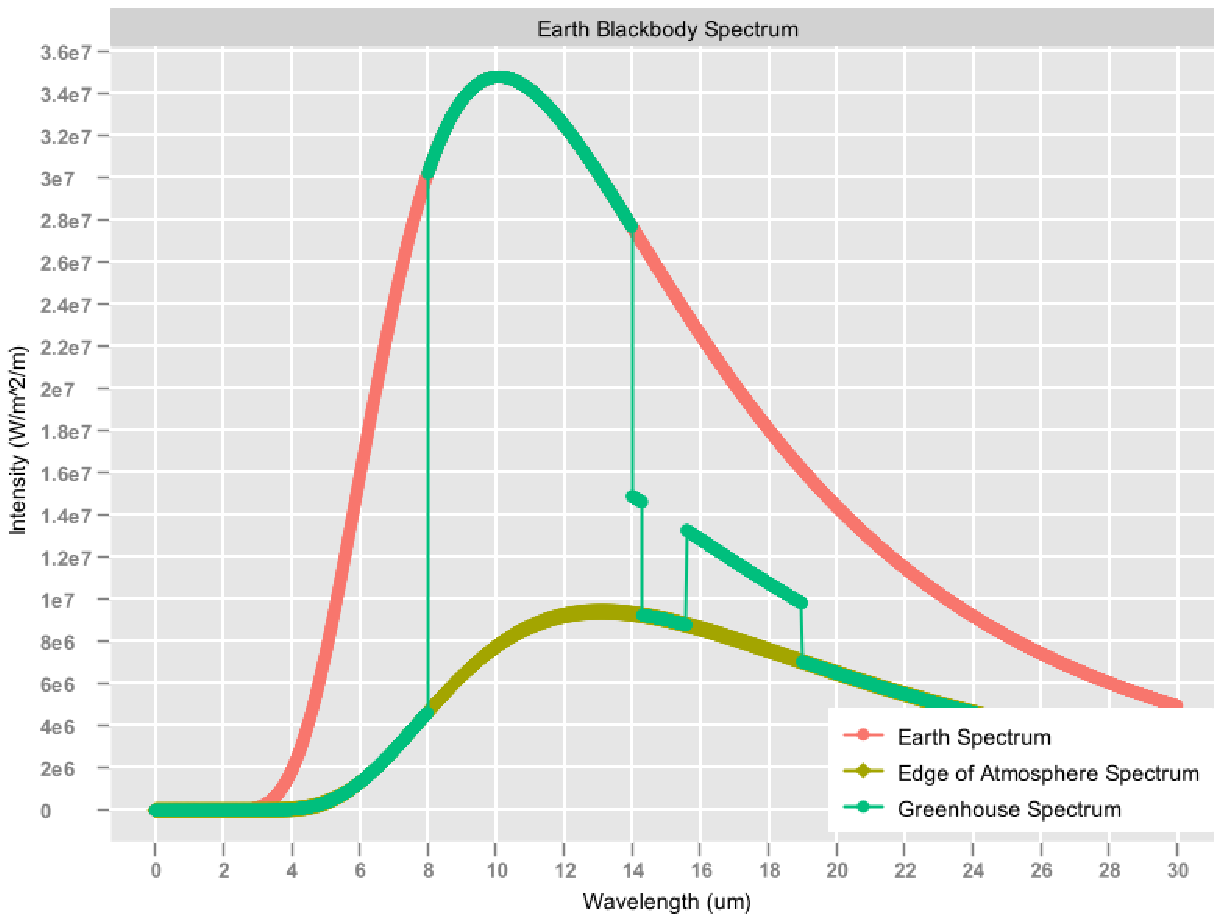

Figure 6: The absorbance spectrum of the Earth with real world temperature and CO_2 concentration. Earth and TOA spectra also graphed for reference.

3.3 Effect of CO2

Finally, the last interesting result of the model is to investigate the effect of CO_2 has on the final equilibrium temperature of the Earth. Instead of graphing one specific model vs. time, instead many models with different quantities of CO_2 were run and their resulting equilibrium temperatures were graphed as seen in figure 7. As can be seen, as the molar density of CO_2 increases the temperature also increases, but the effect this has levels off as the molar density of CO_2 gets yet higher. Interestingly, the real concentration of CO_2 in the atmosphere of $1.6 \cdot 10^{-8}$ isn't even on the graph, meaning that under the current model CO_2 is completely saturated already and wouldn't produce any additional change in temperature. The reason that this is not the case in the real world is described in the discussion section.

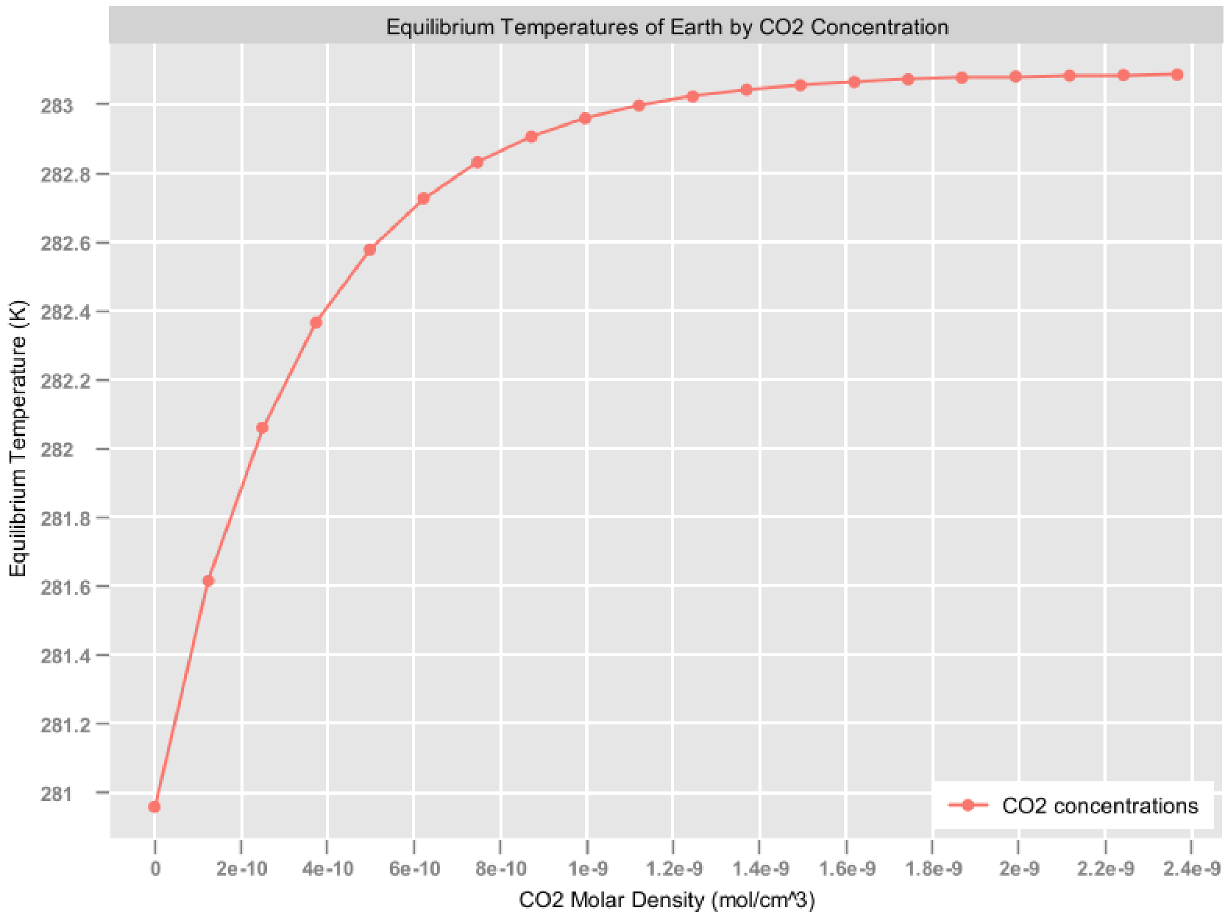

Figure 7: Equilibrium temperature of Earth as the molar density of CO_2 changes. Adding small amounts of carbon dioxide initially changes the temperature significantly, but the effect gradually levels off over time.

4 Discussion

This model is intended to determine the effect that a small concentration of CO_2 has on the greenhouse effect. With concentration of CO_2 close to what is at the surface of Earth currently, the model predicts an equilibrium temperature that is 28K higher than the equilibrium temperature without any greenhouse gas as seen is Figure 5. This demonstrates the significant effect that water vapour and CO_2 has as greenhouse gas on the average surface temperature of Earth. This is smaller than the 32K increase due to greenhouse gas currently measured at the surface of Earth. This is expected as the effect of other greenhouse gases such as methane and nitrous oxide is ignored in this model and also due to the saturation effect that our model displays for CO_2 beyond a very small concentration. The saturation effect do not occur in reality and is an artifact of the simplifying assumptions made in this model. The source of the saturation effect in this model is described later in this section.

The blackbody spectrum predicted by this model in Figure 6 shows some of the major feature seen in the measured TOA emission spectrum of Earth in Figure 8. Notably, both the predicted and the actual measured TOA emission spectrum displays major absorption feature due to water vapour below 8μm and above 18μm, as well as the absorption peak at 15μm due to CO_2. While there was almost no atmospheric absorption between 8-9μm, our model overestimated the transparency of the atmosphere to LWR within the atmospheric window between 9-14μm and ignored the absorption from O_3 at 10μm.

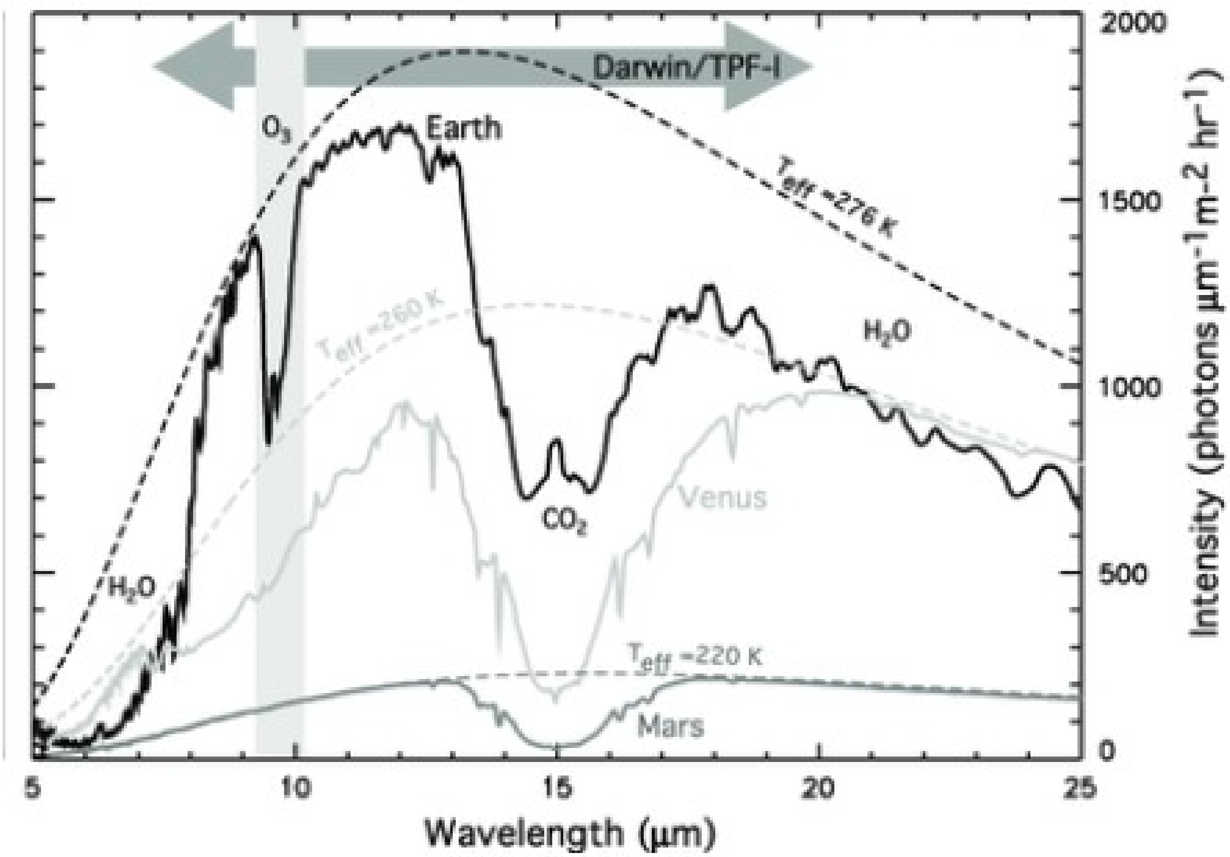

Figure 8: Emission Spectrum of Earth seen at the TOA. This shows major regions of absorption due to water vapor below 8μm and above 20μm, where the emission spectrum matches with the blackbody spectrum higher up in the atmosphere at 260K. There is also a large peak of absorption at 15μm due to CO_2 and at 10μm due to O_3. Figure taken from [Qua+19].

This model predicts that most of the greenhouse effect came from water vapour, with the equilibrium temperature without any CO_2 being at 280.8K, compared to an equilibrium temperature of 283.5K at the low saturating concentration of 1×10^{-9} mol/cm^3 as seen in Figure 7. While it is true that most of the greenhouse effect comes from water vapour, this model greatly underestimates the effect of CO_2 on the greenhouse effect at moderate to high concentrations of CO_2. To fully describe the contribution that CO_2 have on the greenhouse effect require much more sophisticated modeling, and the limitation of this model is described below.

The validity of this model is limited at concentrations of CO_2 beyond the concentrations at which CO_2 saturates in this model. This is due to the assumption that the absorption spectrum of CO_2 is independent of the concentration of CO_2 and that all absorbed radiation is emitted at the same height no matter the concentration. This model shows a saturation effect, where at high concentration of CO_2, the effect of additional CO_2 on average temperature becomes extremely small due to the fact that almost all radiation that can be absorbed by CO_2 is already absorbed. This is due to the fact that the Beer-Lambert's law gives a transmission that decays exponentially with density of CO_2.

In reality, the height at which the absorptivity of the atmosphere above becomes low enough for LWR to be able to escape to space increases with concentration of CO_2, where the temperature is lower. This leads to the LWR that escapes to space being emitted from layers of the atmosphere that are at even lower temperature, and therefore an even lower intensity of radiation will be emitted with higher concentration of CO_2 [WP07]. To simplify the model, only a single fixed height at which absorbed radiation is emitted to space was chosen, and we chose the height to be the height of the top of troposphere. This is because water vapor is only present at a significant concentration within the troposphere, and therefore the majority of LWR escapes near the top of the troposphere since there is little water vapour above [IPC13]. The stratosphere, the layer above the troposphere, has a very low concentration of water vapor compared to the due troposphere [11]. At higher parts of the atmosphere, due to lower pressure, the absorption spectrum of water vapor also becomes less continuous due to decreased effect of pressure broadening and therefore its absorption spectrum displays less overlap with CO_2. This lead to CO_2 having a larger region of wavelength where it dominates absorption [WP07]. This effect is also not modeled as we only used a single absorption spectrum. Detailed simulation also shows that the absorption band of CO_2 at 15 μm broadening at higher concentration. Therefore, even though the center of the absorption band at 15 μm would saturate at moderate concentration (current concentration of CO_2), the wings of the absorption band would not saturate even for a pure CO_2 atmosphere due to increased broadening with higher concentration. Changes in CO_2 absorption spectrum and the height at which LWR escapes means that a saturation effect never occurs in reality, and that the contribution of CO_2 to the greenhouse effect is greater than this model suggest.

The temperature of Earth also responds to an initial change in net flux in a complicated way that our model does not capture. Our model is based on a first order approximation of how temperature responds to a change in net flux based on the effective heat capacity of Earth. The effective heat capacity used was calculated by [Sch07] based on the heat capacity of the parts of Earth whose temperature changes significantly in the timescale of climate change, from 10-100 year, which is dominated by the heat capacity of the upper parts of the ocean. This heat capacity would increase over a longer time scale as a greater part of Earth responds to the change in temperature.

Climate feedback cycle also greatly changes the effect of an initial change in net flux. The water vapour feedback cycle, where higher temperature increases water vapour concentration which further intensifies the greenhouse effect, is one of the most significant feedback cycles. Water vapour feedback cycle was estimated to double the amount of warming caused by changes in net flux that would otherwise occur [DS09]. This feedback cycle is incorporated in our model due to its high importance. Other feedback cycles not included in the model include the ice-albedo feedback cycle, in which an increase in surface temperature causes less surface to be covered by ice, thereby decreasing the average albedo of the surface of Earth, leading to an increase in input flux. Changes in the height and prevalence of different types of cloud due to changes in evaporation is also thought to potentially amplify the effect of an initial change in net flux [AH18].

5 Conclusion

A simple model based on the selective absorptivity of CO_2 and water vapor was presented. This model predicted an increase in the average surface temperature of Earth by 28K compared to the average surface temperature of Earth without any greenhouse effect. This model was able to predict the strong effect that water vapor has on the equilibrium temperature of Earth. This model also demonstrated that CO_2 is also a significant greenhouse, and is predicted by the model to lead to an increase in the average surface temperature by over 2K from even a low density of 1×10^{-9} mol/cm^3. The top of atmosphere spectrum predicted shares major feature with observed data, further supporting that the absorption due to water vapour and CO_2 contributes very significantly to the greenhouse effect on Earth. The simplifying assumption that the absorption spectrum of CO_2 and that the height of the atmosphere at which absorbed radiation are emitted both being constant with concentration lead to the model to deviate from reality and predict a much lower greenhouse effect of CO_2 beyond a CO_2 concentration of 1×10^{-9} mol/cm^3.

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