PHYS 400 Homework 1

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Question a. Let $z = x + iy$, $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ for all the below. Then we have:

$$\frac{1}{2}(z + z*) = \frac{1}{2}(x + iy + x - iy) = x = \text{Re } z.$$

$$\frac{1}{2i}(z - z*) = \frac{1}{2}(x + iy - x + iy) = y = \text{Im } z.$$

$$(\text{Re } z_1)(\text{Re } z_2) - (\text{Im } z_1)(\text{Im } z_2) = x_1x_2 - y_1y_2 = \text{Re } (x_1x_2 - y_1y_2 + x_1y_2i + x_2y_1i) = \text{Re } (z_1z_2).$$

Question b. Again using $z = x + iy$:

$$\text{Im } \left|z^2\right| = \text{Im } \left(x^2 + ixy - ixy + y^2\right) = 0.$$
$$\text{Im } z^2 = \text{Im } \left(x^2 + 2ixy - y^2\right) = 2xy.$$

Question c. Expanding using the Taylor series for the exponential and recalling those for the trigonometric functions:

$$e^{ix} = \sum_{n=0}^{\infty}(-1)^n\frac{x^n}{n!} = \sum_{n \text{ odd}}^{\infty}(-1)^n\frac{x^n}{n!} + \sum_{n \text{ even}}^{\infty}(-1)^n\frac{x^n}{n!} = \cos x + i\sin x.$$

Question d.

$$\text{Re } z = \text{Re } (A\cos\theta + iA\sin\theta) = A\cos\theta.$$
$$\text{Im } z = \text{Im } (A\cos\theta + iA\sin\theta) = A\sin\theta.$$
$$z* = A\cos\theta - A\sin\theta = Ae^{-i\theta}.$$
$$|z| = \sqrt{zz*} = \sqrt{A^2e^{-i\theta}e^{i\theta}} = A.$$

Question e. Expanding:

$$e^{i\alpha}e^{i\beta} = \cos\alpha\cos\beta + i\cos\beta\sin\alpha + i\cos\alpha\sin\beta - \sin\alpha\sin\beta = e^{i(\alpha+\beta)} = \cos(\alpha+\beta) + i\sin(\alpha+\beta).$$

Taking the real and imaginary parts of the two sides gives us the standard trig identities:

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta.$$
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta.$$