

# PHYS 408 Homework 5

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08/04/24

**Question 1a.** From in class, we have that the density of cavity modes is  $\frac{8\pi}{\lambda^3\nu}$ . For a spectral range of 0.1nm, we that the number of cavity modes is approximately

$$\# \text{ Modes} = \frac{8\pi}{\lambda^3\nu} \cdot 120 \cdot 10^9 \cdot 0.2 \cdot 0.1 = 8.04 \cdot 10^{14}.$$

**Question 1b.** The spectral width range goes from 499.95 to 500.05nm, so it is of width  $\Delta\nu = c \left( \frac{1}{499.95 \cdot 10^{-9}} - \frac{1}{500.05 \cdot 10^{-9}} \right) = 120\text{GHz}$ . The number of  $\text{TEM}_{q,0,0}$  modes will be approximately the spectral width divided by the FSR, where  $\nu_F = \frac{c}{2d} = 750\text{MHz}$ , so  $2\frac{\Delta\nu}{\nu} = 320$ .

**Question 1c.** Assuming each mode is equally likely, we get that the probability is  $\frac{320}{8.04 \cdot 10^{14}} = 3.98 \cdot 10^{-13}$ .

**Question 1d.** Using the cross section formula:

$$\sigma = \frac{\gamma_{sp}\lambda^2}{4\Delta\omega} = 5.2 \cdot 10^{-18}\text{m}.$$

**Question 2a.** From in class, we have the following relation:

$$\log \frac{I_2}{I_1} + \frac{I_2 - I_1}{I_{sat}} = g_0 l_g.$$

Since  $g_0 l_g$  are the same in both cases, we can solve for  $I_{sat}$ :

$$\log 10 + \frac{9}{I_{sat}} = \log 7.5 + \frac{13}{I_{sat}} \implies I_{sat} = 13.9\text{W/cm}^2.$$

**Question 2b.** Using this same equation to solve for  $g_0$ :

$$g_0 = \frac{1}{l_g} \left( \log \frac{I_2}{I_1} + \frac{I_2 - I_1}{I_{sat}} \right) = 2.95\text{cm}^{-1}.$$

**Question 2c.** Again using the formula derived in class:

$$I_{ss} = I_{sat} \frac{g_0}{a} = 3.01 \frac{5.29}{0.01/1} = 4100.5\text{W/cm}^2.$$

**Question 2d.** Using  $I_2 = I_{ss}/2$  and solving for  $I_1$ , we can solve the following equation numerically:

$$\log \frac{I_{ss}}{2I_1} + \frac{I_{ss} - 2I_1}{2I_{sat}} = g_0 l_g \implies I_1 = 2009.5\text{W/m}^2.$$

**Question 2e.** Again, we derived expressions for both of these values in class:

$$T_{oc}^{(opt)} = \sqrt{g_0 l_g a} - a = 0.162.$$

$$I_{out}^{(opt)} = I_{sat} \left( \sqrt{g_0 l_g} - \sqrt{a} \right)^2 = 36.4.$$

**Question 3a.** For question 3, as question suggests we use the weak pump approximation. This means that  $R_p \tau \ll 1$ , and so  $g(I) = \frac{g_{0,weak}}{1+I/I_{sat}}$ . Note that  $g \propto N_2$  and  $P \propto N_2$ , so  $\frac{g(I)}{g_{0,weak}} = \frac{P_l}{P_0}$  (since  $g_{0,weak}$  and  $P_0$  are when no lasing occurs, while  $g(I)$  and  $P_0$  are for when it does). Plugging this into the original equation:

$$P_l = \frac{P_0}{1 + I/I_{sat}}.$$

**Question 3b.** We can relate  $I_{out}$  with  $I_2$  with  $I_2 = \frac{I_{out}}{T_{oc}} = 100\text{W/cm}^2$ . Plugging into part a:

$$1 = \frac{2}{1 + 100/I_{sat}} \implies I_{sat} = I_{sat} = 100\text{W/cm}^2.$$

**Question 4a.** Rate equations:

$$\dot{N}_1 = A_{21}N_2 + R_{13}(N_3 - N_1)$$

$$\dot{N}_2 = A_{32}N_3 - A_{21}N_2$$

$$\dot{N}_3 = -A_{32}N_3 - R_{13}(N_3 - N_1).$$

Using  $N = N_3 + N_2 + N_1$ :

$$\dot{N}_1 = A_{21}N_2 + R_{13}(N - 2N_1 - N_2)$$

$$\dot{N}_2 = A_{32}(N - N_1 - N_2) - A_{21}N_2.$$

Substituting given constants:

$$\dot{n}_1 = n_2 + R(1 - 2n_1 - n_2)$$

$$\dot{n}_2 = aR(1 - n_1 - n_2) - n_2.$$

**Question b.** Setting  $\dot{n}_{1,2} = 0$  and solving is a system of two linear equations with two unknowns, which gives:

$$n_1 = \frac{a+1}{aR+a+2}, n_2 = \frac{aR}{aR+a+2} \implies \Delta n = \frac{a(R-1)-1}{a(R+1)+2}.$$

**Question c.** It is a similar form as  $\frac{R-1}{R+1}$ , although with extra terms of  $-1$  and  $+2$ . When  $a$  is large it reduces to the form we saw in class though. To calculate  $a_{10\%}$ , we can solve directly (because the new form makes the numerator smaller and the denominator bigger we know that the real value will be lower than the in class estimate):

$$\frac{a(R-1)-1}{a(R+1)+2} = 0.9 \frac{R-1}{R+1} \implies \frac{a-1}{3a+2} = 0.3 \implies a_{10\%} = 16.$$

**Question d.** See figure 1, the parameters used were  $R = 2$  and  $a = 16$  as calculated. As expected, the equilibrium settles to within 10% of the theoretical value. The code used for it was:

```

import numpy as np
from scipy.integrate import solve_ivp
import matplotlib.pyplot as plt

R = 2
a = 16

F = lambda t,n: np.array([n[1]+R*(1-2*n[0]-n[1]), a*R*(1-n[0]-n[1])-n[1]])
t_eval = np.arange(0, 4.01, 0.01)

sol = solve_ivp(F, [0,4], [1, 1], t_eval=t_eval)

plt.plot(t_eval, sol.y.T[:,0], label='$n_1$')
plt.plot(t_eval, sol.y.T[:,1], label='$n_2$')
plt.axhline(y=(R-1)/(R+1)+sol.y.T[:,0][-1], color='g', linestyle='--', label='$\frac{R-1}{R+1}$')
plt.axhline(y=(R-1)/(R+1)*0.9+sol.y.T[:,0][-1], color='y', linestyle='--', label='$\pm 10\%$')
plt.legend()
plt.title('Numerical Solution to Three Level Lasing System')
plt.xlabel('Time (s)')
plt.ylabel('Scaled number in level')
plt.show()

```

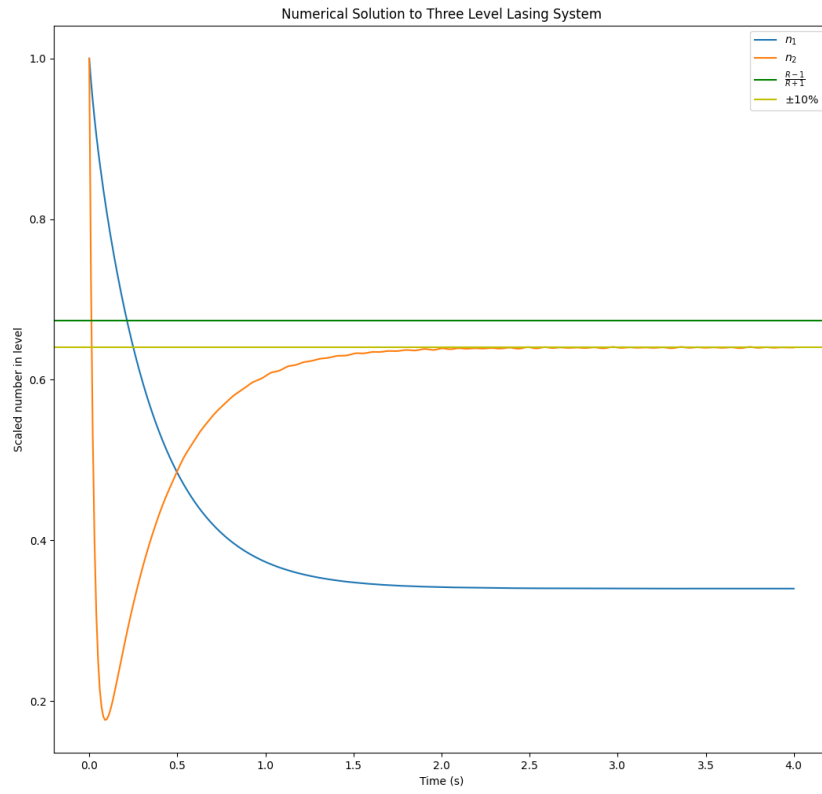


Figure 1: Numerical solution for question 4d. Arbitrary scaling for time and number, parameters used were  $R = 2$  and  $a = 16$ .