PHYS 408, 2023W2

Problem Set 5: Laser Physics

Posted: Fri, March 22 $\longrightarrow$ *Due*: Fri, April 12.

1. Consider a gas of molecules inside a cylindrical cavity, which is 20 cm long and has a cross-sectional area of 0.1 cm^2. Assume that the molecules have an optical resonance at $\lambda = 500$ nm with a spectral width of 0.1 nm.

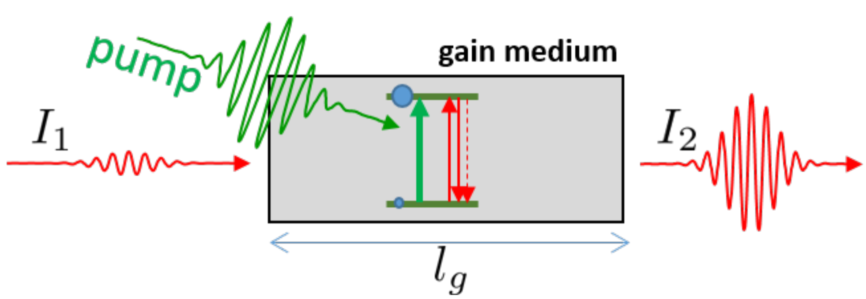
   (a) How many cavity modes exist in the frequency band of this molecular resonance?

   (b) How many longitudinal TEM$_{q,0,0}$ cavity modes fall under the same bandwidth? (Include both polarizations.)

   (c) Using the results from (a) and (b), estimate the probability of a spontaneous photon appearing in one of the polarized longitudinal modes.

   (d) If the spontaneous emission rate for this molecular transition is $\gamma_{sp} = 10^7$ s^{-1}, what is the absorption cross-section?

2. In Lectures 17 and 18, we have discussed the amplification properties of a typical gain medium. Consider a single-pass amplifier (illustrated in the figure below), for which you know the following: an input intensity $I_1 = 1\,\mathrm{W/cm^2}$ results in the output intensity of $I_2 = 10\,\mathrm{W/cm^2}$. If the input intensity is doubled to 2 W/cm^2, the output rises to $15\,\mathrm{W/cm^2}$. The length of the gain medium is $l_g = 1\,\mathrm{cm}$ and 1% of input light intensity is lost in a single pass through.



   (a) What is the saturation intensity I_{sat} of this mdium?

   (b) What is the small-signal gain g_0 of this amplifier?

   (c) What is the maximum intensity that can be extracted from this amplifier (in the limit of very high input I_1)?

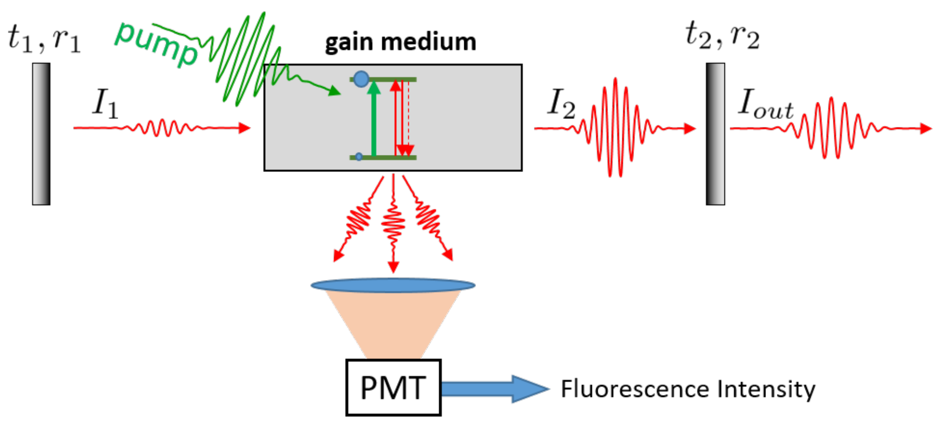
(d) What value of I_1 will result in extracting half of that maximum intensity (i.e. in addition to, or excluding, the input intensity I_1)?

(e) If you wanted to build a laser with this gain medium, what output coupler (in terms of its intensity transmission coefficient T_{oc}) would you use for the highest output power? And what would that output power be if the laser beam had an area of $1\,\text{mm}^2$?

3. When one aligns a real laser, it is often very useful to monitor the intensity of the spontaneous emission (fluorescence) escaping from the side of the cavity (see figure below). You have explored the effect of lasing on the fluorescence intensity in the HeNe lab. For this problem, use the four-level laser model which we discussed in Lecture 17. As we did in class, assume that the lower level population is negligible ($N_1 \approx 0$) and use the weak-pump approximation (explain what it means).

(a) If P_0 is the fluorescence power density (in W/cm^3) that is observed with one of the cavity mirrors blocked and P_l is measured when the laser is operating normally, derive an expression that relates P_0 and P_l to the intensity of the laser light I_l and the saturation intensity I_{sat} of the gain medium.

(b) If the side fluorescence is observed to get suppressed by 50% when the laser is working and its output intensity is $I_{out} = 10\,\text{W/cm}^2$, what is the saturation intensity of the gain medium, I_{sat}? Assume that the intensity transmission of the output coupler is 10% (i.e. $T_{oc} = |t_2|^2 = 0.1$).



4. Examine the validity of the adiabatic approximation in the three-level laser model discussed in Lecture 17.

(a) Write down the rate equations for N_1, N_2 and N_3 without the adiabatic elimination of level 3, i.e. without assuming (as was done in class) that $N_3, \dot{N}_3 \approx 0$. Then modify them in two ways. First, eliminate N_3 by letting N being the total number density of the atoms, and use the conservation of atom number. Second, write the remaining two equations for $\dot{N}_{1,2}$ in terms of the normalized populations $n_{1,2} := N_{1,2}/N$, scaled time $t' := A_{21}t$, scaled pumping rate $R := R_{13}/A_{21}$, and the adiabaticity parameter $a := A_{32}/R_{13}$.

(b) Calculate the steady-state population inversion $\Delta n_{exact} = (n_2 - n_1)$ by setting $\dot{n}_{1,2} = 0$ in the differential equations obtained in (a).

(c) Compare the results from (b) with the approximate adiabatic-limit inversion obtained in class [$\Delta n_{adiabat} = (R-1)/(R+1)$]. Using $R = 2$, calculate the adiabaticity factor $a_{10\%}$ for which the exact solution is within 10% from the approximate adiabatic limit.

(d) For bonus points, solve the system of two differential equations obtained in (a) using a numerical ode solver of your choice. Plot your result and confirm that with the adiabaticity parameter obtained in (c) the numerically calculated population inversion indeed reaches within 10% from the adiabatic limit.