Math 443 Homework 10

Xander Naumenko

12/04/23

Question 1. The chromatic coloring is 4. As seen in figure 1, the graph is planar, so it has $\chi(G) \leq 4$. Since K_3 is an induced subgraph we also have that $\chi(G) \geq 3$. To see why it can't be 3, consider figure 1. By contradiction assume that it was 3-colorable. Without loss of generality color vertices a, b, c with red, green and blue respectively. Then d must be green, e must be blue and f is green, but this is a contradiction since b is also green. Thus it couldn't have been 3-colorable and the only possibility is that $\chi(G) = 4$.
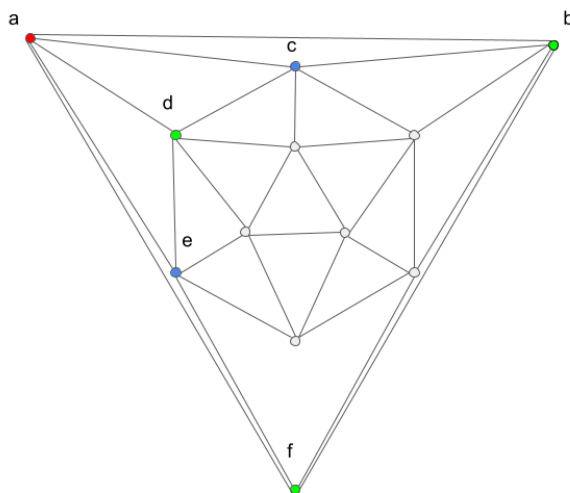

Figure 1: Contradiction proof sketch for question 1.

Question 2. The map is 3 colorable, see figure 2. Since there is a K_3 is a subgraph it must also be at least 3, so $\chi(\text{Canada}) = 3$.

Question 3a. $\chi(G) = 3$. G contains K_3 as a subgraph, so $\chi \geq 3$. As seen in figure 3 we also have that $\chi(G) \leq 3$ since we found an example of a 3 coloring. Thus $\chi(G) = 3$.

Question 3b. The minimum number is 2. Clearly it's at least 2, since there is more than two regions. We can also see that there is an example of a 2-region-coloring in figure 4. Thus the minimum number is 2.

Question 3c. The answer is $\chi'(G) = 6$. Although justification isn't required, one such coloring is in figure 5.

Figure 2: Map of Canada, vertex colors are graph coloring, not region coloring.
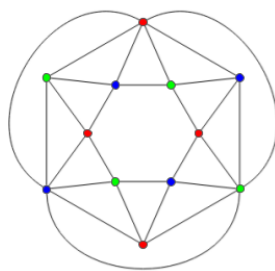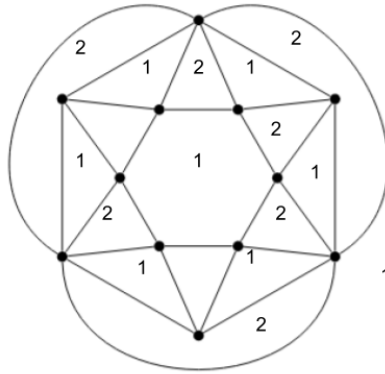
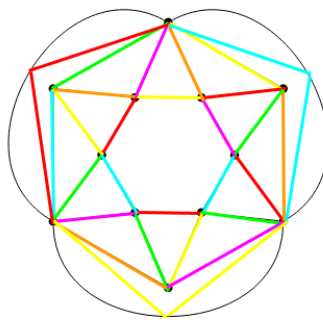

Figure 3: Coloring for question 3a.

Figure 4: Coloring for regions of 3b.

Figure 5: Coloring for question 3c.