HOMEWORK 1: DUE SEPTEMBER 28TH

MATH 437/537: PROF. DRAGOS GHIOCA

Problem 1. (4 points.) For each real number x, we let $[x]$ be the integer part of x, i.e., the largest integer less than or equal to x; for example, $[2.3] = 2$, $[5] = 5$ and $[-3.6] = -4$.

For each $m \in \mathbb{N}$, prove that there exists $n \in \mathbb{N}$ such that

$$m = \left[\frac{n}{\sqrt{n+1}}\right].$$

Problem 2. (5 points.) An (infinite) arithmetic progression in $\mathbb{N}$ is a set of the form $\{an+b\}_{n\geq 0}$ for some given $a, b \in \mathbb{N}$.

If the set S is the complement in $\mathbb{N}$ of a union of finitely many arithmetic progressions, then prove that S is a union of a finitely many arithmetic progressions along with a finite set.

Problem 3. (10 points.) Find all odd positive integers m and n with the property that

$$n \mid (3m+1) \text{ and } m \mid (n^2+3).$$