Math 220 Quesion 9 Question 8

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Question 8. There is only one such function: $f(n) = n$. To show this we will us proof by strong induction on n.

Base Case (n=1): Since $f(n) \leq n \forall n \in \mathbb{N}$, the only value of $f(1)$ such that $f(1) \leq n = 1$ is $f(1) = n = 1$.

Inductive Step: Suppose that $f(m) = m \forall m \leq n$. Then since $f(n)$ is injective it must be the case that $\forall m \in \{1, 2, \ldots n - 1, n\}$, $f(n + 1) \neq f(m)$. Thus it must be the case that $f(n + 1) > n$. However we also have that $f(n + 1) \leq n + 1$ by the original hypothesis, so these two inequalities together mean that the only possible case is that $f(n + 1) = n + 1$.