Problem set 1 (due on Jan. 26 by 4pm)

1.Let $\vec{b} = b_x\vec{e}_x + b_y\vec{e}_y + b_z\vec{e}_z$ and $\vec{c} = c_x\vec{e}_x + c_y\vec{e}_y + c_z\vec{e}_z$. If $\vec{a} = \vec{e}_x$, verify that $\vec{a}\times(\vec{b}\times\vec{c}) = \vec{b}(\vec{a}\cdot\vec{c}) - \vec{c}(\vec{a}\cdot\vec{b})$. Now argue that this identity holds for any vector $\vec{a} = a_x\vec{e}_x + a_y\vec{e}_y + a_z\vec{e}_z$.

2. Assume that the trajectory of an object of mass m is described (in appropriate units) by:

$$\begin{cases} x(t) = \frac{3}{5}t\cos(t^2) \\ y(t) = \frac{3}{5}t\sin(t^2) \\ z(t) = \frac{4}{5}t \end{cases}$$

Find the components of the speed and acceleration both in cartesian and in cylindrical coordinates. What is the net force acting on this object?

3. For the following functions, compute $\frac{\partial f}{\partial q}$, $\frac{\partial f}{\partial \dot{q}}$, $\frac{df}{dt}$ and $\frac{d}{dt}(\frac{\partial f}{\partial \dot{q}})$. Find the expression for the total derivative $\frac{df}{dt}$ if $q(t) = t/(t+1)$.

(i) $f(q,\dot{q},t) = q\exp(\dot{q}t)$

(ii) $f(q,\dot{q},t) = tq^2 + q\cos(\dot{q}^2)$

Check that if you use $q = t/(t+1)$ to replace for q and $\dot{q}$ directly in f, you find the same expression for df/dt.

4. Geometric meaning of vector dot and cross product: (i) take two non-collinear (not parallel) vectors $\vec{a}$ and $\vec{b}$, and show that the area of the surface generated by the two (see Fig. 1a) is $\mathcal{A} = |\vec{a}\times\vec{b}|$. (ii) take a third vector $\vec{c}$ which is not coplanar with $\vec{a}$ and $\vec{b}$ and show that the volume generated (see Fig. 1b) is $\mathcal{V} = \vec{a}\cdot(\vec{b}\times\vec{c})$. Can you use this to show that $\vec{a}\cdot(\vec{b}\times\vec{c}) = \vec{b}\cdot(\vec{c}\times\vec{a}) = \vec{c}\cdot(\vec{a}\times\vec{b})$?

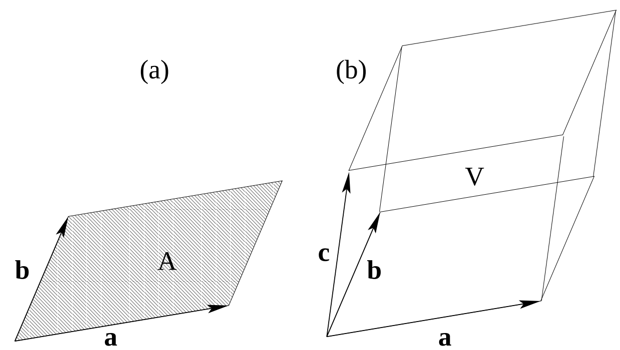

Fig 1.

5. Consider an incline of angle θ and mass M which can slide along the horizontal and has a mass m placed on it. There is no friction. Using Newtonian formalism, find the horizontal acceleration a_x of M, and the acceleration a of m *with respect to* M.

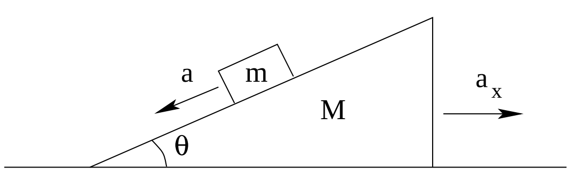

Fig 2.