

Math 220 Homework 5 Problem 8

October 22, 2021

Question 8. The integers that fulfill $n^3 > 2n^2 + n$ are only those that are greater than or equal to 3. Since I'm pretty sick of proof by induction by this point I'll instead prove this by cases for any positive integer n : either $0 < n < 3$ or $n \geq 3$. For $n < 3$ the cases are $n = 1$ and $n = 2$, so explicitly computing we get

$$1^3 = 2 \cdot 1 + 1 = 3$$

$$2^3 = 8 < 2 \cdot 2^2 + 2 = 10$$

Therefore positive integers less than 3 do not fulfill the inequality. The only remaining case is that $n \geq 3$. In this case we get

$$2n^2 + n < 2n^2 + n^2 = 3n^2 \leq n^3$$

Since all cases have been covered and confirm our proposed solution set we have that only the positive integers greater than or equal to 3 fulfill the inequality. \square