PHYS 304 Homework 7

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Question 1. Expanding in terms of energy eigenstates:

$$c_n(t) = \langle n|S(t)\rangle = \langle n| \left(\int |x\rangle \langle x|\right) dx \, |S(t)\rangle = \int \langle n|x\rangle \langle x|S(t)\rangle \, dx = \int \langle x|n\rangle^* \, \Psi(x,t)dx.$$

Since the potential is time independent, we can calculate the energy eigenfunctions:

$$\hat{H}f_n(x) = E_n f_n(x) \implies \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right) f_n = E_n f_n \implies f_n(x) = Ae^{i\frac{\sqrt{2m(E_n-V)}}{\hbar}} + Be^{-i\frac{\sqrt{2m(E_n-V)}}{\hbar}}.$$

Putting the energy eigenstates into the equation above:

$$c_n(t) = \int f_n(x)^* \Psi(x,t)dx.$$

Question 2. Expanding:

$$\langle n|\hat{x}|S(t)\rangle = \sum_{n'}\sum_{n''} \langle n| \, |n'\rangle \langle n'| \, \hat{x} \, |n''\rangle \langle n''| \, |S(t)\rangle \, .$$

Applying equation 3.114:

$$= \sum_{n'}\sum_{n''} \delta_{n,n'} \sqrt{\frac{\hbar}{2m\omega}} \left(\sqrt{n''}\delta_{n',n''-1} + \sqrt{n'}\delta_{n'',n'-1}\right) c_{n''}(t).$$

$$= \sum_{n''} \sqrt{\frac{\hbar}{2m\omega}} \left(\sqrt{n''}\delta_{n,n''-1} + \sqrt{n}\delta_{n'',n-1}\right) c_{n''}(t).$$

$$= \sqrt{\frac{\hbar}{2m\omega}} \left(\sqrt{n+1}c_{n+1}(t) + \sqrt{n}c_{n-1}(t)\right).$$

Question 3. Expanding:

$$\langle S(t)|x|S(t)\rangle = \iint \langle S(t)|p'\rangle \langle p'| \, \hat{x} \, |p''\rangle \langle p''|S(t)\rangle \, dp'dp''.$$

$$= \iint \Phi(p',t)i\hbar\frac{d}{dp'}\delta(p'-p'')\Phi(p'',t)dp'dp'' = \int \Phi(p',t)i\hbar\frac{d}{dp'}\Phi(p',t)dp'.$$

Question 4. Potential: $\Psi(x,0) = A\left(\psi_1(x) + \psi_2(x)\right)$