PHYS 304 Homework 2

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Question 1a. To normalize, integrate:

$$1 = \int_{-\infty}^{\infty} \psi^* \psi dx = \int_{-a}^{a} A^2(a^2 - x^2)^2 dx = \int_{-a}^{a} A^2(x^4 - 2a^2x^2 + a^4)dx = A^2\left(\frac{2}{5}a^5 - \frac{4}{3}a^5 + 2a^5\right) = A^2\frac{16}{15}$$

$$\implies A = \frac{\sqrt{15}}{4}.$$

Question 1b. To find the average value, apply the $\langle x \rangle$ operator:

$$\int_{-\infty}^{\infty} \psi^* x \psi dx = \int_{-a}^{a} A^2(a^2 - x^2)^2 x dx = \int_{-a}^{a} A^2(x^5 - 2a^2x^3 + a^4x)dx.$$

Because all the x terms are odd and we're integrating around 0, the integral goes to zero and the expectation value is $\langle x \rangle = 0$.

Question 1c. To calculate the momentum we use the momentum operator:

$$\langle p \rangle = -i\overline{h} \int_{-\infty}^{\infty} \psi^* \frac{\partial \psi}{\partial x} dx = -i\overline{h} \int_{-a}^{a} A^2(a^2 - x^2)2x dx.$$

Because all the terms in the integral are once again odd, the integral goes to zero so $\langle p \rangle = 0$.

Question 1d. Taking the integral:

$$\int_{-\infty}^{\infty} \psi^* x^2 \psi dx = \int_{-a}^{a} A^2(x^6 - 2a^2x^4 + a^4x^2)dx = A^2a^7\left(\frac{2}{7} - \frac{4}{5} + \frac{2}{3}\right) = A^2a^7\frac{16}{105} = \frac{1}{7}a^7.$$

Question 1e. Again taking the integral:

$$\langle p \rangle = -\overline{h} \int_{-\infty}^{\infty} \psi^* \frac{\partial^2}{\partial x^2} \psi dx = \overline{h} \int_{-a}^{a} 2A^2(a^2 - x^2)dx = 2\overline{h}A^2a^3\left(2 - \frac{2}{3}\right) = \frac{8}{3}A^2a^3\overline{h}.$$