PHYS 304 Homework 5

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Question 1.
Question 4a.

$$A + B = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 3 \\ 3i & 3-2i & 4 \end{pmatrix}.$$

Question 4b.

$$AB = \begin{pmatrix} -3 & 1+3i & 3i \\ 4+3i & 9 & 6-2i \\ 6i & 6-2i & 6 \end{pmatrix}.$$

Question 4c.

$$[A,B] = AB - BA = \begin{pmatrix} -3 & 1+3i & 3i \\ 4+3i & 9 & 6-2i \\ 6i & 6-2i & 6 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 3 \\ 6+3i & -3i & 12 \end{pmatrix} = \begin{pmatrix} -3 & 1+3i & 3i \\ 2+3i & 9 & 3-2i \\ -6+3i & 6+i & -6 \end{pmatrix}.$$

Question 4d.

$$\tilde{A} = \begin{pmatrix} -1 & 2 & 2i \\ 1 & 0 & -2i \\ i & 3 & 2 \end{pmatrix}.$$

Question 4e.

$$A^* = \begin{pmatrix} -1 & 1 & -i \\ 2 & 0 & 3 \\ -2i & 2i & 2 \end{pmatrix}.$$

Question 4f.

$$A^\dagger = \begin{pmatrix} -1 & 2 & -2i \\ 1 & 0 & 2i \\ -i & 3 & 2 \end{pmatrix}.$$

Question 4g.

$$\det(B) = 2 \cdot 1 \cdot 2 - i(-i) = 4 - 1 = 3.$$

Question 4h. Row reducing:

$$\begin{pmatrix} 2 & 0 & -i & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ i & 3 & 2 & 0 & 0 & 1 \end{pmatrix} \implies \begin{pmatrix} 1 & 0 & -\frac{i}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{3}{2} & -\frac{i}{2} & -3 & 1 \end{pmatrix}.$$

$$\implies B^{-1} = \begin{pmatrix} a & & \\ 0 & 1 & 0 \\ -\frac{i}{3} & -2 & \frac{2}{3} \end{pmatrix}.$$