Math 220 Homework 3 Question 8

October 08, 2021

Question 8. Show that:
Question 8a. $f(x) = x^3 + 3x + 4$ is increasing.

Let $a, b \in \mathbb{R}$ with $a < b$. This assumption also gives us that $3a < 3b$ and $a^3 < b^3$ (the power of three doesn't change sign so the inequality is preserved). Putting this together and comparing terms elementwise we have

$$f(a) = a^3 + 3a + 4 < b^3 + 3b + 4 = f(b)$$

Since this is true regardless of the choice of a and b it is f is increasing. □
Question 8b. $g(x) = \sin x$ is not increasing.

Let $a = \frac{\pi}{2}$ and $b = \frac{3\pi}{2}$. Then $a < b$ but

$$f(a) = \sin\frac{\pi}{2} = 1 > -1 = \sin\frac{3\pi}{2} = f(b)$$

Since there is an a, b that fulfill the requirements but do not fulfill the result, the original statement must be false. □