Physics 408

Optics Laboratory

Department of Physics & Astronomy
UBC

2023/2024 Winter Term 2
(Last edited January 4, 2024 by V. Milner)

Chapter 1

Rules and Resources

1.1 Safety Rules – READ THIS IN FULL!

Please do not be apprehensive of these labs. If you are careful, the danger involved in working with laser beams used here is extremely minimal. However, if you fail to heed the following warnings, bad things may happen. Because of the importance of these safety rules, we will be penalizing (subtracting marks!) those who don't follow them.

Do Not enter the lab without permission

Even if you have card access, you are not allowed in the Optics Lab without one of the lab supervisors, i.e. one of the TAs, Dr. Van Dongen or Dr. Milner being present! No supervisor - no entry, no exceptions!

Do Not enter a curtained section without permission

If you see that the curtain around a particular section is closed, do not poke your head in! The occupants might be either doing some sensitive data collection, which requires darkness, or they may be aligning their laser beam and send it inadvertently towards the curtain. Either way, you will ruin their data or your eyes, so always ask from outside before entering.

Do Not touch the high voltage electrodes in the HeNe lab

There is a plastic casing surrounding the laser tube, there is no reason for you to remove this casing or to place your hands within the confines of it. People have died by mishandling laser power supplies. You won't, but there is no reason to test this theory.

Do Not stare into the laser beam

In this lab we use class 3R (1-5 mW) and 3B (5-500 mW) HeNe lasers. If for some reason the beam does impact near the area of your eyes your "blink reflex" should be enough to protect you for a very short exposure. That being said, you should never place your head/eye in the path of the beam to see where it is going or for any reason at all. This might also happen by accident if, for example, you bend down to the beam level or an optic is being moved or removed with the laser not being blocked. A small index card (provided) is a much better means of observing the path of the beam. It is also important to watch for stray reflections off of mirrors or other reflective surfaces. This means that any **watches, rings, or bracelets should be removed** before beginning this experiment.

If you do get a laser beam or scatter in your eye please warn your partner and do not try to repeat the process to figure out what happened. If possible block the beam or turn off the laser and report the incident to lab staff. If you want to check for laser scatter one method is to put your BACK to the experiment. With the room lights off place a white sheet of paper in FRONT of you where you would like to check if the laser light is hitting the paper.

Laser goggles are provided and should be used when you are aligning the optical components in your setup. If you feel uncomfortable with the light intensity or anything else laser related, please seek out assistance. Curtains and doors are provided to protect others from your laser equipment. Please be mindful to appropriately block out access to random bystanders so they do not have direct line of sight to your optical equipment.

Do Not touch any optical surface

It does not take many impurities on the optical surfaces (e.g. scratches, fingerprints, and even dust) to prevent lasing action from occurring in the HeNe lab, to ruin the cavity mode in the Cavity lab, to mess up the interference pattern in the Michelson lab, or to destroy the Fourier image in the Fourier lab. Either hold optical elements by their mounts or on their edges as applicable.

Do Not eat or drink in the lab rooms

If you must eat, do so in the hallway.

Do Not move optics mounts or other hardware fixed to the bench with screws painted in red

These are elements that are already in their proper place and moving them will make doing the lab impossible. If you don't have a tool to loosen a screw, you probably shouldn't be trying to move that component. If you do move an optical element held with red screws or suspect that such an element is not in its proper place, please immediately contact the lab TA or professor to help you reset this element - obtaining good data and completing the lab may depend on it.

Do Not place foreign objects close to the optical setup

It feels very convenient to place your laptop, or your paper notebook, right by your setup and type/write your report as you work. Do not do it! A computer monitor or a pen can easily scatter light into your eyes and cause serious injuries. Plastic covers, e.g. in the HeNe lab, were not designed or tested to hold extra weight and will easily brake if used as desk surfaces.

1.2 Logistics and Resources

How will the labs be marked?

All questions in the lab manual labeled with red upper-case letters, e.g. (A), (B), (C), etc., are for marks. Those which are labeled with blue lower-case roman numerals, e.g. (i), (ii), (iii), etc., are for bonus points. Although the weights of each point may be slightly different, you may expect that all of them will be pretty similar.

Working with lab desktop computers

Although you are welcome to bring and use your own laptop computer, you will need lab desktop computers for data collection. After you login to the lab computers with your CWL account, you will be able to store your files and data under `C:\Users\yourUsername\Desktop\yourFolders`. Please be mindful of the (relatively) limited space on the local hard drive and clean after yourself when you are done with a particular lab.

On every lab desktop, you will also see the following folders with read-only files:

- `C:\Users\public\Desktop\labName`
- `C:\Users\public\Desktop\hardware`
- `C:\Users\public\Desktop\software`

The `labName` directories, with `labName`=**Cavity**, **HeNe**, **Michelson** or **Fourier**, contain useful materials pertaining to each particular lab, including this manual and videos explaining the relevant optical alignment procedures.

The `hardware` folder contains subdirectories with technical spec sheets and manuals of all electronic, optical and opto-mechanical components. These are useful for various calibration procedures, which you are asked to do in these labs. Computer communication protocols can also be found in these folders.

For several experiments in this course you will use the Raspberry Pi CCD cameras to capture images or movies – please read Appendix A for details on how to operate these cameras in real time.

Finally, the `software` directory contains a number of examples of MATLAB scripts, demonstrating the way you can communicate with the hardware components used in all four labs, e.g. `raspiCamera.m` for capturing images and movies with a Raspberry Pi camera or `thorlabsStage.m` for controlling the position of a Thorlabs translation stage.

Feel free to copy these files (especially MATLAB scripts) to your own folders and modify them for your own purposes.

Pacing yourself

If you find yourself spending more than 10 or 15 minutes trying (unsuccessfully) to get the proper alignment of any element in any part of any experiment, you should seek the advice of a TA or lab instructor (or a friend!) to get you past this hurdle. The point of the lab is both to learn lab techniques (such as optical alignment) and to perform a certain set of experiments. Make sure you don't spend too much time on any given task since you are expected to complete each experiment (typically, up to six separate experimental tasks).

File sharing platform and feedback

As you work on each experiment within your lab, you will collect experimental data. To share the data with your lab partner, as well as with your TA, who may provide you with important feedback, we will use the Microsoft Teams platform. Look on Canvas for the link to your lab Team, appropriately called 'PHYS 408 L2X 2012W2 Optics', where X=A,B,C or D. Inside your Team, you will find multiple channels. The main public channel called 'General' will be used for general announcements, notifications and all labs related discussions within your section. On top of that, you and your lab partner will have a private channel with a name similar to 'Lab1_Cavity_Optics3' or 'Lab2_Fourier_Optics5', where the first part 'LabX' (X=1,2 or 3) says whether this is your first, second or third lab in this course; the second part is the type of the lab; and the third part 'OpticsYY' (YY=1,...,12) is the name of the desktop which serves the specific setup you are working on. These private channels will be pre-set by your TA and used for storing and sharing data within your group (the files are stored on your UBC OneDrive and can be accessed from anywhere), as well as for seeking feedback from your TA by showing your intermediate results and asking questions pertaining to your specific lab and group.

Lab report format and submission

As you work on each experiment in your lab, it is critical to keep a detailed and organized *electronic* lab notebook. Since the provided examples of communicating with experimental hardware and acquiring data are written in MATLAB (see `C:\Users\public\Desktop\software` on your lab desktop), and because MATLAB is an extremely powerful tool for data processing available to every UBC student, you are encouraged to use MATLAB "Live Scripts" as a means of keeping your electronic notes[1]. In the course of each lab, keep your electronic notebook in your private MS Teams channel, where it will be accessible to your TA and used for giving you feedback on your progress, providing help and guidance. At the end, you will submit your final lab report as a pdf file on Canvas. It is your responsibility to keep track of the deadlines, outlined on the main Canvas page.

1.3 Collaborations and academic integrity

Although you and your partner will be working in the lab together, and although we do encourage scientific collaboration among students working on the same project, it goes without saying that everybody is expected to complete their work independently. Most data acquisitions are relatively quick; hence, you and your partner should be using distinct data sets (probably, taken one after another) in your individual lab reports. If for some reason you want to share data with your partner (e.g. due to the very long data collection time), please get

[1] Python scripts and Jupiter notebooks will be accepted as well

an approval from your TA first. Close similarities between any two lab reports will be considered as plagiarism and will be treated with utmost seriousness following an official UBC policy on academic misconduct.

Chapter 2

Fourier Optics

2.1 Objective

Optical Fourier transform arising from the wave propagation through a lens is an amazing phenomenon which plays an important role in pattern recognition, image restoration, and optical image processing. For example, high-pass or low-pass filters can be placed in the Fourier plane of the lens to clean up the image from noise. Better yet, carefully tuned filters can be designed to block or to accentuate certain image features, such as a face or a letter, no matter where it is in the overall image. Because the Fourier transform happens as part of image formation, even large and complicated two-dimensional transforms are essentially instantaneous – they truly happen at the light speed. This allows to carry out complex image processing operations which would be exceptionally difficult computationally. In this lab you will explore optical Fourier transforms and play with their powerful capability of image filtering. Specific objectives are:

1. To explore and study Fourier transformations using a laser and lenses.
2. To learn the method of spatial filtering and Fourier filtering.
3. To investigate the diffraction of light from a slit.

2.2 Pre-lab Study

Before you start this lab, familiarize yourself with the following material from the course textbooks, "Fundamentals of Photonics" by Saleh & Teich and "Classical and Modern Optics" by Steck, which provide essential background for completing the required experimental tasks. Note that all this material has been covered (or will be covered - depending on your lab schedule), in class before (or after) your completion of this lab. In the latter case, you do not need to understand every single detail in the text (this will be thoroughly discussed in lectures), but you should form the general conceptual picture and become familiar with the mathematical tools used in the relevant topics. The following list refers to the **Third Edition (2019)** of the Saleh & Teich book (in red) as well as the Steck book (in blue).

- **Experiment 1:** Section 12.5.1 (Spatial filtering of a gaussian beam).
- **Experiment 2:** Section 4.3 Diffraction of Light, Section 4.2A (Fourier transform in the far field), Sections 4.3A and B (Fraunhofer and Fresnel diffraction from a slit).
- **Experiment 3:** Section 4.4C (Wave optics of a single-lens imaging system).
- **Experiment 4:** Section 12.3.2 (Thin lens as a Fourier transform computer).
- **Experiment 5:** Section 12.5 (Spatial filters).
- Appendices A, B, C.

2.3 Experiments

Familiarize yourself with the setup shown in figure 2.1. This is part of the basic optical configuration for this lab. The beam from the laser is first attenuated with a polarizer, and then expanded and collimated by the spatial filter and collimating lens (CL). The beam then passes through one of the objects used in this lab (e.g. a mesh). The optical wave transmitted by the object passes through the Fourier Transform lens (FTL) and, through the magic of wave propagation, the Fourier transform of the object is generated at the focal distance (f) after the FTL lens. That optical Fourier transform can then be (a) modified with various apertures (as you will do in this lab) and/or (b) imaged onto the image screen with a Fourier Plane Imaging Lens (FPIL), where it can be recorded with a Raspberry Pi CCD camera.

2.3.1 Aligning the spatial filter

A spatial filter is an optical tool which "cleans up" the beam from a light source, typically a laser, and produces a gaussian beam at the output. It consists of a very short focal length lens, a pinhole, and a mechanism to align the pinhole precisely at the lens focus.

Naively, a lens focuses a parallel beam of light to a point, but that is not a solution of the three-dimensional wave equation, even for a perfect beam and a perfect lens. As the beam gets small near the focus, it "self-diffracts" and fans out again never collapsing to a point. Stated a bit more formally, the solution to the paraxial wave equation near the focus is a gaussian beam, like that shown in Fig. 2.2.

Imagine that the laser beam is not perfect, and that a wavefront is a bit wrinkled, as is shown by the black curve at the left in Fig. 2.2. The main curved shape of the wave is focused in the center of the focal plane, whereas the wrinkled component is focused off-center. Think of the wave at the left as a superposition of a large-amplitude E-field with a smooth gaussian shape and a smaller-amplitude perturbation field, whose *wrinkles* correspond to high *spatial frequencies*. These high spatial frequency components are also focused in the focal plane (i.e. $z = 0$), but farther away from the optical axis. A pinhole properly placed near the focus blocks most of the perturbation beam

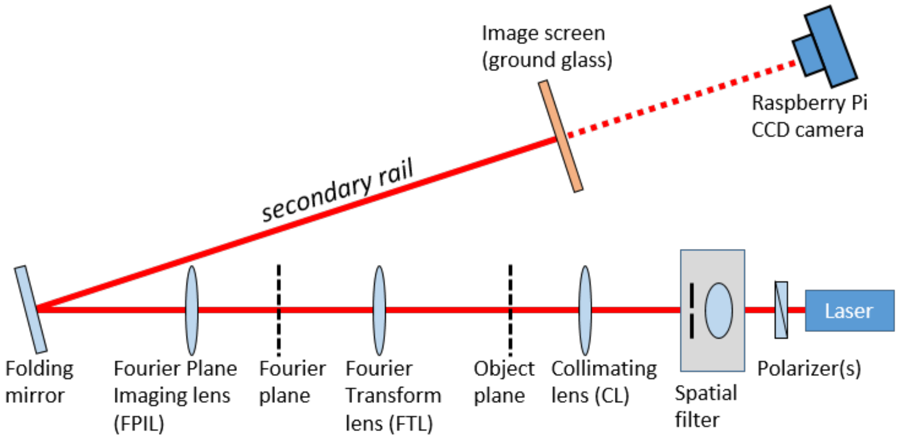

Figure 2.1: An optical configuration to explore Fourier Transform capabilities of lenses, used in this lab. Note: depending on the particular experiment, not all of the depicted components will be present at the same time.

(black noise below and above the focus), but does not intercept the main smooth beam (blue gaussian in the middle). In essence, a spatial filter is a Fourier filter, which utilizes the very same Fourier-transform property of a lens that you will be studying throughout this lab!

Typically (and in this lab as well), the spatial filter is comprised of a microscope objective (which is just a complex lens with a short focal length on the order of a few mm) and a pinhole mounted on translation stages. It should be placed just after the attenuating polarizers. The ideal solution of the wave equation provides a relation between the input beam diameter D, the focal length of the lens f, and the focal beam diameter $d_0 \equiv 2w_0$:

$$d_0 = \frac{4\lambda f}{\pi D}. \tag{2.1}$$

For the HeNe lasers we use in this lab, $\lambda = 0.63\ \mu$m and $D \approx 1$ mm, meaning that the microscope objective lens focuses the beam down to the size of a few microns. Hence, cleaning the beam calls for a tiny pinhole, a few microns in size, to be placed in the focal plane of the microscope objective. After passing through the pinhole, the (now clean) gaussian beam keeps diverging rapidly. To keep the size of this outgoing beam within reasonable limits, so that it can be used in the further experiments, a collimating lens is placed right after the spatial filter, as shown in both Fig. 2.1 and Fig. 2.3. Note, that the collimated beam is still a gaussian beam with a much smaller convergence angle (effectively converging at infinity).

(A) Remove everything from the main rail except the laser, and the polarizers right in front of the laser (the polarizers serve as a variable attenuator of the beam). Leave the large mirror at the end of the rail and the ground glass screen together with the raspberry pi camera on the secondary rail. **CAUTION:** Please place the white blocking screen (not the ground glass

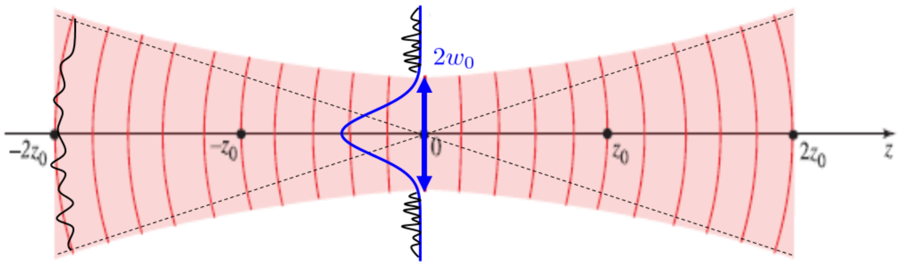

Figure 2.2: The profile of a gaussian beam near its focus. The dashed crossing straight lines show the naive ray-optics picture of focusing down to an infinitesimal point. In reality, the beam narrows down to the smallest diameter of $2w_0$. The red concentric lines represent the spherical wavefronts of an ideal gaussian beam, resulting in a "clean" gaussian intensity profile in the focus (blue curve). Adding phase noise to the incoming phase front (black line on the left) adds noise to the wings of the Gaussian in the focal plane, which can be removed if a pinhole is placed at the ideal focus.

screen) between the mirror and the laser so that the laser will not be reflected by that mirror (if it is not properly aligned) into someone's eyes. Before removing the blocking screen, make sure that you know where the beam will be going and make sure you are not sending it towards your own eyes or in the direction of other people around you.

Check that the laser is aligned so that it produces a beam which is centered over the optical rail and parallel to it. To check the alignment of the laser, place the pointed metal rod on the optical rail in a holder which allows no horizontal motion. Only if it is needed make small adjustments to the laser mount so that the beam remains on the tip of the rod as it is moved along the entire rail, approximately 20 cm above the rail. If you find that there is a place where the rail rolls a bit and the beam misses the tip, try to avoid placing lenses or objects at that position. If you feel like the beam is grossly misaligned, please consult with a TA before moving the laser too much.

Next, insert and align the spatial filter. With the white screen on the rail, you want to see a large uniform beam without diffraction rings. There are six adjustments which you can make on the spatial filter assembly. It can be rotated about a vertical axis by turning the clamped rod in the holder. This is not a critical adjustment. The assembly as a whole can be raised or lowered, and moved side to side by turning the black knobs on the optical mount. The pinhole itself can be moved up or down, and side to side using the silver knobs attached to it. These are critical adjustments. Finally, the microscope objective lens can be focused on the pinhole using the knob near it parallel to the optic axis.

With this many knobs, aligning the spatial filter is not a trivial process. Before starting it, please watch the video `FourierOptics-Alignment.mp4` in the `Fourier` folder on your lab desktop. Although the setup has changed

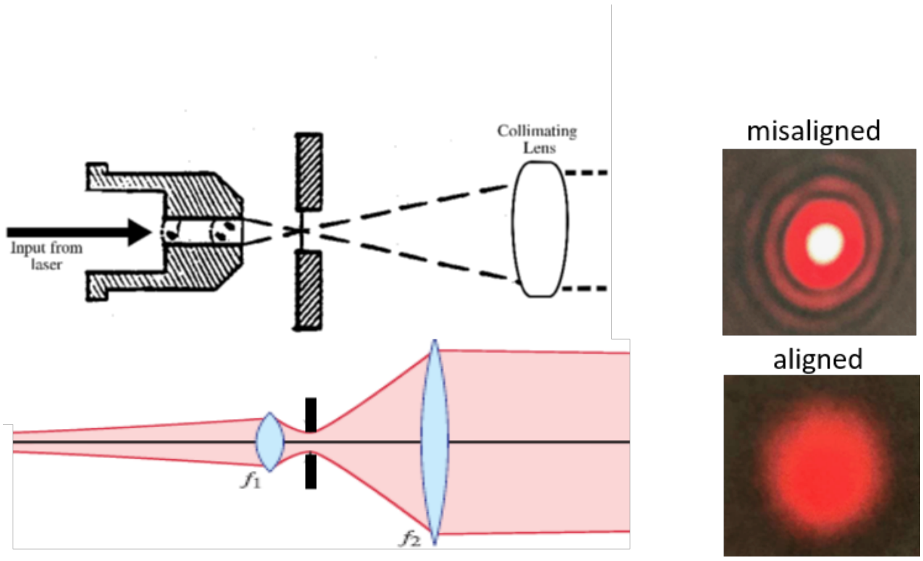

Figure 2.3: A spatial filter consists of a microscope objective (top left), which can be adjusted to focus a laser beam through a small pinhole, followed by a collimating lens. Dashed lines represent the simplistic ray-optics picture, whereas the more realistic gaussian-beam picture is shown below. Note that the outgoing collimated beam is still a gaussian beam slowly converging to its focal point located far away. Two pictures on the right show the examples of two beams originated from a misaligned (wrong focusing on the pinhole) and a well aligned spatial filter.

slightly since that video was recorded, the process of aligning the spatial filter is the same, so make sure you understand it and follow the instructions closely.

When adjusting the spatial filter, or any other device, assume that the previous user has left it close to the proper alignment. So remember how you found it, start with small adjustments in both directions, and do not make changes without observing their effect. Check that the beam still runs parallel to the rail. Before you move on, ask a TA to check your spatial filter. If you can't get a uniform circle in ten minutes, ask a TA to help. It is often easier to absorb a demonstration than written instructions. Useful tip in case of difficulty: If you don't see any light through the pinhole, increase the distance between the lens and the pinhole until you can see where the beam hits the area around the pinhole. You can now center the beam at the pinhole and see the light transmitted. Proceed with readjusting the distance until the diffraction pattern disappears (see Fig. 2.3). *Hint: a useful trick for the spatial filter alignment is to slowly move the pinhole in the transverse direction while watching the transmitted beam. If the focus of the objective lens is in front of the pinhole, the beam exiting the pinhole will move in the same direction as the pinhole. However, if the focus of the objective lens is behind the pinhole, the trans-*

mitted beam moves in the opposite direction. Draw a picture and convince yourself that this is the case.

Take a few pictures of the outgoing beam on a white beam blocking screen with your smartphone to explain the alignment process in your lab report.

Finally, place the collimating lens (CL) behind the spatial filter. Adjust the CL position so as to have the outgoing beam converge very slowly to a point "at infinity", i.e. very far away (see Fig. 2.3). *Hint: a useful signature of a proper alignment is the almost constant beam size along the entire length of the primary rail. Be careful and do not rely on observing the beam size at two locations only, e.g. close to the spatial filter and far away from it, as the beam may be focusing between those points and, therefore, is not even close to being collimated.* By aligning the spatial filter and the collimating lens correctly, you generate a large and defect-free gaussian beam.

2.3.2 Diffraction from a slit

Let's start with the simplest optical setup, in which a collimated beam, originating from the collimating lens CL, hits an object and then propagates in free space all the way to the image screen (i.e. without encountering any lenses or other objects in its way). As different parts of the propagating wave interfere with one another, the intensity distribution changes – a phenomenon widely known as **wave diffraction**. It turns out that if the observation plane is located far away from the object, the result of such wave diffraction can be described by a Fourier transform (FT)! That is, the distribution of light intensity "in far field", i.e. on a distant screen, is proportional to the modulus squared of the FT of either the field distribution in the object plane (the so-called *Fraunhofer* limit), or that field distribution multiplied by an extra phase factor (the *Fresnel* limit). Note that both limits imply a specific threshold distance, beyond which the respective approximation holds. However, in the Fraunhofer limit, this threshold is much farther away. In this part of the lab, you will explore this remarkable Fourier-transform property of wave diffraction.

To make the experiments and their interpretation easier, you will use a one-dimensional object – a vertical slit, whose width w along the x axis can be varied with a micrometer. Assuming the illuminating beam is larger than the slit (as is the case here), the field distribution in the object plane is a rectangle of width w:

$$E(x) = E_0(x)\mathrm{rect}(x/w) = \sqrt{I_0}\mathrm{rect}(x/w), \tag{2.2}$$

where the second equality implies uniform illumination by a plane wave, which is a good approximation for a clean and collimated gaussian beam (the one you have obtained in the previous section using the spatial filter!).

What does "far field" mean? The less stringent far field approximation simply requires that the field distributions in both the object xy-plane and the observation $x'y'$-plane be much smaller than the distance z between those planes. This is the Fresnel limit, which in our case implies $w \ll z$. Then, the intensity distribution along the relevant x'-axis in the observation plane can be calculated as:

$$I(x', z) \approx \frac{1}{\lambda z} \left| \int dx\, E(x) \exp\left[i\frac{\pi}{\lambda z}x^2\right] \exp\left[-i2\pi\frac{x'}{\lambda z}x\right] \right|^2. \tag{2.3}$$

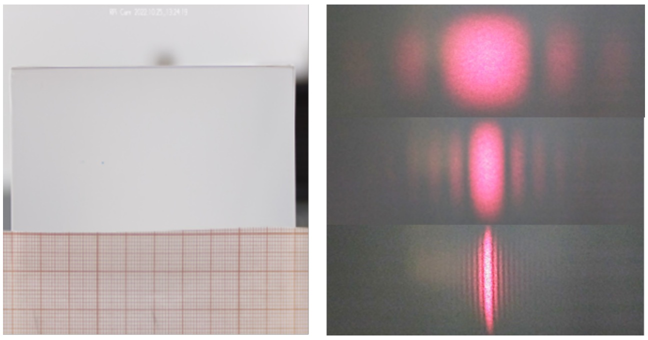

Figure 2.4: Left: using the graph paper to calibrate the length scale on the camera images. Right: Examples of diffraction patterns from a slit of various widths, increasing from top to bottom.

Note that this is a 1D equivalent of Equation (4.3-12) in the textbook (we will derive it step by step in class). The integral in the above formula, loosely known in optics as the *Fresnel integral*, can be seen as the Fourier transform of the initial field $E(x)$ multiplied by the phase factor $\exp\left[i\pi/(\lambda z)x^2\right]$. Note that in the case of a plane wave diffracting from a slit, the above formula reduces to the sum of two functions $C(x)$ and $S(x)$, which are the proper Fresnel integrals (as defined in mathematics).

The more stringent far field approximation, known as the Fraunhofer limit, demands $w \ll \sqrt{\lambda z/\pi}$. Given the smallness of the optical wavelength, this limit requires either much smaller slit width w for a given distance to the observation plane z, or other way around – much larger z for a given w. In this case, the first exponent under the integral in Eq. 2.3 can be neglected and the intensity distribution becomes:

$$I(x', z) \approx \frac{1}{\lambda z}\left|\int dx\, E(x) \exp\left[-i2\pi\frac{x'}{\lambda z}x\right]\right|^2, \tag{2.4}$$

which is simply a modulus-squared Fourier transform of the field in the object plane. Note that the ratio $f_{x'} \equiv k_{x'}/2\pi \equiv x'/\lambda z$ in the Fourier exponent is known as the *spatial frequency*.

Substituting $E(x)$ from Eq. 2.2 in either Eq. 2.3 or Eq. 2.4, one can calculate the diffraction pattern in the observation plane in either Fresnel or Fraunhofer limits, respectively.

In this lab, you will move between the Fresnel and Fraunhofer limits by adjusting the slit width w, since this is more convenient than varying the distance to the observation screen (though the latter is equally acceptable). In each measurement, when assessing what limit you are in, recall that the wavelength of the HeNe laser is 632.8 nm.

(A) Focus the camera on the observation screen (ground glass plate) and calibrate the length scale in your images – you will need it for further analysis. To do both of these tasks, use the provided graph paper, positioning it on the ground glass as shown in Fig. 2.4. Calculate and document the scale (e.g. in mm/pixel) in your lab report. Note that for safety reasons, the

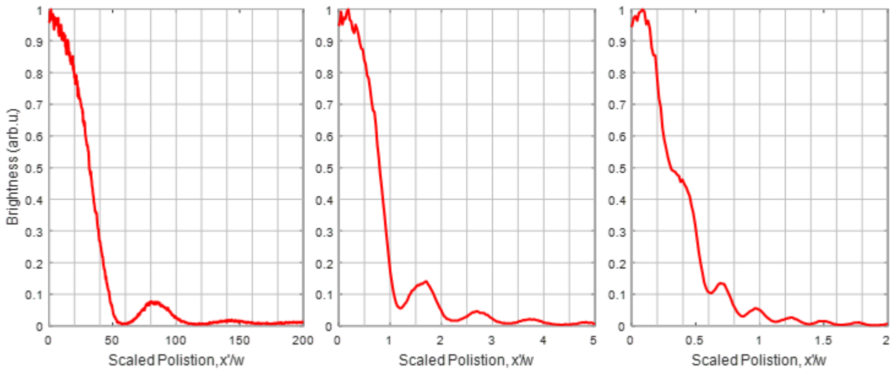

Figure 2.5: Vertically integrated diffraction patterns from a slit for various values of the dimensionless parameter $w/\sqrt{\lambda z/\pi}$: lower to higher from left to right. Notice the difference in the horizontal scale, expressed in terms of a dimensionless ratio x'/w.

ground glass plate should be oriented with its dull side towards the incoming beam, not its glossy side. This eliminates back reflections towards people working near the main optical rail.

(B) Place the slit after the collimating lens (in this section, the exact location of the object plane is not important) and reflect the transmitted light with the folding mirror at the end of the rail onto the imaging screen. Start by closing the slit with the micrometer until you see absolutely no light passing through. Now open it slowly until you get the very first glimpse of light (you should be doing this with the room lights off). Read out the micrometer – this is your zero width setting. From here, always open it in one direction to avoid backlash. Make sure you understand the micrometer scale. Measure the distance z between the slit and the observation screen. With that value in hand, determine what slit width would satisfy the Fraunhofer approximation (explain your thinking in the lab report). Open the slit to that value of w and record the diffraction pattern with the Raspberry Pi camera. *Hint: since the camera has an auto gain circuit, it may help to turn on the room lights so that the diffraction maxima are visible but not saturated. Even further, you can turn auto-gain off using the camera's web interface: change "Exposure Mode" from Auto to Off, and then tweak the exposure time for best fringe visibility. You can also turn off the Automatic White Balance to have a more accurate depiction of the laser's red color. Finally, remember that you can always adjust the overall intensity of the laser light by rotating one of the polarizers.* Does the experimental observation match your expectations, and in which way?

(C) Now slowly widen the slit width, while watching the diffraction pattern. You should be able to see the transition from Fraunhofer to Fresnel diffraction, indicated by the shrinking distance between consecutive maxima and minima, as well as by the increase of the secondary maxima (top to bottom images in Fig. 2.4). Record a series of images of the Fresnel patterns corresponding to the slit width values of $w/\sqrt{\lambda z/\pi} \simeq 0.25, 1, 2$ and 4, thus crossing over from the Fraunhofer to the Fresnel limit. *Hint: to be safe, take several images of the same pattern at different laser intensities. To change the latter, rotate the supplied polarizer in front of the laser. For some patterns, reducing the intensity so that the central maximum does not saturate the camera makes the "wings" of the pattern invisible. In this case, having two or more images at different intensities may be useful for further analysis.*

(D) Import your two-dimensional images into your favorite software package and integrate them along the vertical axis by summing up all vertical pixel values for each horizontal location on the image. This will provide you with the one-dimensional cross-section of the intensity distribution along the horizontal x'-axis of the diffraction pattern at a given distance z, $I(x', z)$. For example, in MATLAB it will look similar to:

```
>> colorImage=imread('My Image.jpg'); % load the file
>> bwImage = rgb2gray(colorImage); % convert to black-and-white
>> xSection=sum(bwImage); % integrate
```

Hint: it makes sense to sum up only in the region of interest containing the image, not the whole CCD area. When doing this partial summation, and if the image is saturated in the center portion, be sure to choose a section of the image which is not saturated, e.g. its upper or lower edge. Plot the obtained intensity distributions $I(x', z)$ expressing the distance from the center of the pattern in terms of a dimensionless parameter x'/w. For each plot, indicate the corresponding value of $w/\sqrt{\lambda z/\pi}$. A few examples of such intensity cross-sections are shown in Fig. 2.5. Discuss whether a ray-optics limit is being reached at very wide slit widths.

(i) Compare the above experimental results with numerical calculations. To carry out the latter, substitute the field in the plane of the slit $E(x)$ from Eq. 2.2 (for convenience, use $I_0 \equiv 1$) into either Fresnel or Fraunhofer integrals (Eq. 2.3 or 2.4, respectively). As discussed in the introduction to this section, both integrals can be evaluated using the standard Fast Fourier Transform (FFT) function, rather than much more laborious numerical integration. This is the beauty of Fourier optics, so make sure to use it! For example, the Fraunhofer integral (2.4) can be calculated as:

$$I(x', z) \propto |FFT\{\mathrm{rect}(x/w)\}|^2 \propto \mathrm{sinc}^2(x'w/\lambda z), \tag{2.5}$$

where the second proportionality stems from the fact that the 'sinc' function is the known Fourier transform of the 'rect' function. *Hint: do not use this analytical result blindly, because it won't apply in all cases. Rather, carry out an honest numerical FFT and compare the result with 'sinc' (as a sanity check) in the case when you expect Fraunhofer approximation to hold.* Plot the calculated intensity distributions $I(x', z)$ vs the dimensionless parameter x'/w for each experimentally used value of $w/\sqrt{\lambda z/\pi}$, together with the corresponding experimental distribution from task (D). For each value of $w/\sqrt{\lambda z/\pi}$, which integral does a better job of modeling your data? Explain your results.

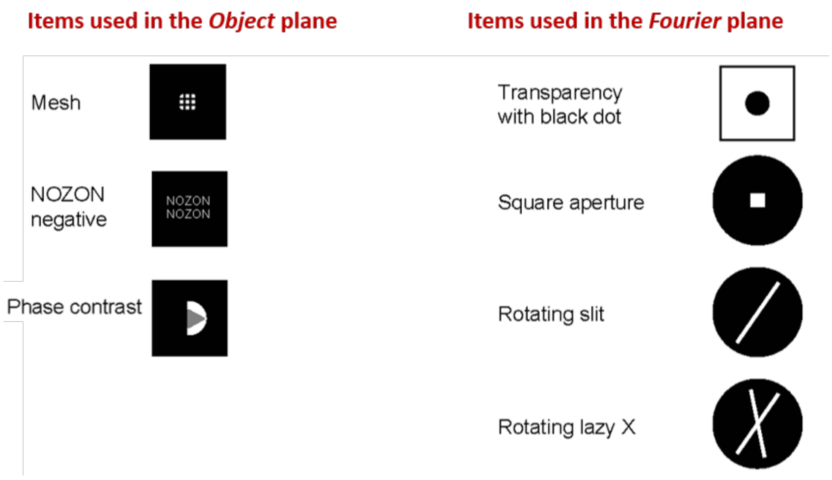

Figure 2.6: Various objects and apertures used in this lab.

(ii) Before moving to the next section, substitute the slit with a mesh and a NOZON object and record the images appearing on the observation screen. For the mesh object, investigate what happens with the image when you aperture the collimated beam in front of the object with an iris. As well, investigate the effect of moving the collimating lens (CL). For now, just record the images. It will be instructive to compare them with those images you will obtain in the later sections of this lab.

2.3.3 Setting up the FTL and FPIL lenses

To continue with the next series of experiments in this lab, you now have to correctly identify the locations of other optical components (and planes) in your optical setup. Before starting this task, first check the alignment of the spatial filter and make fine adjustments if needed to ensure that the output beam is round, symmetrical and collimated. Make sure the collimating lens transverse (x and y) position is correct so that the beam propagates along the rail (along z) at a constant height. Position and tilt the end mirror so that the beam runs parallel to the secondary rail towards the image screen (see Fig. 2.1).

Next, place the Fourier transform lens (FTL) on the bench about 70 cm from the collimating lens and lock it. Avoid the junction of the two rail components. Center the transform lens, checking that the reflections from its front surface return approximately in the direction of the spatial filter.

(A) In the following sections, you will have to place various apertures and objects, shown in Fig. 2.6, at the object plane. The best way to find that plane is with a “convenient object” and then change to other objects while leaving the object mount locked down in the object plane position. Use the NOZON negative object for this purpose. Place it in between the collimating lens and the Fourier transform lens and move it along the rail (i.e. along z) until a clear and sharp image appears on the screen at the end of the secondary rail. You can have your lab partner watch carefully (either directly or using the CCD camera) for the exact position at which the image becomes sharp and in focus without any additional structure arising from diffraction. Lock down the position of this mount – from now on, this is your object plane. Document your work by taking a picture with a Raspberry Pi camera. Explain why using NOZON is more convenient than other objects to find the object plane. *Hint: you might want to try doing it with the mesh object and analyze the difference.*

(B) Place the mesh object at the object plane (by inserting it into the mount you attached to the rail in the previous task). From the number of wires per cm given on the mesh aperture, and the spacing of the image wires on the screen, calculate the magnification ratio of this system. *Hint: use the provided graph paper placed in the plane of the image screen to determine the spacing of the wire images.* Does this magnification agree with the thin lens formula? Include a picture of the magnified image. Be sure to indicate the scale.

(C) Let's now proceed to finding the exact location of the Fourier plane. The easiest way to do this is to start with the mesh object in the object plane since it gives a nice FT pattern. As in point (A) above, the goal is to place a mount, equipped with an XY translation stage, at the FT plane and fix it in that location for further experiments. Hence, take an appropriate mount, insert an opaque observation screen in it, and place it some distance behind the Fourier transform lens. For an opaque screen, you can use the "square aperture" object; just move the translation stage in the horizontal direction x far enough that the beam emerging from the FTL lens hits the side of the aperture (rather than passing through the aperture!) on the black metal surface, which will serve as an observation screen. Now move the entire mount in the z direction until the Fourier transform pattern on the screen is sharp and crisp. *Hint: if the crispness of the pattern is not obvious, start by observing it on a piece of white paper placed in front of the aperture mount.*

A typical intensity pattern of the Fourier transform of the mesh object is shown in Fig. 2.7. When you obtain a similarly sharp picture, lock down the mount – it now sits exactly at the Fourier plane of the FTL. Did you find the Fourier plane where you expected it? Explain where that location is.

(D) Since the intensity pattern at the Fourier plane is typically quite small, you will use yet another lens – the Fourier plane imaging lens (FPIL, see Fig. 2.1), to image and magnify the FT pattern onto the same imaging screen at the end of the secondary rail. Insert the FPIL and adjust its position along the main rail until the FT pattern of the mesh (similar to the one in Fig. 2.7) is properly imaged on the ground glass, i.e. is as sharp and crisp as what you obtained in the Fourier plane in the previous task. What is the magnification ratio of this imaging system?

(E) Create a sketch in your lab book (similar to Fig. 2.1) showing the location of each of the *lenses* and *planes* in your setup, and indicating the exact relative distances between them, measured with the supplied ruler.

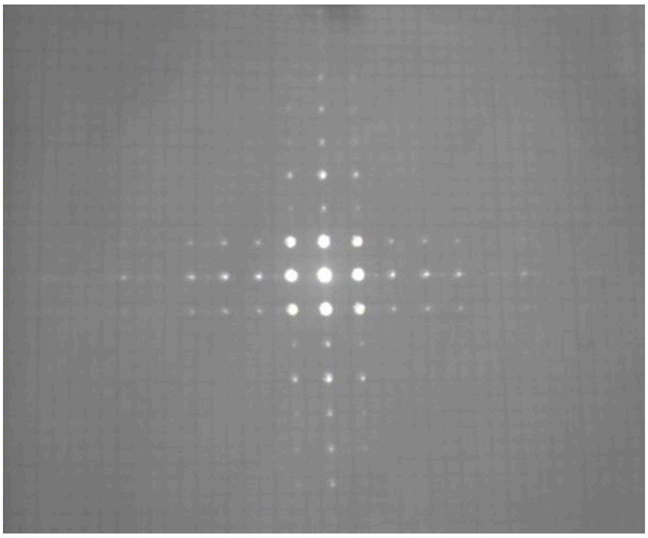

Figure 2.7: An optical Fourier transform of a mesh object, used in this lab.

2.3.4 Fourier transform of a mesh

Continue with the wire mesh object and all three lenses (CL, FTL and FPIL) in place. Check that the FT plane is imaged on the imaging screen, as achieved in point (D) of the previous section. If you have completed that section last week, repeat the necessary alignment, since proper positioning of all the components is critical in the following experiments. Attach a sheet of black graph paper to the screen. Position the Raspberry Pi camera such that the relevant portion of the screen is visible (typically, about two thirds of the whole ground glass area). Turn off the lights, but leave the curtain open so that the white lines of the graph paper are still recognizable in the image. You should be seeing a picture similar to Fig. 2.7.

(A) Explain what the bright spots in the Fourier transform plane represent.

(B) Rotate the mesh around in the mount and lift it up and down. How does moving the mesh change the magnified image of its Fourier transform? Why do some types of movement change that image, whereas others don't?

(C) Take a picture of the Fourier transform image. Use it to calculate the spatial frequencies that are present. Be careful with units! Do these frequencies agree with what you expect, given the wire spacing of the object? *Hint: you can compensate for any distortions in the image due to the camera lens by using the grid on the graph paper. Appendix B might be useful in completing the required analysis.*

(i) Compare the Fourier transform images obtained with the FTL lens above with those you have seen without using any lenses in task (ii) of Section 2.3.2. What can you conclude from this comparison? How can you explain the effect of aperturing the collimated beam? Why did you obtain the sharpest FT image by moving the lens towards the screen?

2.3.5 Optical Fourier filtering

Continue with the same optical setup as in the previous section, i.e. imaging the FT pattern of the mesh onto the imaging screen. Now, insert the "rotating slit" in a holder and place it at the Fourier transform plane.

(A) By using the XY translation stage on the slit mount and observing the image, center the slit with respect to the FT pattern. Confirm that by rotating the slit, it can be aligned so as to pass the central vertical series of dots in the FT pattern. Record the image from the Raspberry Pi camera and use it to explain the procedure in your lab report.

(B) Now remove the Fourier plane imaging lens. Without FPIL *and if there was no slit at the FT plane*, the image on the screen should have been that of the mesh in the object plane (remember that this is how you defined that plane in the first place in Section 2.3.3). By allowing the slit to pass only the central *vertical* dots of the Fourier transforms, you erased nearly all of the information about the *horizontal* separation of the mesh wires. What does the resulting image look like? What happens if you align the slit horizontally? Record your observations and explain them in reference to the mathematical expression of the Fourier transform.

(C) In light of what you have learned about filtering in the Fourier plane of a lens, explain how the spatial filter immediately after the HeNe laser works.

(D) Strange as it may seem, an image pattern containing lines at 45° to the vertical may be produced by rotating the slit-filter around an axis parallel to the light propagation direction by 45 degrees with respect to the vertical axis. Verify this and take a picture. Explain why this is the case.

(i) It is possible to analyze the effects of the optical Fourier filtering transform through image processing. Using your favorite software package (e.g. MATLAB) take the inverse Fourier transforms of the mesh images recorded with the Raspberry Pi camera with and without the slit in the Fourier plane. Does your numerical analysis confirm your experimental observations?

(E) Let's do some exciting stuff – an optical **character recognition**! Use optical Fourier filtering techniques to pass through only certain characters from the NOZON pattern (e.g. all the "N"'s), while blocking others. Place the negative NOZON object in the object plane and observe its magnified image on the image screen. Now bring back the Fourier plane imaging lens FPIL and record the corresponding Fourier transform pattern. Include these images and their description in your lab report. Why is the FT pattern more complex than the one from the mesh?

(F) Based on the obtained Fourier transform and your knowledge from the earlier experiments, can you filter out all the letters except the "N"'s from the original object using one of the available apertures (Fig. 2.6)? Document your approach with pictures of the filtered images corresponding to different filters and/or their orientations. Explain what worked best and why. *Hint: try the "lazy X" aperture. If the filtering isn't perfect, your* *spatial filter might not be exactly at the Fourier plane. The position of the filter can be optimized by observing the FT pattern formed on the back side of the filter, and adjusting the filter position to make the pattern as sharp as possible. This fine adjustment of the aperture in the FT plane may be critical to getting a good result for this part.* Note that in general, when you are trying to align a filter in the FT plane it is best to look at the image of the FT plane on the Raspberry Pi camera, rather than at the filter itself.

In the last task below, you will learn about yet another powerful capability of the optical Fourier filtering technique, used for image enhancement. It is known as "dark-field imaging". A "dark-field image" is obtained when (typically the brightest) central spot in the Fourier transform, corresponding to the zeroth spatial harmonic, is blocked. The field distribution in the unfiltered image is a result of constructive and destructive interferences between all the light waves coming from the object. If the zero-frequency component is removed, the interferences – and hence the intensity distribution in the image plane, will change. Initially bright areas may become darker, whereas initially dark spots may light up. Importantly, since the high spatial frequency components required to define the edges of the object are still transmitted, those edges do not lose their sharpness, but may now appear more clearly, given the re-arranged intensity distribution. Note that this is opposite of the spatial filtering you studied at the beginning of this lab. The spatial filter passes predominantly low frequency components, which results in a uniform illumination with no sharp features.

(G) For this experiment, you should use the "phase contrast" object which consists of a razor blade with a wedge of clear glass glued to it. Place it in the object plane (where the wire mesh was) and translate it until you see the image on the imaging screen. Being almost perfectly transparent, the glass wedge is barely visible in the image, blending in with the bright background. Now place the transparency with the black dot on it into the FT plane. Describe your observations and take a picture for your lab book. *Hint: here it is essential that the image be in focus in the image plane. A difference in focussing of even a few millimeters will destroy the effect.*

Appendix A

Raspberry Pi Camera Operation

For several experiments in this course you will use the Raspberry Pi CCD cameras to capture images or movies. While you will be taking your final data using Matlab (or Python) scripts (see an example in the `Software` directory), it is often convenient to look at the camera image in real time, e.g. when you search for various cavity modes or optimize the interference pattern. For a real time view of the raspberry pi camera it is useful to use the Raspberry Pi Camera Web Interface. This interface simply needs a web browser and to see the interface type in the web url: `http://142.103.238.21/html/` where the IP address is different for different desktop stations and can be found on the corresponding monitor.

If you want to view the camera on your laptop, then you need to be connected to UBC VPN. The web interface has many configurable options but for us there are only a few to be concerned with. When you first open the page, it will look similar to the one shown in Fig. A.1.

On this page the items of interest are:

1. The stop camera/ start camera button. You must press the stop camera button to free up the camera when you want to take images with MATLAB instead. Press the start camera button when you want to use the web interface again.

2. Record image or record video start. These will record an image or video saved on the raspberry pi.

3. Download Videos and Images will allow you to see images or videos you have taken and you can download them to the computer you are using the web interface on. You can also use this to delete picture files saved on the raspberry pi. **Please clean up after yourself as the RasPi memory is limited!**

4. The Camera Settings menu provides many options which we will refer to later in this document.

5. The System menu has options. **Please do not touch any of those except the Reset Settings button!** The reset settings button can

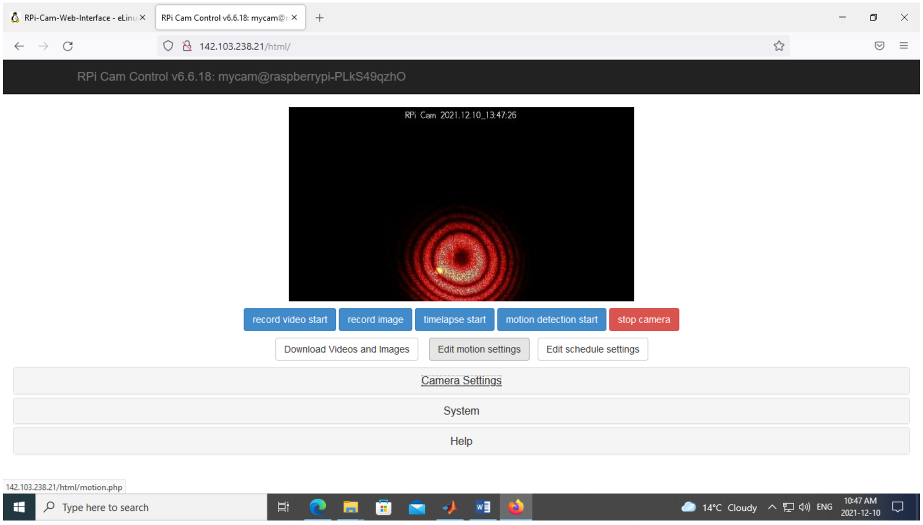

Figure A.1: An example of the web interface for operating Raspberry Pi cameras in real time.

help if the web interface is left in an odd state or if you want to return to the default settings. The web interface saves the last used settings even if the web browser is closed.

Please note that more than one person can have a web interface open at the same time to see the camera but it is not advisable to be tweaking settings on different computers at the same time.

Camera Settings Menu. There are many many options in the camera settings. You do not need to and should not play with most of them. The main settings of use are:

1. Exposure mode to 'off' or 'auto'. 'Off' allows you to set the shutter speed yourself.

2. Shutter speed (in micro seconds) can be set when the exposure mode is set to off. Note that a shutter speed of 0 is auto exposure regardless of the exposure mode setting.

3. White Balance to 'off' of 'auto'. This is useful when you want to correctly collect the red pixel values for data analysis. For the most part we are using HeNe lasers whose wavelength is red (633 nm) so it makes sense to set the white balance to 'off', for example, for taking cross sections of images where the red pixel value only is needed.

4. Other settings such as ISO may be useful. Sharpness, contrast, brightness, saturation can be played with but typically the shutter speed is the most valuable.

5. Image quality changes the amount of image compression and is mildly useful.

6. If you want truly uncompressed data for analysis, for example, for cross section analysis then the raw layer can be set to 'On'. The raw Bayer information is appended to the end of the jpeg image file and must be extracted. This option makes the file size large so only use if needed for analysis.

7. Annotation and size can be changed if you want to have a descriptor written on your image.

We do not recommend altering other settings unless you have a very good reason to. If you find that the camera view is strange, first try resetting the settings. If buttons are not working at all or the preview is updating only sporadically, the web interface will need to be reinstalled on the raspberry pi. **Please ask for assistance if that is the case and do not do that on your own.**

Resources for more information: `https://elinux.org/RPi-Cam-Web-Interface`, which is based on the picamera module urlhttps://picamera.readthedocs.io/en/release-1.13/.

Raspberry Pi High Q camera image resolution and effective pixel size. It is important to realize that the effective pixel size of the RasPi camera changes depending on the image resolution specified. The specifications for the Raspberry Pi High Q camera are given as:
`Sony IMX477R stacked, back-illuminated sensor, 12.3 megapixels, 7.9 mm sensor diagonal, 1.55` μ`m` $\times$ `1.55` μ`m pixel size.`

The maximum resolution of this camera is 4056×3040 pixels. Just to make sense of these numbers it means that there are more pixels along one direction than another which you can tell from looking at the rectangular shape of the CCD chip. If we multiply (4056×3040) we get 12330240 total pixels which agrees with the 12.3 megapixel specification. Each physical pixel is 1.55 μm $\times$ 1.55 μm in size which gives us an approximate 7.9 mm diagonal, as shown in Fig. A.2.

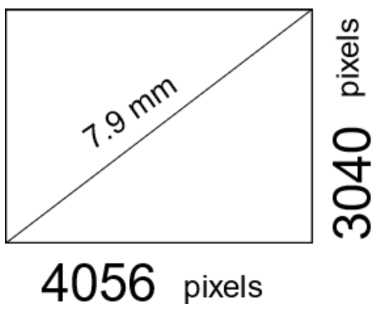

Figure A.2: The CCD chip of the raspberry pi high Q camera has 4056 by 3040 pixels with a 7.9 mm diagonal dimension.

Note that the image resolution can be specified by the user. For example, the web interface we use to collect raspberry pi images has a default image resolution of (2592×1944). This means that images collected have 2592×1944 elements so that the effective pixel size has increased in comparison to the 1.55 μm $\times$ 1.55 μm size of the physical pixel elements. In that case the effective pixel size is instead 2.42 μm $\times$ 2.42 μm as calculated by:

$$\begin{aligned}(4056/2592) \times 1.55\,\mu\text{m} &= 2.42\,\mu\text{m},\\ (3040/1944) \times 1.55\,\mu\text{m} &= 2.42\,\mu\text{m}.\end{aligned}$$

This is important for the HeNe lab where you are asked to measure the beam diameter with a camera. In that case it is important to know the separation distance between the elements in your image array. You need to take into account your image resolution used when saving your images to calculate the correct beam diameter.

Appendix B

Image Formation

In the figure below a mesh object is illuminated with collimated coherent radiation (produced by a laser) and a magnified image of the mesh is formed by the second lens, referred to as the transform lens. The magnified image is located in the plane a distance s_i away from the lens (at position $z = B$) which is itself a distance s_0 from the object. In the thin lens approximation, the magnification is equal to the ratio

$$m = \frac{s_i}{s_o}, \tag{B.1}$$

and the object and image distances (s_o, s_i) are related to the focal length f by the image equation

$$\frac{1}{s_i} + \frac{1}{s_0} = \frac{1}{f} \tag{B.2}$$

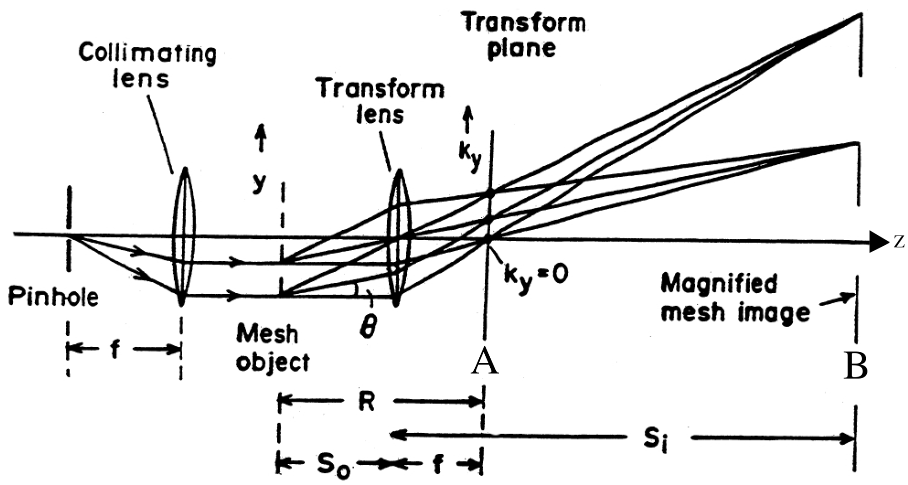

Figure B.1: Magnified image formation with parallel coherent incident radiation.

In contrast, if the screen is placed at $z = A$, something else is produced. In particular, as will be shown below, the spatial 2-D Fourier transform [1] of the object image will appear in the plane at $z = A$

Optical Fourier Transform Produced by a Lens

In order to understand how a lens generates the Fourier Transform of the light wave emanating from the object, let's imagine that we break up this object wave into a superposition of plane waves (this can always be done for linear optical systems). Now, the lens simply focuses each of these plane waves to a different spot in the focal plane. So, the optical wave at the focal plane behind the lens is the 2-D Fourier transform of the optical wave leaving the object.

Let's see how this works. Consider a plane wave of complex amplitude $U(x, y, z) = A \exp[i(k_x x + k_y y + k_z z)]$ with wavevector $\mathbf{k}= (k_x, k_y, k_z)$, wavelength λ, wavenumber $k = (k_x^2 + k_y^2 + k_z^2)^{1/2} = 2\pi/\lambda$, and complex envelope A. The vector $\mathbf{k}$ makes angles $\theta_x = \sin^{-1}(k_x/k)$ and $\theta_y = \sin^{-1}(k_y/k)$ with the $y - z$ and $x - z$ planes, respectively, as illustrated in Fig. B.2. The complex amplitude in the $z = 0$ plane, $U(x, y, 0)$, is a spatial harmonic (i.e. sinusoidal) function $f(x, y) = A \exp[2i\pi(v_x x + v_y y)]$ with spatial frequencies $v_x = k_x/2\pi$ and $v_y = k_y/2\pi$ (cycles/mm). The angles of the wavevector are therefore related to the spatial frequencies of the harmonic function by

$$\begin{aligned} \theta_x &= \sin^{-1} \lambda v_x \\ \theta_y &= \sin^{-1} \lambda v_y \end{aligned} \tag{B.3}$$

Recognizing $\Lambda_x = 1/v_x$ and $\Lambda_y = 1/v_y$ as the spatial periods of the harmonic function in the x and y directions, we see that the angles $\theta_x = \sin^{-1}(\lambda/\Lambda_x)$ and $\theta_y = \sin^{-1}(\lambda/\Lambda_y)$ are governed by the ratios of the wavelength of light to the period of the harmonic function in each direction. These geometrical relations follow from matching the wavefronts of the wave to the periodic pattern of the harmonic function in the $z = 0$ plane, as illustrated in Fig. B.2.

If $k_x \ll k$ and $k_y \ll k$, so that the wavevector $\mathbf{k}$ is paraxial, the angles θ_x and θ_y are small (therefore $\sin\theta_x \approx \theta_x$ and $\sin\theta_y \approx \theta_y$) and

$$\begin{aligned} \theta_x &\approx \lambda v_x \\ \theta_y &\approx \lambda v_y \end{aligned} \tag{B.4}$$

Thus the angles of inclination of the wavevector are directly proportional to the spatial frequencies of the corresponding harmonic function.

Now, the different plane-wave components that constitute a wave may be spatially separated (i.e. isolated) by the use of a lens. Recall that a thin spherical lens transforms a plane wave into a paraboloidal wave focused to a point in the lens focal plane. If the plane wave arrives at small angles θ_x and θ_y, the paraboloidal wave is centered about the point$(\theta_x f, \theta_y f)$, where f is the focal length (see Fig. B.3). The lens therefore focuses (maps) each plane wave propagating in the direction (θ_x, θ_y) onto a single point $(\theta_x f, \theta_y f)$ in the focal plane and thus spatially separates the contributions of the different harmonic functions.

[1] If you don't remember what a Fourier transform is, now is a good time to review it. Also see *http://www.falstad.com/fourier/* for an instructional java applet.

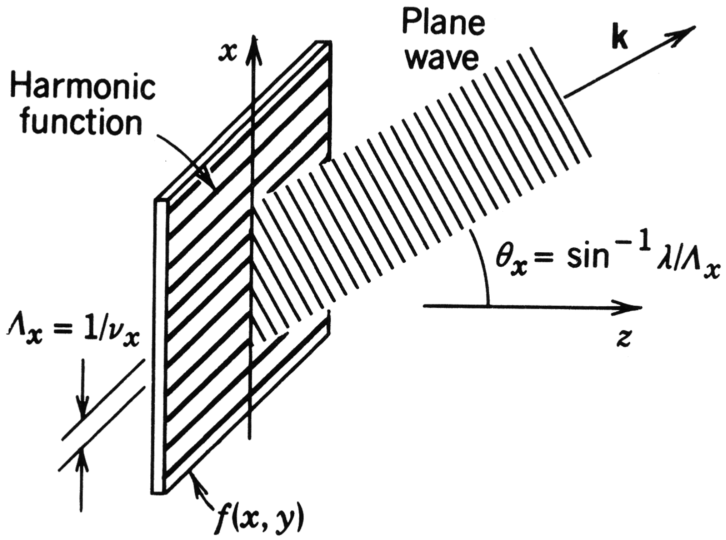

Figure B.2: A harmonic function of spatial frequencies v_x and v_y at the plane $z = 0$ is consistent with a plane wave traveling at angles $\theta_x = \sin^{-1} \lambda v_x$ and $\theta_y = \sin^{-1} \lambda v_y$.

In reference to the optical system shown in Fig. B.1, let $f(x, y, z)$ be the complex amplitude of the optical wave in the x–y plane at some position z. The optical wave emanating from the object in the $z = 0$ plane located to the left had side of the lens is then $f(x, y, 0)$. Think of the light coming from the object as a superposition of plane amplitude waves, with each wave component traveling at a small angle $\theta_x = \lambda v_x$ and $\theta_y = \lambda v_y$ having a complex amplitude proportional to the Fourier transform of $f(x, y, 0)$, namely $F(v_x, v_y)$. Each of these plane wave component is then focused by the lens to a unique point (x', y') in the focal plane where $x' = \theta_x f = \lambda f v_x$, $y' = \theta_y f = \lambda f v_y$. The complex amplitude of the wave at point (x', y') in the output or focal plane (located at $z = A$) is therefore proportional to the Fourier transform of $f(x, y, 0)$ evaluated at $v_x = x/\lambda f$ and $v_y = y/\lambda f$. Thus we have

$$f(x, y, z = A) \propto F\big(\frac{x}{\lambda f}, \frac{y}{\lambda f}\big) \tag{B.5}$$

Short range irregularities in $f(x, y, 0)$ (sharp edges) contribute primarily to the diffraction field away from the optic axis, *i.e.* for large (x', y') or (k_x, k_y). Long range irregularities (soft edges) contribute primarily to the small spatial frequencies (k_x, k_y) near the optic axis in the diffraction pattern. Obviously, spatial filtering of the beam is performed in the transform plane. This filtering can be of simple low-pass or high-pass type, or it can involve selective phase-change of any particular spatial frequency component. Later we will investigate the filtering properties.

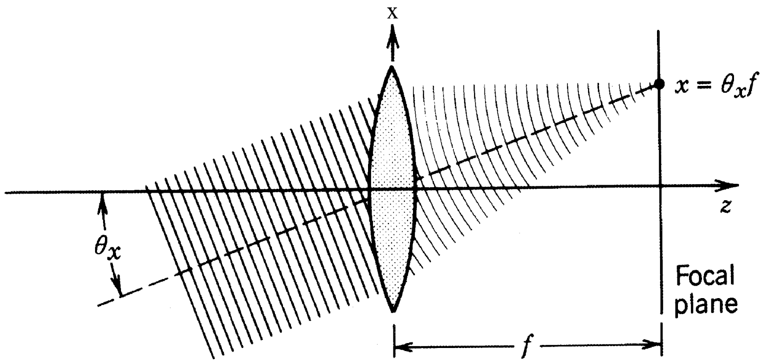

Figure B.3: Focusing of a plane wave into a point. A direction (θ_x, θ_y) is mapped into a point $(x, y) = (\theta_x f, \theta_y f)$ at $z = f$.

Inverse Fourier Transform

So if the Fourier transform appears at $z = A$, how does an image appear at $z = B$? Very simply, propagation of the wave through free space from $z = A$ to $z = B$ generates the inverse Fourier transform! In this way, the object wave is modified by the lens so that propagation of the wave after the lens produces the Fourier transformed at the transform plane $z = A$, and then, by additional propagation through free space, the inverse Fourier transform is created at $z = B$. This process is what produces an image of the object at $z = B$.

To see how propagation generates the inverse transform, consider this: the electromagnetic wave which crosses the plane at $z = A$ can be thought of as originating from a distribution of point emitters in that plane, and each of those point emitters illuminates the image plane (at $z = B$) after propagation by the distance d between A and B. If you write the image wave at $z = B$ as the sum of spherical waves generated by all these point emitters, you get something which, after some approximations, is exactly the inverse Fourier Transform!

Here's the same argument but with the math too. First consider just a single point source of light located on the z-axis in the Transform plane at $z = A$. This point source generates a spherical wave propagating outward (i.e. $E = \frac{1}{r}e^{i(kr-\omega t)}$ where r is the length of the vector pointing from the point source to the observation point). If we examine this spherical wave far from the source but close to the z axis, we can safely approximate it as a paraboloidial wave (i.e. $E \simeq \frac{1}{z}e^{ikz}e^{ik(x^2+y^2)/2z}$). If the point source emitter weren't located exactly *on* the z-axis (i.e. at $x = 0, y = 0$) but rather just off the z-axis, say at (x_i, y_i), then we simply replace $x \to x - x_i$ and $y \to y - y_i$ since the position variables come from the vector pointing from the source at $\vec{r}_i$ to the observation point at $\vec{r}$. In this case, the electric field produced in the plane at $z = B$, a distance d from the point emitter at $z = A$ is then

$$E(x, y, z)|_{z=B} \simeq \frac{1}{d}e^{ikd}e^{ik((x-x_i)^2+(y-y_i)^2)/2d} \tag{B.6}$$

Okay, now if we have a whole distribution of point sources of different amplitudes (brightnesses) in the plane $z = A$ described by the function $E(x, y, z = A)$, then the electric field observed in the plane $z = B$ far from $z = A$ will just be the sum of all these point sources each located at some initial position (x_i, y_i) and with some amplitude $E(x_i, y_i, z = A)$. This sum is just the integral

$$E(x,y,z)|_{z=B} \simeq \frac{1}{d}e^{ikd} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x_i,y_i,z)|_{z=A}\, e^{ik((x-x_i)^2+(y-y_i)^2)/2d}\, dx_i\, dy_i. \tag{B.7}$$

This equation expresses the form of the diffracted field after propagation by a distance d. In the far field limit this reduces to the Fraunhofer diffraction expression

$$E(x,y,z)|_{z=B} \simeq \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x_i,y_i,z)|_{z=A}\, e^{-i2\pi(xx_i+yy_i)/\lambda d}\, dx_i\, dy_i. \tag{B.8}$$

where k has been replaced by it's value $2\pi/\lambda$, we have dropped the unimportant phase prefactor $(\frac{1}{d}e^{ik(d+[x^2+y^2]/2d)})$ which doesn't matter if we only look at the field amplitude. Also, we have dropped the quadratic terms inside the exponent $(kx_i^2/2d)$. This last step is okay so long as we're only interested in the field generated by point emitters lying close to the z-axis such that $kx_i^2/2d \ll \pi$. After these simplifications, we are left with just the integral which is the inverse Fourier Transform of the wave at $z = A$. The result is that if we modify the wave at $z = A$ by placing there a mask, the image which results at $z = B$ will be the convolution of the original image and the Fourier transform of the mask.

The Abbé theory of image formation arose from Abbé's consideration of these effects when trying to view small objects through a microscope. It became clear to him that a lens (the objective lens of the microscope) always acts as a low-pass filter, since its finite size will admit only spatial frequencies up to an obvious geometrical limit. The bigger the lens and the larger the range of spatial frequencies admitted, the greater the image fidelity. In the limit of admitting only the undeflected central order ($\theta = 0$), no detail of the object at all will be present in the image. Only a uniform illumination (the so-called d.c. level) will be obtained in the image plane. This, therefore determines the size of the smallest objects which can be viewed by a particular microscope. This relationship is exploited to "clean" up the laser beam coming from the HeNe laser. By focussing the beam through a pinhole, we spatially filter out the high spatial frequencies (i.e. the transverse spatial structures) of the laser to give a clean uniform beam without structure.

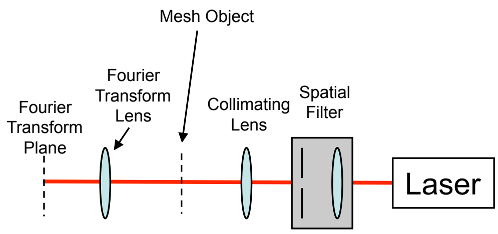

Figure B.4: Part of the imaging setup for Fourier Optics lab. The spatial filter and collimating lens expand and collimate the laser beam which then illuminates the mesh object. The optical wave transmitted by the mesh passes through the Fourier Transform lens and at the focal distance (f) after that lens, the Fourier transform of the transmitted beam appears. A picture of the mesh Fourier transform intensity pattern is shown in Fig. B.5

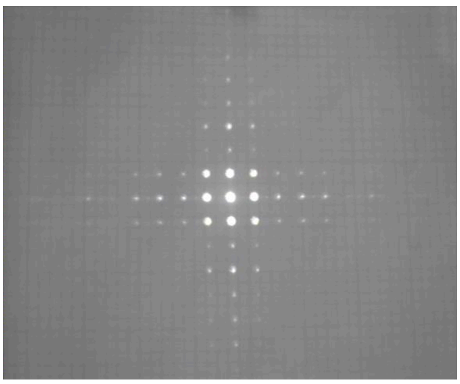

Figure B.5: Picture of the intensity profile at the Fourier Transform plane produced by a wire mesh object. See Fig. B.4 for the optical setup.

Appendix C

Diffraction

The term diffraction is generally understood to mean the propagation of light "around" the edges of obstacles or apertures. Diffraction phenomena are traditionally divided into two classes - Fresnel and Fraunhofer diffraction - although they are both manifestations of the same phenomenon. Fraunhofer diffraction refers to what happens to an optical pattern after propagation over a very large distance z in the paraxial approximation. Fraunhofer diffraction is the *far field* limiting case of Fresnel diffraction that arises when both source and observer are at very large distances from the diffracting screen such that the incident and diffracted waves are effectively plane. In this limit, the curvatures of the incident and diffracted waves can be safely neglected and the Fraunhofer diffraction pattern is just the Fourier transform of the initial pattern. If, however, either the source or receiving point is close to the diffracting aperture such that the wave front curvature is significant, then the result will be Fresnel diffraction [see Fig. C.1 [1]].

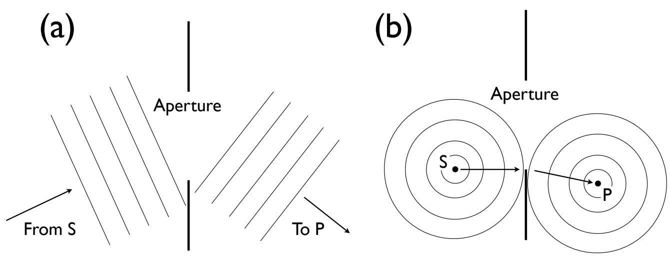

Figure C.1: Diffraction by an aperture in the (a) Fraunhofer limit and (b) Fresnel limit

In order to be in the Fraunhofer limit, the distances between the source and diffracting aperture and between the diffracting aperture and receiving point should be large compared to a characteristic distance, R_0 given by;

$$R_0 = \frac{2\mathrm{w}^2}{\lambda} \tag{C.1}$$

[1]figure adapted from "Introduction to Modern Optics" by Grant R. Fowles

where λ is the wavelength of the light, and w is a dimension of the aperture or obstacle. This relation for R_0 guarantees that the phase change across the width of the aperture is less than $\frac{\lambda}{4}$. Perfect Fraunhofer diffraction can only be achieved with collimated incident and diffracted beams.

Diffraction Pattern of a Slit

In this experiment, you will investigate the two (Fraunhofer and Fresnel) diffraction regimes by studying the diffraction pattern from a slit aperture of variable width. You should start by inserting the collimating lens after the spatial filter so that the beam wave fronts are parallel. Thus the source of illumination is effectively at infinity. Place the slit after the collimating lens and reflect the transmitted light with the secondary mirror onto the screen with the image visible by the CCD camera.

The diffraction pattern observed can be characterized by a dimensionless parameter, Δv, defined as;

$$\Delta v = \mathrm{w}\left(\frac{2}{R\lambda}\right)^{1/2} = \left(\frac{R_0}{R}\right)^{1/2} \tag{C.2}$$

where w is the width of the slit, and R is the slit-screen distance. Fresnel diffraction occurs for $\Delta v \geq 1$. Fraunhofer diffraction, the so called far-field approximation is characterized by $\Delta v \ll 1$. This condition can be established with either a large slit-screen distance as the name implies, or a narrow slit. In the lab, the slit width is the more convenient adjustment to vary the Δv value, since a large range of w is easily achieved with a turn of the micrometer. However, varying R is equally acceptable. For each measurement you make, you should compute your Δv value on the spot to insure that you are in the proper limit (recall that the wavelength of the He-Ne laser is 632.8 nm). Because the Fraunhofer diffraction pattern is essentially the Fourier Transform of the input wave, the Fraunhofer diffraction can also be observed at the focal plane of a converging lens (see Figure C.2)

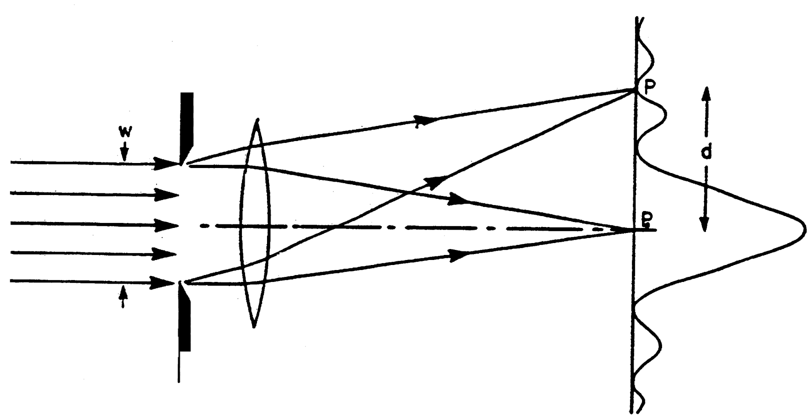

Figure C.2: Fraunhofer diffraction pattern of a slit formed at the focal plane of a lens. In this part of the lab, you will generate your Fraunhofer patterns *without* the aid of a "Fourier Transform" lens placed after the aperture.

Start by adjusting the slit width to be very narrow until you see the characteristic Fraunhofer pattern. Record and reproduce in your lab write-up the image of the observed diffraction pattern.

Now slowly widen the slit width watching the diffraction pattern as you do. You should be able to see the transition from Fraunhofer to Fresnel diffraction, with the initial loss of the zeros, and subsequent increase of secondary maxima. Record the image of the Fresnel patterns at values of $\Delta v \simeq 0.5, 1,$ and 4. For each measurement you make, you should compute your Δv value to insure that you are in the proper limit. Be sure that the image is not saturated so that your data is good. To be safe, take several images of the same pattern at different laser intensities. There are two polarizers available to reduce the intensity of the laser so that the patterns are not saturated. Note: for some patterns, reducing the intensity so that the central maximum does not saturate the camera makes the "wings" of the pattern invisible. In this case, having two or more images at different intensities may be important. Also, the line out that you take of the image of the slit diffraction for comparison with theory can be chosen in a region where the image is not saturated.

Your images should look something like those in Fig. C.3. Since the CCD has an auto gain circuit, it may help to turn on the room lights so that the patterns are visible and not completely saturated. Your analysis will be done by taking a line out of the image, so as long as there is a part of the image that is not saturated, you can extract the data. Because the CCD has a limited dynamic range, the image may not show clearly the low intensity wings of the diffraction pattern that will be obvious to you by eye.

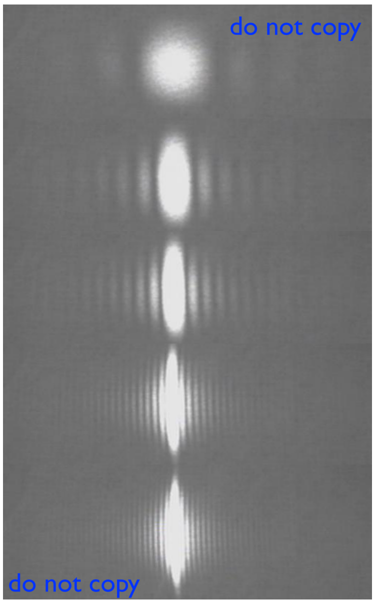

Figure C.3: Diffraction patterns from a slit of various widths.

Analysis of the slit diffraction

Take your images and import them into an array in MATLAB, maple, mathematica, octave, or your favourite math tool. Plot the intensity as a function of position along an axis perpendicular to the slit axis (let's call that the x-axis, and this axis is horizontal in the images shown in Fig. C.3). To improve the signal to noise of your data, you may need to integrate (add pixel intensities) the image along the y direction. Since there might be some distortion of the image along the vertical (y) direction (as seen in Fig. C.3), you should probably only sum up the 10 adjacent pixels along a vertical line for each sample along x. Be sure that you choose an area of the image that is not saturated. In Fig. C.3, this area would correspond to the top of the images above the point where the central spot is completely saturated. The problem of saturation is probably most apparent in the lowest image where the interference fringe spacing is the smallest. Calculate your values of Δv, and compare your diffraction patterns to theory by plotting your experimentally measured data against the predicted diffraction patterns at the corresponding Δv. For this you should numerically evaluate the form of the diffraction intensity using Matlab or Mathematica using the theory presented.

It is convenient to represent the intensity of the diffraction pattern at a distance, d from the optic axis in terms of the dimensionless parameter,

$$z = \frac{d}{\mathrm{w}} \tag{C.3}$$

Thus the edges of the geometric shadow correspond to $z = \pm 0.5$.

In the Fraunhofer limit ($\Delta v \ll 1$), the intensity variation in the diffraction pattern is given by the Fourier transform of a square function,

$$I(z) \propto (\Delta v)^2 \sin^2 \beta / \beta^2 \tag{C.4}$$

where $\beta = (z\pi/2)(\Delta v)^2$. The pattern has a global maximum at $\beta = 0$, with subsidiary maxima on both sides. These maxima are separated by zeroes in the intensity for

$$\pm\beta = n\pi \ \text{ or } \ \pm z = \frac{2n}{(\Delta v)^2} \quad n = 1, 2, 3, \ldots$$

Notice that the intensity maximum is proportional to $(\Delta v)^2$ and therefore to w^2, while the width of the central maximum is proportional to $(\Delta v)^{-2}$ or to w^{-2}.

For Fresnel diffraction, ($\Delta v > 1$), the general expression for the intensity is given by;

$$I(z) \propto \Big[C^2(v) + S^2(v)\Big]_{v_1}^{v_2} \tag{C.5}$$

where $C(v)$ and $S(v)$ are the Fresnel cosine and sine integrals, respectively, given by

$$C(v) = \int_{v_1}^{v_2} \cos(\frac{\pi x^2}{2}) \mathrm{d}x$$

and

$$S(v) = \int_{v_1}^{v_2} \sin(\frac{\pi x^2}{2}) \mathrm{d}x$$

The limits on the variable of integration are

$$v_1 = -(z + 0.5)\Delta v \qquad \text{and} \qquad v_2 = -(z - 0.5)\Delta v$$

In the limit of small Δv, equation C.5 reduces to equation C.4.

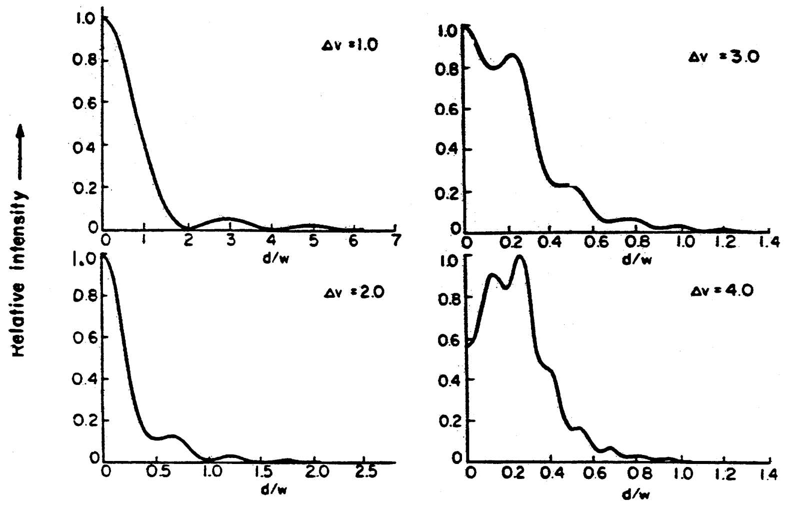

Figure C.4: Line intensities of Fresnel diffraction patterns of a slit for various values of the parameter Δv. Only one half of the symmetrical pattern is shown.