Math 220 Homework 2 Question 8

September 27, 2021

Question 8. Let $a, b, k \in \mathbb{Z}$ and assume that a, b are not zero. Then, using Bézout's Identity, show that if $k \nmid \gcd(a, b)$, then $k \nmid a$ or $k \nmid b$.

We will prove the contrapositive, i.e. that if $k|a$ and $k|b$ then $k|\gcd(a, b)$. Applying Bézout's Identity, we have that $\exists x, y \in \mathbb{Z}$ s.t. $\gcd(a, b) = ax + by$. Applying the divisibility requirements we also have that $\exists m, n \in \mathbb{Z}$ s.t. $a = km, b = kn$. Putting these together we get

$$\gcd(a, b) = ax + by = xmk + ynk = k(xm + yn)$$

This is clearly divisible by k so we're done. $\square$