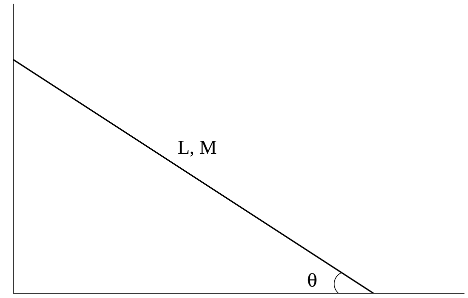
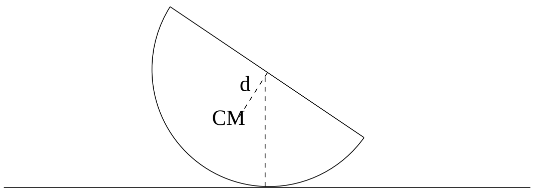
The last problem set - due Monday, March 28, by 8pm

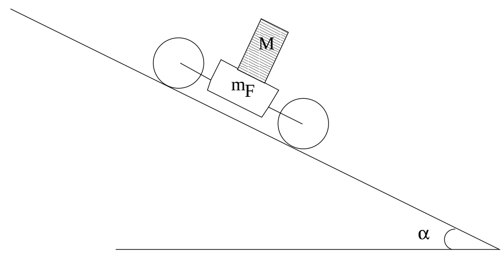
1. A homogeneous rod of length L and mass M is sliding down a frictionless wall (see Fig. $\rightarrow$; the floor is also frictionless). Using θ as a generalized coordinate, find the Lagrangian and the equation of motion. Assuming that at $t = 0$, $\theta(0) = \theta_0, \dot{\theta}(0) = 0$, find after how much time will the rod hit the ground? Write the answer as an integral, do not try to integrate it. (You could numerically evaluate this integral for a few values of θ_0, if you wish to).

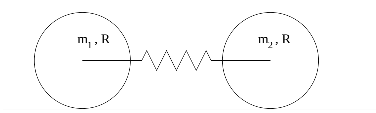




2. Find the frequency of small oscillations of a half-cylinder. Assume that it can roll without slipping, and it has a total mass M (uniform distribution) and radius R. The position of the center of mass is at a distance d from its axis.
For fun: what is the value of d?

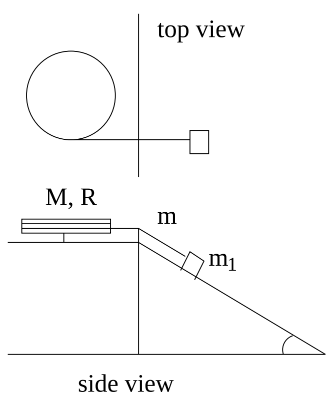
3. Find the equation of motion for the two-wheel cart shown in the Fig. $\rightarrow$ Assume that each wheel is a uniform cylinder of mass m_w and radius R, that the mass of the frame is m_F while the mass of the rider is M. The cart is rolling without slipping down the plane. How can one optimize the cart (increase its acceleration) - for example, should the wheels be made heavier or lighter as compared to the frame?

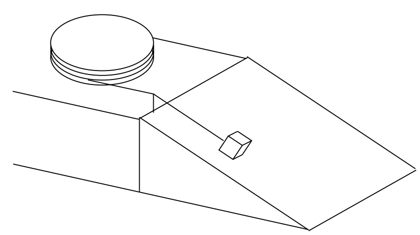




4. Two homogeneous disks of masses $m_1 = m$ and $m_2 = 2m$, both of radius R, have their centers connected by a spring such that they can roll freely without slipping. At the initial moment, the centers of the disks are l_0 apart and moving towards left with speed v_0 (disk 1), and towards left with speed $2v_0$ (disk 2). These initial conditions are such that the disks never collide. Find their positions at all later times. The spring has an unstretched length l_0, and a spring constant k.

5. Consider the system shown $\rightarrow$. The object of mass m_1 is tied, through a homogeneous, very thin string of total length L and total mass m to a wheel of total mass M and radius R. At the initial moment, the mass m_1 is released from the top of the incline of angle α, with zero initial speed. If l is the distance measured from the top of the incline to m_1, find $l(t)$.





Hint: You may find the following integral useful: For $a > 0$, $\int \frac{dx}{\sqrt{x+ax^2}} = \frac{2}{\sqrt{a}}\text{arcsinh}(\sqrt{ax}) + const$; where if $\text{arcsinh}(u) = v \rightarrow u = \sinh(v) = \frac{1}{2}\left(e^v - e^{-v}\right)$

6. **This problem is optional, but you will get 3 bonus points for any reasonable attempt:** Draw a "concept map" for the material covered in this course. For examples of a concept

maps google the term or see

http://cmap.ihmc.us/Publications/ResearchPapers/TheoryCmaps/Fig5CmapSeasons-large.png

Basically, start by writing down things like “Lagrangian”, “action”, “Euler-Lagrange equations” and other concepts we learned in this course, and make a map showing how they are related to one another. This should be a good exercise to help you prepare for the second midterm in terms of systematizing the material covered in this class, and the various strategies that we used for solving various types of problems.

THE END