PHYS408 Homework 2

Xander Naumenko

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Question 1a. In class, we derived the following result in the Fraunhofer limit:

$$E(x', y') \approx \frac{e^{ikz}}{i\lambda z} e^{ik\frac{x'^2+y'^2}{2z}} \tilde{E}(k_x, k_y).$$

Thus all we have to do is compute the Fourier transform of the two slits. We can describe the slits as two shifted rectangles (assuming the input field E is uniform and has magnitude 1):

$$E(x, y) = \text{rect}\left(\frac{x}{d_x}\right)\left(\text{rect}\left(\frac{y-\Delta/2}{d_y}\right) + \text{rect}\left(\frac{y+\Delta/2}{d_y}\right)\right).$$

Also recall the following transform:

$$\mathcal{F}(\text{rect}(ax)) = \frac{1}{|a|}\text{sinc}\left(\frac{k_x}{a}\right).$$

Using this we get the following:

$$\tilde{E}(k_x, k_y) = d_x d_y \text{sinc}\left(\frac{d_x k_x}{2\pi}\right)\text{sinc}\left(\frac{d_y k_y}{2\pi}\right)\left(e^{i\Delta k_y/2} + e^{-i\Delta k_y/2}\right).$$

Intensity is proportional to electric field squared:

$$I(x', y') = \frac{1}{2}\epsilon_0 |E(x', y')|^2 c = \frac{c\epsilon_0 d_x d_y}{\lambda z}\text{sinc}\left(\frac{d_x k_x}{2\pi}\right)\text{sinc}\left(\frac{d_y k_y}{2\pi}\right)\sin\left(\frac{\Delta k_y}{2}\right)$$

where as usual $k_x = \frac{kx'}{z}$ and $k_y = \frac{ky'}{z}$.

Question 1b. See figure 1. The code used to produced the graphs is here:

```
import numpy as np
import matplotlib.pyplot as plt

dx = 0.01
dy = 0.001
delta = 0.005
lam = 500e-9
z = 50

k = 2*np.pi/lam
```

```
c = 3e8
eps = 8.85e-12

x = np.linspace(-0.05, 0.05, 1000)
y = np.linspace(-0.05, 0.05, 1000)

kx = k*x/z
ky = k*y/z

Ix = c*eps*dx*dy/lam/z*np.sinc(dx*kx/(2*np.pi))
Iy = c*eps*dx*dy/lam/z*np.sinc(dy*ky/(2*np.pi))*np.sin(delta*ky/2)

plt.plot(x, Ix)
plt.title("Intensity of Double Slit for y=0 Axis")
plt.xlabel("x' (m)")
plt.ylabel("I (W/m$^2$)")
plt.show()

plt.plot(y, Iy)
plt.title("Intensity of Double Slit for x=0 Axis")
plt.xlabel("y' (m)")
plt.ylabel("I (W/m$^2$)")

plt.show()
```

Question 1c. The Fraunhofer limit applies only if $\frac{x^2}{\lambda}, \frac{y^2}{\lambda} \ll \frac{z}{\pi}$. In our case, these are $\frac{(d_x/2)^2}{\lambda} = 50$ and $\frac{(d_y/2)^2}{\lambda} = \frac{1}{2}$. In comparison to $\frac{z}{\pi} \approx 15.9$, we see that the x-axis is no in the Fraunhofer approximation, so our results above are not totally valid.

Question 1d.

Question 2a. Width the addition of the finite length, the thickness function is

$$d(x) = \left(\overline{d} + \frac{d_0}{2}\sin\left(2\pi x/\Lambda\right)\right)\operatorname{rect}(x/d_x).$$

The Fourier transform of the left hand side is

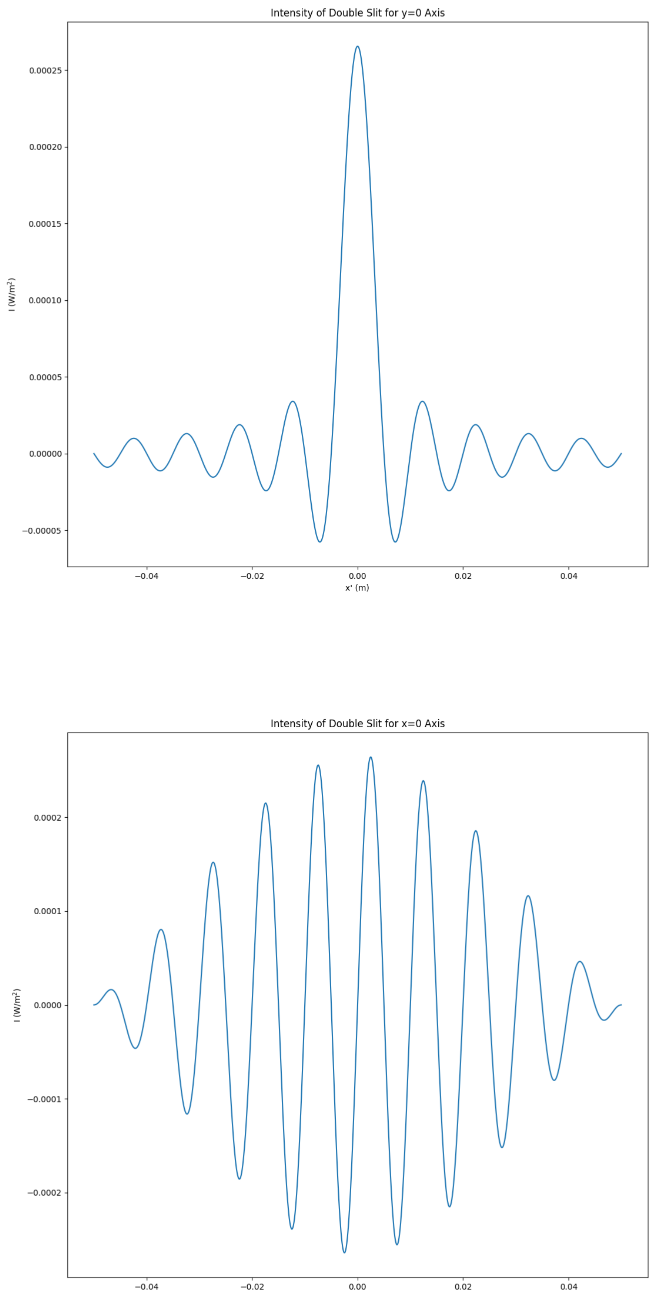

Figure 1: Graphs for question 1b.