PHYS 350 Homework 3

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Question 1a. Taking the Euler Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) = \frac{d}{dt}\left(\alpha q^2 \dot{q}\right) = 2\alpha q \dot{q}^2 + \alpha q^2 \ddot{q} = \frac{\partial \mathcal{L}}{\partial q} = \alpha q \dot{q}^2 - 2\beta q$$

$$\implies \ddot{q} = -\frac{\dot{q}^2}{q} - \frac{2\beta}{q}.$$

Question 1b. There is no time dependence, so energy is conserved. There is q dependence in the lagrangian, so there is no momentum conserved. Computing the energy:

$$E = \frac{\partial \mathcal{L}}{\partial \dot{q}}\dot{q} - \mathcal{L} = \alpha q^2 \dot{q}^2 - \frac{\alpha}{2} q^2 \dot{q}^2 + \beta q^2 = \alpha q^2 \dot{q}^2 + \beta q^2.$$

Question 1c. From the initial conditions we have the energy is $E = \beta q_0^2$. Rearranging the energy equation we then have that:

$$\beta q_0^2 = \alpha q^2 \dot{q}^2 + \beta q^2 \implies \dot{q} = \sqrt{\frac{\beta\left(q_0^2 - q^2\right)}{\alpha q^2}} = \frac{dq}{dt}$$

$$\implies \int_0^T dt = T = \int_{q_0}^0 \sqrt{\frac{\alpha q^2}{\beta\left(q_0^2 - q^2\right)}} dq.$$

Let $p = q_0^2 - q^2 \implies dp = -2q dq$. Also note that the sign could be either positive or negative when we took the square root, so to get a positive time we choose it appropriately. Then the integral becomes:

$$T = \sqrt{\frac{\alpha}{\beta}} \int_0^{q_0^2} \frac{1}{2\sqrt{p}} dp = \sqrt{\frac{\alpha}{\beta}} \sqrt{p}\Big|_0^{q_0^2} = \sqrt{\frac{\alpha}{\beta}} q_0^2.$$

Question 2a. Based on the constraints of the system $s = 2$. Let ϕ be the angle of m_2 around the z axis, while θ is the angle between the z axis and m_2. Then the kinetic terms of the lagrangian for m_2 are simple $\frac{m_2}{2} a^2 \dot{\theta}^2 \sin^2 \phi + \frac{m_2}{2} a^2 \dot{\phi}^2$, while the kinetic term for m_1 is $\frac{m_1}{2}\left(4a^2 \dot{\phi}^2 \sin^2 \phi\right)$. The potential terms combined are then