# PHYS 408, 2023W2

## Problem Set 4:

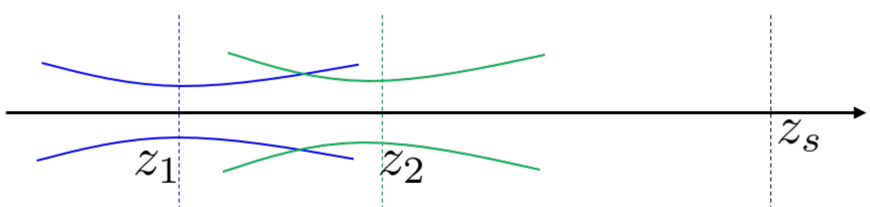
## Cavities, Periodic Structures, Interferometers

*Posted*: Fri, March 1 $\longrightarrow$ *Due*: Fri, March 22.

1. Two identical Gaussian beams are propagating along the z axis. Their complex amplitude is given by equation 3.1-7 in the textbook (discussed in detail in Lecture 3).

   (a) Suppose the locations of two beam waists are at position $z = z_1$ and $z = z_2$ respectively. If we put a screen that is normal to the $z$ axis at $z = z_s$, as shown in the figure below, what is the intensity distribution on the screen? Describe the shape of the pattern.

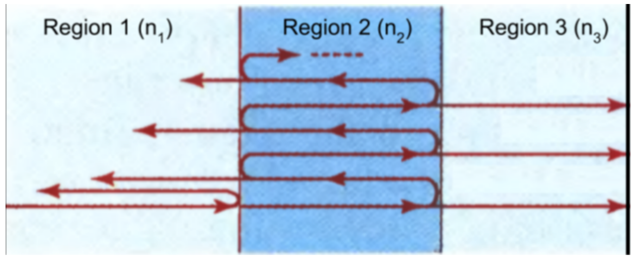
   (b) Suppose the beams are produced by a HeNe laser (just like in your lab, $\lambda = 633\,\text{nm}$), their waist radius is 4 $\mu$m, and the distance between the two beam waists is 5 cm. Plot the normalized interference pattern on the screen placed 1 m away from the midpoint between the two waists. Assume the square screen 2 cm $\times$ 2 cm. You may plot either a one dimensional cross section through the center of the screen, or (for bonus points) the full 2d distribution.




2. In Lecture 13, we have discussed anti-reflective (AR) coating implemented with a quarter-wave film, schematically shown below. Using the ABCD matrix approach, we derived the condition for a perfect AR with zero reflection, which was $n_2 = \sqrt{n_1 n_3}$. Here, instead of the eigenmode solution of the system, let us consider the propagation of light step by step.

   (a) Using the Fresnel equations we derived in Lecture 9, write down the single-interface ($1 \leftrightarrow 2$ and $2 \leftrightarrow 3$) transmittance and reflectance that are involved in this system: $t_{12}, t_{23}, t_{21}, r_{12}, r_{23}$ and $r_{21}$.

   (b) Write down the infinite series of reflected waves in Region 1 ($E_r$), and transmitted waves in Region 3 ($E_t$) – see the figure below, in terms of $E_i$




and the single-interface transmittance and reflectance. Don't forget the phases which accumulate on each pass between the two interfaces.

(c) Sum the (geometric!) series and calculate the overall transmittance $t_{13}$ and reflectance $r_{13}$ of this system.

(d) Finally, confirm the result we have derived in class, i.e. that if we carefully make the thin film width $d$ to be exactly one quarter of the wavelength (in that material), the reflection can be reduced to zero as long as $n_2 = \sqrt{n_1 n_3}$.

3. Suppose we want to construct an optical cavity using two concave spherical mirrors, with radii of curvature $R_1 = -8$ cm and $R_2 = -15$ cm, separated by distance $d$.

   (a) Find out the range (or ranges) of $d$ which will make the cavity stable.

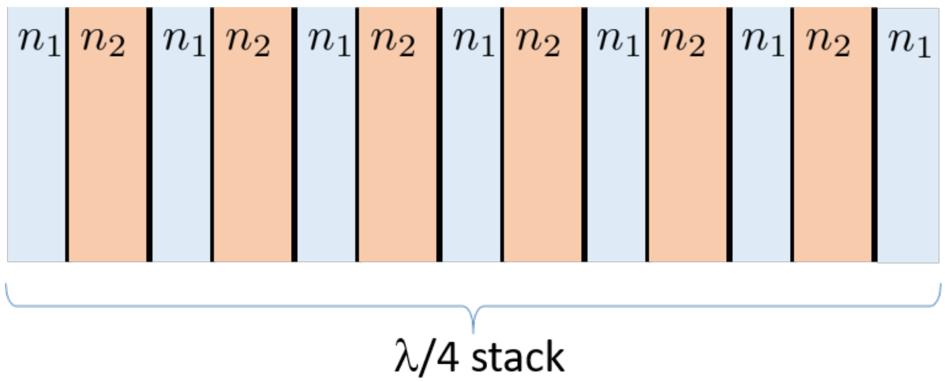
   (b) Calculate the q-parameters of Gaussian modes in cavities of $d = 5$ cm and $d = 17$ cm. (You can use your results from Homework 1 Problem 2.)

   (c) If we set $d = 10$ cm and try to fit a Gaussian into the cavity, what would be the beam's Rayleigh length $z_0$ of that Gaussian? Explain why this $z_0$ leads to an unstable mode.

   (d) If we set $d = 5$ cm, excite it using a source of frequency $\nu = 4.74$ THz, and assume that the speed of light is $c = 3 \times 10^8$ m/s, what longitudinal mode numbers could be excited if only transverse modes with $l$ or $m$ less than or equal to 2 could be excited in practice? (Refer to Steck Chapter 7.5-7.6)

4. In Lecture 13, we discussed wave propagation through periodic dielectric structures. We showed that a quarter-wave stack can work as a highly efficient mirror – a *high reflector* (HR). We discussed an algorithm to calculate the field inside the dielectrics by starting from the transmission side and using inverse **M** matrices to propagate the transmitted field layer-by-layer backwards. Here, you will implement this procedure *numerically* and investigate the properties of the dielectric quarter-wave stack.

   (a) Consider a period stack shown in the figure below. Take $n_1 = 1.5$ (glass), $n_2 = 1.38$ ($MgF_2$), 20 $\{n_1, n_2\}$ segments (40 layers in total), with all the layers being quarter-wave plates (i.e. $kd_1 = kd_2 = \pi/2$). Calculate and plot the intensities of forward and backward-propagating waves inside the structure (i.e. as a function of the layer number). Does your calculation confirm that the stack is working as a high reflector? If not, how can you make its reflectivity higher?




(b) Introduce a half-wave layer of Sapphire ($n_3 = 1.77, kd_3 = \pi$) in the middle of the stack as shown below. Repeat the calculation of the intensity distribution inside the structure. Describe your observations.