Problem Set 4 - Blackbody radiation

(Dated: PHYS403, Spring 2024)

I. Photon numbers

Consider a gas of photons in thermal equilibrium. Show that the average number of photons grows with temperature as $\sim T^3$. *Hint: first consider the mean number of photons, $\langle N_{\vec{p}}\rangle$ with a single momentum $\vec{p}$, then compute $\langle N\rangle = \sum_{\vec{p}}\langle N_{\vec{p}}\rangle$. Then, use all the "usual" tricks of turning sums to integrals and rescaling variables to get dimensionless integrals. Don't forget about polarization!*

II. Global warming

Recall from the lectures that a black body at temperature T absorbs all the radiation that falls on it and emits radiation at the rate (power) $P = \sigma T^4$, where $\sigma = 5.67\times 10^{-8}\mathrm{W/m^2K^4}$ is Stefan's constant.

Suppose the earth were a black, perfectly (thermally) conducting sphere that absorbed all of the light from the sun that it absorbs (luckily this is not the case! although it may soon be a better approximation than we would like if we keep dumping greenhouse gases into the atmosphere!).

The sun has radius 7×10^5km and is at a distance of 1.5×10^8km from the earth. The sun radiates like a black body at temperature 6000K.

1. Write an equation for the rate of change of energy of the Earth with time: $\frac{dE_{earth}}{dt}$ as a function of the temperature of the Earth, T_e, and the temperature of the sun, T_s. Your equation should include two contributions: i) the increase in energy from absorbing light from the sun, and ii) the decrease in energy from Earth radiating light (as a blackbody) back into space.

2. After a very long time, what is the steady state temperature of this "black-body earth" (assume the Sun's temperature stays constant, as it burns a continuous supply of nuclear fuel)?

III. Blackbody radiation in different numbers of spatial dimensions

Suppose our universe had d spatial dimensions (where d is a positive integer). Consider a universe filled with a gas of photons (e.g. the cosmic microwave background) in thermal equilibrium at temperature T.

1. How does the average energy, $\langle E\rangle$ of the gas scale with temperature? (I am just looking for the proportionality with T, not paying attention to various numerical or fundamental constant factors).

2. String theory (a candidate theory to explain quantum mechanical origins of gravity) requires $d = 11$ dimensions, but postulates that all but 3 of them are rolled up tightly (those dimensions have periodic boundary conditions with a very short length, $\ell_P \approx 10^{-35}$m). If string

theory is right (the jury is still out!), why do we see that thermal photons in our universe have the $d = 3$ Planck distribution? Are there conditions under which one would instead see the $d = 11$ distribution?