Assignment - 2
Course: PHYS 304 - Introduction to Quantum Mechanics

Term: Winter 2022
Instructor: Dr. Ke Zou

Note: The solutions of the homework submitted need to include steps showing how to solve the problems.

Problem 1

Griffiths, Problem 1.16. A particle is represented (at time $t = 0$) by the wave function

$$\Psi(x,0) = \begin{cases} A(a^2 - x^2), & -a \le x \le a \\ 0, & \text{otherwise} \end{cases}$$

a) Determine the normalization constant A.
b) What is the expectation value of x?
c) What is the expectation value of p? (Note that you *cannot* get it from $< p >= md < x > /dt$. Why not?)
d) Find the expectation value of x^2.
e) Find the expectation value of p^2.
f) Find the uncertainty in x (σ_x).
g) Find the uncertainty in p (σ_p).
h) Check that your results are consistent with the uncertainty principle.

Problem 2

Griffiths, Problem 1.18

Very roughly speaking, quantum mechanics is relevant when the de Broglie wavelength of the particle in question (h/p) is greater than the characteristic size of the system (d). In thermal equilibrium at (Kelvin) temperature T, the average kinetic energy of a particle is

$$\frac{p^2}{2m} = \frac{3}{2}k_B T$$

(where k_b is Boltzmann's constant), so the typical de Broglie wavelength is $\lambda = \frac{h}{\sqrt{3mk_bT}}$. The purpose of this problem is to determine which systems will have to be treated quantum mechanically, and which can safely be described clasically.

a) **Solids.** The lattice spacing in a typical solid is around $d = 0.3$ nm. Find the temperature below which the unbound *electrons* in a solid are quantum mechanical. Below what temperature are the *nuclei* in a solid quantum mechanical? (Use silicon as an example.)

Moral: The free electrons in a solid are *always* quantum mechanical; the nuclei are generally *not* quantum mechanical. The same goes for liquids (for which the interatomic spacing is roughly the same), with the exception of helium below 4 K.

b) **Gases.** For what temperatures are the atoms in an ideal gas at pressure P quantum mechanical? *Hint:* Use the ideal gas law ($PV = NK_bT$) to deduce the interatomic spacing.

Answer: $T < (1/k_B)(h^2/3m)^{3/5}P^{2/5}$. *Obviously (for the gas to show quantum behavior)* we want m to be as *small* as possible, and P as *large* as possible. Put in the numbers for helium at atmospheric pressure. Is hydrogen in outer space (where the interatomic spacing is about 1 cm and the temperature is 3 K) quantum mechanical? (Assume it's monoatomic hydrogen, not H_2)

Problem 3

Griffiths, Problem 2.2 Show that E must exceed the minimum value of $V(x)$, for every normalizable solution to the time-independent Schrödinger equation. What is the classical analog to this statement?

Hint: Rewrite equation 2.5 in the form

$$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}\left[V(x) - E\right]\psi;$$

if $E < V_{min}$, then ψ and its second derivative always have the *same sign*–argue that such a function cannot be normalized.

Problem 4

If $\Psi_1(x,t), \Psi_2(x,t)$ are two solutions for the full Schrödinger equation prove

$$\frac{d}{dt}\int_{-\infty}^{\infty} \Psi_1^* \Psi_2 dx = 0.$$