PHYS 350 Homework 5

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Question 1. For this problem $s = 1$, and choose $q = \theta$. We tie the IRF to the bottom left corner of the wall. Then for the center of mass of the rod, we have:

$$x_{CM} = \frac{1}{2}\cos\theta, \dot{x}_{CM} = -\frac{1}{2}\dot{\theta}\sin\theta.$$

$$y_{CM} = \frac{1}{2}\sin\theta, \dot{y}_{CM} = \frac{1}{2}\dot{\theta}\cos\theta.$$

For the rod's kinetic energy, note that the moment of inertia of the rod is $I = \frac{ML^2}{12}$. Then we have:

$$\mathcal{L} = T - U = \frac{1}{2}mv_{CM}^2 + \frac{1}{2}\vec{\Omega}\hat{I}\vec{\Omega} - \frac{L}{2}mg\sin\theta = \frac{1}{6}mL^2\dot{\theta}^2 - \frac{L}{2}mg\sin\theta.$$

Because there's no time dependence we can find the energy in the system:

$$E = \frac{d\mathcal{L}}{d\dot{\theta}}\dot{\theta} - \mathcal{L} = \frac{1}{6}mL^2\dot{\theta}^2 + \frac{L}{2}mg\sin\theta = \frac{L}{2}mg\sin\theta_0.$$

$$\implies \dot{\theta}^2 = \frac{3g(\sin\theta_0 - \sin\theta)}{L}.$$

$$\implies T = \int_0^T dt = \int_{\theta_0}^0 \sqrt{\frac{L}{3g(\sin\theta_0 - \sin\theta)}}d\theta.$$

Question 2. For this problem $s = 1$, and choose $q = \theta$ where θ is the angle between the vertical and the line marked d on the diagram. First we calculate the moment of inertia by noting that with the parallel axis theorem, two times the moment of inertia displaced by distance d to the center should be equal to the moment of inertia of the semicircle, i.e.

$$2(I_{CM} + Md^2) = \frac{1}{2}MR^2 \implies I_{CM} = \frac{1}{4}MR^2 - Md^2.$$

Next we consider the kinetic energy at the point O at the circle touches the ground, with r' being the distance from the CM to O.

$$r'^2 = d^2 + R^2 - 2dR\cos\theta.$$

$$I_O = I_{CM} + r'^2 M = \frac{5}{4}MR^2 - 2dRM\cos\theta.$$

$$\mathcal{L} = T - U = \left(\frac{5}{8}R - dM\cos\theta\right)R\dot{\theta}^2 - Mgd\cos\theta.$$