Assignment - 4 Term: Winter 2022
Course: PHYS 304 - Introduction to Quantum Mechanics Instructor: Dr. Ke Zou

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Problem 1
Griffiths, Problem 2.13. A particle in the harmonic oscillator potential starts out in the state

$$\Psi(x,0) = A\left[3\psi_0(x) + 4\psi_1(x)\right] \tag{1}$$

a) Find A.
b) Construct $\Psi(x,t)$ and $|\Psi(x,t)|^2$. Don't get too excited if $|\Psi(x,t)|^2$ oscillates at exactly the classical frequency; what would it have been had I specified $\psi_2(x)$, instead of $|\psi_1(x)|$?
c) Find $< x >$ and $< p >$. Check that Ehrenfest's theorem (Equation 1.38) holds, for this wave function.
d) If you measured the energy of this particle, what values might you get, and with what probabilities?
Hint: In this and other problems involving the harmonic oscillator it simplifies matters if you introduce the variable $\xi = \sqrt{m\omega/\hbar}x$ *and the constant* $\alpha = (m\omega/\pi\hbar)^{1/4}$.

Problem 2
Griffiths, Problem 2.14. In the ground state of the harmonic oscillator, what is the probability (correct to three significant digits) of finding the particle outside the classically allowed region? *Hint* : Classically, the energy of an oscillator is $E = (1/2)ka^2 = (1/2)m\omega^2a^2$, where a is the amplitude. So the 'classically allowed region' for an oscillator of energy E extends from $-\sqrt{2E/m\omega^2}$ to $+\sqrt{2E/m\omega^2}$. Look in a math table under 'Normal Distribution' or 'Error Function' for the numerical value of the integral, or evaluate it by computer.

Problem 3
Griffiths, Problem 2.21. **The gaussian wave packet.** A free particle has the initial wave function

$$\Psi(x,0) = Ae^{-ax^2}$$

where A and a are (real and positive) constants.
(a) Normalize $\Psi(x,0)$.
(b) Find $\Psi(x,t)$. Hint: Integrals of the form

$$\int_{-\infty}^{+\infty} e^{-\left(ax^2+bx\right)}dx$$

can be handled by "completing the square": Let $y \equiv \sqrt{a}[x + (b/2a)]$, and note that $\left(ax^2 + bx\right) = y^2 - \left(b^2/4a\right)$. Answer: $\Psi(x,t) = \left(\frac{2a}{\pi}\right)^{1/4}\frac{1}{\gamma}e^{-ax^2/\gamma^2}$, where $\gamma \equiv \sqrt{1 + (2i\hbar at/m)}$.
(c) Find $|\Psi(x,t)|^2$. Express your answer in terms of the quantity $w \equiv \sqrt{a/\left[1 + (2\hbar at/m)^2\right]}$ Sketch $|\Psi|^2$ (as a function of x) at $t = 0$, and again for some very large t. Qualitatively, what happens to $|\Psi|^2$, as time goes on?
(d) Find $\langle x\rangle, \langle p\rangle, \left\langle x^2\right\rangle, \left\langle p^2\right\rangle, \sigma_x$, and σ_p. Partial answer: $\left\langle p^2\right\rangle = a\hbar^2$, but it may take some algebra to reduce it to this simple form.
(e) Does the uncertainty principle hold? At what time t does the system come closest to the uncertainty limit?

Problem 4

Griffiths, Problem 2.41. Find the allowed energies of the half harmonic oscillator

$$V(x) = \begin{cases} (1/2)m\omega^2 x^2, & x > 0 \\ \infty, & x < 0 \end{cases}$$

(This represents, for example, a spring that can be stretched, but not compressed.) Hint: This requires some careful thought, but very little actual calculation.