PHYS 408, 2023W2

Problem Set 1: Wave Optics

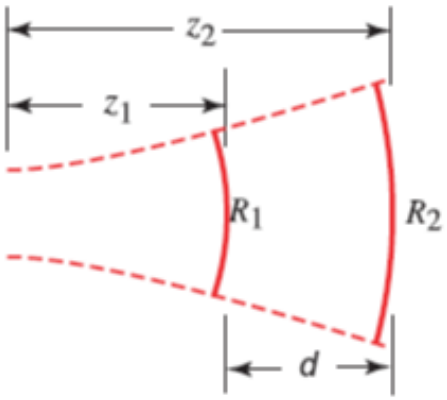
Posted: Fri, January 12 $\longrightarrow$ *Due*: Fri, January 26.

1. A laser of wavelength λ emits a Gaussian beam, where the focus of the Gaussian beam occurs at the front of the laser. The laser beam hits a screen, which is a distance d from the front of the laser.
   (a) What beam-waist parameter w_0 produces the smallest spot on the screen, and what is the corresponding $1/e^2$ beam radius on the screen?
   (b) What is the $1/e^2$ beam radius on the screen for $\lambda = 600$ nm and $d = 10$ cm?
2. A Gaussian beam has radii of curvature R_1 and R_2 at two points on the beam axis separated by a distance d, as illustrated in the figure below. Verify that the location of the beam center and its depth of focus may be determined from the relations

$$z_1 = \frac{-d(R_2 - d)}{R_2 - R_1 - 2d} \tag{1}$$

$$z_0^2 = \frac{-d(R_1 + d)(R_2 - d)(R_2 - R_1 - d)}{(R_2 - R_1 - 2d)^2} \tag{2}$$

$$w_0 = \sqrt{\frac{\lambda z_0}{\pi}} \tag{3}$$

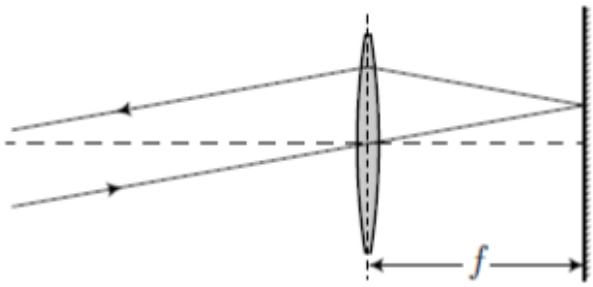


3. Check that the paraboloidal wave

$$\vec{E}(r) = \frac{\vec{E}_0}{z} \exp\left[ik\frac{x^2+y^2}{2z}\right] \tag{4}$$

is indeed a solution of the paraxial wave equation.

4. A retroreflector is any optic that reflects an incident ray, such that the exiting ray is parallel (but opposite) to the incoming ray. One version of a "cat's eye" retroreflector uses a thin lens of focal length f and a mirror as shown.



Set up the ray matrix for this optical system to prove that it is, indeed, a retroreflector.

5. In Lecture 4, we wrote down the following ABCD matrix for a single spherical interface between two dielectric media:

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{(n_2-n_1)}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix} \tag{5}$$

Check that this is correct by considering a ray making an angle θ_1 with the z axis, meeting the spherical boundary at a point of height y, and subsequently changing direction so that the refracted ray makes an angle $-\theta_2$ with the z axis, as shown.