Math 220 Homework 4 Question 8

Xander Naumenko

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Question 8. Let $L \in \mathbb{R}$ and choose $\epsilon = \frac{1}{2}$. Next let $N \in \mathbb{N}$. If $L \geq 0$ then select $n = 2N + 1 > N$. Otherwise if $L < 0$ then select $n = 2N > N$. Note that in either case $n \geq 2$ and L has the opposite sign as $(-1)^n$. Then we get

$$|x_n - L| = |(-1)^n + \frac{1}{n} - L| \geq \left| |(-1)^n - L| - |\frac{1}{n}| \right| \geq |1 - \frac{1}{n}| \geq |1 - \frac{1}{2}| = \frac{1}{2} = \epsilon$$

The middle step is an application of the triangle inequality. Thus the (x_n) doesn't converge. $\square$