

## Assignment - 5

Course: PHYS 304 - Introduction to Quantum Mechanics

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### Problem 1

**Griffiths 2.29.** Analyze the *odd* bound state wave functions for the finite square well. Derive the transcendental equation for the allowed energies, and solve it graphically. Examine the two limiting cases. Is there always an odd bound state?

### Problem 2

**Griffiths 2.33** Determine the transmission coefficient for a rectangular *barrier* (same as Equation 2.148, only with  $V(x) = +V_0 > 0$  in the region  $-a < x < a$ ). Treat separately the three cases,  $E < V_0$ ,  $E = V_0$  and  $E > V_0$  (note that the wave function inside the barrier is different in the three cases).

### Problem 3

**Griffiths 2.34** Consider the ‘step’ potential,

$$V(x) = \begin{cases} 0 & x \leq 0, \\ V_0, & x \geq 0. \end{cases}$$

- a) Calculate the reflection coefficient, for the case  $E < V_0$ , and comment on the answer.
- b) Calculate the reflection coefficient for the case  $E > V_0$ .
- c) For a potential (such as this one) that does not go back to zero to the right of the barrier, the transmission coefficient is *not* simply  $|F|^2/|A|^2$  (with  $A$  the incident amplitude and  $F$  the transmitted amplitude), because the transmitted wave travels at a different *speed*. Show that

$$T = \sqrt{\frac{E - V_0}{E}} \frac{|F|^2}{|A|^2}, \quad (1)$$

for  $E > V_0$ . *Hint:* You can figure it out using Equation 2.99, or more elegantly, but less informatively, from the probability current (Problem 2.18). What is  $T$ , for  $E < V_0$ ?

- d) For  $E > V_0$ , calculate the transmission coefficient for the step potential, and check that  $T + R = 1$ .

### Problem 4

**Griffiths A.8 (page 472)** Given the following two matrices:

$$A = \begin{pmatrix} -1 & 1 & i \\ 2 & 0 & 3 \\ 2i & -2i & 2 \end{pmatrix}, B = \begin{pmatrix} 2 & 0 & -i \\ 0 & 1 & 0 \\ i & 3 & 2. \end{pmatrix} \quad (2)$$

compute: a)  $A+B$ , b)  $AB$ , c)  $[A,B]$ , d)  $\tilde{A}$ , e)  $A^*$ , f)  $A^\dagger$ , g)  $\det(B)$ , and h)  $B^{-1}$ . Check that  $BB^{-1} = 1$ . Does  $A$  have an inverse?