HW2_2021

October 7, 2021

1 Motion of a charge through a charged ring

Write down the formula for the electric field along the axis of a charged ring of radius R that carries a total charge $-Q$. Imagine that a charge $Q = 1\,\mu\mathrm{C}$ of mass $m = 100\,g$ is released from a point x_0 on the axis at time $t = 0$. Compute the trajectory of the charge assuming $R = 1m$ and that the ring carries the opposite charge $-Q$ up to $t = 1000\,s$ for $x_0 = 1\,m$ and $x_0 = 10\,m$.

In order to do this, we need to numerically integrate Newton's equation of motion $\mathbf{F} = m\mathbf{a}$. This is a second order ordinary differential equation. Without getting into any details about numerical methods for integrating ODEs, all of them are essentially based on a Taylor expansion in time for a small time time increment Δt. A suitable algorithm for this particular kind is the "velocity Verlet" method. Let us denote position, velocity and acceleration at time t with $x(t)$, $v(t)$ and $a(t)$ then these quantities after a small time step Δt can be obtained as follows:

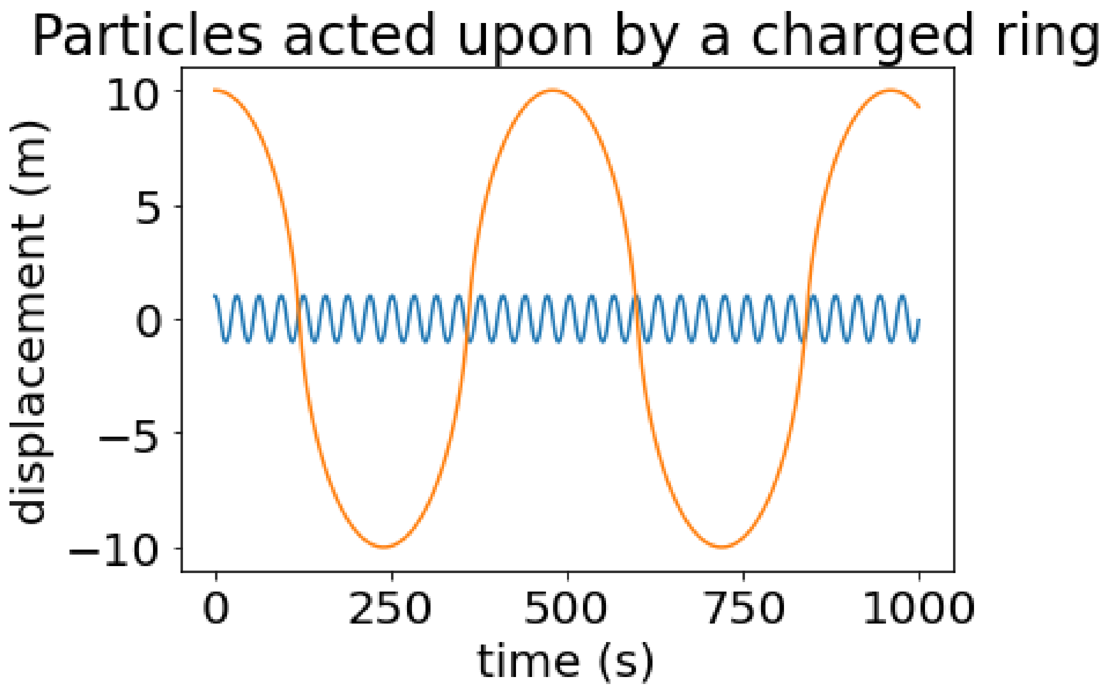
$$
\begin{aligned}
v(t+\Delta t/2) &= v(t) + a(t)\Delta t/2 \\
x(t+\Delta t) &= x(t) + v(t+\Delta t/2)\Delta t \\
v(t+\Delta t) &= v(t+\Delta t/2) + a(t+\Delta t)\Delta t/2
\end{aligned}
$$

These equations are in a form that can be directly implemented in a computer program. First define all relevant constants and a function that returns the force or acceleration as a function of position $x(t)$, and initialize the position to $x(0) = x_0$ and the velocity to $v(0) = 0$. Then advance $x(t)$ and $y(t)$ according to the algorithm above and store the trajectory in an array. Your answer should consist of a plot of this trajectory (position versus time) for the two initial positions given above. Comment on the differences between these trajectories. Make sure you integrate long enough to see the characteristic pattern of the motion in each case.

Hints: By considering the symmetry of the problem, you should notice that the charge moves in one dimension only. You can try out different values of the timestep Δt (what happens qualitatively), but a possible value for the parameters given above is $\Delta t = 1s$. Also, ignore gravity. If you have time, experiment with the paramenters of the problem (mass, charge, size of ring, etc.)

[14]:
```python
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.pyplot import rcParams
rcParams.update({'font.size':20}) #optional to change default font size

# define constants
k = 8.98755e9
```

```python
m=0.1
Q=1e-6
delta_t = 0.1
R=1

# set up arrays for position and time
time = np.linspace(0, 1000, int(1000/delta_t))
pos1 = np.zeros_like(time)
vel1 = np.zeros_like(time)
pos2 = np.zeros_like(time)
vel2 = np.zeros_like(time)

# define a function that returns the force or acceleration
def get_e_field(z):
    return -k*Q*z/(z**2+R**2)**(3/2)

# set initial position and velocity
pos1[0] = 1
pos2[0] = 10

# integrate equation of motion

def update(pos, vel):
    half_v = vel[i] + get_e_field(pos[i]) * delta_t * Q / m / 2
    pos[i+1] = pos[i] + half_v * delta_t
    vel[i+1] = half_v + get_e_field(pos[i+1]) * delta_t * Q / m / 2

for i, t in enumerate(time):
    try:
        update(pos1, vel1)
        update(pos2, vel2)
    except IndexError:
        break


# plot trajectories
plt.plot(time, pos1, label="x=1m")
plt.plot(time, pos2, label="x=10m")
plt.title("Particles acted upon by a charged ring")
plt.xlabel("time (s)")
plt.ylabel("displacement (m)")
plt.show()
```


Particles acted upon by a charged ring
10
5
0
−5
−10
displacement (m)
0
250
500
750
1000
time (s)

```
[ ]:
```