Math 322 Homework 2

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Question 1. Simply using the definition of the maps and manually carrying through where each number gets mapped:

$$\alpha\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 5 & 4 & 3 \end{pmatrix}.$$

$$\beta\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 1 & 2 & 5 \end{pmatrix}.$$

$$\alpha^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{pmatrix}.$$

Question 4. It is clearly closed, since by definition the operation always produces a tuple of reals, and since the first entry can never be zero the product can't either. For associativity, let $(a,b),(c,d),(e,f) \in G$. Then $((a,b)(c,d))(e,f) = (a,b)((c,d)(e,f)) = (ace, ad+b+acf)$. The inverse of $(a,b) \in G$ is just $(\frac{1}{a}, -\frac{b}{a})$, since $(a,b)(\frac{1}{a}, -\frac{b}{a}) = (1,0) = I$. Finally for any $(a,b) \in G$ we have $(a,b)(1,0) = (1,0)(a,b) = (a,b)$. Thus G is a group.

Question 7. If we apply c to both sides of $ab = 1$, we get $cab = 1 \cdot b = b = c$, as required. Since $b = c$ is a left and right inverse of a, we have $a^{-1} = b$.

For the forward direction of the second part, let $b = a^{-1}$. Then we have $aba = aa^{-1}a = a$ and $ab^2a = a(a^{-1})^2a = 1$ as required. For the backward direction, assume that $aba = a$ and $ab^2a = 1$. Then we have that ab^2 is a left inverse of a and b^2a is a right inverse, so by the first part of the question we have $ab^2 = b^2a$