HOMEWORK 2: DUE OCTOBER 19TH

MATH 437/537: PROF. DRAGOS GHIOCA

Problem 1. (6 points.) Find all integers $n > 1$ with the property that for each positive divisor d of n, we also have that

$$(d+2) \mid (n+2).$$

Problem 2. (4 points.) Find all positive integers m and n such that

$$2^m - 3^n = 7.$$

Problem 3. (5 points.) Let $k \in \mathbb{N}$. Show that there exist k consecutive positive integers with the property that no integer from this set is of the form $a^2 + b^2$ for some $a, b \in \mathbb{Z}$.

Problem 4. (12 points.) As always, for each positive integer m, we have that $d(m)$ is the number of positive divisors of m; also, we let $\phi(m)$ be the corresponding value of the Euler-ϕ function. Then compute the following limits:

- $$\lim_{n\to\infty} \frac{n!}{d(n!)\cdot\phi(n!)}$$
- $$\lim_{n\to\infty} \frac{n!}{2^{d(n!)}}$$