

MATH 406, HWK 6, Due 1 December 2023

1. Alter the program `Laplace.m` (a copy of `Laplace.m`, `V.m`, and `VN.m` can be found on the course web site) to be able to solve the following ‘crack-like’ mixed boundary value problem for Laplace’s equation on a semi-circular domain:

$$\Delta u = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

subject to

$$u(r, 0) = 0 \quad \frac{\partial u}{\partial \theta}(r, \pi) = 0 \quad (1)$$

$$u(a, \theta) = f(\theta) = \sin\left(\frac{1}{2}\theta\right). \quad (2)$$

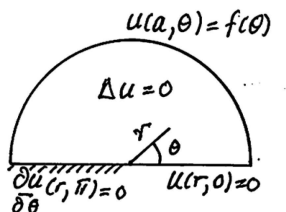


Figure 1:

The exact solution is:

$$u(r, \theta) = \left(\frac{r}{a}\right)^{\frac{1}{2}} \sin\left(\frac{1}{2}\theta\right)$$

Assuming that $a = 10$ compare the exact with the numerical solution for $N = 32$ elements along the line of benchmarks

$$x = 0 \quad y = 0.1 : 0.1 : a - 0.1$$

Also plot the error in a separate plot. Now provide a plot that compares the exact solution along $-a < x < 0, y = 0$ to the values of u computed by the BEM at the element centres.