Assignment - 6

Course: PHYS 304 - Introduction to Quantum Mechanics Instructor: Dr. Ke Zou

Problem 1

Griffiths 3.5 a) and b).

a) Find the hermitian conjugates of x, i, and $\frac{d}{dx}$.

b) Show that $(\hat{Q}\hat{R})^\dagger = \hat{R}^\dagger\hat{Q}^\dagger$ (note the reversed order), $(\hat{Q}+\hat{R})^\dagger = \hat{Q}^\dagger + \hat{R}^\dagger$, and $(c\hat{Q})^\dagger = c^*\hat{Q}^\dagger$ for a complex number c.

Problem 2

Griffiths 3.11 Find the momentum-space wave function $\Phi(p,t)$, for a particle in the ground state of the harmonic oscillator.

No probability calculation.

Problem 3

Griffiths 3.13 Show that

$$< x >= \int \Phi^* \left(i\hbar \frac{\partial}{\partial p} \right) \Phi dp.` \tag{1}$$

Hint: Notice that $x \exp(ipx/\hbar) = -i\hbar(\partial/\partial p)\exp(ipx/\hbar)$, *and use Equation* 2.147. *In momentum space, then, the position operator is* $i\hbar\partial/\partial p$. *More generally,*

$$\langle Q(x,p,t)\rangle = \begin{cases} \int \Psi^* \hat{Q}\left(x, -i\hbar\frac{\partial}{\partial x}, t\right)\Psi dx, & \text{in position space;} \\ \int \Phi^* \hat{Q}\left(i\hbar\frac{\partial}{\partial p}, p, t\right)\Phi dp, & \text{in momentum space.} \end{cases}$$

In principle you can do all calculations in momentum space just as well (though not always as easily) as in position space.

Problem 4

Griffiths 3.25 The Hamiltonian for a certain two-level system is

$$\hat{H} = \epsilon(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|) \tag{2}$$

where $|1\rangle$, $|2\rangle$ is an orthonormal basis and ϵ is a number with the dimensions of energy. Find its eigenvalues and eigenvectors (as linear combinations of $|1\rangle$ and $|2\rangle$). What is the matrix H representing $\hat{H}$ with respect to this basis?

Problem 5

Griffiths 3.33 An operator $\hat{A}$, representing observable A, has two (normalized) eigenstates ψ_1 and ψ_2, with eigenvalues a_1 and a_2, respectively. Operator $\hat{B}$, representing observable B, has two (normalized) eigenstates ϕ_1 and ϕ_2, with eigenvalues b_1 and b_2. The eigenstates are related by

$$\psi_1 = (3\phi_1 + 4\phi_2)/5 \; ; \; \psi_2 = (4\phi_1 - 3\phi_2)/5. \tag{3}$$

a) Observable A is measured, and the value a_1 is obtained. What is the state of the system (immediately) after this measurement?

b) If B is now measured, what are the possible results, and what are their probabilities?

c) Right after the measurement of B, A is measured again. What is the probability of getting a_1? (Note that the answer would be quite different if I had told you the outcome of the B measurement).