Assignment - 8

Course: PHYS 304 - Introduction to Quantum Mechanics Instructor: Dr. Ke Zou

Problem 1

Griffiths 4.1 .

a) Work out all of the canonical commutation relations for components of the operators r and p : $[x, y], [x, p_y], [x, p_x], [p_y, p_z]$, and so on. Answer:

$$[r_i, p_j] = -[p_i, r_j] = i\hbar\delta_{ij}, \quad [r_i, r_j] = [p_i, p_j] = 0$$

where the indices stand for x, y, or z, and $r_x = x, r_y = y$, and $r_z = z$.

(b) Confirm the three-dimensional version of Ehrenfest's theorem,

$$\frac{d}{dt}\langle \mathbf{r} \rangle = \frac{1}{m}\langle \mathbf{p} \rangle,$$

and

$$\frac{d}{dt}\langle \mathbf{p} \rangle = \langle -\nabla V \rangle.$$

(Each of these, of course, stands for three equations-one for each component.) Hint: First check that the "generalized" Ehrenfest theorem, Equation 3.73, is valid in three dimensions.

(c) Formulate Heisenberg's uncertainty principle in three dimensions. Answer:

$$\sigma_x\sigma_{p_x} \geq \hbar/2, \quad \sigma_y\sigma_{p_y} \geq \hbar/2, \quad \sigma_z\sigma_{p_z} \geq \hbar/2$$

but there is no restriction on, say, $\sigma_x\sigma_{p_y}$.

Problem 2

Griffiths 4.2. Use separation of variables in cartesian coordinates to solve the infinite cubical well (or "particle in a box"):

$$V(x, y, z) = \begin{cases} 0, & x, y, z \text{ all between 0 and } a \\ \infty, & \text{otherwise} \end{cases}$$

(a) Find the stationary states, and the corresponding energies.

(b) Call the distinct energies $E_1, E_2, E_3, \ldots$, in order of increasing energy. Find E_1, E_2, E_3, E_4, E_5, and E_6. Determine their degeneracies (that is, the number of different states that share the same energy). Comment: In one dimension degenerate bound states do not occur (see Problem 2.44), but in three dimensions they are very common.

(c) What is the degeneracy of E_{14}, and why is this case interesting?

Problem 3

Griffiths 4.3

(a) Suppose $\psi(r, \theta, \phi) = Ae^{-r/a}$, for some constants A and a. Find E and $V(r)$, assuming $V(r) \to 0$ as $r \to \infty$.

(b) Do the same for $\psi(r, \theta, \phi) = Ae^{-r^2/a^2}$, assuming $V(0) = 0$.

Problem 4

a) What examples that we treated in 1D motion resulted in 'degenerate' eigenstates for a given energy eigenvalue? Recall that for the infinite square well, there was only one eigen function associated with each energy eigen value.

b) Explain how in 3D infinite well example (Problem 4.2) at least some of the degeneracies are a direct result of the symmetry of the potential.

c) Can you identify a 1D symmetry that might be responsible for the degeneracies you identified in 4 a)?