PHYS 304 Homework 6

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Question 1a. From the definition of operators:

$$\int \psi^* x \psi dx = \int (x\psi)^* \psi dx \implies x^\dagger = x.$$

$$\int \psi^* i \psi dx = \int (-i\psi)^* \psi dx \implies i^\dagger = -i.$$

$$\int \psi^* \frac{d}{dx}\psi dx = \psi\psi^* \Big|_{-\infty}^{\infty} - \int (\frac{d}{dx}\psi)^* \psi dx = \int (-\frac{d}{dx}\psi)^* \psi dx \implies \left(\frac{d}{dx}\right)^\dagger = -\frac{d}{dx}.$$

Question 1b. Expanding:

$$\left\langle f \mid \left(\hat{Q}\hat{R}\right)^\dagger g \right\rangle = \left\langle \hat{Q}\hat{R} f \mid g \right\rangle = \left\langle \hat{R} f \mid \hat{Q}^\dagger g \right\rangle = \left\langle f \mid \hat{R}^\dagger \hat{Q}^\dagger g \right\rangle \implies \left(\hat{Q}\hat{R}\right)^\dagger = \hat{R}^\dagger \hat{Q}^\dagger.$$

$$\left\langle f \mid \left(\hat{Q}+\hat{R}\right)^\dagger g \right\rangle = \left\langle \hat{Q} f \mid g \right\rangle + \left\langle \hat{R} f \mid g \right\rangle = \left\langle f \mid \left(\hat{Q}^\dagger + \hat{R}^\dagger\right) g \right\rangle \implies \left(\hat{Q}+\hat{R}\right)^\dagger = \hat{Q}^\dagger + \hat{R}^\dagger.$$

$$\left\langle f \mid \left(c\hat{Q}\right)^\dagger g \right\rangle = \left\langle c\hat{Q} f \mid g \right\rangle = \left\langle f \mid c^* \hat{Q} g \right\rangle \implies \left(c\hat{Q}\right)^\dagger = c^* \hat{Q}^\dagger.$$

Question 2. Note that from what we derived in the textbook, the space dependent wave function for the ground state of the harmonic oscillator is

$$\psi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{\frac{m\omega}{2\hbar}x^2} e^{-i\omega t/2}.$$

Plugging this into equation 3.54:

$$\Psi(p) = \frac{1}{\sqrt{2\pi\hbar}} e^{-i\omega t/2} \int_{-\infty}^{\infty} e^{-ipx/\hbar} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{\frac{m\omega}{2\hbar}x^2}.$$