Math 220 Question 8

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Question 8. First we will show the forward definition. Assume R is reflexive and circular, we must show it is symmetric and transitive for it to be a equivalence relation. For symmetric, let $b = a$. Then $(aRb \wedge bRc) \Leftrightarrow (aRa \wedge aRc) \Leftrightarrow aRc \implies cRa$, which gives us symmetry. Combining the fact that R is symmetric and circular, we get that $aRb \wedge bRc \implies cRa \implies aRc$, which is the definition of transistivity.

For the backwards direction, assume that R is an equivalence relation and we will show that R is reflexive and circular. Reflexive is given automatically by the definition of an equivalence relation. To show circular, similarly to before we combine the transitive and symmetric property of R:

$$aRb \wedge bRc \implies aRc \implies cRa \implies R \text{ is circular}$$

Thus any relation R is an equivalence relation if and only if it is reflexive and circular. $\square$