HOMEWORK 3: DUE NOVEMBER 16TH

MATH 437/537: PROF. DRAGOS GHIOCA

Problem 1. (10 points.) Let $\{a_n\}_{n\geq 0}$ be a sequence defined as follows:

$$a_0 = 0;\ a_1 = 1;\ a_2 = 2 \text{ and}$$

$$a_{n+3} = 5^n \cdot a_{n+2} + n^2 \cdot a_{n+1} + 11a_n \text{ for } n \geq 0.$$

Prove that there exist infinitely many $n \in \mathbb{N}$ such that $2023 \mid a_n$.

Problem 2. (5 points.) Let $n \in \mathbb{N}$. Find the number of solutions for the congruence equation:

$$x^3 \equiv 1 \pmod{n}.$$

As always, $\phi(\cdot)$ is the Euler-ϕ function.

Problem 3. (14 points.) Let α be any real number in the interval $[0, 1]$. Prove that there exists an infinite sequence $\{n_k\}_{k\geq 1} \subset \mathbb{N}$ such that

$$\lim_{k\to\infty} \frac{\phi(n_k)}{n_k} = \alpha.$$