PHYS 403 Homework 6

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Question 2a. In class, we derived N_F for a Fermi gas, and we can simplify it using $E_C - \mu \gg k_B T$:

$$N_F = \frac{1}{e^{\beta(\varepsilon + E_C - \mu)} + 1} \approx e^{-\beta(\varepsilon + E_C - \mu)}.$$

We can then calculated n_e:

$$n_e = n_d = g_c \int_0^\infty d\varepsilon D_C(\varepsilon) N_F\left(\varepsilon + E_C, \mu\right) = g_c e^{-\beta(E_C - \mu)} \int_0^\infty d\varepsilon D_C(\varepsilon) e^{-\beta\varepsilon}.$$

$$\implies n_e = g_c \frac{1}{\lambda(T)^3} e^{-\beta(E_C - \mu)} \implies \mu = E_C - \frac{1}{\beta} \log \frac{n_d \lambda^3}{g_c}.$$

Here $\lambda = \left(\frac{2\pi\hbar^2}{m_c k_B T}\right)^{3/2}$ as usual.

Question 2b. Ionization has energy $E_C - \mu$, whereas being occupied has energy ε_b. Thus computing probability:

$$p_{\text{ionized}} = 1 - p_{\text{occ}} = 1 - \frac{1}{e^{\beta(E_C - \mu - \varepsilon_b)} + 1} = \frac{1}{\frac{n_d \lambda^3}{g_c} e^{-\beta(E_C - \varepsilon_b)} + 1}.$$

When n_d is large or $E_C - \varepsilon_b \gg k_B T$, the probability of ionization goes to 1. Of course because of the exponential, the $E_C - \varepsilon$ condition will asymptotically outpace the magnitude of n_d.

Question 2c.