Problem set 3 - due on Monday, Feb. 28

Upload your work to Canvas by 4pm, as usual.

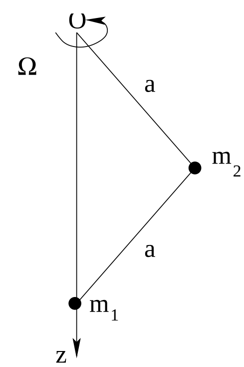
1. Consider the Lagrangian

$$\mathcal{L} = \frac{\alpha}{2} q^2 \dot{q}^2 - \beta q^2$$

where α, β are known, positive constants. (a) Find the Euler-Lagrange equation. (b) Find the conserved quantities. (c) If $q(0) = q_0 > 0$ and $\dot{q}(0) = 0$, find the time T_c when $q(T_c) = 0$ for the first time. What is the period of oscillation?

For fun: Can you solve the EL equation directly?

2. The system shown in Fig. $\rightarrow$ is rotated about the z-axis with a constant angular speed Ω. The mass m_1 is constrained to move vertically.
(a) Find the Lagrangian.
(b) Are there any conserved quantities? Briefly explain if the answer makes sense to you.
(c) Is it possible that the motion of the mass m_2 is a circle? If yes, for what value(s) of Ω does that happen?



3. Two bodies of masses m_1 and m_2 are linked through a spring of constant k and unstretched length l_0. Motion is restricted to the x-axis (horizontal). At $t = 0$, we know that $x_1(0) = 0$, $x_2(0) = 2l_0$ and $v_1(0) = v_0$, $v_2(0) = -v_0$. Find $x_1(t)$ and $x_2(t)$ (we assume the the IC conditions are such that the objects never collide).

4. **From last year's midterm 2:** A satellite of mass m orbits a planet of mass M on a circular trajectory of radius R (very much larger than the radius of the planet, so we can take the radius of the planet to be zero). The trajectory of the satellite must be changed such that its new trajectory has a minimum distance $R/2$ from the planet, while the maximum distance remains R. The engines can only be fired once to achieve this. By how much must the speed of the satellite be changed (give both magnitude and direction) when the engines are fired, in order for the satellite to move onto the correct new trajectory?

5. Two stars of masses m_1 and m_2 orbit about their center of mass, which is at rest, on ellipses. Their relative trajectory is described by $p/r = 1 + e\cos\phi$, with p and e having known values. Assume that by some miracle, at the moment when the two stars are closest to one another, the mass of the first one doubles: $m_1' = 2m_1$, but all else (velocities, distances) remain the same. Find by how much will the center of mass move during a full rotation of the new star system.