Math 318 Homework 6

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Question 1a. Since X is the sum of normal variables, we have that $\mu_X = 200$ and $(9\sigma_X)^2 = 9\mu^2 \implies \sigma_X = \frac{1}{3}\sigma$ and X is normal. Let $Y = \frac{X-\mu_X}{\sigma_X} = 3\frac{X-200}{\sigma} = N(0,1)$. Then we have that:

$$P(|X-200| > 5|\sigma = 5) = 2P(N(0,1) < -3\frac{5}{5}) = 2\Phi(-3) = 0.0027.$$

$$P(|X-200| > 5|\sigma = 5) = 2P(N(0,1) < -3\frac{10}{5}) = 2\Phi(-1.5) = 0.1336.$$

$$P(|X-200| > 5|\sigma = 5) = 2P(N(0,1) < -3\frac{15}{5}) = 2\Phi(-1) = 0.3173.$$

Question 1b. From the probabilities above, case i would be rejected at 5% level of confidence since it is the only one with $p < 0.05$.

Question 1c. Again from the probabilities above, only case i would be rejected at 1% level of confidence since it is the only one with $p < 0.01$.

Question 2a. This is equivalent to a series of Bernoulli trials, each with mean $\frac{1}{5}$ and variance $pq = \frac{4}{25} \implies \sigma = \frac{2}{5}$. Then the sum S approximates a normal distribution with mean $\frac{n}{5}$ and variance $\frac{4n}{25}$. As such we have that

$$\frac{S-\mu}{\sqrt{n}\sigma^2} = \frac{S-\frac{n}{5}}{\frac{2}{5}n^{3/2}} \approx N(0,1).$$

Using tables we know that $\Phi(-3.09) = 0.001$ which is what we want to achieve, so to find the required n we must solve the equation:

$$3.09 = \frac{\frac{n}{3} - \frac{n}{5}}{\frac{2}{5}n^{3/2}} \implies .$$