Computer Assignment 4.1.1
MMP Finite Element Methods (WI4243AP-FE)

1 Introduction

The following is a report containing the evaluation of the heat equation with the finite element method. First the problem is defined followed by the derivation of the element matrix and vector.

2 Problem formulation

Consider a rectangular plate, with width, b, and height, h. In the center of the plate is a rectangular part, of a different material, with width, b_0 and height, h_0. A denotes the outer part of the plate while B denotes the inner part.

Figure 1 shows a schematic of the rectangular plate. The stationary heat equation is given by the two-dimensional equation

$$-\nabla \cdot (k\nabla T) = 0 \tag{1}$$

with heat conductivity k and temperature T. The thermal conductivity is different in each material, i.e. $k|_A = k_A$ and $k|_B = k_B$.

The boundary of A, ∂A, is divided in four segments such that $\partial A = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4$. On these boundaries the following conditions hold:

$$T|_{\Gamma_1} = T_0 \tag{2}$$

$$\nabla T|_{\Gamma_2} = T_0 \tag{3}$$

$$T|_{\Gamma_3} = T_0 \tag{4}$$

$$k\frac{\partial T}{\partial n}|_{\Gamma_4} = -\alpha(T_w - T_\infty) \tag{5}$$

In words, the temperature at boundaries Γ_1 and Γ_3 are held constant at $T = T_0$. The flux at Γ_2 is zero while Γ_4 obeys Newton's heat transfer relation with T_∞ the environment temperature and T_w the temperature at the wall.

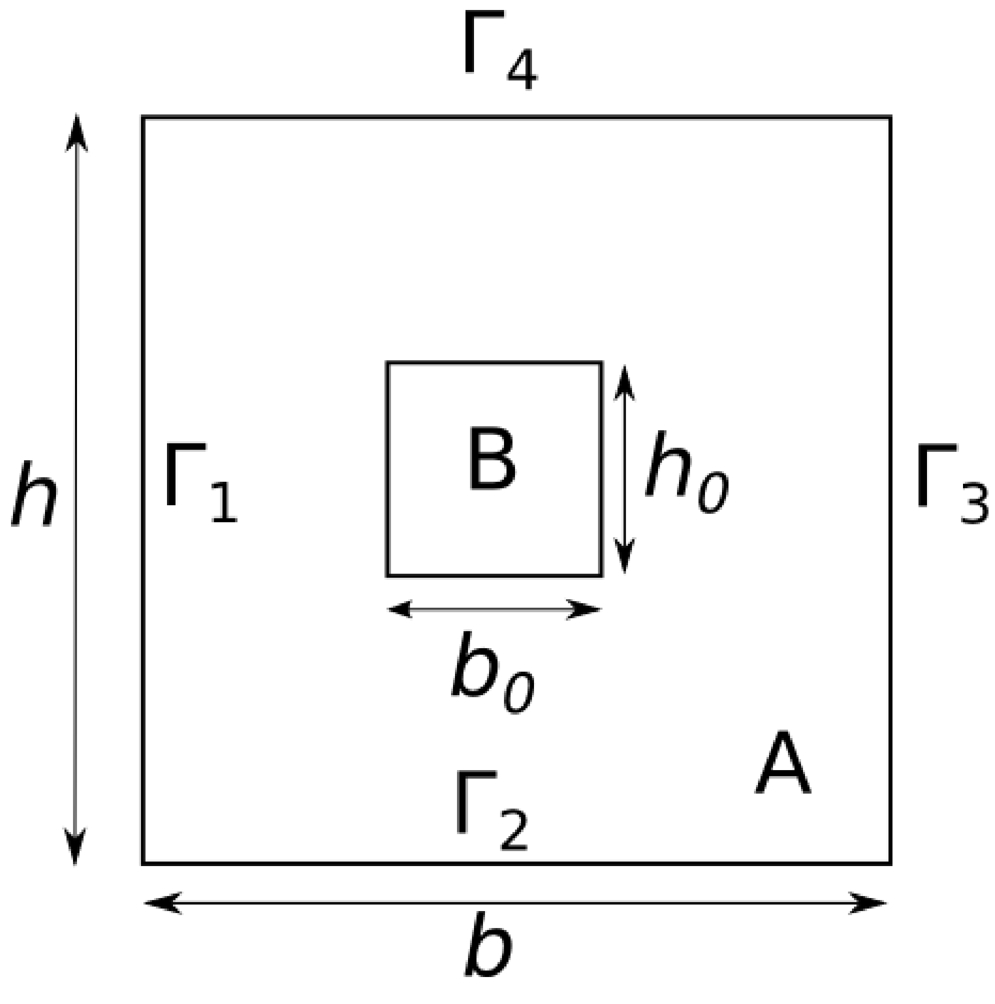

Figure 1: caption