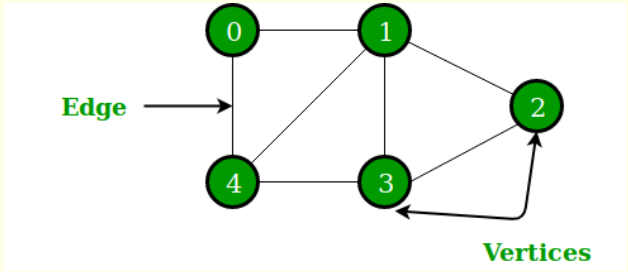
Graph Terminology

GRAPH



Modeling connections or relationships between items

Vertex /node
Edge is the line
Each vertex has a '**degree**' – number of connections
NO UNIQUE ROOT

Type of graphs:
Undirected graph
Directed graph
Weighted graph

Breadth-First and Depth-First Traversal

The Applications of Graphs
Graphs have many uses in the world of Computer Science, for example:
•Social Networks
•Transport Networks
•Operating Systems

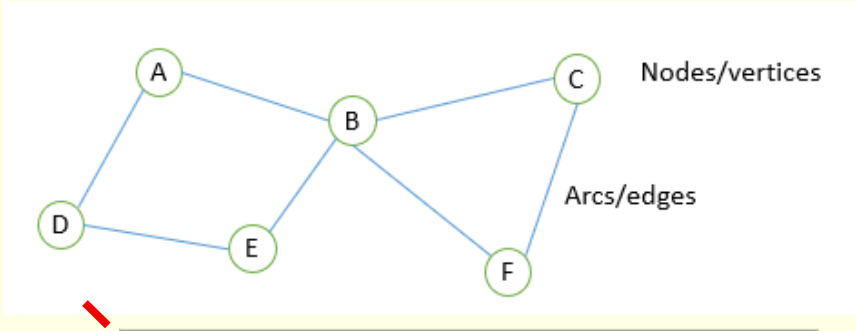
Key words

Keyword	Definition
Directed Graph	A directed graph is a set of objects that are connected together, where the edges are directed from one vertex to another.
Undirected Graph	An undirected graph is a set of objects that are connected together, where the edges do not have a direction.
Weighted Graph	A weighted graph is a set of objects that are connected together, where a weight is assigned to each edge.
Adjacency Matrix	Also known as the connection matrix is a matrix containing rows and columns which is used to present a simple labelled graph.
Adjacency List	This is a collection of unordered lists used to represent a finite graph.
Vertices/Nodes	A vertex (or node) of a graph is one of the objects that are connected.
Edges/Arcs	An edge (or arc) is one of the connections between the nodes (or vertices) of the network.
Dictionary	A collection of names, definitions, and attributes about data elements that are being used or captured in a database, information system, or part of a research project.

Implementing a graph

Two possible implementations of a graph are the **adjacency matrix** and the **adjacency list.**

No Weighting, Undirected graph



	A	B	C	D	E	F
A	-	1	-	1	-	-
B	1	-	1	-	1	1
C	-	1	-	-	-	1
D	1	-	-	-	1	-
E	-	1	-	1	-	-
F	-	1	1	-	-	-

Adjacency Matrix

- A two-dimensional array can be used to store information about a directed or undirected graph.
- Each of the rows and columns represents a node, and a value stored in the cell at the intersection of row i, column j indicates that there is an edge connecting node i and node j.
- Graphs with no weights are given a value of 1 for connected nodes
- In the case of an undirected graph, the adjacency matrix will be symmetric, with the same entry in (0,1) as in (1,0), for example.

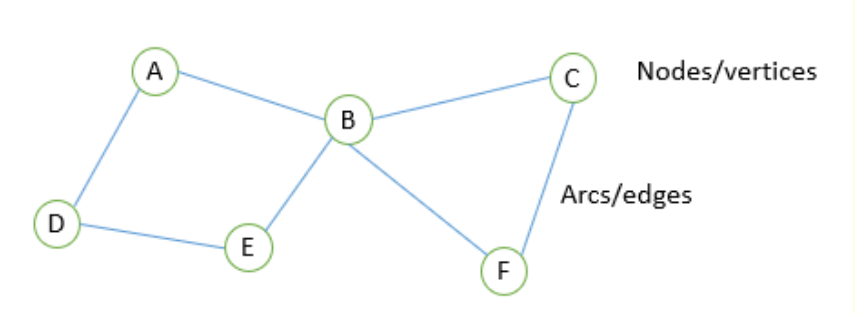
Implementing a graph

Two possible implementations of a graph are the **adjacency matrix** and the **adjacency list.**

A list of all the nodes is created, and each node points to a list of all the adjacent nodes to which it is directly linked.

The adjacency list can be implemented as a list of dictionaries, with the key in each dictionary being the node and the value, the edge weight.

No Weighting, Undirected graph



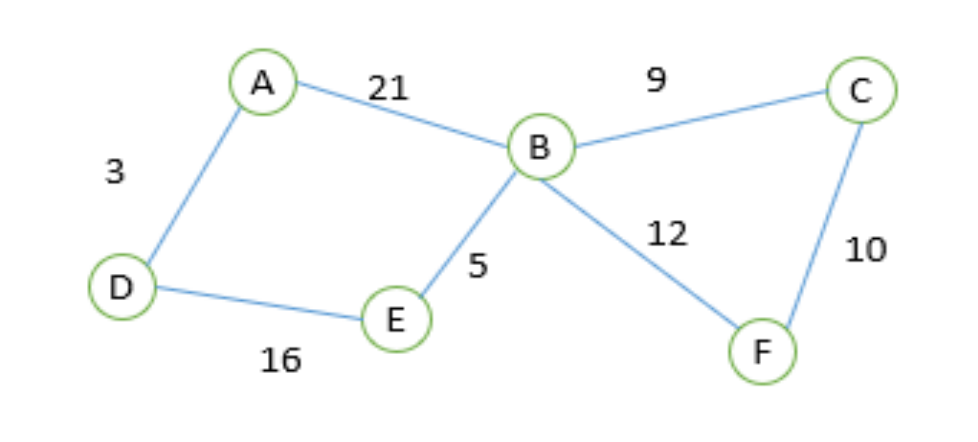
A	[B, D]
B	[A, C,E, F]
C	[B, F]
D	[A, E]
E	[B, D]
F	[B, C]

The next nodes should be in alphabeti cal order

Weighted graph

Weighted graphs add a value to an arc.

This might represent the distance between places or the time taken between train stations



Adjacency List

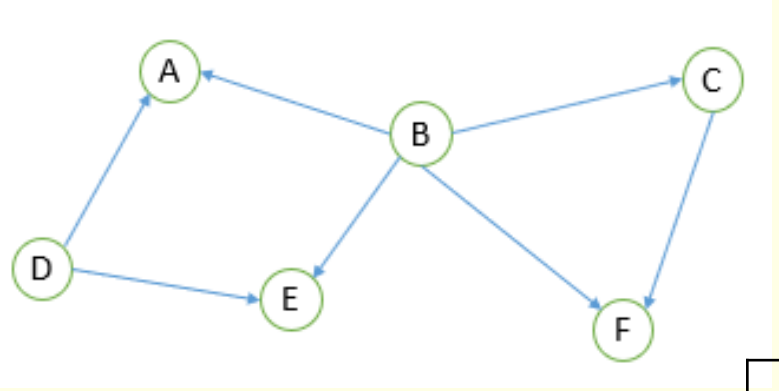
A	{B:21, D:3}
B	{A:21, C:9, E:5, F:12}
C	{B:9, F:10}
D	{A:3, E:16}
E	{B:5, D:16, }
F	{B:12, C:10}

Adjacency Matrix

	A	B	C	D	E	F
A	-	21	-	3	-	-
B	21	-	9	-	5	12
C	-	9	-	-	-	10
D	3	-	-	-	16	-
E	-	5	-	16	-	-
F	-	12	10	-	-	-

Directed Graph

- Directed graphs only apply in one direction and are represented with edges with arrow heads on one end.



Adjacency List

A	
B	[A, C, E, F]
C	[F]
D	[A, E]
E	
F	

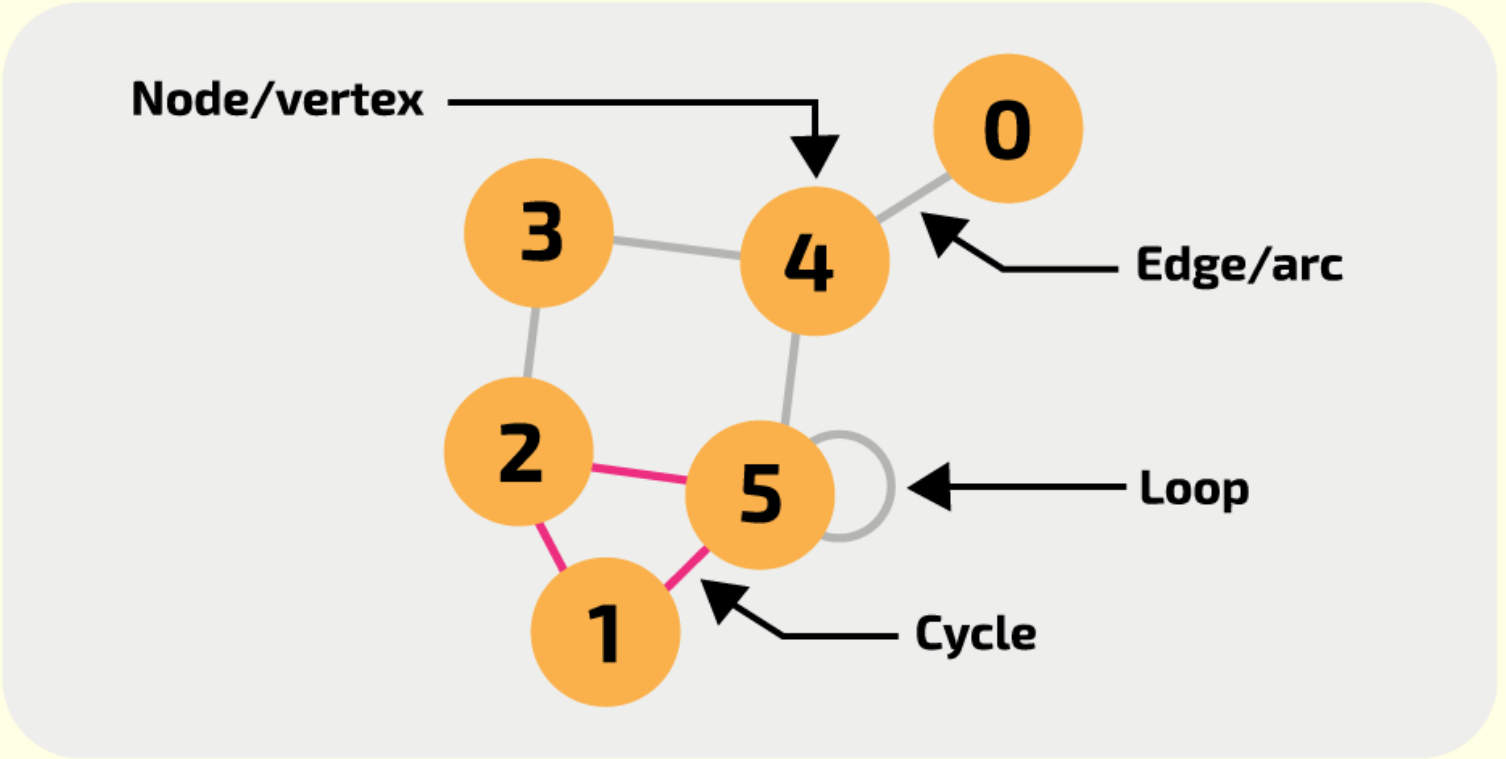
Adjacency Matrix

		To					
		A	B	C	D	E	F
From	A						
	B	1		1		1	1
	C						1
	D	1				1	
	E						
	F						

Graphs allow cycles and loops

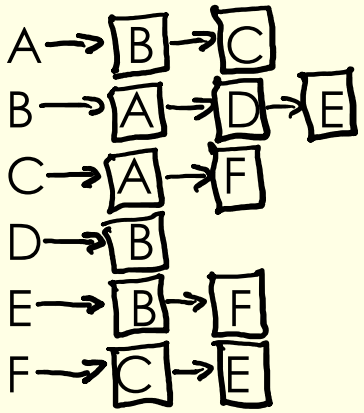
- A **cycle** is a closed path, i.e. a path that starts and ends at the same node (and no node is visited more than once)

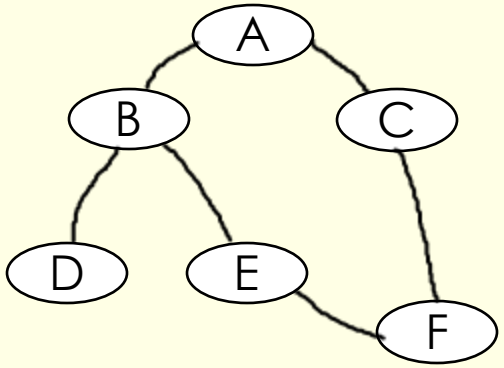
- A **loop** is an edge that connects a node to itself



Graph representation

Adjacency List





Adjacency Matrix

	A	B	C	D	E	F
A	0	1	1	0	0	0
B	1	0	0	1	1	0
C	1	0	0	0	0	1
D	0	1	0	0	0	0
E	0	1	0	0	0	1
F	0	0	1	0	1	0

Adjacency list can be coded as a dictionary

```
graph = {
'A': ['B', 'C'],
'B': ['A', 'D', 'E'],
'C': ['A', 'F'],
'D': ['B'],
'E': ['B', 'F'],
'F': ['C', 'E']}
```

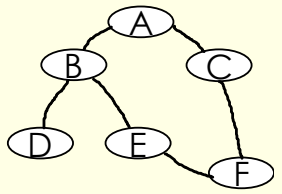
Adjacency Matrix can be coded as a 2D list

```
graph =
[[0,1,1,0,0,0],
[1,0,0,1,1,0],
[1,0,0,0,0,1],
[0,1,0,0,0,0],
[0,1,0,0,0,1],
[0,0,1,0,1,0]]
```

Depth-first search

Uses a stack to manage traversal
Visits nodes by exploring as far down one branch as possible before backtracking Typically explores neighbours in a depth-wise manner

Current node	Stack	Visited
A	C B	A
B	C E D	A B
D	C E	A B D
E	CF	A B D E
F	C	A B D E F
C		A B D E F C



A B D E F C

This method makes use of a stack data structure to push each visited node onto the stack and then pop them from the stack when there are no nodes left to visit

1. Set the **current node** to the **start node**.
2. If the **current node** has **unvisited neighbours**, **push** them to the stack **in reverse order**.
3. If the **current node** is **not in visited**, add it to visited.
4. **Pop** the next node from the stack and set it as the new **current node**.
5. Repeat until the **stack is empty**.

```
Procedure depth_first_search (graph, start_node)

    visited = []
    s = Stack()
    current_vertex = start_node
    while current_vertex != None  #while there is a node in the stack

        for vertex in reversed(graph[current_vertex])
            if not vertex in visited then
              s.push(vertex) #adds the vertex to the stack if not already in visited
            end if
        next vertex

         if not current_vertex in visited then
            visited.append(current_vertex) # current_vertex is added to visited
         endif

        current_vertex = s.pop() #next current_vertex
    endwhile

    print(visited)

end procedure
```

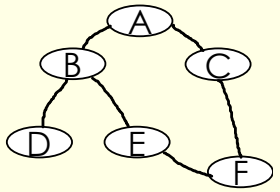
Trinket Python Implementation:
https://trinket.io/python3/90a4a3516460

```
graph = {
'A': ['B', 'C'],
'B': ['A', 'D', 'E'],
'C': ['A', 'F'],
'D': ['B'],
'E': ['B', 'F'],
'F': ['C', 'E']
}
```

Breadth-First search

Uses a queue to manage traversal
Visits nodes level-by-level, exploring all neighbours before moving to the next level Ideal for finding the shortest path in an unweighted graph

Current node	Queue	Visited
	A	
A	BC	A
B	C D E	A B
C	D E F	A B C
D	E F	A B C D
E	F	A B C D E
F		A B C D E F



A B C D E F

1. Set **current node** to **stat node**
2. If the **current node** has **unvisited neighbours**, e**nqueue unvisited neighbour** nodes to the **queue**
3. If the **current node** is **not in visited**, add it to visited.
4. **Dequeue** node from queue and set to **current node.**
5. Repeat until the **queue is empty**

Algorithm

```
Procedure bfs(graph,start_node)
    visited = []
    q = queue() #create queue
    current_vertex = start_node# currentNode
    while current_vertex != None #while the queue is not empty

        for vertex in graph[current_vertex] #get all nodes that are adjacent nodes
            if not vertex in visited then # if vertex is in visited do not add to queue
                q.enqueue(vertex) #add vertex to queue
        next vertex

        if not current_vertex in visited then #if the current node has not been visited
            visited.append(current_vertex) #add node to visited

        current_vertex = q.dequeue() #next current_vetex

       print(visited)

End procedure
```

Trinket Python Implementation:
https://trinket.io/python3/6bf76ad43d10

```
graph = {
'A': ['B', 'C'],
'B': ['A', 'D', 'E'],
'C': ['A', 'F'],
'D': ['B'],
'E': ['B', 'F'],
'F': ['C', 'E']
}
```

Recursive implementation

```
procedure dfs(graph, current_vertex)

    #Take the top item of the stack and add it to the visited list.
    visited.append(current_vertex)

    #Create a list of that vertex's adjacent nodes. Add the ones
which aren't in the  visited list to the top of stack.

    for vertex in graph[current_vertex]

        if not vertex in visited then
                dfs(graph, vertex)
        end if
    #Keep repeating until the stack is empty.
    next vertex
end procedure
```

```
#Main program starts here
graph = {'A': ['B', 'C'],
         'B': ['A', 'D', 'E'],
         'C': ['A', 'F'],
         'D': ['B'],
         'E': ['B', 'F'],
         'F': ['C', 'E']}

visited = []

dfs(graph,"A")

print visited
```

Backtracking

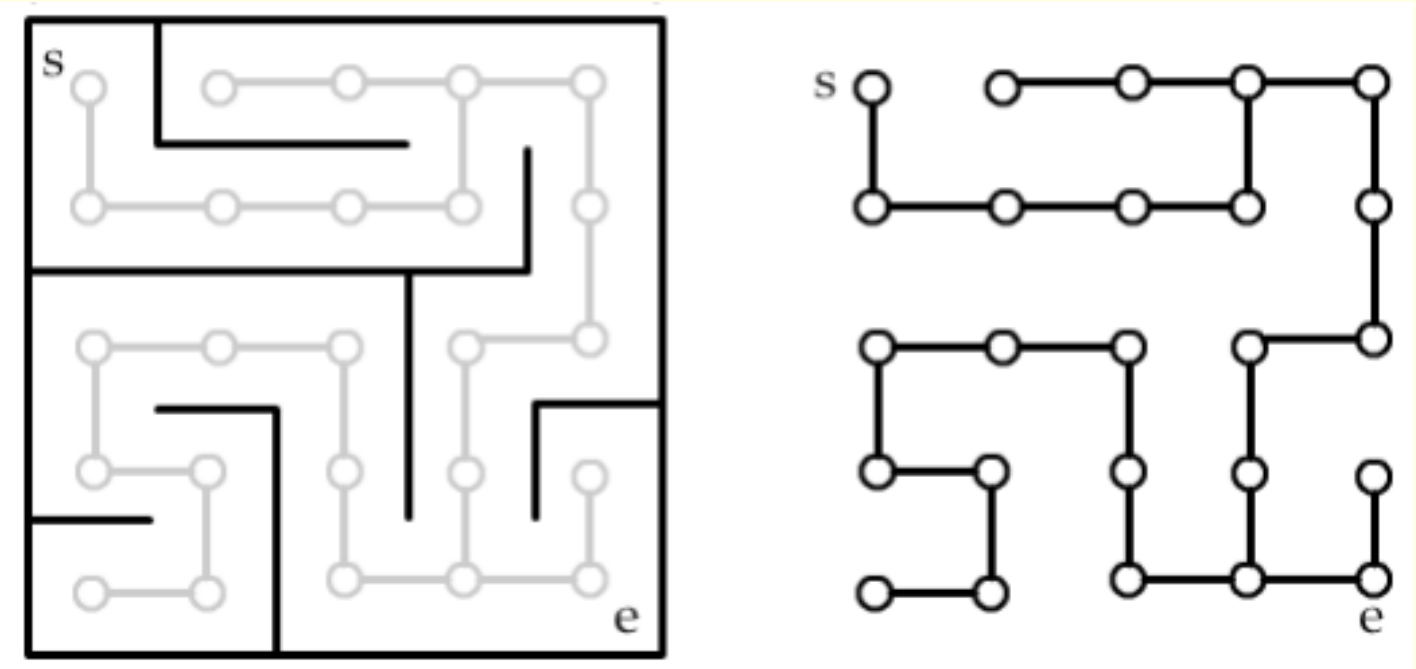
- The algorithm used to demonstrate a DFS does not show step-wise backtracking.
- It is an algorithm that shows you the order in which nodes will be visited in the graph.
- If you wanted to show step-wise backtracking the algorithm would need to be extended.
- An example of a Graph DFS and backtracking is available for you to look at on Trinket.
- This approach used to implement the graph – it is using a graph class.

Explain the differences between depth-first search and breadth-first search in terms of data structures and traversal order.

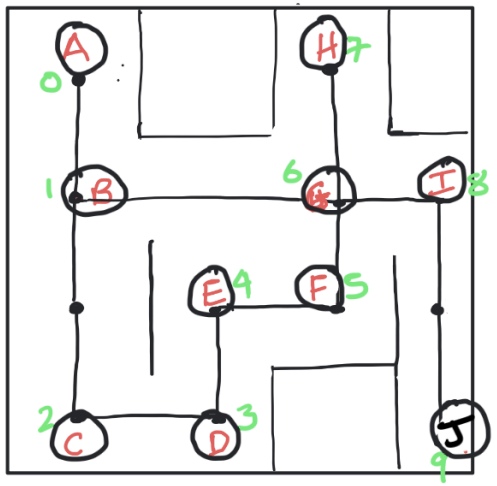
Feature	**DFS**	**BFS**
Data Structure	Stack (or recursion)	Queue
Traversal Order	Goes deep before backtracking	Visits all neighbors level by level
Memory Usage	Can use less memory in wide graphs	Uses more memory for wide graphs
Use Case	Solving mazes, topological sort	Shortest path in unweighted graph

Assignment

- Kira is creating a text-based adventure game whereby the user can explore the world by moving from one location to another.
- The game map is stored in a graph structure, if we consider each location on the map to be a vertex, and we add edges to the graph between adjacent junctures to indicate a valid route.



trinket



Index	Vertex
0	A
1	B
2	C
3	D
4	E
5	F
6	G
7	H
8	I
9	J

Vertex List

A B C D E F G H I J

	0	1	2	3	4	5	6	7	8	9
A	0									
B	1									
C	2									
D	3									
E	4									
F	5									
G	6									
H	7									
I	8									
J	9									

Adjacency Matrix

Adjacency List

A → [B]
B → [A, C, G]
C → [B, D]
D → [C, E]
E → [D, F]
F → [E, G]
G → [B, F, H, I]
H → [G]
I → [G, J]
J → [I]

Index	Vertex	Location
0	A	Shoreline
1	B	Beach
2	C	Forest
3	D	Waterfall
4	E	Cave
5	F	Village
6	G	Hilltop
7	H	Cliffs
8	I	River
9	J	Waterfall

Treasure Chest
Key
} Randomly allocate to a node

Must find the key before you can open the chest

- In the game the user has to navigate from one location to another to find the key to open the treasure chest. Every time the user plays the game the key and treasure chest are allocated to a single node but not the same node, obviously. The user must find the key before the chest will open.

- Create a **function** that returns the graph using an adjacency list

- Create a **function** that returns an adjacency matrix for the graph using a 2D array

- Create a **function** that returns a vertex list that stores the locations associated with each vertex

Using the adjacencylist() for implementing the game:

- Set key location
- Set treasure chest location
- Start at vertex A
- Print the current location and the next possible locations based on the adjacency list.
- Check for a key or treasure chest at the current node.
- When selection has been made change current vertex
- Display next available vertex locations.
- Continue until key is found and and the treasure chest opened to win the game.