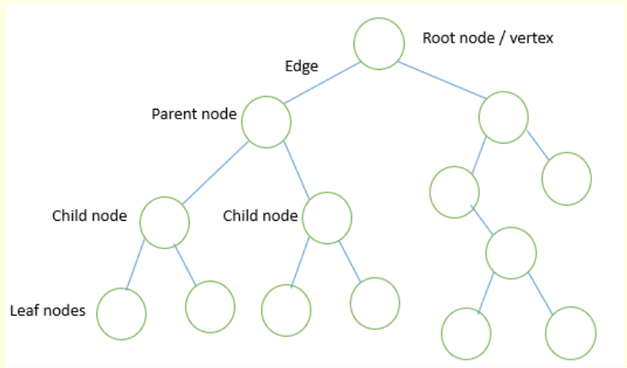
## Non-Linear Data Structures

**TREE**

*Hierarchical data structure*




*Root Node* stores the first data item
*Node* can have up to 2 descendants
*Leaf Nodes* have no descendants
Edge/ branches is the line

Type of Tree:
A common kind of tree is a Binary Tree
Every node can have at the most two child nodes

Depth-First Traversal
- Pre-Order
- In-Order
- Post-Order

Breadth-First

**The Applications of Trees**

•Manipulating hierarchical data, such as folder structures or moves in a game

•making information easy to search

• manipulating sorted lists of data

**Tree**

- A tree is a data structure used to represent **non-linear** data.
- A hierarchical data structure made up of nodes, with one root node and sub-nodes called children.
- A tree consists of **nodes** that are connected by **edges.**
- Each node contains some data, and each edge indicates a relationship between the nodes it connects.

**Binary Tree**

A binary tree is a rooted tree where every node has at most two child nodes. This means that a tree where every nodes has either two, one or none child nodes is a binary tree.

**Binary Search Tree**

A binary search tree (BST) is a **rooted tree** where the nodes of the tree are **ordered.**

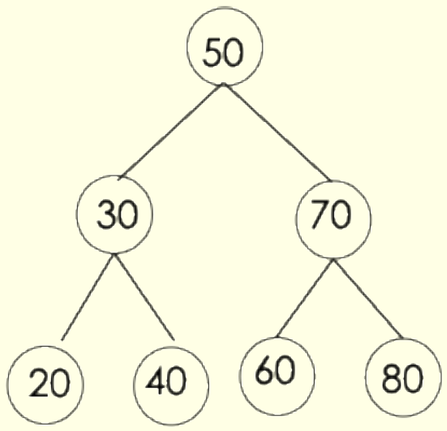
Binary Search Trees can have a **maximum of 2 children** for each parent node, but it is not a requirement that they have 2.

An advantage of storing data in a binary search tree instead of a 1-dimensional array: searching is faster ( O(log n), inserting new tasks is faster and do not need to sort the structure (each time a new task is inserted)

If the order is ascending (low to high), the nodes of the left subtree have values that are lower than the root, and the nodes of the right subtree have values that are higher than the root.

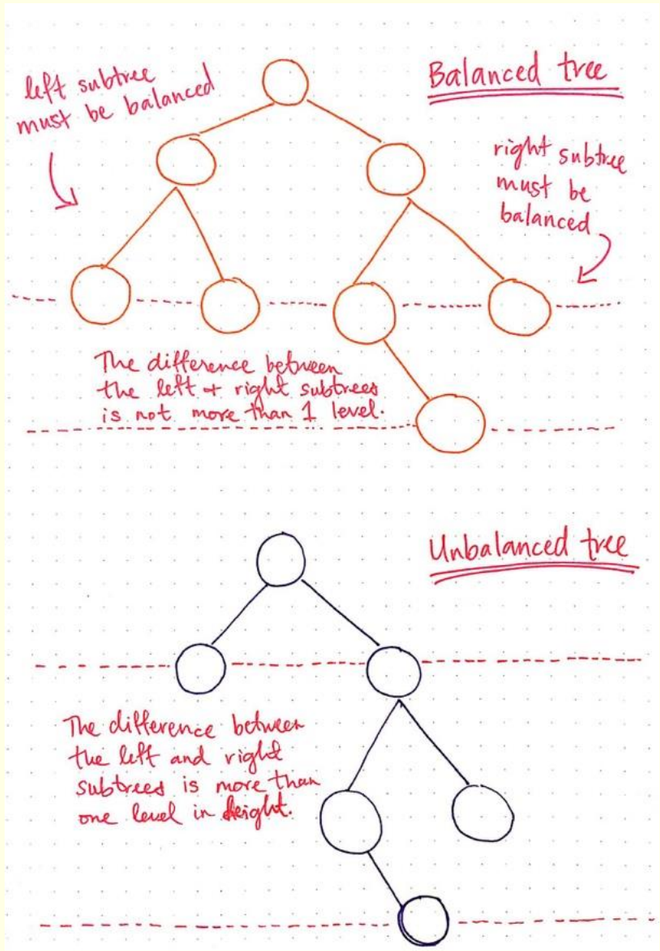
This property is true for any node of the tree; the nodes of its left subtree will have values that are lower, and the nodes of its right subtree will have values that are higher.

Imagine that you are searching for the number 40:




- Start at the root 50
- Compare 40 with 50 - 40 is lower, so check the left child
- Compare 40 with 30 - 40 is higher, so check the right child
- Compare 40 with 40 - success!

**You have found the item with 3 comparisons**.

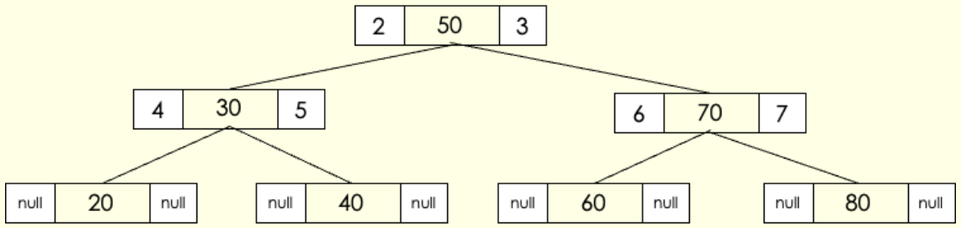



Drawback of an unbalanced binary search tree when searching for a value

- If the tree is unbalanced, nodes may form a long chain.
- **Time complexity** worsens from **O(log n)** (balanced) to **O(n)** (unbalanced), meaning more comparisons are needed to find a value.
- Searching may require visiting almost every node in the tree.

**OCR Exam Abstract Representation of a Binary Tree using an array representation**

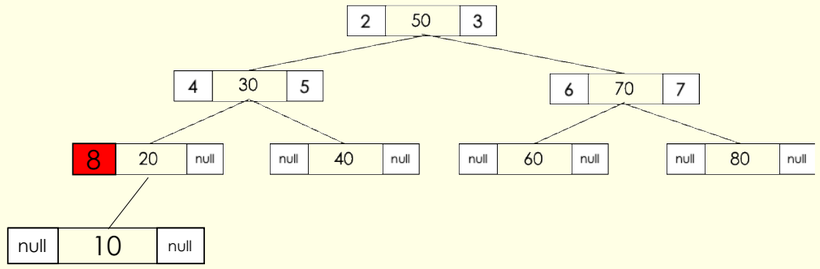
**Binary tree created using the data: 50, 30, 70, 20, 60, 80**




| Position | Left | Value | Right |
|---|---|---|---|
| 1 | 2 | 50 | 3 |
| 2 | 4 | 30 | 5 |
| 3 | 6 | 70 | 7 |
| 4 | null | 20 | null |
| 5 | null | 40 | null |
| 6 | null | 60 | null |
| 7 | null | 80 | null |

**Adding a node**

- Create a new node
- Start at the root node
- Search the tree to find the location in of the new node
- Once a null pointer is found insert the new node at that position by updating the appropriate left or right of the parent node to point to the new node.




**After**

| Position | Left | Value | Right |
|---|---|---|---|
| 1 | 2 | 50 | 3 |
| 2 | 4 | 30 | 5 |
| 3 | 6 | 70 | 7 |
| 4 | 8 | 20 | null |
| 5 | null | 40 | null |
| 6 | null | 60 | null |
| 7 | null | 80 | null |
| 8 | null | 10 | null |

| **Implementing binary tree** | **Binary Tree Class** |
|---|---|

Remember that Binary Search Trees can have a maximum of 2 children for each parent node, but it is not a requirement that they have 2.

A binary tree can be stored as objects

Each node has a left pointer, a right pointer and the data being stored.
This is the node class and this is used to create a new_node object.

**Node object**

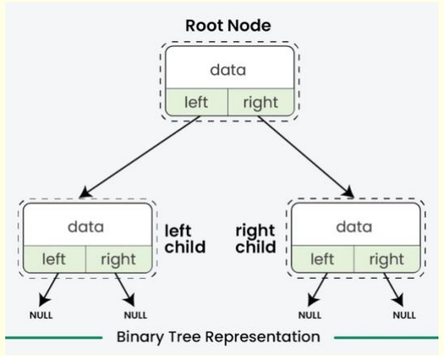
- **data** which in this case is a simple value attached to the node
- **right** which is a reference to the node to the right of the node
- **left** which is a reference to the node to the left of the node

| class: node |
|---|
| attributes:<br>private data<br>private left: Integer<br>private right: Integer |
| methods:<br>new (new_data)<br>getData()<br>setData()<br>getLeft ()<br>getRight ()<br>setLeft()<br>setRight () |

```
class Node
   private data
   private left
   private right

   public procedure new (new_data)
      data = new_data # instance variable to store the data
      left = null
      right = null
   end procedure
end class
```

**A binary tree has a root node**

```
class BinaryTree

   private root

   public procedure new ()

      root = null

   …

end class
```




This implementation is using **linked-node representation** of a binary tree instead of using an array representation.

| **Linked-node representation** | **Array representation** |
|---|---|
| Each node stores:<br>• Data value<br>• Pointer (reference) to left child<br>• Pointer (reference) to right child<br><br>The tree is connected via pointers rather than array indexes | • The tree's nodes are stored in a sequential array.<br>• Each node's position is determined by its index in the array.<br><br>We can store each node's **data**, **left index**, and **right index** in a **2D array**<br><br>• Each **row** represents a node.<br>• **Column 0** = node data.<br>• **Column 1** = index of left child<br>• **Column 2** = index of right child |

**Inserting a node into binary tree**
**Step 1: Create a New Node**

- Assign the new data to the node.
- Set the left and right of the node to null.

**Step 2: Check if the Tree is Empty**

- If the root is null:
    - Set the root to the new node.

**Step 3: Traverse the Tree to find insertion point**

- If the tree is not empty:
    - Start from the **root node.**
    - Repeat until you find **null:**
        - Compare the new data with the current node's data:

            If new_data < current_node.data

            - Follow the **left**

            Else

            - Follow the **right**

        - Keep track of the **parent node** during traversal.

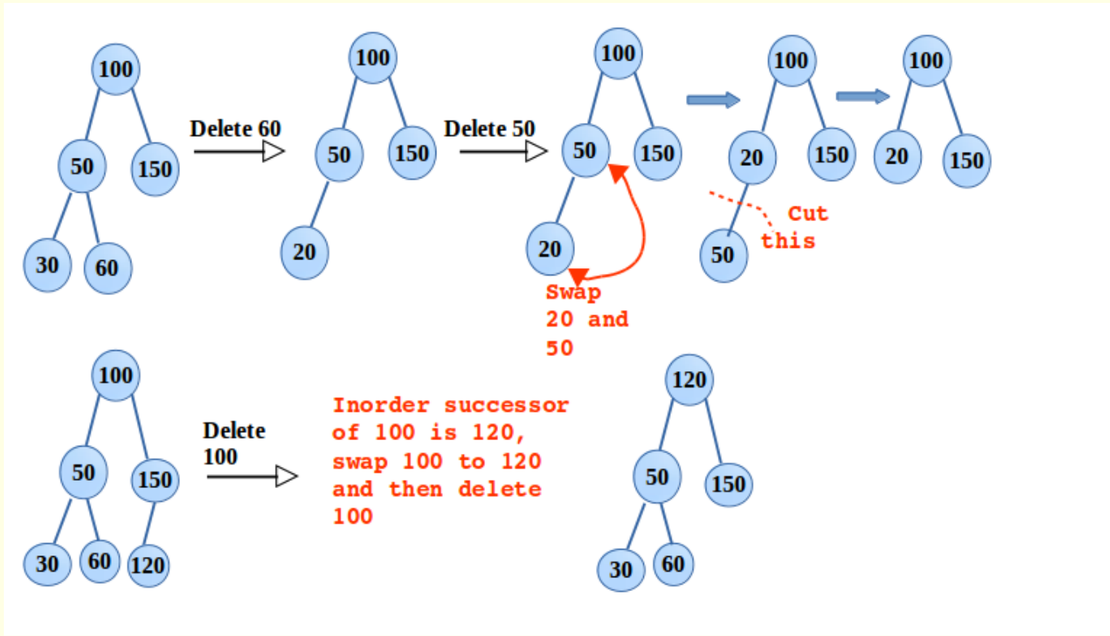
    - Once a null pointer is found:
        - Insert the new node at that position by updating the appropriate left or right of the parent node to point to the new node.

```
Procedure insert(new_value)
    # Step 1: Create a New Node
    new_node = new Node(new_value)
    # Step 2: Check if the Tree is Empty
    if root == null then
      root = new_node
      return
    # Step 3: Traverse the Tree to find insertion point
    current = root
    parent = null
    while current != null:
       parent = current
       if new_value < current.getData()
          current = current.getLeft()
       else
          current = current.getRight()
# Step 4: Insert the Node
if new_value < parent.getData() then
   parent.setLeft(new_node)
else
   parent.setRight(new_node)
```

**Removing an item from a binary tree**

## Step 1: Find the Node to Delete

- **Start at the root.**
- While the current node does not match the target value
  - Keep track of the **parent** node.
  - If the value to delete is less than the current node's value, go left.
  - If greater, go right.




**Step 2: Handle Deletion Cases**

**Case 1: Node has No Children (Leaf Node) -** simply remove it from the tree

- If the node to delete is a leaf
  - Update the parent's pointer to null.

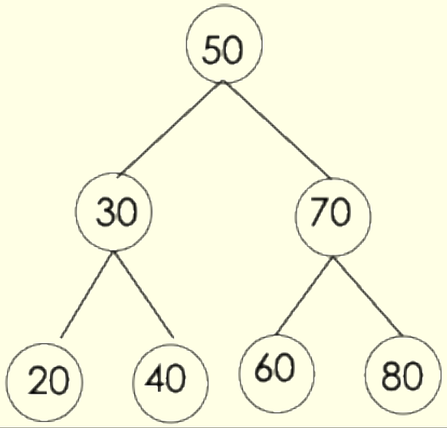
**Case 2: Node has One Child –** replace the node with its child

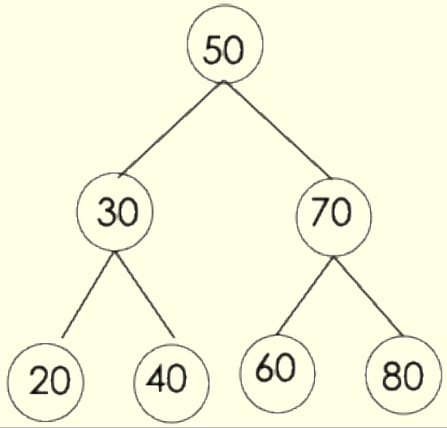
**Case 3: Two Children**

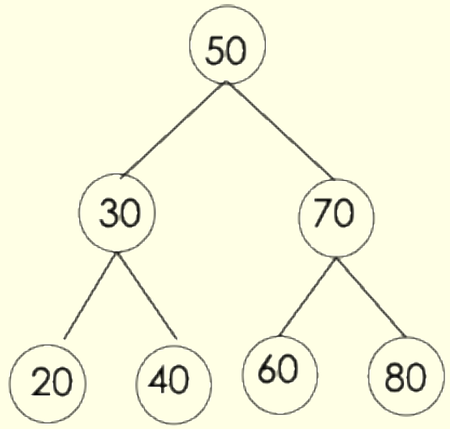
- Find the **smallest value** in the node's **right side** (this is called the in-order successor).
- Copy that smallest value into the node you want to delete.
- Then delete that smallest node (which will now be easy — it will have 0 or 1 child).

**Step 3: Add the deleted node to the free storage list** (leave for garbage clear up)

**Binary Tree Traversals**



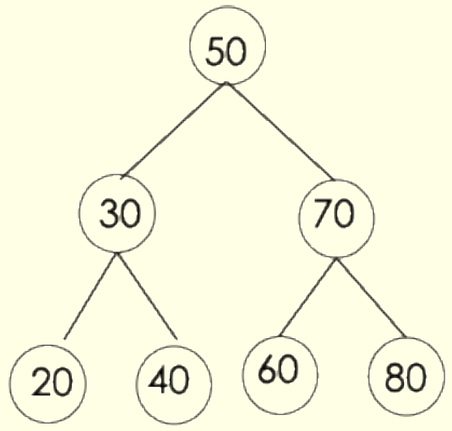




| **Pre-order**<br>Create a copy of a tree.<br>1. Visit the root.<br>2. Traverse the left subtree,<br>3. Traverse the right subtree | **In-order traversal**<br><br>1. Traverse the left subtree,<br>2. Visit the root.<br>3. Traverse the right subtree | **Post-order**<br>Delete a tree<br>1. Traverse the left subtree<br>2. Traverse the right subtree<br>3. Visit the root. |
|---|---|---|
| 50, 30, 20, 40, 70, 60, 80 | 20, 30, 40, 50, 60, 70, 80 | 20, 40, 30, 60, 80, 70, 50 |
| Algorithm Preorder<br><br>procedure preorder(current_node)<br>if current_node != None:<br>#Visit each node: NLR<br>**print(current_node.getData())**<br>if current_node.getLeft() != None:<br>preorder(current_node.getLeft())<br>if current_node.getRight() != None:<br>preorder(current_node.getRight ()) | Algorithm Inorder<br><br>procedure inorder(current_node)<br>if current_node != None:<br>#Visit each node:<br>if current_node.getLeft() != None:<br>inorder(current_node.getLeft())<br>**print(current_node.getData())**<br>if current_node.getRight() != None:<br>inorder(current_node. getRight()) | Algorithm Postorder<br><br>procedure postorder(current_node):<br>if current_node != None:<br>#Visit each node: LRN<br>if current_node.getLeft() != None:<br>postorder(current_node.getLeft())<br>if current_node.getRight() != None:<br>postorder(current_node. getRight())<br>**print(current_node.getData())** |
| Useful for saving a tree. | Outputs values in sorted order for a BST | Useful for deleting nodes. |

**Breadth-first traversal**

Level by level (top-down, left to right)




50 30 70 20 40 60 80

First add the root node into the **queue**
while the queue is not empty.

Get the first node in the queue, and then print its value

Add both left and right children into the queue (if the current node has children)

We will print the value of each node, level by level.

```
procedure breadthFirst(currentnode)

  if (currentnode ! = None)
    q = new Queue()
    q.enqueue(currentnode)
    while NOT q.isEmpty()

      currentnode = q.dequeue()

      print (currentnode.getData())


      if(currentnode.getLeft() !=null)
         q.enqueue(currentnode.getLeft())
      if(currentnode.getRight()!=null)
        q.enqueue(currentnode.getRight())
    endwhile
end procedure
```

*Check root node is not null*

*Add root node to queue*

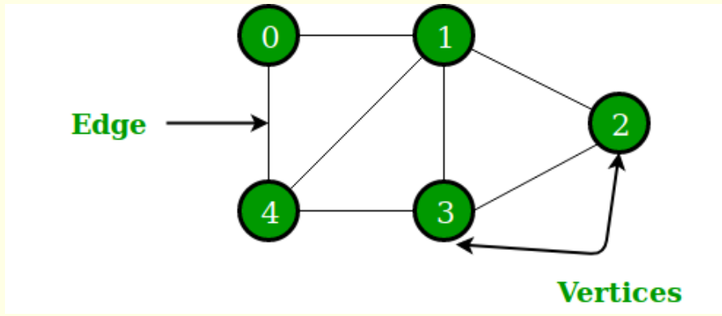
*While the queue is not empty*

print data

*Add the left and right child nodes to the queue*

**GRAPH**

*Modeling connections or relationships between items*




A graph is a set of vertices or nodes connected by edges or arcs.

Each vertex has a 'degree' – number of connections

No unique Root

Type of graphs:

Undirected graph

Directed graph

Weighted graph

Breadth-First and Depth-First Traversal

**The Applications of Graphs**

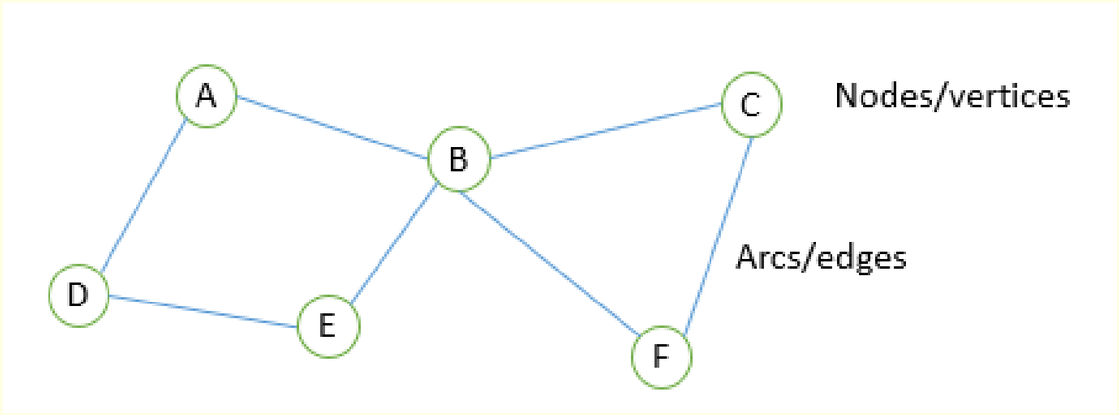
Graphs may be used to represent, for example:

- Computer networks, with nodes representing computers and weighted edges representing the bandwidth between them
- Roads between towns, with edge weights representing distances, rail fares or journey times
- Tasks in a project, some of which have to be completed before others

**Implementing Graphs**

Two possible implementations of a graph are the **adjacency matrix** and the **adjacency list.**

**No Weighting, Undirected graph**




## Adjacency List

A list of all the nodes is created, and each node points to a list of all the adjacent nodes to which it is directly linked.

The adjacency list can be implemented using a dictionary, with the key being the node and the value, the edges.

| | |
|---|---|
| A | [B, D] |
| B | [A, C,E, F] |
| C | [B, F] |
| D | [A, E] |
| E | [B, D] |
| F | [B, C] |

The next nodes should be in alphabetical order

**Weighted Graph**

## Adjacency Matrix

A two-dimensional array can be used to store information about a directed or undirected graph.

Each of the rows and columns represents a node, and a value stored in the cell at the intersection

of row i, column j indicates that there is an edge connecting node i and node j.

Graphs with no weights are given a value of 1 for connected nodes

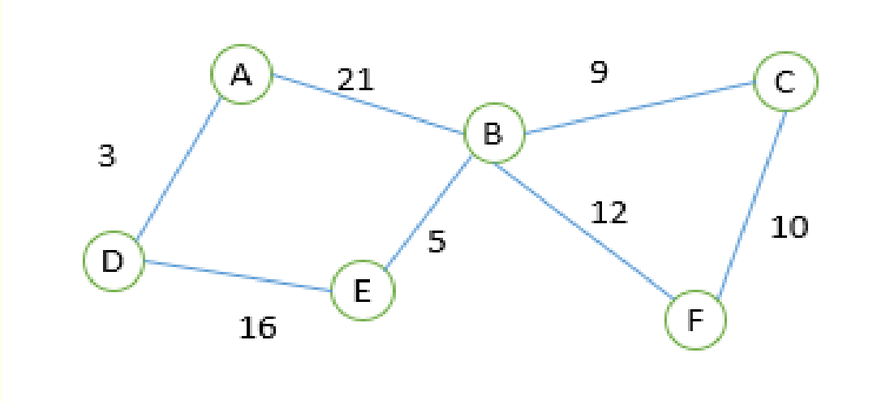
In the case of an undirected graph, the adjacency matrix will be symmetric, with the same entry in (0,1)

as in (1,0), for example.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | - | 1 | - | 1 | - | - |
| B | 1 | - | 1 | - | 1 | 1 |
| C | - | 1 | - | - | - | 1 |
| D | 1 | - | - | - | 1 | - |
| E | - | 1 | - | 1 | - | - |
| F | - | 1 | 1 | - | - | - |

Weighted graphs add a value to an arc.

This might represent the distance between places or the time taken between train stations.




**Adjacency Matrix**

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | - | 21 | - | 3 | - | - |
| B | 21 | - | 9 | - | 5 | 12 |
| C | - | 9 | - | - | - | 10 |
| D | 3 | - | - | - | 16 | - |
| E | - | 5 | - | 16 | - | - |
| F | - | 12 | 10 | - | - | - |

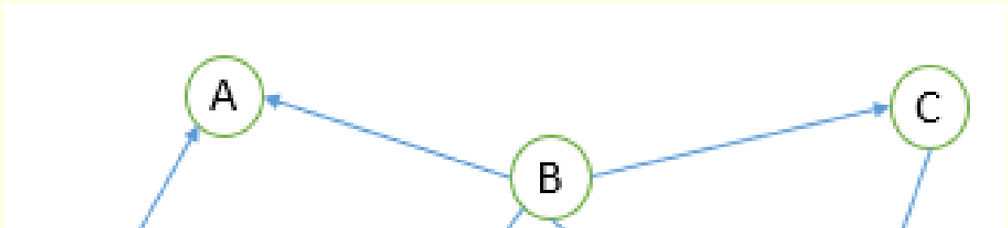
**Adjacency List**

The adjacency list can be implemented as a list of dictionaries, with the key in each dictionary being the node and the value, the edge weight.

| | |
|---|---|
| A | {B:21, D:3} |
| B | {A:21, C:9, E:5, F:12} |
| C | {B:9, F:10} |
| D | {A:3, E:16} |
| E | {B:5, D:16, } |
| F | {B:12, C:10} |

**Directed Graph**

Directed graphs only apply in one direction and are represented with edges with arrow heads on one end.



Adjacency List

| | |
|---|---|
| A | |
| B | [A, C, E, F] |
| C | [F] |
| D | [A, E] |
| E | |
| F | |

Adjacency Matrix

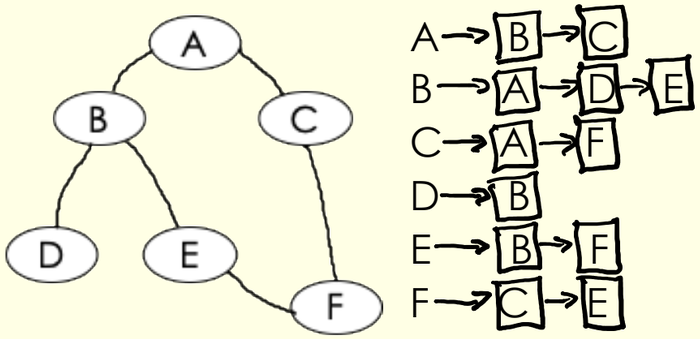
| | | To | | | | | |
|---|---|---|---|---|---|---|---|
| | | A | B | C | D | E | F |
| From | A | | | | | | |
| | B | 1 | | 1 | | 1 | 1 |
| | C | | | | | | 1 |
| | D | 1 | | | | 1 | |
| | E | | | | | | |
| | F | | | | | | |

**Depth-first search**

Uses a stack to manage traversal

Visits nodes by exploring as **far down one branch** as possible before **backtracking**

Typically explores neighbours in a depth-wise manner







```
graph = {
'A': ['B', 'C'],
'B': ['A', 'D', 'E'],
'C': ['A', 'F'],
'D': ['B'],
'E': ['B', 'F'],
'F': ['C', 'E']
}
```

| Current node | Stack | Visited |
|---|---|---|
| A | C B | A |
| B | C E D | A B |
| D | C E | A B D |
| E | CF | A B D E |
| F | C | A B D E F |
| C | | A B D E F C |

A B D E F C

This method makes use of a stack data structure to push each visited node onto the stack and then pop them from the stack when there are no nodes left to visit:

1. Set the **current node** to the **start node**.
2. If the **current node** has **unvisited neighbours**, **push** them to the stack **in reverse order**.
3. If the **current node** is **not in visited**, add it to visited.
4. **Pop** the next node from the stack and set it as the new **current node**.
5. Repeat until the **stack is empty**.

**Algorithm**

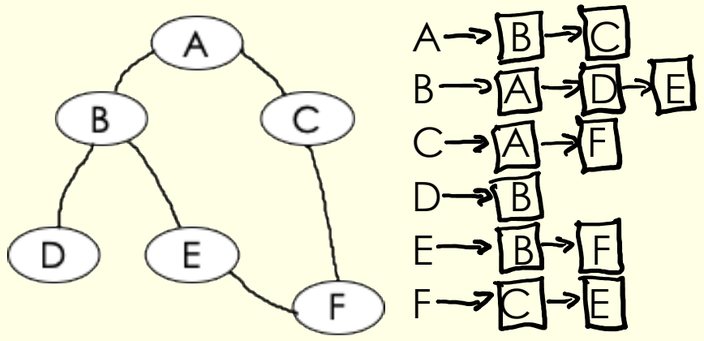
```
Procedure depth_first_search (graph, start_node)
    visited = []
    s = Stack()
    current_vertex = start_node
    while current_vertex != None  #while there is a node in the stack


        for vertex in reversed(graph[current_vertex])
            if not vertex in visited then
              s.push(vertex) #adds the vertex to the stack if not already in visited
            end if
        next vertex


         if not current_vertex in visited then
              visited.append(current_vertex) # current_vertex is added to visited
         endif
        current_vertex = s.pop() #next current_vertex
    endwhile
    print(visited)
end procedure
```

**Breadth first Traversal**

Uses a queue to manage traversal

Visits nodes **level-by-level,** exploring all neighbours before moving to the next level

Ideal for finding the shortest path in an unweighted graph.







```
graph = {
'A': ['B', 'C'],
'B': ['A', 'D', 'E'],
'C': ['A', 'F'],
'D': ['B'],
'E': ['B', 'F'],
'F': ['C', 'E']
}
```

A B C D E F

| Current node | Queue | Visited |
|---|---|---|
| | A | |
| A | BC | A |
| B | C D E | A B |
| C | D E F | A B C |
| D | E F | A B C D |
| E | F | A B C D E |
| F | | A B C D E F |

1. Set **current node** to **stat node**
2. If the **current node** has **unvisited neighbours**, e**nqueue unvisited neighbour** nodes to the **queue**
3. If the **current node** is **not in visited**, add it to visited.
4. **Dequeue** node from queue and set to **current node.**
5. Repeat until the **queue is empty**

**Algorithm**

```
Procedure bfs(graph,start_node)
    visited = []
    q = queue() #create queue
    current_vertex = start_node# currentNode
    while current_vertex != None #while the queue is not empty


        for vertex in graph[current_vertex] #get all nodes that are adjacent nodes
            if not vertex in visited then # if vertex is in visited do not add to queue
                q.enqueue(vertex) #add vertex to queue
        next vertex


        if not current_vertex in visited then #if the current node has not been visited
            visited.append(current_vertex) #add node to visited


        current_vertex = q.dequeue() #next current_vetex
        print(visited)
End procedure
```

| **Depth-first search** | **Breadth-First search** |
|---|---|
| Uses a stack to manage traversal<br>Visits nodes by exploring as far down one branch as possible before backtracking<br>Typically explores neighbours in a depth-wise manner | Uses a queue to manage traversal<br>Visits nodes level-by-level, exploring all neighbours before moving to the next level<br>Ideal for finding the shortest path in an unweighted graph |