Problem 1:

$$\nabla f(x) = \begin{bmatrix} 3{x_1}^2 - x_2 & -x_1 + 2 \\ 1 & 2x_2 \end{bmatrix}$$

```
f=@(x)[x(1)^3-x(1)*x(2)+2*x(2)-2
    x(2)^2+x(1)-2]
df=@(x)[3*x(1)^2-x(2), -x(1)+2
    1,2*x(2)]
```

```
>> [x,xs,fs]=newton(f,df,[1/2;2],1e-10)

x =

    0.4669
    1.2382


xs =

    0.5000    0.6154    0.4457    0.4663    0.4669    0.4669
    2.0000    1.3462    1.2504    1.2385    1.2382    1.2382


fs =

    1.1250    0.0970    0.0320    0.0008    0.0000    0.0000
    2.5000    0.4275    0.0092    0.0001    0.0000    0.0000
```

The solution is x=0.4669, y=1.2382.

Four iterations are needed. The method does converge quadratically.

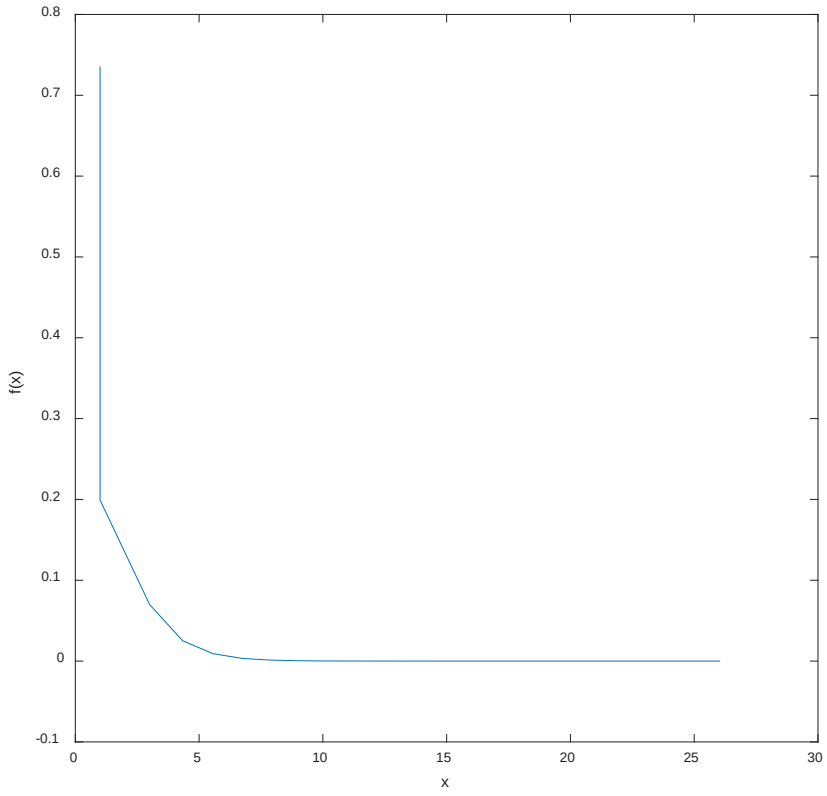
Problem 2:

$$df(x) = (1 + x) - e^{-x} + e^{-x} = e^{-x}(1 - 1 - x) = -xe^{-x}$$

```
f=@(x)(1+x)*exp(-x)
df=@(x)-x*exp(-x)


[x,xs,fs]=gnewton(f,df,1,1e-10)
```

```
x =

   27.0850


xs =

  Columns 1 through 9

    1.0000    1.0000    3.0000    4.3333    5.5641    6.7438    7.

  Columns 10 through 18

   11.2284   12.3175   13.3987   14.4733   15.5424   16.6067   17.

  Columns 19 through 24

   20.8275   21.8755   22.9212   23.9649   25.0066   26.0466
```

```
fs =

  Columns 1 through 7

    0.7358    0.1991    0.0700    0.0252    0.0091    0.0033    0.0012

  Columns 8 through 14

    0.0004    0.0002    0.0001    0.0000    0.0000    0.0000    0.0000

  Columns 15 through 21

    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000

  Columns 22 through 24

    0.0000    0.0000    0.0000
>> plot(xs,abs(fs))
>> axis([0,30,-0.1,0.8])
>> xlabel('x')
>> ylabel('f(x)')
```



For an error of 1e-10, it takes 23 steps to get the final answer. The ideal solution to this problem is ∞.

The function value (fs) converges to 0.

For the given solution 27.085, $f(x) = 4.86e^{-11}$.