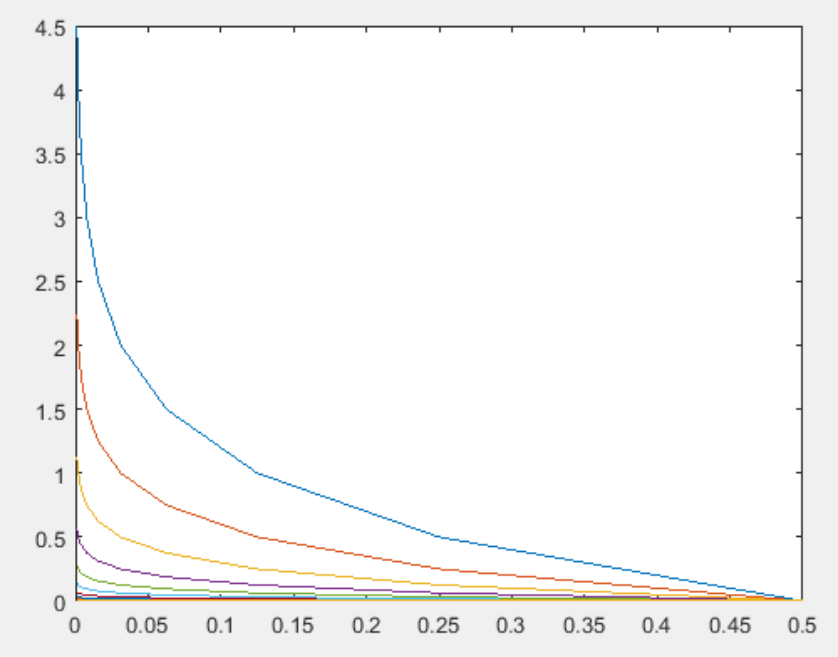
Problem 1:

The rate of convergence is $O(h)$.

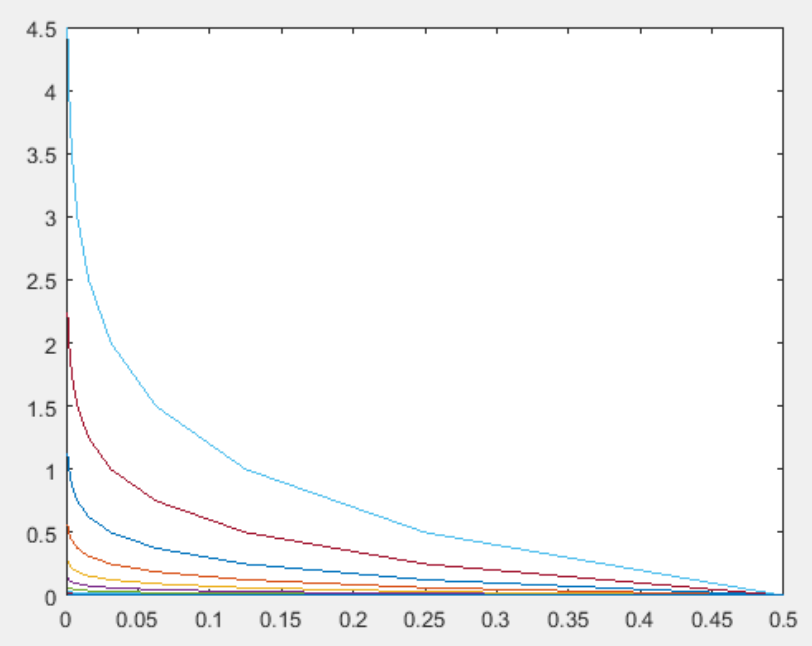
```
y0 = 0;
t0 = 0;
hlist = 2.^(-(1:10));
f = @(t,y) 1+y.^2;
for k = 1:10
    h = 2.^(-k);
    [ylist,tlist] = euler(f,y0,t0,h,10-1);
    plot(hlist,tlist);
    hold on
end
```



Problem 2:

Heun: rate of convergence is $O(h^2)$.

```
y0 = 0;
t0 = 0;
hlist = 2.^(-(1:10));
f = @(t,y) 1+y.^2;
for k = 1:10
    h = 2.^(-k);
    [ylist,tlist] = heun(f,y0,t0,h,10-1);
    plot(hlist,tlist);
    hold on
end
```



RK4: rate of convergence is $O(h^4)$.

```
y0 = 0;
t0 = 0;
hlist = 2.^(-(1:10));
f = @(t,y) 1+y.^2;
for k = 1:10
    h = 2.^(-k);
   [ylist,tlist] = rk4(f,y0,t0,h,10-1);
    plot(hlist,tlist);
    hold on
end
```

Problem 3:

```
dx = @(x,y,z) sigma*(y-x);
dy = @(x,y,z) x*(rho-z)-y;
dz = @(x,y,z) x*y-beta*z;
sigma = 10; beta = 8/3; rho = 28;

h = 10^(-3);
x0 = [1;-1;1]; %x0 = [1;-1+10^-6;1];
xlist = 1:50;
hlist = h.^-(1:50)
[ylist,tlist] = rk4(f,x0,t0,h,50-1);
plot(xlist,ylist);
hold on
```

```
>> ylist

ylist =

  Columns 1 through 8

    1.0000    1.0020    1.0040    1.0060    1.0080    1.0101    1.0121
   -1.0000   -0.9980   -0.9960   -0.9940   -0.9920   -0.9900   -0.9881
    1.0000    1.0020    1.0040    1.0060    1.0080    1.0101    1.0121

  Columns 9 through 16

    1.0161    1.0182    1.0202    1.0222    1.0243    1.0263    1.0284
   -0.9841   -0.9822   -0.9802   -0.9782   -0.9763   -0.9743   -0.9724
    1.0161    1.0182    1.0202    1.0222    1.0243    1.0263    1.0284
```

Problem 4:

```
1      function dx = vdppar(t,x,flages,mu)
2 -    dx = zeros(2,1);
3 -    dx(1) = x(2);
4 -    dx(2) = mu*(1-x(1)^2)*x(2)-x(1);
5
6 -    [t,x] = ode45('VDPPAR',[0 20],[2 0],[],1); %mu = 2
7      function phasport(equations,timespan,plotrange,solver)
8 -        clf;
9 -        axis(plotrange);
10 -       hold on;
11 -       button = 1;
12 -       [xinit(1),xinit(2),button] = ginput(1);
13 -       if button ~=1 break; end;
14 -       [T,Y] = feval(solver, equations, timespan, xinit);
15 -       plot(Y(:,1),Y(:2));
16 -       [T,Y] = feval(solver, equations, -timespan, xinit);\
17 -       plot(Y(:,1),Y(:2));
18 -   end;
```