

Problem 1:

Since $b^2 \gg 4ac$, $\sqrt{b^2 - 4ac}$ will be very close to $|b|$. To get the smaller root, we subtract the $|b|$.

When b is positive, for the x_+ , the numerator would be close to $-b + (b)$, and subtracting a nearly equal quantity will result in large relative errors.

When b is negative, for the x_- , the numerator would be close to $-b - (-b)$, and subtracting a nearly equal quantity will result in large relative errors.

```
%problem 1
function [xp, xm] = solve_quadratic(a, b, c)
if (b >= 0)
    xp = (2*c) / (-b - sqrt(b^2 - 4*a*c));
    xm = (-b - sqrt(b^2 - 4*a*c)) / (2*a);
else
    xp = (-b + sqrt(b^2 - 4*a*c)) / (2*a);
    xm = (2*c) / (-b + sqrt(b^2 - 4*a*c));
end
```

$$(a) \ x^2 - 10^6 x + 1 = 0$$

The x^+ is $1.0 * 10^6$, the x^- is $1.0 * 10^{-6}$.

The error in the smaller root is $-5.0593 * 10^{-11}$

```
>> xp          a=1;b=-10^6;c=1;          >> x_m - xm
                z = -4*a*c/b^2;
xp =            delta = abs(b)*(1+z/2-z^2/4);    ans =
                x_m = (-b-delta)/(2*a);
                x_m =
1.0000e+06      -5.0593e-11
>> xm          9.9995e-07;
xm =
1.0000e-06
```

$$(b) \ x^2 + 10^6 x + 1 = 0 \quad (\text{codes and results are on the next page})$$

The x^+ is $-1.0 * 10^{-6}$, the x^- is $-1.0 * 10^{+6}$.

The error in the smaller root is 0

```

>> xp          a=1;b=10^6;c=1;          >> x_m - xm
                z = -4*a*c/b^2;
xp =            delta = abs(b)*(1+z/2-z^2/4);  ans =
                x_m = (-b-delta)/(2*a);
                x_m =
-1.0000e-06    -1.0000e+06;
                0
>> xm
xm =
-1.0000e+06

```

Problem 2:

```

function [costheta, sintheta] = cos_and_sin(x,y)
costheta = (x/abs(y))/(sqrt((x/y)^2+1));
sintheta = costheta*y/x;
end

>> cos_and_sin(10^200,10^200)

ans =

    0.7071

>> cos_and_sin(10^(-200),10^(-200))

ans =

    0.7071

```

The answers are 0.7071. By reducing the power of x and y, we prevent the overflow.

Problem 3:

```

function y = mytanh(x)
y = 1 - 2/(exp(2*x)+1);
end

for k=[-2, -1, 0, 1, 2, 3]
x = 10^k
[mytanh(x)]
end

```

The results are on the next page.

k = -2, -1:

k = 0, 1:

k = 2, 3:

x =

0.0100

x =

1

x =

100

ans =

0.0100

ans =

0.7616

ans =

1

x =

0.1000

x =

10

x =

1000

ans =

0.0997

ans =

1.0000

ans =

1

```

for k=[-2, -1, 0, 1, 2, 3]
x = -10^k
[mytanh(x)]
end

```

k = -2, -1:

k = 0, 1:

k = 2, 3:

x =

-0.0100

x =

-1

x =

-100

ans =

-0.0100

ans =

-0.7616

ans =

-1

x =

-0.1000

x =

-10

x =

-1000

ans =

-0.0997

ans =

-1.0000

ans =

-1

Problem 4:

```
%1st
xbar = mean(hw2q4datafile)
n = length(hw2q4datafile)
var_1 = 1/(n-1)*sum((hw2q4datafile-xbar).^2)

%2nd
var_2 = 1/(n-1)*(sum(hw2q4datafile.^2) - n*xbar^2)

var_1 =

    1.2303

var_2 =

    1.3240e+03
```

The second variance = $1.324e^{+03}$ is more accurate. Since all entries are the same, we expect the variance to be 0. In the first function, we keep adding $(x_i - \bar{x})^2$, the subtracting will lead to a large relative error.