Problem 1:

```
>> x = bisect(@(x)x-2*sin(x),pi/4,pi,1e-6,1)
Iter # 0: a =    0.785398163397448, f(a) =  -0.628815, b =     3.14159265358979, f(b) =    3.14159
Iter # 1: a =    0.785398163397448, f(a) =  -0.628815, b =     1.96349540849362, f(b) =   0.115736
Iter # 2: a =     1.37444678594553, f(a) =  -0.587124, b =     1.96349540849362, f(b) =   0.115736
Iter # 3: a =     1.66897109721958, f(a) =  -0.321398, b =     1.96349540849362, f(b) =   0.115736
Iter # 4: a =      1.8162332528566, f(a) =  -0.123829, b =     1.96349540849362, f(b) =   0.115736
Iter # 5: a =     1.88986433067511, f(a) = -0.00919203, b =     1.96349540849362, f(b) =   0.115736
Iter # 6: a =     1.88986433067511, f(a) = -0.00919203, b =     1.92667986958437, f(b) =  0.0520018
Iter # 7: a =     1.88986433067511, f(a) = -0.00919203, b =     1.90827210012974, f(b) =  0.0210852
Iter # 8: a =     1.88986433067511, f(a) = -0.00919203, b =     1.89906821540242, f(b) = 0.00586639
Iter # 9: a =     1.89446627303877, f(a) = -0.0016829, b =     1.89906821540242, f(b) = 0.00586639
Iter # 10: a =     1.89446627303877, f(a) = -0.0016829, b =     1.8967672442206, f(b) = 0.00208673
Iter # 11: a =     1.89446627303877, f(a) = -0.0016829, b =     1.89561675862968, f(b) = 0.000200661
Iter # 12: a =     1.89504151583422, f(a) = -0.000741433, b =     1.89561675862968, f(b) = 0.000200661
Iter # 13: a =     1.89532913723195, f(a) = -0.000270464, b =     1.89561675862968, f(b) = 0.000200661
Iter # 14: a =     1.89547294793082, f(a) = -3.49212e-05, b =     1.89561675862968, f(b) = 0.000200661
Iter # 15: a =     1.89547294793082, f(a) = -3.49212e-05, b =     1.89554485328025, f(b) = 8.2865e-05
Iter # 16: a =     1.89547294793082, f(a) = -3.49212e-05, b =     1.89550890060553, f(b) = 2.39707e-05
Iter # 17: a =     1.89549092426817, f(a) = -5.47559e-06, b =     1.89550890060553, f(b) = 2.39707e-05
Iter # 18: a =     1.89549092426817, f(a) = -5.47559e-06, b =     1.89549991243685, f(b) = 9.24745e-06
Iter # 19: a =     1.89549092426817, f(a) = -5.47559e-06, b =     1.89549541835251, f(b) = 1.88591e-06
Iter # 20: a =     1.89549317131034, f(a) = -1.79484e-06, b =     1.89549541835251, f(b) = 1.88591e-06
Iter # 21: a =     1.89549317131034, f(a) = -1.79484e-06, b =     1.89549429483143, f(b) = 4.55335e-08

x =

   1.8955
```

```
>> format long
>> x-2*sin(x)

ans =

     4.553347365821026e-08
```

The initial interval is $[\frac{\pi}{4}, \pi]$. The computed solution is **1.8955**. Error is $\mathbf{4.5533e^{-8}}$.

Problem 2:

1. $x_{n+1} = 2\sin(x_n)$:

$g(z) = 2\sin(z)$; $\quad g'(z) = 2\cos(z)$;

$x^* = 1.8955$; $\quad g'(x^*) = -0.6380$;

$|g'(x^*)| = 0.6380$

Since $|g'(x^*)| < 1$, it **converge** to a solution and the rate of convergence is controlled by **0.6380**.

```
x =

   1.895494294831430

>> 2*cos(x)

ans =

  -0.638045100975142
```

2. $x_{n+1} = 2(x_n - \sin(x_n))$:

$g(z) = 2\,(z - \sin(z));\quad g'(z) = 2 - 2\cos(z);$

$x^* = 1.8955;\qquad g'(x^*) = 2.6380;$

$|g'(x^*)| = 2.6380$

Since $|g'(x^*)| > 1$, it **diverges.**

```
%iteration x[n+1] = 2*sin(x[n])
%accuracy of 10 digits

x(1) = 1.8955;
n = 0;  %iteration number

for k = 1:30
    x(k+1) = 2*sin(x(k));
    x(k) = x(k+1);
    n = n+1;
    [n,x(k)]
end

```

ans =

1.000000000000000   1.895490609112251

2.000000000000000   1.895496600940147

…

25.000000000000000   1.895494266958182

26.000000000000000   1.895494267082344

27.000000000000000   1.895494267003123

The solution is **1.8954942670**, with an accuracy of 10 digits.

<u>Problem 3:</u>

The solution is approximately **1.8955**.

The number of iterations is **5**.

The value is **0.**

```
>> x-2*sin(x)

ans =

     0
```

```
>> f = @(x) x-2*sin(x);
>> df = @(x) 1-2*cos(x);
>> x = pi/2;
>> x = x-f(x)/df(x)

x =

     2

>> x = x-f(x)/df(x)

x =

   1.900995594203909
```

```
x =

   1.895511645379595

>> x = x-f(x)/df(x)

x =

   1.895494267208713

>> x = x-f(x)/df(x)

x =

   1.895494267033981
```

Problem 4:

The solution is **4.4934**. It's computed using the **bisection method**.

The stopping criterion is when $|a - b|$ is small enough --- ε less than $\mathbf{1 * e^{-6}}$.

The value of x – tan(x) is $\mathbf{7.1591 * e^{-6}}$.

```
>> x = bisect(@(x)x-tan(x),pi,3*pi/2,1e-6,1)
Iter # 0: a =     3.14159265358979, f(a) =    3.14159, b =     4.7
Iter # 1: a =     3.92699081698724, f(a) =    2.92699, b =     4.7
Iter # 2: a =     4.31968989868597, f(a) =    1.90548, b =     4.7
Iter # 3: a =     4.31968989868597, f(a) =    1.90548, b =     4.5
Iter # 4: a =     4.41786466911065, f(a) =    1.12131, b =     4.5
Iter # 5: a =     4.46695205432299, f(a) =   0.474728, b =     4.5
Iter # 6: a =     4.49149574692916, f(a) =  0.0382935, b =     4.5
Iter # 7: a =     4.49149574692916, f(a) =  0.0382935, b =     4.5
Iter # 8: a =     4.49149574692916, f(a) =  0.0382935, b =      4.
Iter # 9: a =     4.49149574692916, f(a) =  0.0382935, b =     4.4
Iter # 10: a =     4.49302972771704, f(a) = 0.00765332, b =      4.
Iter # 11: a =     4.49302972771704, f(a) = 0.00765332, b =      4.
Iter # 12: a =     4.49302972771704, f(a) = 0.00765332, b =      4.
Iter # 13: a =     4.49322147531553, f(a) = 0.00379214, b =      4.
Iter # 14: a =     4.49331734911477, f(a) = 0.00185894, b =      4.
Iter # 15: a =     4.49336528601439, f(a) = 0.000891677, b =      4
Iter # 16: a =      4.4933892544642, f(a) = 0.000407883, b =      4
Iter # 17: a =     4.49340123868911, f(a) = 0.000165946, b =      4
Iter # 18: a =     4.49340723080156, f(a) = 4.49665e-05, b =      4
Iter # 19: a =     4.49340723080156, f(a) = 4.49665e-05, b =      4
Iter # 20: a =     4.49340872882968, f(a) = 1.47206e-05, b =      4

x =

   4.493409103336704
```

```
>> x-tan(x)

ans =

     7.159062301198560e-06
```