Problem 1.1:

For example, we want to test the probability of the U.S economy facing a recession or a boost. However, there's too little past experience for us to use and it's not an event that will happen frequently. Therefore, the long-frequency interpretation of the probability of facing an economy recession is not quite applicable.

Problem 1.2:

Model	Prior Probabilities	Likelihood for M-	Prior × Likelihood	Posterior Probabilities
Breast cancer	0.0045	0.276 (=1-0.724)	0.001242	0.00128
No breast cancer	0.9955	0.973 (=1-0.027)	0.968622	0.99872

The posterior probability that my friend will be diagnosed with breast cancer is **0.00128**.

Problem 1.3:

Model	Prior Probabilities	Likelihood for S+	Prior × Likelihood	Posterior Probabilities
Breast cancer	0.107	0.89 (=1-0.11)	0.09523	0.63994
No breast cancer	0.893	0.06 (=1-0.94)	0.05358	0.36006

(The prior probabilities are from posterior probabilities of Table 1.3 with M+.)

The posterior probability that my friend has breast cancer in this case is **0.63994**.

Problem 1.6:

The partner I picked is my classmate. I want to know her subjective probability of the event D, that is the Hawkeye football team will score three straight wins in the next three matches. The calibration experiment is to pick 1 card from an ordinary 52-count poker cards (J,Q,K stands for 11, 12, 13; the poker is w/o 2 jokers).

Step 1:

Game 1 If the color of the card is red, then she wins. Otherwise, I win.

Game 2 If the Hawkeye football team win all games for next three games, she wins. Otherwise, I win.

She chose Game 1, then I conclude that $0 < P_s(D) < 0.5$

Step 2:

Game 1 If the card is spade, she wins. Otherwise, I win. (redefined)

Game 2 The same.

She chose Game 2, then I conclude that $0.25 < P_s(D) < 0.5$

Step 3:

Game 1 If the number on the card is larger than 8, she wins. Otherwise, I win.

Game 2 The same.

She chose Game 2, then I conclude that $0.3846 < P_s(D) < 0.5$

Since the width of the interval is 0.1154, which is smaller than 1/8. I have succeeded in finding the subjective P(D). The probability of Hawkeye football team will win the next three games is between **0.3846 and 0.5**.

<u>Problem 2.2:</u>

$P(15\&16)=0.2$; $P(15\&16^c)=0.1$; $P(15^c\&16)=0.1$; $P(15^c\&16^c)=0.6$

1. P(15) = 0.2+0.1 = **0.3**

2. P(15 U 16) = 0.3 + 0.3 – 0.2 = **0.4**

3. P(16) = 0.2+0.1 = **0.3**

4. P(16/15) = P(15&16)/P(15) = 0.2/0.3 = **0.6667**

Problem 2.3:

(a) (1) Prior Prob: P(carrier)=**0.5** P(not carrier)=**0.5**

(2) P(y=0 given carrier) = **0.5** P(y=0 given not carrier) = **0.5**

P(y=1 given carrier) = **1** P(y=1 given not carrier) = **0**

(b) y1 = 0

Model	Prior Prob	Likelihood for 0	Product	Posterior Prob
Carrier	0.5	0.5	0.25	0.3333
Not carrier	0.5	1	0.5	0.6667

y2 = 1

Model	Prior Prob	Likelihood for 1	Product	Posterior Prob
Carrier	0.3333	0.5	0.1667	1
Not carrier	0.6667	0	0.0	0

y3 = 0

Model	Prior Prob	Likelihood for 0	Product	Posterior Prob
Carrier	1	0.5	0.5	1
Not carrier	0	1	0	0

(c) (1) P(carrier given y1=0) = **0.3333**

(2) **Yes, it changed**. That's because only a carrier can give birth to a son with hemophilia. Therefore, the probability of Danielle is a carrier become 1. She must be a carrier.

(3) **No, it didn't**. That's because Danielle is determined to be a carrier and with prior probability of 0, the posterior probability of that event could only be 0 thereafter.

Problem 2.4:

P(Stat) = 0.35 P(Bio) = 0.25 P(other) = 0.4

P(w/Stat) = 0.6 P(w/Bio) = 0.75 P(w/other) = 0.4

(a) P(w) = P(Stat)*P(w/Stat) + P(Bio)*P(w/Bio) + P(other)*P(w/other) = **0.5575**

(b) P(o/w) = $\frac{P(other)*P(w \vdots other)}{P(women)} = \frac{0.16}{0.5575} = \mathbf{0.2870}$