Problem 3.1:

1. $\alpha = 2, \ \beta = 7$

The mean is $\frac{\alpha}{\alpha+\beta} = \frac{2}{2+7} = \mathbf{0.222}$

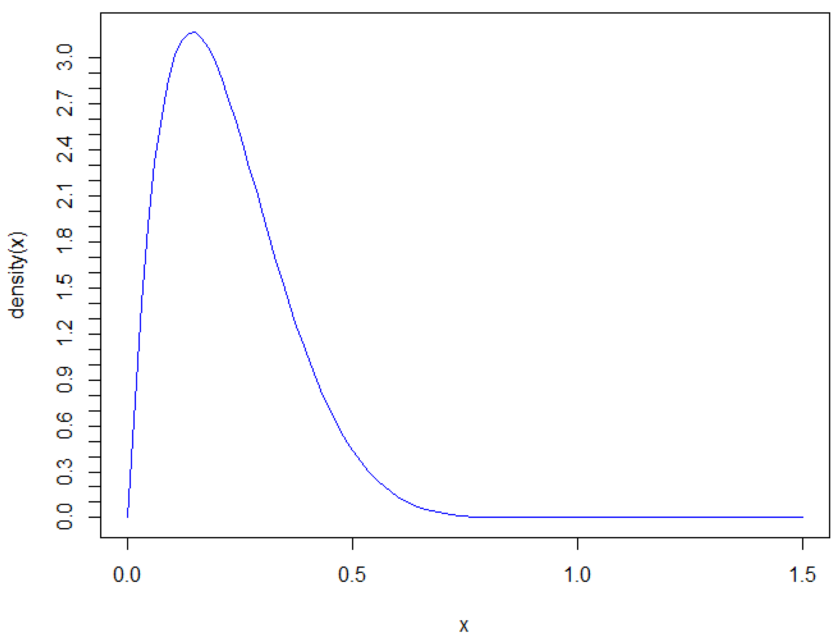
The mode is $\frac{\alpha-1}{\alpha+\beta-2} = \frac{2-1}{2+7-2} = \mathbf{0.143}$

2. The median is **0.2011**

The 90% central interval is **[0.0464, 0.4707]**

```
> qbeta(0.5,2,7)
[1] 0.2011312
> qbeta(0.05,2,7)
[1] 0.04638926
> qbeta(0.95,2,7)
[1] 0.4706794
```

3. The beta(2,7) plot is:



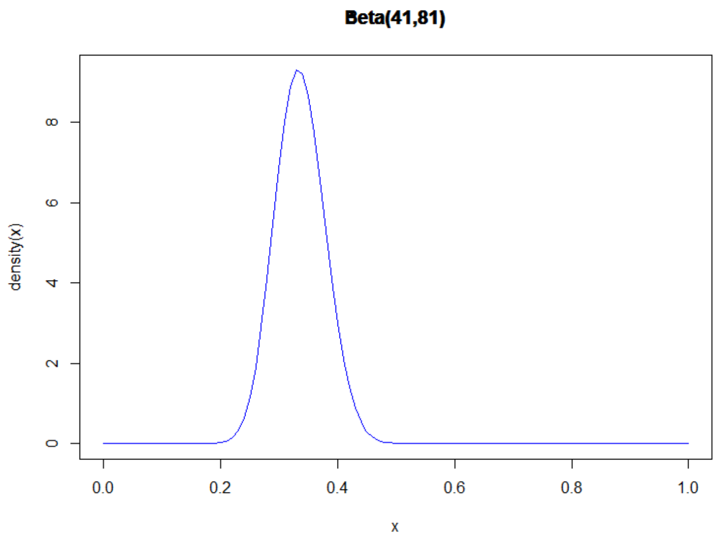
Problem 3.3:

1. uniform(0,1) = **beta (1,1)**

2. Since the desired sample size for beta (1,1) is 1+1-2 = 0. The desired equivalent prior sample size for U(0,1) is also 0.

Problem 3.4:

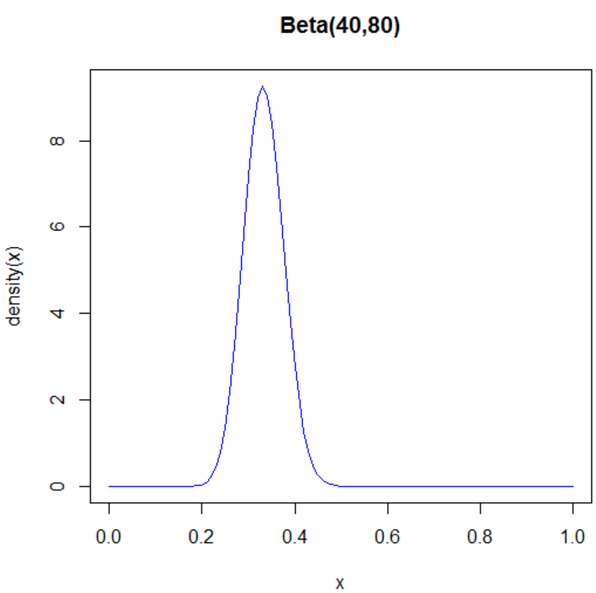
1. $\alpha - 1 = 40,\ \beta - 1 = 80$, we plot the density function of beta (41, 81):

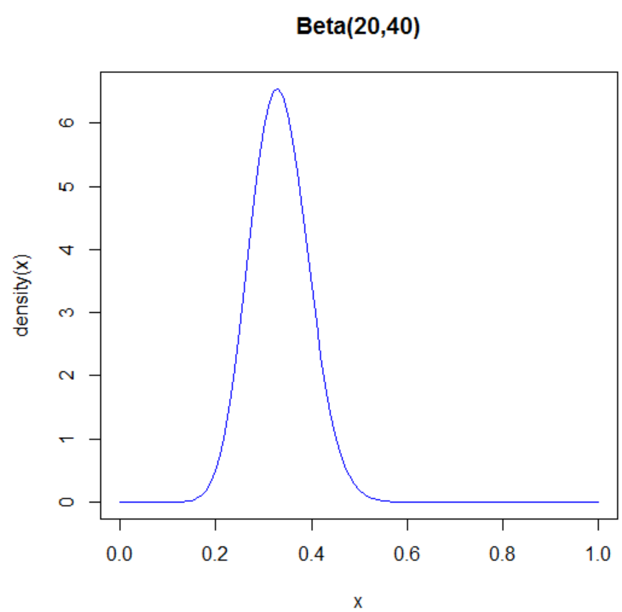


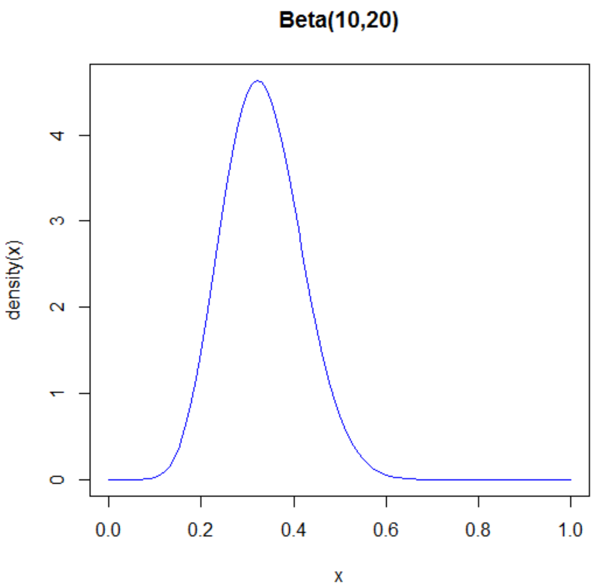
We notice that the distribution is concentrated between 0.3 to 0.4.We need to use the probability of a sample high school student getting hit to estimate θ.

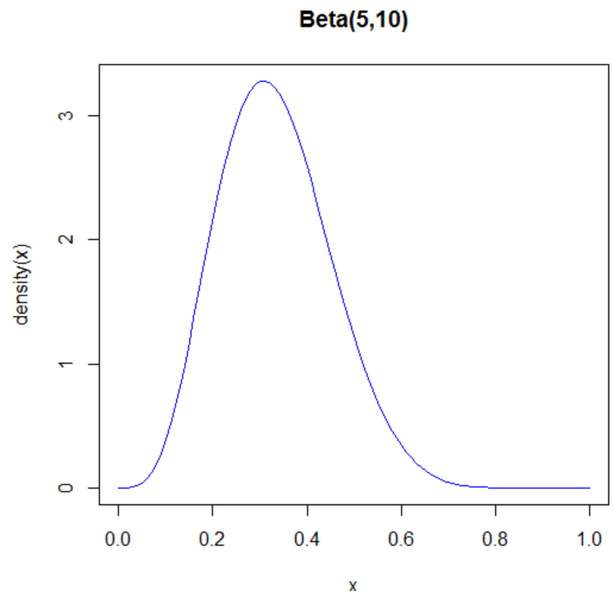
The probability P(getting hit) $= \frac{40}{120} = 0.333$. It's within the interval of [0.3, 0.4]. Therefore **the guess of beta(41,81) is good**.

2. The player's mother's belief would probability want a steady hit, therefore we want the prior mean to be 0.333. We could try beta(40,80), beta(20,40), beta (10,20) and beta(5,10).









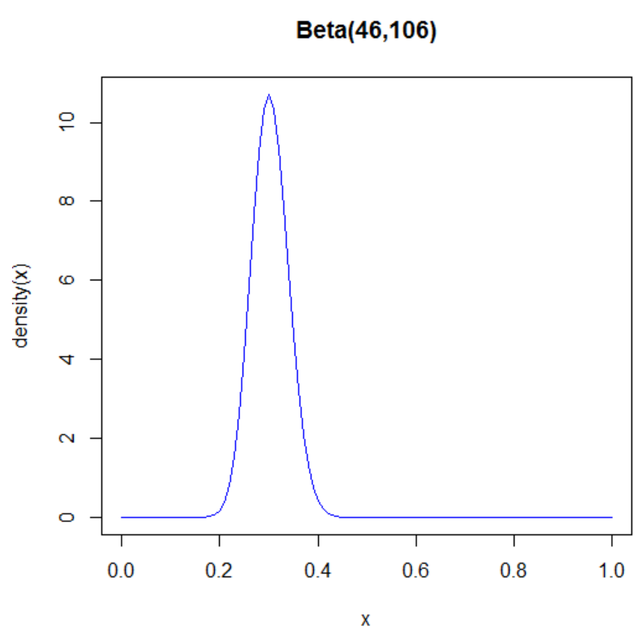
The plots indicate that, when the sum of $\alpha\ and \beta$ is larger, the distribution is more concentrated. To get a more certain outcome of getting a hit at bat, the **beta(40,80) would be the best fit** in all.

3. One of the reason of dependence might be that the players' performances would go down since the eight college-levels games are very exhausting. **The model might not be independent**.

4. (a) Posterior: probability of getting a hit at bat based on player's latest record.

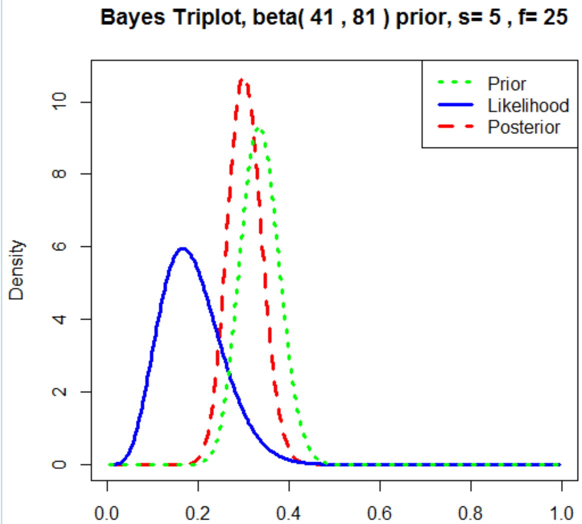
The likelihood function is $L(\theta) = \binom{30}{5}\theta^5(1-\theta)^{25}$. The posterior $P(\theta \vdots y)$ is proportional to beta$(\alpha + y,\ \beta + n - y)$.

(b) Here we plot **Beta(46,106).**

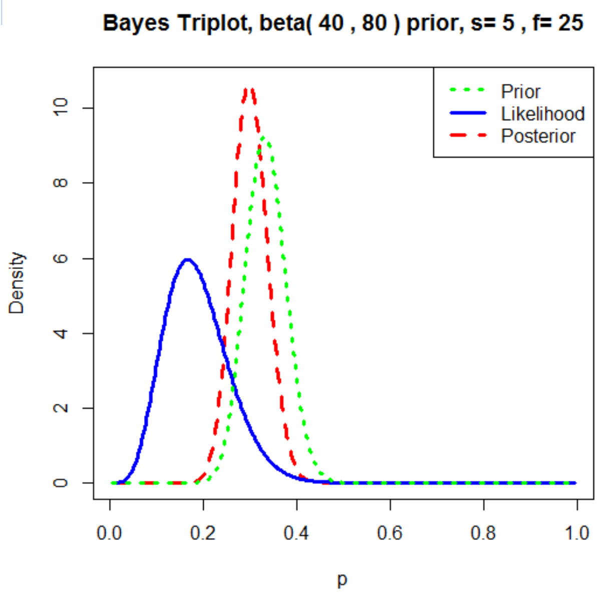


(c)

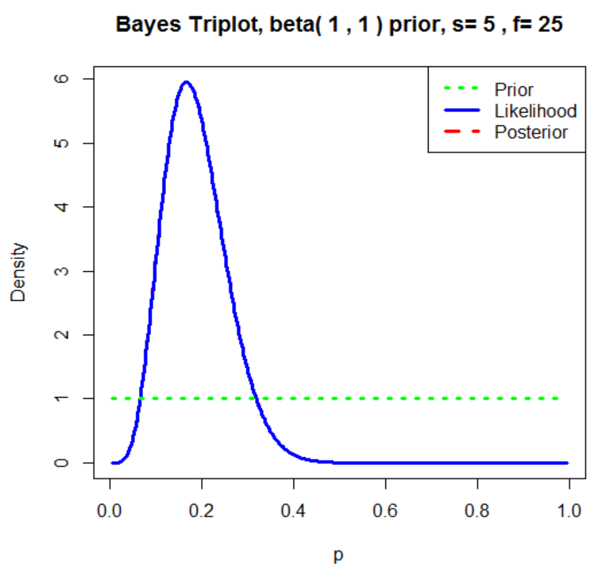
(1)


Bayes Triplot, beta(41 , 81) prior, s= 5 , f= 25
Prior
Likelihood
Posterior
Density
0
2
4
6
8
10
0.0
0.2
0.4
0.6
0.8
1.0

(2)


Bayes Triplot, beta(40 , 80) prior, s= 5 , f= 25
Prior
Likelihood
Posterior
Density
0
2
4
6
8
10
0.0
0.2
0.4
0.6
0.8
1.0
p

(3)


Bayes Triplot, beta(1 , 1) prior, s= 5 , f= 25
Prior
Likelihood
Posterior
Density
0
1
2
3
4
5
6
0.0
0.2
0.4
0.6
0.8
1.0
p