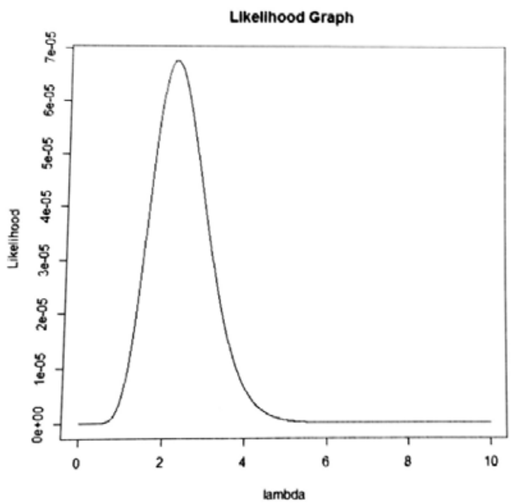
Problem 5.4:

1. $fy(y1 \ldots yn|\lambda) = \prod_{i=0}^{n} \frac{e^{-\lambda}\lambda^{yi}}{yi!} = \frac{\boldsymbol{e^{-n\lambda}\lambda^{n\bar{y}}}}{\boldsymbol{\prod_{i=1}^{n}(yi!)}}$

2. $\bar{y} = \frac{1}{5}(2+5+1+0+3) = 2.2$

$$L(\lambda) = \prod_{i=0}^{5} \frac{e^{-\lambda}\lambda^{yi}}{yi!} = \frac{e^{-5\lambda}\lambda^{5*2.2}}{2!\,5!\,1!\,0!\,3!} = \frac{\boldsymbol{e^{-5\lambda}\lambda^{11}}}{\mathbf{1440}}$$



3. $\frac{\partial L(*)}{\partial x} = \frac{\partial}{\partial x}\left(\frac{e^{-5\lambda}\lambda^{11}}{1440}\right) = \frac{-5e^{-5\lambda}\lambda^{10}(-5\lambda+11)}{1440} = 0$

$$\frac{\partial^2}{\partial x^2}\left(\frac{e^{-5\lambda}\lambda^{11}}{1440}\right) < \mathbf{0}$$

Therefore, $\hat{\lambda} = \frac{11}{5} = 2.2 = \bar{y}$ is **the mle of λ**.

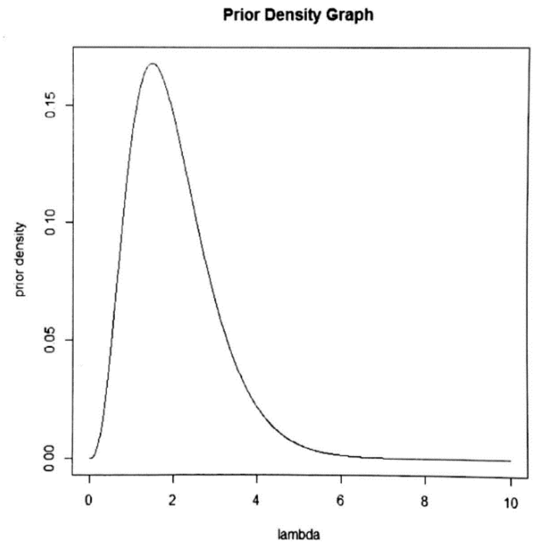
```
> poisson.test(11,5,alternative = c("two.sided","less","greater"),
+ conf.level=0.95)

        Exact Poisson test

data:  11 time base: 5
number of events = 11, time base = 5, p-value = 0.02043
alternative hypothesis: true event rate is not equal to 1
95 percent confidence interval:
 1.098232 3.936408
sample estimates:
event rate
       2.2
```

The R output has a same **mle of 2.2**. The 95% CI for λ is **(1.0982, 3.9364)**

Problem 5.5:

1. The prior is a member of **Gamma (4,2).**

2. The plot indicates that the president's prior belief about rate parameter is maximized around somewhere between 1.6 to 2.5. The majority of expected number of breakdowns are distributed around 0 to 5.



3. $p(\lambda|y) = p(\lambda)L(\lambda) \propto \lambda^3 e^{-2\lambda}\left(\frac{e^{-5\lambda}\lambda^{11}}{1440}\right) = \frac{\lambda^{14}e^{-7\lambda}}{1440} = \boldsymbol{Gamma}(\mathbf{15}, \mathbf{7})$

4. Mean is 15/7 = **2.143**

5. **Yes**, it was. It's a Poisson distribution because they have the same likelihood as the one in problem 5.4.

Problem 5.6:

(1). $L(\lambda|y) = \frac{e^{-\lambda}\lambda^y}{y!}$ $\quad l(\lambda|y) = -\lambda + y\log\lambda - \log(y!)$

$\frac{\partial l}{\partial \lambda} = -1 + \frac{y}{\lambda}$; $\frac{d^2 l}{d\lambda^2} = -\frac{y}{\lambda^2}$

$\Rightarrow I(\lambda|y) = -E\left(\frac{\partial^2 l(\lambda|y)}{\partial \lambda^2}\right) = E\left(\frac{y}{\lambda^2}\right) = \frac{1}{\lambda}$

$I(\lambda|y^*) = n \times I(\lambda|y) = \frac{n}{\lambda}$

$P(\lambda) \propto \sqrt{I(\lambda|y)} = \sqrt{\frac{n}{\lambda}} \propto \sqrt{\frac{1}{\lambda}}$

(2) $p(\lambda|y) = p(\lambda)p(y|\lambda) \propto \sqrt{\frac{1}{\lambda}}\,\frac{e^{-5\lambda}\lambda^{11}}{1440} = \frac{e^{-5\lambda}\lambda^{10.5}}{1440} = Gamma(11.5, 5)$

(3). mean $= \frac{11.5}{5} = 2.3$

The 95% credible set is **(1.1689, 3.8076).**

```
> qgamma(c(0.025,0.975),11.5,rate=5)
[1] 1.168855 3.807563
```

4. The Bayesian point estimate is **larger** than the classical estimate (2.3 > 2.2). In addition, the interval estimate is **wider** than the classical interval estimate.

5. The employee's estimate is **larger** than that of the president since the employee has larger λ.

6. Employee:

```
> pgamma(2,11.5,rate=5,lower.tail=FALSE)
[1] 0.6419118
```

7. President:

```
> pgamma(2,15,rate=7,lower.tail=FALSE)
[1] 0.5704367
```

(problems in Section 6 are on next page)