MAT128A: Numerical Analysis, Section 2
Project Five: Adaptive Integration
Due Date: December 7, 2018

1. Project Description

In this project, you will write two C functions to perform adaptive integration. Recall that the N^{th} order Chebyshev expansion of the function f on the interval $[a, b]$ is the polynomial

$$p(x) = {\sum_{n=0}^{N}}' a_n T_n \left(\frac{2}{b-a} x - \frac{b+a}{b-a} \right), \tag{1}$$

where the coefficients $\{a_n\}$ are defined via the formula

$$a_n = \frac{2}{N+1} \sum_{j=0}^{N} f\left(\frac{b-a}{2} x_j + \frac{b+a}{2} \right) T_n(x_j)$$

with $x_0, x_1, \ldots, x_N$ given by

$$x_j = \cos\left(\frac{j + \frac{1}{2}}{N+1} \pi \right).$$

The N^{th} order piecewise Chebyshev expansion of the function $f : [a, b] \to \mathbb{R}^n$ given on the partition

$$a = a_0 < a_1 < a_2 < \ldots < a_m = b \tag{2}$$

consists of m polynomials of degree N

$$p_0, p_1, \ldots, p_{m-1}. \tag{3}$$

The i^{th} polynomial p_i is the N^{th} order Chebyshev expansion of f on the interval $[a_i, a_{i+1}]$. It is defined by the formula

$$p_i(x) = {\sum_{n=0}^{N}}' c_n^i \; T_n \left(\frac{2}{a_{i+1} - a_i} x - \frac{a_{i+1} + a_i}{a_{i+1} - a_i} \right),$$

where the coefficients $\left\{ c_n^i \right\}$ are

$$c_n^i = \frac{2}{N+1} \sum_{j=0}^{N} f\left(\frac{a_{i+1} - a_i}{2} x_j + \frac{a_{i+1} + a_i}{2} \right) T_n(x_j). \tag{4}$$

Your task is to implement two functions, "chebint" and "chebadap_int". These functions will be placed in a C file called "chebint.c". The first of them, "chebint", will evaluate the definite integral

$$\int_a^b p(x) \; dx$$

of an expansion p of the form (1) given the coefficients $a_0, a_1, \ldots, a_N$. The second of these functions, "chebadap_int" will, given a piecewise Chebyshev expansion of a function f consisting of the partition (2) and the expansions (3), approximate the integral

$$\int_a^b f(x)\ dx$$

by computing the sum

$$\sum_{j=0}^{m-1} \int_{a_j}^{a_{j+1}} p_j(x)\ dx. \tag{5}$$

Presumably, each term in the sum (5) will be evaluated via a call to your "chebint" function. The calling syntaxes for both of these functions are in the "chebint.c" file.

2. Testing and grading

A public test code is given in the file inttest1.c. Another test code, called inttest2.c, will be used to test your function as well. Half of the project grade will come from the first test file, and the second half will come from the second. The commands

```
gcc -o inttest1 chebexp.c chebadap.c chebint.c inttest1.c -lm
./inttest1
```

can be used to compile and execute your program. There are five tests of your function in "inttest1.c", and the program will tell your score out of 5. We will also test your code by compiling against inttest2.c, which we will not release until after the projects are due. You will get a 0 on your project if it does not compile and run. Please start work on your project early and come see either myself or our TA, Karry Wong, if you are having difficulties getting it to compile.

You should feel free to use either my versions of the files "chebexp.c" and "chebadap.c", or your own.

3. Submitting your project

You will submit your project using canvas. You should submit only your "chebint.c" fule. You must submit your file by 11:59 PM on the due data. Late assignments will not be accepted.