MAT128A: Numerical Analysis, Section 2
Homework for the week of November 26, 2018

1. Let $x_0 = 0$, $x_1 = 1/2$ and $x_2 = 1$. Find weights w_0, w_1, and w_2 such that the formula

$$\int_0^1 f(x)\ dx = w_1 f(x_0) + w_2 f(x_1) + w_3 f(x_2) \tag{1}$$

holds whenever f is a polynomial of degree less than or equal to 2.

2. Let $x_0 = -\sqrt{3/5}$, $x_1 = 0$ and $x_2 = \sqrt{3/5}$. Find weights w_0, w_1 and w_2 such that

$$\int_{-1}^1 f(x)\ dx = w_1 f(x_0) + w_2 f(x_1) + w_3 f(x_2) \tag{2}$$

holds whenever f is a polynomial of degree less than or equal to 2. Show that the formula in fact holds when f is a polynomial of degree less than or equal to 5.

3. Let $x_0 = 0$, $x_1 = 1/2$ and $x_2 = 1$ Find weights w_0, w_1 and w_2 such that

$$\int_0^1 f(x)\sqrt{x}\ dx = w_0 f(x_0) + w_1 f(x_1) + w_2 f(x_2) \tag{3}$$

when f is a polynomial of degree less than or equal to 2. Use this quadrature rule to approximate

$$\int_0^1 \cos(x)\sqrt{x}\ dx.$$

How accurate is your approximation?

4. Suppose that

$$f(x) = 2T_0(x) + 4T_1(x) - 6T_2(x) + 12T_3(x) - 14T_4(x).$$

Find

$$\int_{-1}^1 f(x)\ dx.$$