MAT128A: Numerical Analysis, Section 2
Project Four: Piecewise Chebyshev Expansions
Due Date: November 19, 2018

## 1. Project Description

In this project, you will write C functions to construct and evaluate piecewise Chebyshev expansions. Recall that the $N^{th}$ order Chebyshev expansion of the function $f$ on the interval $[a, b]$ is the polynomial

$$p(x) = {\sum_{n=0}^{N}}' a_n T_n \left( \frac{2}{b-a} x - \frac{b+a}{b-a} \right),$$

where the coefficients $\{a_n\}$ are defined via the formula

$$a_n = \frac{2}{N+1} \sum_{j=0}^{N} f\left( \frac{b-a}{2} x_j + \frac{b+a}{2} \right) T_n(x_j)$$

with $x_0, x_1, \ldots, x_N$ given by

$$x_j = \cos\left( \frac{j+\frac{1}{2}}{N+1} \pi \right).$$

The $N^{th}$ order piecewise Chebyshev expansion of the function $f : [a, b] \to \mathbb{R}^n$ given on the partition

$$a = a_0 < a_1 < a_2 < \ldots < a_m = b \tag{1}$$

consists of $m$ polynomials of degree $N$

$$p_0, p_1, \ldots, p_{m-1}.$$

The $i^{th}$ polynomial $p_i$ is the $N^{th}$ order Chebyshev expansion of $f$ on the interval $[a_i, a_{i+1}]$. It is defined by the formula

$$p_i(x) = \sum_{n=0}^{N} c_n^i \; T_n \left( \frac{2}{a_{i+1}-a_i} x - \frac{a_{i+1}+a_i}{a_{i+1}-a_i} \right),$$

where the coefficients $\left\{c_n^i\right\}$ are

$$c_n^i = \frac{2}{N+1} \sum_{j=0}^{N} f\left( \frac{a_{i+1}-a_i}{2} x_j + \frac{a_{i+1}+a_i}{2} \right) T_n(x_j).$$

I have provided you with a function called "chebadap" which, given a user-specified function $f : [a, b] \to \mathbb{R}$ and an integer $N$, attempts to determine a partition of $[a, b]$ of the form (1) such that for each $i = 0, 1, \ldots, m-1$, $p_i$ approximates $f$ on the interval $[a_i, a_{i+1}]$ to specified precision.

Your task is to implement two functions, "chebadap_coefs" and "chebadap_eval". The first of these computes the coefficients $\left\{c_n^i\right\}$ in a piecewise Chebyshev expansion of a user-specified function $f$. The second uses the coefficients in the piecewise Chebyshev expansion of $f$ to approximate $f$ at a specified point. More explicitly, it finds the interval $[a_i, a_{i+1})$ containing the point $x$ and then evaluates $p_i(x)$ in order to approximate $f(x)$.

The file "chebdap.c" contains the "chebadap" routine and gives the calling syntax for the "chebadap_coefs", "chebadap_eval" . Your task is to implement the functions as described there. Your code should rely on the "chebexp.c" code you wrote for Project 3 (or, if you wish, you can use the version of "chebexp.c" which I wrote and posted to the course website).

## 2. Testing and grading

A public test code is given in the file adaptest1.c. Another test code, called adaptest2.c, will be used to test your function as well. Half of the project grade will come from the first test file, and the second half will come from the second. The commands

```
gcc -o adaptest1 chebexp.c chebadap.c adaptest1.c -lm
./adaptest1
```

can be used to compiler and execute your program. There are five tests of your function in "adaptest1.c", and the program will tell your score out of 5. We will also test your code by compiling against adaptest2.c, which we will not release until after the projects are due. You will get a 0 on your project if it does not compile and run. Please start work on your project early and come see either myself or our TA, Karry Wong, if you are having difficulties getting it to compile.

## 3. Submitting your project

You will submit your project using canvas. You should submit only your chebadap.c file. You must submit your file by 11:59 PM on the due data. Late assignments will not be accepted.