MAT128A: Numerical Analysis, Section 2
Homework for the week of November 5, 2018

1. Go over the midterm problems and the provided solutions!

2. Show that for all nonnegative integers n, $T_n(1) = 1$ and $T_n(-1) = (-1)^n$.

3. Show that for all integers $n \geq 2$ and all $-1 < t \leq 1$,

$$\int_{-1}^{t} T_n(x)\, dx = \frac{1}{2}\left(\frac{T_{n+1}(t)}{n+1} - \frac{T_{n-1}(t)}{n-1}\right) - \frac{(-1)^n}{n^2-1}.$$

4. Let $x_0, x_1, \ldots, x_N, w_0, w_1, \ldots, w_N$ denote the nodes and weights of the $(N+1)$-point Gauss-Legendre quadrature rule. Suppose that $f : [-1, 1] \to \mathbb{R}$ is continuously differentiable, and that $c_0, c_1, \ldots, c_N$ are defined by the formula

$$c_n = \sum_{j=0}^{N} f(x_j) P_n(x_j) w_j.$$

Show that the polynomial

$$p_N(x) = \sum_{n=0}^{N} c_n P_n(x)$$

interpolates f at the points $x_0, x_1, \ldots, x_N$.