MAT128A: Numerical Analysis, Section 2
Project Three: Chebyshev Expansions
Due Date: November 2, 2018

1. Project Description

In this project, you will write C functions to evaluate Chebyshev polynomials, to calculate the Chebyshev coefficients of a function and to evaluate a function given its Chebyshev coefficients.

The first of these functions will be called "chebpolys," and it will, given an integer $N > 0$ and a point x in the interval $[-1, 1]$, return the values of

$$T_0(x), T_1(x), T_2(x), \ldots, T_N(x),$$

where $T_n(x) = \cos(n \arccos(x))$ is the Chebyshev polynomial of degree n. You should compute the Chebyshev polynomials using the formulas

$$T_0(x) = 1, \quad T_1(x) = x, \quad T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x).$$

The second function will be called "chebcoefs," and it will, given the capability of evaluating a function at any point on an interval $[a, b]$ and an integer $N > 1$, return approximations of the first $N + 1$ Chebyshev coefficients of f. The Chebyshev expansion of f on $[a, b]$ is

$$f(y) = {\sum_{n=0}^{\infty}}' a_n T_n \left(\frac{2}{b-a} y - \frac{b+a}{b-a} \right),$$

where the coefficients $\{a_n\}$ are given by the formula

$$a_n = \frac{2}{\pi} \int_{-1}^{1} f \left(\frac{b-a}{2} x + \frac{b+a}{2} \right) T_n(x) \, dx.$$

You will use the Chebyshev extrema quadrature rule to approximate the coefficients $a_0, a_1, \ldots, a_N$. That is, if, for each $j = 0, 1, \ldots, N$, we let

$$x_j = \cos \left(\frac{j + \frac{1}{2}}{N+1} \pi \right),$$

then you should use the quadrature formula

$$a_n \approx \frac{2}{N+1} \sum_{j=0}^{N} f \left(\frac{b-a}{2} x_j + \frac{b+a}{2} \right) T_n(x_j).$$

to approximate the Chebyhev coefficients.

The third function will be called "chebeval," and it will, given an integer $N > 1$, a point y on the interval $[a, b]$ and the approximations of the coefficients $a_0, \ldots, a_N$ computed by "chebcoefs", compute the value of the expansion

$$\sum_{n=0}^{N}{}' a_n T_n \left(\frac{2}{b-a} y - \frac{b+a}{b-a} \right),$$

at the point y.

The file "chebexp.c" gives the calling syntax for the "chebcoefs", "chebpolys" and "chebval" routines. Your task is to implement the functions as described there.

2. Testing and grading

A public test code is given in the file chebtest1.c. Another test code, called chebtest2.c, will be used to test your function as well. Half of the project grade will come from the first test file, and the second half will come from the second. The commands

```
gcc -o chebtest1 chebexp.c chebtest1.c -lm
./chebtest1
```

can be used to compiler and execute your program. The first command compiles your program and creates an executable file named chebtest1. The -lm option tells the compiler to include the math library which implements cosine, sine and other basic functions. The second command runs the fourtest1 executable. There are five tests of your function in chebtest1, and the program will tell your score out of 5. We will also test your code by compiling against chebtest2.c, which we will not release until after the projects are due. You will get a 0 on your project if it does not compile and run. Please start work on your project early and come see either myself or our TA, Karry Wong, if you are having difficulties getting it to compile.

3. Submitting your project

You will submit your project using canvas. You should submit only your chebexp.c file. You must submit your file by 11:59 PM on the due data. Late assignments will not be accepted.