MAT128A: Numerical Analysis, Section 2
Fall Quarter 2018
MWF 1:10 - 2:00 PM in Young 184

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1. Overview

This course is the first quarter of a three-quarter long introduction to numerical analysis. Numerical analysis is a broad field concerned with solving mathematics problems using finite precision arithmetic on computers. Our principal focus will be on techniques for the representation of piecewise smooth functions of one variable. That is, we will discuss how we can, given the ability to evaluate a function $f(x)$, calculate a finite amount of information which enables us to evaluate $f(x)$ at any point we wish. This is a basic and important task which must be undertaken by essentially all numerical procedures. We will also discuss the numerical integration and differentiation of piecewise smooth functions of one variable, topics which are closely connected to schemes for representing them.

2. Prerequisites

MAT21C and ECS30; Knowledge of C programming

3. Textbook

The textbook for the course is

> Sauer, T. "Numerical Analysis," 2nd Edition, 2011. Published by Pearson. ISBN: 978-0321783677.

Note that the book does not cover all of the material I will present in class. I will refer you to additional references and make lecture slides available on the course website.

4. Grading

Your grade will be based on a set of 5 programming projects worth a total of 30 percent of your grade, a midterm worth 30 percent of your grade and a final worth 40 percent of your grade. Your grades will be posted on Canvas.

Final grades will be "curved." That is, once all of your exams and projects have been graded, I will look at the distribution of grades and assign cutoffs for "A," "B," etc.

5. Homework

I will assign – but not collect – homework every week. The homework assignments will be posted on the course website at the beginning of the week and solutions will be posted at the end of the week. **I strongly encourage you to do the homework in a timely fashion**.

6. Midterm and Final

Your midterm will be held during class on

November 3, 2018.

Our final is scheduled for

Wednesday, December 12 from 8:00-10:00 AM.

Only university approved excuses for missing these exams will be accepted. A practice midterm and a practice final are currently posted on the course website.

7. Programming Projects

Projects descriptions will be posted on the course website. We will discuss expectations for projects in detail when the first project is assigned. They must be written in the C language and they will be submitted electronically to Canvas. No late projects will be accepted. Please submit whatever you have completed at the time the project is due.

8. Accommodation for Disabilities

Please arrange any necessary accommodations through the Student Disability Center at the beginning of the course.

9. Notice of Code of Academic Conduct

You are expected to abide by the code of academic conduct, which can be found online at

http://sja.ucdavis.edu/files/cac.pdf

10. Outline of the topics which will be covered

1. Finite Precision Arithmetic
   - Basics of finite precision arithmetic and the IEEE format
   - Typical problems which occur: underflow, overflow, cancellation
   - Problems which arise when performing operations on quantities of difficult scales
   - The condition number of evaluation of a function; examples:
     - $f(x) = x^2$; $f(x) = \cos(\lambda x)$
     - especially a comparison of $f(x) = 1/x$ with $f(x) = 1/(1-x)$ and a discussion of the consequences of the logarithmic distribution of finite precision numbers and the desirability of placing singularities at 0
   - Forward and backward error estimates
2. Representing Piecewise Smooth Functions

- The Weierstrauss Theorem
- Taylor Series
  - applicability to C^k functions
  - rapid convergence for analytic functions
  - instability of power series representations
- Expansions of Smooth Periodic Functions in Exponentials
  - Fourier series of periodic functions
  - uniform convergence of Fourier series for differentiable functions
  - improved convergence for periodic C^k functions
  - rapid convergence for periodic functions which are analytic in a strip
  - the trapezoidal rule as a Gaussian quadrature rule for exponentials
  - computation of the coefficients of an expansion of the form $f(x) = \sum_{j=-N}^{N} a_j \exp(ijx)$
  - aliasing and the error incurred in representing $f(x) = \sum_{j=-\infty}^{\infty} a_j \exp(ijx)$.
- Chebyshev Expansions of Smooth Functions
  - definition of Chebyshev polynomials via the change of variable $x = \cos(\theta)$
  - definition of Chebyshev polynomials via the recurrence relations
  - convergence results for Chebyshev expansions
  - bounds on the L^∞ norm of the Chebyshev polynomials
  - orthogonality relation; i.e., the value $\int_{-1}^{1} \frac{T_n(x)T_m(x)}{\sqrt{1-x^2}}\, dx$
  - Gauss-Chebyshev quadrature rules and the computation of coefficients in the expansion $f(x) = \sum_{j=0}^{N} a_j T_j(x)$
  - aliasing and the error incurred when representing $f(x) = \sum_{j=0}^{\infty} a_j T_j(x)$
- Legendre Expansions of Smooth Functions
  - definition of Legendre polynomials via the Gram-Schmidt procedure
  - definition of Legendre polynomials via the recurrence relations
  - convergence results for Legendre expansions
  - orthogonality relation
  - Gauss-Legendre quadrature rules and the computation of coefficients in the expansion $f(x) = \sum_{j=0}^{N} a_j P_j(x)$
  - aliasing and the error incurred when representing $f(x) = \sum_{j=0}^{\infty} a_j P_j(x)$
- Expansions of Piecewise Smooth Functions
  - advantages of piecewise expansions
  - strategies for representing singular functions (e.g., bisection, dense discretization, variable substitutions)
  - adaptive discretization

3. Polynomial Interpolation
   - the Weierstrauss Theorem on the density of polynomials in $C(X)$
   - Lagrange polynomials and the Lagrange interpolation formula

- The unsuitability of equispaced nodes, even in the case of well-behaved functions
- The suitability of Chebyshev and Legendre nodes and the barycentric form of the Lagrange formula
- Minimax polynomials and their relation to Chebyshev expansions

4. Integration and Differentiation

    - Review of the trapezoidal rule, Gauss-Legendre and Gauss-Chebyshev quadrature rules
    - Curtis-Clenshaw quadrature rules
    - Adaptive integration
    - Spectral integration and differentiation
    - Finite differences approximations of derivatives