Math 4553, Homework 2, Due on 2/18/2011

1. Consider the problem

$$\begin{aligned} \min \quad & f = \mathbf{p}^t \mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \geq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

   where

$$A = \begin{bmatrix} 0 & -1 \\ -1 & -1 \\ -1 & 2 \\ 1 & -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -5 \\ -9 \\ 0 \\ -3 \end{bmatrix}, \quad \mathbf{p} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

   (a) (2 points) Solve the problem using the simplex method.

   (b) (3 points) Solve the problem using graphical optimization. In the graph, denote the vertices corresponding to each basic feasible solution in the simplex method, and trace the path of the simplex method.

2. (a) (2 points) Solve the following problem

$$\begin{aligned} \min \quad & f = x_1 - 2x_2 - 4x_3 + 4x_4 \\ \text{subject to} \quad & x_2 - 2x_3 - x_4 \geq -4 \\ & 2x_1 - x_2 - x_3 + 4x_4 \geq -5 \\ & -x_1 + x_2 - 2x_4 \geq -3 \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

   (b) (3 points) Find a feasible point $\mathbf{x}$ such that $f(\mathbf{x}) = -415$.

3. (4 points) Use phase I process to demonstrate that

$$\begin{aligned} \min \quad & f = -3x_1 + x_2 \\ \text{subject to} \quad & -x_1 - x_2 \geq -2 \\ & 2x_1 + 2x_2 \geq 10 \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

   is infeasible.

4. (6 points) Solve the following problem:

$$\begin{aligned} \min \quad & f = \mathbf{p}^t \mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \geq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

   where

$$A = \begin{bmatrix} -1 & -3 & 0 & -1 \\ -2 & -1 & 0 & 0 \\ 0 & -1 & -4 & -1 \\ 1 & 1 & 2 & 0 \\ -1 & 1 & 4 & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -4 \\ -3 \\ -3 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{p} = \begin{bmatrix} -2 \\ -4 \\ -1 \\ -1 \end{bmatrix}$$