

167 Homework 1

Due Wednesday October 11 in class.

It is OK to work in teams on this homework but you must list who you worked with so the TA does not think you are copying somebody else's work. It is OK to use any reference (Google, Wikipedia, Wolfram Alpha, books, research papers, your brother/sister in law etc...) to solve these problems, but you MUST properly cite your sources!

The first five of the following questions are based on topics that might be found on a nasty 22A final exam so you can treat them as review. However, note that we will treat many of these topics in more detail in this class, so don't worry if you find them challenging.

Question 1: Let M be an $m \times n$ matrix, X a column vector of n unknowns and V a given column vector with m entries. How many solutions could the linear system of equations $MX = V$ have? If you need to break your answer into several cases, give an example of each. Also, explain and demonstrate what happens when you try to solve each of these cases using Gaussian elimination.

Question 2: What is the determinant of a 20×20 matrix whose entries (read from left to right) are consecutive integers ranging from 1 to 400?

Question 3: Let (x_1, \dots, x_n) be a real numbers and M be an $n \times n$ matrix whose i th column is given by the entries $x_i, x_i^2, x_i^3, \dots, x_i^n$. Compute the determinant of M .

Question 4: Let M , X and V be as given in Question 1. Moreover, suppose V is not in the span of the columns of M . Does $MX = V$ have any solutions? If not, explain how you might find a vector \bar{X} such that $M\bar{X} - V$ is as small as possible. (*Hint:* You may assume that only the zero vector is in the kernel of M and your answer should first address the question “what does it mean for a vector to be *small*”).

Question 5: Suppose the matrix S is symmetric. Explain how you would compute the matrix exponential $\exp S$. Give a non-trivial example of your method.

Question 6: How many numbers are there of the form $a \times 10^c$ where c is an

integer between -38 and 38 and a is a number between 0 and 10 stated to 20 digit accuracy?

Question 7: Let $\vec{x} \in \mathbb{R}^2$ and suppose $\|\vec{x}\|_2 = 1$. What is the smallest possible value of $\|\vec{x}\|_1$ and which vectors \vec{x} achieve this value? (Recall $\|(x, y)\|_2 = \sqrt{x^2 + y^2}$ and $\|(x, y)\|_1 = |x| + |y|$.)

Question 8: Conoil operates two wells (well A and well B) in southern Grease (a small Mediterranean country). You have been employed to figure out how many barrels of oil they should pump from each well to maximize their profit (all of which goes to shareholders, not operating costs). The quality of oil from well A is better than from well B, so is worth 50% more per barrel. The Greasy government cares about the environment and will not allow Conoil to pump in total more than 6 million barrels per year. Well A costs twice as much as well B to operate. Conoil's yearly operating budget is only sufficient to pump at most 10 million barrels from well B per year. Using both a graphical method and then (as a double check) Dantzig's algorithm, determine how many barrels Conoil should pump from each well to maximize their profits.