MAT 168 **Programing Project 2** **Missael Colin Cuevas 914373182**

Path-Following Interior-Point Algorithm Code

```
iter = 0;
[m,n] = size (A);
d = 0.1;
r = 0.8;
tol = 10^-12;
M = 10^8;
kmax = 1000;
w = ones(m,1);
x = ones(n,1);
y = ones(m,1);
z = ones(n,1);
zeta = b - A*x - w;
sigma = c- A'*y + z;
gamma = (x'*z +y'*w)/(n+m);
normzeta = norm(zeta, inf);
normsigma = norm(sigma, inf);
normgamma = norm(gamma, inf);

for iter=1:kmax
Z = diag(z);
X = diag(x);
Y = diag(y);
W = diag(w);
delta = d*gamma;
Xz = X*z;
Yw = Y*w;

D=[A eye(m); zeros(n,n+m); diag(z) zeros(n,
m);zeros(m,n) Y];
E=[zeros(m,m+n); A', -eye(n); zeros(n,m) X; W,
zeros(m,n)];
F = [D E];

G = [zeta; sigma;delta - Xz; delta - Yw];

dv = linsolve (F,G);

dx = dv(1:n);
dw = dv(n+1:m+n);
dy = dv(n+m+1:2*m+n);
dz = dv(2*m+n+1:2*m+2*n);

xi = max(max(-dx./x));
yi = max(max(-dy./y));
wi = max(max(-dw./w));
zi = max(max(-dz./z));
di = [xi yi wi zi];
di = max(di);
theta = min( 1,r*(1./di));

x = x + theta * dx
w = w + theta * dw;
y = y + theta * dy;
z = z + theta * dz;

zeta = b -A*x - w;
sigma =c- A'*y+z;
gamma = (x'*z +y'*w)./(n+m);

convzeta = norm(zeta, inf);
convsigma = norm(sigma, inf);
convgamma = norm(gamma, inf);

if(convzeta <= tol*normzeta && convsigma <=
tol*normsigma && convgamma <=
tol*normgamma)
break;
end
if(norm(x,inf) > M || norm(y,inf) > M)
   fprintf('unbounded\n')
   break;
end
iter = iter+1
end
```

Problem 1.-

Use your program to solve the following linear programs:

(1) The linear program with data

$$c = \begin{bmatrix} 1 \\ 4 \\ 1 \\ 3 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 2 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 \end{bmatrix}, \quad \text{and} \quad b = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}.$$

δ = 0.1 r = 0.8

```
x =
    9.999999999994104e-01
    3.054529874030829e-13
    4.363066609319624e-14
    2.999999999999368e+00
x =
   1.0000
   0.0000
   0.0000
   3.0000
```

iteration steps= 23

δ = 0.5 r = 0.8

x =
9.999999999987430e-01
6.518373794981269e-13
9.311962564258971e-14
2.999999999998650e+00
x =
1.0000
0.0000
0.0000
3.0000 iteration steps = 42

δ = 0.1 r = 0.99

x =
9.999999999996648e-01
1.738763565536712e-13
2.483947950816092e-14
2.999999999999640e+00
x =
1.0000
0.0000
0.0000
3.0000 iteration steps = 16

δ = 0.5 r = 0.99

x =
9.999999999994104e-01
3.054529874030829e-13
4.363066609319624e-14
2.999999999999368e+00
x =
1.0000
0.0000
0.0000
3.0000 iteration steps = 23

Simplex method Pivot Rule

x =
1.000000000000000e+00
0
3.000000000000000e+00
0

||x – xsimplex||/||x||

fnorm =
1.000000000000598e+00

As can be noticed, once “format long e” is applied, we can see that the interior point method is approximately 14 digits accurate compared to the simplex method result.

Problem 2.-

The linear program

$$\begin{aligned} \max \quad & \sum_{j=1}^{5} 10^{5-j} x_j \\ \text{s.t.} \quad & x_i + 2\sum_{j=1}^{i-1} 10^{i-j} x_j \le 100^{i-1}, \quad i = 1,2,3,4,5 \\ & x_j \ge 0, \quad j = 1,2,3,4,5. \end{aligned}$$

δ = 0.1 r = 0.8

x =
9.588471285334104e-17
5.046776667977240e-17
4.818673844543579e-17
4.801623419855780e-17
9.999999999999979e+07
x =
1.0e+08 *
0.0000
0.0000
0.0000
0.0000
1.0000 iteration steps= 175

δ = 0.5 r = 0.8

x =
8.335766551875397e-17
4.387245553618630e-17
4.188827413002712e-17
4.169968260067733e-17
1.000000000000000e+08
x =
1.0e+08 *
0.0000
0.0000
0.0000
0.0000
1.0000 iteration steps= 180

δ = 0.1 r = 0.99

x =
1.556643549145544e-17
8.192860784976523e-18
7.822329392691152e-18
7.787111301378389e-18
1.000000000000000e+08
x =
1.0e+08 *
0.0000
0.0000
0.0000
0.0000

1.0000 iteration steps= 70

δ = 0.5 r = 0.99

x =
9.588471285334104e-17
5.046776667977240e-17
4.818673844543579e-17
4.801623419855780e-17
9.999999999999979e+07
x =
1.0e+08 *
0.0000
0.0000
0.0000
0.0000
1.0000 iteration steps= 175

Simplex method Pivot Rule

x=
1.000000000000000e+00
1.000000000000000e+02
1.000000000000000e+04
9.999999999999999e+05
1.000000000000000e+08

||x – xsimplex||/||x||
fnorm =
0.010000000000000

Problem 3.-

The linear program with data

$$A = \begin{bmatrix} 3 & -2 \\ -1 & 1 \\ 4 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 9 \\ 3 \\ 4 \end{bmatrix}, \quad \text{and} \quad c = \begin{bmatrix} 5 \\ 6 \end{bmatrix},$$

δ = 0.1 r = 0.8

x =
1.999999999999851e-01
3.199999999999882e+00
x =
0.2000
3.2000 iteration steps= 24

δ = 0.5 r = 0.8

x =
1.999999999999796e-01
3.199999999999839e+00
x =
0.2000
3.2000 iteration steps= 44

δ = 0.1 r = 0.99

x =
1.999999999999842e-01
3.199999999999876e+00
x =
0.2000
3.2000 iteration steps= 17

δ = 0.5 r = 0.99

x =
1.999999999999754e-01
3.199999999999805e+00
x =
0.2000
3.2000 iteration steps= 42

Simplex Method Pivot Rule

x=
3.200000000000000e+00
2.000000000000000e-01

||x – xsimplex||/||x||
fnorm =
9.375000000000726e-01

Problem 4- “problem4.mat”

δ = 0.1 r = 0.8

x =
1.968831044512699e-13
1.836138658522161e-13
3.137824649778598e-13
7.971351976490360e-13
8.058263083420853e-13
1.619205357554723e-13
1.244430424677948e-13
1.444881959891911e-13
2.948210430806652e-13
6.565999970141670e-13
2.560344669122232e-13
1.856879604277949e-13
1.947198139748195e-13
1.409523093008927e-13
1.252462579756622e-13
3.759945268926606e-13
1.510484038798635e-13
2.718337030647114e-13
3.522194095894743e-13
5.153221863569195e-13
1.869940312667618e-13
4.455423161324354e-03
1.684961021700083e-13
1.671991635650453e-13

4.444154827884930e-13
4.806223661718166e-13
2.008284739269251e-13
1.618901463333867e-13
2.086553724273705e-13
2.747525438425845e-13
7.373685105723060e-03
4.568841868141012e-13
1.480698382944925e-13
9.015130844331128e-13
6.010303088860771e-13
5.833630007244892e-13
1.447240835178347e-13
2.538782875160546e-13
1.705934450689440e-13
1.316910594263975e-13
3.432560008846042e-13
4.824575154734862e-13
1.110733706343185e-13
3.054540115092323e-13
1.901393631035512e-13
6.612829479326720e-13
3.124198007519989e-13
3.347442926685832e-13
5.893966475973716e-12
1.944917035384248e-13

x =

0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0045
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0074
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000
0.0000 iteration steps= 27

δ = 0.5 r = 0.8

x =

3.261099557843700e-13
3.040604499710121e-13
5.196487715166729e-13
1.318207064033580e-12
1.334108305039931e-12
2.682042058785496e-13
2.061237408453723e-13
2.393335915009478e-13
4.882211079176377e-13
1.087152890660763e-12
4.240193205116437e-13
3.075381691316387e-13
3.224823983465598e-13
2.334652546458608e-13
2.074639959450183e-13
6.223220888866438e-13
2.502001619231900e-13
4.501115470359477e-13
5.829635826368629e-13
8.527272075720899e-13
3.096711403788336e-13
4.455423135010408e-03
2.790430716467680e-13
2.768916502649167e-13
7.357261485293564e-13
7.954505401938662e-13
3.325257441338265e-13
2.681759165000437e-13
3.455579998463900e-13
4.548232135171056e-13
7.373685095203058e-03
7.561636092615265e-13
2.452812812877210e-13

1.491546863189468e-12
9.944161412159640e-13
9.651540738904397e-13
2.397123832383058e-13
4.204698153683069e-13
2.825453909702460e-13
2.181332094690070e-13
5.682305943464962e-13
7.984320578983799e-13
1.839936029787158e-13
5.056065881047035e-13
3.149422809798311e-13
1.094169367662960e-12
5.175100752898873e-13
5.541512827734887e-13
9.754420470874641e-12
3.221525740999700e-13 Interaction steps: 48

δ = 0.1 r = 0.99

x =

9.661767084448938e-14
9.008499094567709e-14
1.539580530618070e-13
3.905495506804198e-13
3.952606646147636e-13
7.946174349617353e-14
6.106895958692542e-14
7.090815143364657e-14
1.446468757945141e-13
3.220943693444959e-13
1.256256007685841e-13
9.111534638160692e-14
9.554292227190602e-14
6.916951994596203e-14
6.146604134055372e-14
1.843774148691260e-13
7.412762603094114e-14
1.333560306288585e-13
1.727165407102994e-13
2.526402985417389e-13
9.174728862347761e-14
4.455423181748178e-03
8.267300989061353e-14
8.203560118509446e-14
2.179760097262017e-13
2.356707519356826e-13
9.851849733641412e-14
7.945336210654269e-14
1.023796067484555e-13
1.347519713076180e-13
7.373685113886244e-03
2.240310831000329e-13
7.267029308292629e-14
4.419055035946066e-13
2.946189460715783e-13
2.859493770119263e-13
7.102037732230023e-14
1.245739774083968e-13
8.371065361012247e-14
6.462704438603465e-14
1.683515496419476e-13
2.365540963657450e-13
5.451239072616312e-14
1.497977293767563e-13
9.330898696861436e-14
3.241731623735327e-13
1.533244151432358e-13
1.641802257294480e-13
2.889974490268298e-12
9.544520428165206e-14 Interaction steps: 19

δ = 0.5 r = 0.99

x =

3.979976508655408e-13
3.710875508799240e-13
6.342001728875543e-13
1.608792695634967e-12
1.628199207009807e-12
3.273271545334306e-13
2.515616686609836e-13
2.920923004680483e-13
5.958445935415534e-13
1.326804928644284e-12
5.174901608855628e-13
3.753318986275121e-13
3.935704344833987e-13
2.849303471410035e-13
2.531973696623122e-13
7.595068015108162e-13
3.053542982215826e-13
5.493341591393012e-13
7.114721041606166e-13
1.040702436166085e-12
3.779350621638889e-13
4.455423120369955e-03
3.405553404176118e-13
3.379296595979659e-13
8.979096577739054e-13
9.707996973456744e-13
4.058277359219525e-13
3.272926290429017e-13
4.217328227403043e-13
5.550844656169204e-13
7.373685089350614e-03
9.228523533794252e-13
2.993510992910379e-13
1.820343528848758e-12
1.213625285480238e-12
1.177912686559230e-12
2.925545930750716e-13
5.131582026496804e-13

3.448297111891896e-13
2.662185051529995e-13
6.934913751893634e-13
9.744384609632244e-13
2.245531620881769e-13
6.170625298568634e-13
3.843681732645085e-13
1.335368118190341e-12
6.315900223567961e-13
6.763084194638898e-13
1.190468541086130e-11
3.931679037632759e-13 Interaction steps: 43

Simplex method did not find a solution!

Problem 5.-

(5) The linear program with data

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \text{and} \quad c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

δ = 0.1 and r = 0.8

x =
5.175111808529496e+07
1.035022357106058e+08 Iteration steps = 32

!!UNBOUNDED!!

δ = 0.5 and r = 0.8

x =
7.077073265247031e+07
1.415414639850699e+08 Iteration steps = 32

!!UNBOUNDED!!

δ = 0.1 and r = 0.99

x =
6.783452906676289e+07
1.356690592693092e+08 Iteration steps = 28

!!UNBOUNDED!!

δ = 0.5 and r = 0.99

x =
6.519929067960122e+07
1.303985808329512e+08 Iteration steps = 27

!!UNBOUNDED!!

Simplex Method Pivot Rule

ans =
'unbounded'

Clearly, both methods show that problem 5 is unbounded.