167 Homework 2

Due Wednesday October 25 in class.

It is OK to work in teams on this homework but you must list who you worked with so the TA does not think you are copying somebody else's work. It is OK to use any reference (Google, Wikipedia, Wolfram Alpha, books, research papers, your uncle's niece etc...) to solve these problems, but you MUST properly cite your sources!

Question 1: Maximize $f(x, y) = 2x + 3y$ subject to the constraints

$$x \geq 0\,, \quad y \geq 0\,, \quad x + 2y \leq 2\,, \quad 2x + y \leq 2\,,$$

by:

(i) Sketching the region in the xy-plane defined by the constraints, and then checking the values of f at its corners.

(ii) Using the simplex algorithm (hint: introduce slack variables).

Question 2: Conoil operates two wells (well A and well B) in southern Grease (a small Mediterranean country). You have been employed to figure out how many barrels of oil they should pump from each well to maximize their profit (all of which goes to shareholders, not operating costs). The quality of oil from well A is better than from well B, so is worth 50% more per barrel. The Greasy government cares about the environment and will not allow Conoil to pump in total more than 6 million barrels per year. Well A costs twice as much as well B to operate. Conoil's yearly operating budget is only sufficient to pump at most 10 million barrels from well B per year. Using both a graphical method and then (as a double check) Dantzig's algorithm, determine how many barrels Conoil should pump from each well to maximize their profits.

Question 3: Read and write a short (< 1 page) summary of chapter 2 of "Matrix Methods in Data Mining and Pattern Recognition" by Lars Elden.

Question 4: Computers store numbers in a floating point notation. Each addition or multiplication of a pair of floating point numbers is called a floating point

operation (FLOP). Counting FLOPs gives a measure of how computationally expensive a calculation is. Show that approximately $2n^3/3$ FLOPs are required to perform an LU decomposition of an $n \times n$ (non-singular) matrix.

Question 5: Let $S = S^T$ be a symmetric $n \times n$ matrix with Cholesky decomposition $S = LDL^T$. Prove or disprove the following statements:

(i) The product of the diagonal elements of D equals the product of the eigenvalues of S.

(ii) The sum of the diagonal elements of D equals the sum of the eigenvalues of S.

(iii) The diagonal elements of D are the eigenvalues of S.

Question 6: Captain Conundrum falls off the leaning tower of Pisa and makes three (rather shaky) measurements of his velocity at three different times:

t (s)	v (m/s)
1	11
2	19
3	31

He posits two possible theoretical models to explain his data

$$v = at + b \text{ and } v = a(t - 10/3) + b\,,$$

where a and b are coefficients he plans to find by fitting to data. Use the least squares method to find (a, b) for both models. In each case you will try to approximate a linear system $MX = V$ by $M^T MX = M^T V$ and then invert $M^T M$. Calculate the determinant of $M^T M$ for both cases. Finally compare and plot the results you find for these least square computations.

Question 7: Suppose that the only solution to $MX = 0$ is the zero vector. *Explain* why the matrix $M^T M$ is invertible.

Now let $\varepsilon > 0$. *Compute* the kernel of the matrix

$$M = \begin{pmatrix} 1 & 1 \\ \varepsilon & 0 \\ 0 & \varepsilon \end{pmatrix}\,.$$

Check that the matrix $M^T M$ is invertible (*hint*: compute its determinant). Suppose that $\varepsilon = 10^{-20}$. What will a computer using floating point arithmetic find for the determinant of $M^T M$?