Homework 6: Math 22B - Tavernetti, Fall 2016
Due Monday, Nov 7 in class
(20 points)

Instructions : Solve all problems. Print out your solutions when computer results are asked for, work neatly, label your plots, show your work. Staple your homework together with your name on it. A random subset of the problems will be graded. You are encouraged to work in groups, but everyone must do their own write up.

Warning : Unstapled homework with multiple pages is minimum -5 out of 20 points and if a page is lost from an unstapled homework the default assumption will be it was not turned in.

Reading Assignment : Read Boyce and Diprima Chapter 6.1 - 6.2

Problem 1 : Find the general solution using the method of undetermined coefficients (3.5.5)

$$y'' + 9y = t^2 e^{3t} + 6$$

Problem 2 : Find the general solution using the method of undetermined coefficients (3.5.8)

$$y'' + 2y' + y = 2e^{-t}$$

Problem 3 : Find the general solution using the method of undetermined coefficients (3.5.18)

$$y'' + 2y' + 5y = 4e^{-t}\cos(2t), \quad y(0 = 1, \quad y'(0) = 0$$

Problem 4 : (3.6.1) Use the variation of parameters method from section 3.6 to solve for the general solution

$$y'' - 5y' + 6y = 2e^t$$

Check your answer by the method of undetermined coefficients.

Problem 5 : (3.6.15) Verify that $y_1 = 1 + t$ and $y_2 = e^t$ satisfy the differential equation

$$ty'' - (1 + t)y' + y = t^2 e^{2t}, \quad t > 0$$

then find the general solution using the variation of parameters method to find the particular solution.

Problem 6 : (3.7.1) Find R, ω_0, δ to write $u = 3\cos(2t) + 4\sin(2t) = R\cos(\omega_0 t - \delta)$.

Problem 7 : (3.7.2) Find R, ω_0, δ to write $u = -2\cos(\pi t) - 3\sin(\pi t) = R\cos(\omega_0 t - \delta)$.

Problem 8 : (3.7.26) The position of a certain spring satisfies

$$mu'' + \gamma u' + ku = 0, \quad u(0) = u_0, \quad u'(0) = v_0$$

Solve the initial value problem, assuming $\gamma^2 < 4km$. Then determine R in terms of m, γ, k, u_0, v_0 and write the solution in the form

$$u(t) = Re^{-\gamma t/2m}\cos(\mu t - \delta)$$

Problem 9 : (3.7.29) The position of a certain spring satisfies

$$u'' + \frac{1}{4}u' + 2u = 0, \quad u(0) = 0, \quad u'(0) = 2$$

(a) Find the solution and (b) using MATLAB plot u *vs.* t and u' *vs.* t on the same set of axis.

Problem 10 : (3.8.19) Consider the vibrating system

$$u'' + u = 3\cos(\omega t), \quad u(0) = 1, \quad u'(0) = 1$$

(a) Find the solution for $\omega \neq 1$, (b) using MATLAB plot u *vs.* t for $\omega = 0.7$.