Homework 1: Math 22B - Tavernetti, Fall 2016
Due Wednesday, Sept 28 in class
(20 points)

Instructions : Solve all problems. Print out your solutions when computer results are asked for (do not include the dfield8.m code), work neatly, label your plots, show your work. Staple your homework together with your name on it and be sure to write your name. A random subset of the problems will be graded. You are encouraged to work in groups, but everyone must do their own write up.

Warning : Unstapled homework with multiple pages is minimum -5 out of 20 points and if a page is lost from an unstapled homework the default assumption will be it was not turned in.

Reading Assignment : Read Boyce and DiPrima chapters 1.1-1.3, 2.1-2.2

Problem 1 : Solve the initial value problem and plot the solution by hand

$$y' = 2 - y \text{ subject to } y(0) = 1$$

Problem 2 : Use dfield8.m to draw the direction field for the differential equation

$$y' = 2 - y$$

Plot the particular integral curve satisfying the condition $y(0) = 1$.

Problem 3 : Write down a differential equation of the form $dy/dt = ay + b$ such that all solutions approach $y = -3$ as $t \to \infty$. Verify your solution by using dfield8.m to draw the direction field together with several integral curves.

Problem 4 : A spherical raindrop evaporates at a rate proportional to its surface area. Write a differential equation for the volume of the raindrop as a function of time. Classify your differential equation (linear / nonlinear, order, system/scalar, etc..).

Problem 5 : In class we studied a model for an object falling under gravity with a drag force proportional to velocity. In the case the falling mass has greater surface area (like a parachutist), it is more accurate to assume the drag force is proportional to the square of the velocity.

(a) Write a differential equation for the velocity of the mass m if the magnitude of the drag force is proportional to the square of the velocity (and its direction is of course opposite to gravity).

(b) Determine the terminal velocity.

(c) Does the initial velocity affect the terminal velocity? Why or why not?

(d) Suppose a soldier were to jump out of an airplane (so $v(0) = 0$ m/s initially) and deploy a parachute immediately via static line at an altitude of 1000 meters. If we wanted to use this model to determine the velocity of the soldier as they descend toward the ground (1) what difficulties would we encounter with using this model to make an accurate prediction (2) what possible affects might typically be encountered in practice, that the model fails to account for, and (3) the model only applies in a single vertical direction and the jumper is falling in 3-dimensional space, will this impact the answer? A single consideration for each item (1),(2),(3) is sufficient.

Problem 6 : A radioactive isotope Th-234 decays at a rate proportional to the amount currently present. Let $Q(t)$ be the amount at time t. Then $dQ/dt = -rQ$ where $r > 0$ is the decay rate.

(a) If 100 mg of Th-234 is present initially and the isotope decays a rate of 82 mg per week, determine the decay rate r.

(b) Find an expression for $Q(t)$

(c) Find the half-life of Th-234 (that is, the amount of time it takes for the isotope to decay to one-half the amount present from any given time).

Problem 7 : According to Newton's law of cooling, for a body immersed in a surrounding environment with ambient temperature u_0, the rate of change of temperature of the body in time $u(t)$ obeys the differential equation $(k > 0)$

$$\frac{du}{dt} = -k(u - u_0)$$

(a) Suppose that $u(0) = 10$. Solve for u(t).

(b) For $k > 0$ explain what will happen to $u(t)$ if $u(0) < u_0$ initially. Does this make sense thermodynamically? Explain.

(c) Use dfield8.m to plot the solution and verify your answer to parts (a), (b) using $u(0) = 10$ and $u_0 = 15$ initially.

Problem 8 : Give a single differential equation example for each of the following

(a) A first order nonlinear equation

(b) A second order linear equation

(c) A third order linear equation

(d) A system of coupled linear ordinary differential equations

(e) A partial differential equation

Problem 9 : Find two different solutions to $y'' - y = 0$. Show the work to verify each of your solution candidates solves the differential equation.

Problem 10 : Verify that $y = \cos(t)\ln(\cos(t)) + t\sin(t)$ solves $y'' + y = \sec(t)$ for $0 < t < \pi/2$.