Your Name: _____ **First letter, Last Name**_____
Student ID: _____ **TOTAL:** 100 Points

Practice Midterm 1: Math 22B

Tavernetti, Fall 2016

1. There are N problems. Please make sure you have all N problems BEFORE starting the exam.

2. It is a violation of the university honor code to assist another person to, in any way, complete this exam, copy answers from another student's exam or have another student take the exam, or help take this exam, for you.

3. No notes, books, cel phones, smartphones, electronic devices, or other materials may be used unless specifically allowed.

4. No calculators of any kind may be used on this exam.

5. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will receive little or no credit.

6. Neatness as well as proper use of mathematical notation, such as dx, du, = signs, etc, will be factored into the grading.

``In the sciences, the authority of thousands of opinions is not worth as much as one tiny spark of reason in an individual man."
-Galileo Galilei

``If I were again beginning my studies, I would follow the advice of Plato and start with mathematics."
-Galileo Galilei

``We cannot teach people anything; we can only help them discover it within themselves."
-Galileo Galilei

``Philosophy is written in this grand book — I mean the universe — which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering about in a dark labyrinth."

GOOD LUCK!!

Problem 1 (20 points) : Solve the following Initial Value Problem

(a) $t^3 \dfrac{dy}{dt} + 4t^2 y = \exp(-t), \quad y(-1) = 0, \quad t < 0$

(b) What happens to y in the limit as t approaches $-\infty$?

Problem 2 (20 points) : Solve the following Initial Value Problem

(a) $\dfrac{dy}{dx} = 2y^2 + xy^2, \qquad y(0) = 1$

(b) Determine where the solution attains its minimum value.

Problem 3 (20 points) : Logistic growth with a threshold can be modeled according to the equation

$$\frac{dy}{dt} = -r\left(1 - \frac{y}{T}\right)\left(1 - \frac{y}{K}\right)y$$

for $r > 0$ and $0 < T < K$.

(a) Find all equilibrium solutions and classify their stability.

(b) Sketch the phase plane

(c) Sketch examples of each of the qualitatively different solution curves for several initial values of $y \geq 0$.

Problem 4 (20 points) : A tank with a maximum capacity of 300 gallons is being filled with a solution that contains 1 lbs of salt per gallon, at a rate of 2 gallons per minute. The solution is well mixed in the tank, and the mixture is allowed to leave at a rate of 1 gallon per minute. If the tank initial contains 100 gallons of fresh water, what will the amount of salt be in the tank at the time it starts to overflow?

Problem 5 (20 points) : Consider the differential equation for $a \in \mathbb{R}$

$$\frac{dy}{dt} = ay - y^3$$

(a) Sketch all the qualitatively different phase portraits that occur as a is varied.

(b) Sketch examples of each of the qualitatively different solution curve families depending on a.

(c) Sketch the bifurcation diagram of fixed points y^* vs. a. Be sure to clearly indicate the asymptotic stability of each branch on your plot.

(d) Fully classify the type of bifurcation.