Homework 5: Math 22B - Tavernetti, Fall 2016
Due Monday, Oct 31 in class
(20 points)

Instructions : Solve all problems. Print out your solutions when computer results are asked for, work neatly, label your plots, show your work. Staple your homework together with your name on it. A random subset of the problems will be graded. You are encouraged to work in groups, but everyone must do their own write up.

Warning : Unstapled homework with multiple pages is minimum -5 out of 20 points and if a page is lost from an unstapled homework the default assumption will be it was not turned in.

Reading Assignment : Read Boyce and Diprima Chapter 3.5-3.9

Problem 1 : [Programming in Matlab - section 2.7] Write a simple program in Matlab to solve the initial value problem for $t \in [0, 5]$.

$$\frac{dy}{dt} = -ty + 0.5y^3, \quad y(0) = 1$$

using the Euler Method and an appropriately chosen time step. Make sure to include the code with your homework.

Problem 2 : Solve the given IVP and use Matlab to plot your solution (just plotting, you don't have to solve it numerically).

$$y'' + 3y = 0, \quad y(0) = -2, \quad y'(0) = 3$$

Problem 3 : Solve the IVP

$$2y'' + 3y' - 2y = 0, \quad y(0) = 1, \quad y'(0) = -\beta$$

and (b) plot the solution when $\beta = 1$ and find the coordinates (t_0, y_0) of the minimum point of the solution in this case.

Problem 4 : Verify that $y_1 = e^t$ and $y_2 = te^t$ are solutions of $y'' - 2y' + y = 0$. Do they form a fundamental set of solutions?

Problem 5 :

(a) Consider the equation $y'' + 2ay' + a^2y = 0$. Show that the roots of the characteristic equation are $r_1 = r_2 = -a$, so that one solution of the equation is e^{-at}.
(b) Use Abel's formula to show that the Wronskian of any two solutions of the given equation is

$$W(t) = y_1(t)y_2'(t) - y_1'(t)y_2(t) = c_1e^{-2at},$$

where c_1 is a constant.
(c) Let $y_1(t) = e^{-at}$ and use the result of part (b) to show that a second solution is $y_2(t) = te^{-at}$.

Problem 6 : Use the method of order reduction to find a second solution given $y_1 = t^2$, $t > 0$ and

$$t^2y'' - 4ty' + 6y = 0$$

Problem 7 : Use the method of order reduction to find a second solution given $y_1 = e^x$, $x > 1$ and

$$(x-1)y'' - xy' + y = 0$$

Problem 8 :Find the general solution to $y'' - 2y' - 10y = 0$

Problem 9 : Find the particular solution

$$y'' + 2y' + 2y = 0, \quad y(\pi/4) = 0, \;\; y'(\pi/4) = -2$$

Problem 10 : Find the particular solution

$$y'' - 6y' + 9y = 0, \quad y(0) = 0, \;\; y'(0) = 2$$