Policy Reforms and Self-Employment in Developing Countries: A Multi-Good Approach

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Abstract

Developing countries have more self-employment and less wage employment than developed countries. Along these lines, this paper documents two interesting facts. First, self-employed and wage-employed have distinguishable occupational distributions in low-income countries, with the self-employed concentrating on home-production-related occupations. Second, the decrease in home-production-related self-employment is the primary driver of the decline in the self-employment rate along the development path. Given the enormous amount of the self-employed in developing countries, it is essential to understand how policies affect the size of the wage and the self-employment sectors. This paper builds a simple heterogeneous agent model with occupational choice. My innovation is to assume that the self-employed and the wage-employed produce different goods, in line with the empirics. The model calibrated to Tanzania shows that with a realistic elasticity of substitution between goods produced by two sectors, occupational choice in response to corporate income tax cuts is only 1/3 as elastic as in a case with very high substitutability. When the wage and self-employment sectors provide goods that are harder to substitute, a reduction in the supply of home production substitutes increases its price, making self-employment more attractive, thus weakening the effectiveness of those policies.

1 Introduction

This scene might look familiar to anyone who has ever been to a developing country: bumpy yet busy streets are constantly flooded with street vendors shouting out to sell, contrasting with the silent boulevards in the developed world. In low-income countries, disproportionally, more people set up small informal businesses and become self-employed. In high-income countries, becoming an employee for a formal sector firm is the norm. International Labor Organization (ILO) statistics show that, on average, self-employment takes up 61% of the non-agricultural labor force in low-income countries, while barely 12% in high-income countries. Despite numerous attempts to crack down on informal self-employment and promote employment in more efficient firms, high self-employment seems like a deep-rooted phenomenon in many low-income countries. Why is it so hard to encourage wage employment in developing countries? This paper aims to answer this question by evaluating policy experiments in a quantitative model featuring a realistic characterization of self-employment and wage employment.

Admittedly, the ubiquity of the self-employed might provide some convenience to daily life, but it also poses challenges to developing economies. First, most self-employed work in the informal sector and don't contribute to tax revenue, and their economic activities are notoriously hard to monitor. The substantial presence of tax-avoiding self-employment dramatically reduces the fiscal capacity of a nation, causing severe budgetary strain for governments in developing countries. Second, the literature has shown that self-employment activities are less productive than the corporate sector. Having so many labor forces centering on the low-productivity sector and with scanty earnings to spare may be a source of misallocation, thus worrying the policymakers.

Encouraging wage employment is generally a goal for policymaking in low-income countries. According to the United Nations' Sustainable Development Goals, promoting decent work, fostering industrialization, and reducing inequality is paramount. Hence, it is essential to have a framework to understand the high rate of self-employment in developing countries and how it reacts to policies aimed at reducing it.

A strand of literature studies whether policy reforms are effective at promoting wage employment. Quantitative analysis (Ordonez, 2014; Gollin, 2006; Ihrig and Moe, 2004) shows that tax reforms have a large effect on reducing the informal self-employment rate and increasing the share of employment in formal firms. However, natural experiments in Vietnam (Pham, 2020), China (Li, Liu, and Sun, 2021), India (Hasan, Jiang, and Rafols, 2021), and Brazil (Rocha, Ulyssea, and Rachter, 2018) suggest the opposite. This paper attempts to reconcile the discrepancy between the quantitative and the empirical literature's findings. My main insight is that the quantitative models miss a crucial feature of self-employment: it supplies goods and services very different from the wage sector.

My paper makes several main contributions. Empirically, I establish two interesting stylized facts. First, a remarkable difference exists between the self-employed and the wage-employed occupations in low-income countries. The self-employed tend to concentrate on jobs that provide marketable home production goods or services. Therefore, it is unsurprising that you see many street food vendors, hairdressers, cleaners, and helpers in developing countries. Since the self-employed and the wage-employed take on different professions, it implies the goods or services they bring to the economy will be different. Therefore, it contradicts the assumption used in the quantitative literature that both sectors produce homogeneous goods. Second, the reduction in home-production-related self-employment accounts for around 70% of the decline in self-employment rate with economic development. High-income countries have a much lower self-employment rate than low-income countries, primarily because of a sharp decrease in home-production-related self-employment with income level.

Theoretically, my innovation is to incorporate the empirical evidence in an occupational choice model by assuming the wage-employed and the self-employed produce different goods. In a simple setting, heterogeneous agents choose occupations based on their idiosyncratic earning ability and taste for self-employment. The wage-employed work in a representative firm, which produces manufacturing goods and is subject to corporate income tax. At the same time, the self-employed provide home production substitute goods and escape the tax burden. Modeling all wage-employed produce manufacturing goods and all self-employed make home production substitute goods is a simplification to show two employment sectors bring distinguishable goods and services to the market. Since there are two goods in the economy, the relative price of home production substitute goods directly affects the self-employed income. Thus, when making an occupational choice, not only does the wage rate matter but so does the relative price. Moreover, as there are home-production substitute goods in the economy, I also model households' home production time so that they can make home-production goods themselves, like cooking at home instead of buying lunch from food trucks. It's a general equilibrium model where agents solve the occupational choice problem by choosing the optimal occupation, time use, and consumption bundle.

I calibrate the model to fit Tanzania's economy. Tanzania is a sub-Saharan country that works well in my model setting. Around 60% of the self-employed are doing home-production-related professions for a living. The high informality rate is a pressing issue as the former agricultural workers move to urban areas at the initial stage of structural transformation. Despite a humble start, the nation aspires to leverage industrialization to achieve its development goals and promote wage employment. The country also has a good data source that facilitates calibration. The Tanzania Integrated Labour Force Survey (ILFS) covers a wide range of topics that provide a comprehensive view of Tanzania's economy. More importantly, ILFS contains the Time Use Survey (TUS) data, which is uncommon for low-income countries. TUS data allows me to target moments in the household's time use. ILFS thus provides a useful set of calibration targets.

Using the calibrated model, I study the effect of the corporate tax cuts on promoting wage employment. In a setting where the self-employed and the wage-employed produce different goods with a reasonable elasticity of substitution, wage employment is essentially inelastic to the tax reform, consistent with the natural experiment evidence. According to the model, with lower corporate taxes, the corporate sector demands more labor, and the equilibrium wage rate increases, making wage employment more appealing. The existing literature also captures this effect. However, while the wage incentive moves the self-employed to work in the formal sector firm, my model predicts that fewer people provide home-production-substitute goods; thus, the relative price of these goods will increase, making self-employment still a relatively attractive choice. The policy experiment implies that the elasticity of substitution between the goods produced by two employment sectors is a crucial parameter of the effectiveness of tax reforms. When the two goods are good substitutes, corporate tax cuts are three times as powerful in promoting wage employment as the benchmark scenario because the demand for home-production-substitute goods is subdued. Thus, the relative price effect that makes self-employment equally favorable becomes less influential.

Related Literature. Besides contributing to the literature that theoretically and empirically studies how tax policies affect wage employment, as mentioned above, this paper also contributes to the following strands of literature.

First, the literature on self-employment and development (Poschke, 2023; Gindling and Newhouse, 2014; Gollin, 2008) shows that the self-employment rate negatively correlates with the national income level. I contribute by investigating the occupations of the self-employed in low-income countries and finding out that the self-employed and wage-employed take different professions. Moreover, I identify that the self-employed that provide home-production-substitute goods account for around 70% of the decline in the self-employment rate with economic development.

Second, the model is linked to the literature on occupational choice and entrepreneurship (Bento, Shao, and Sohail, 2023; Feng and Ren, 2023; Gu, 2021; Buera, 2009) and more closely to "necessity entrepreneur" (Herreno and Ocampo, 2023; Fairlie and Fossen, 2018; Poschke, 2013b), who are more likely to have low-skills, be own-account workers, and take jobs mainly for subsistence needs. I contribute by looking deeper into the nature of these necessity entrepreneurs and explicitly modeling them as providing home production substitute goods, an imperfect substitute of goods made by the wage-employed.

Third, this paper is closely related to the literature studying the informal sector and its regulation (Abras et al., 2018; Ulyssea, 2018; Ordonez, 2014; Almeida and Carneiro, 2012). Most self-employed work in the informal sector in developing countries, so the quantitative exercise provides a rigorous foundation to explain why formalization efforts were unsuccessful in some experiments (De Mel, McKenzie, and Woodruff, 2013; Kaplan, Piedra, and Seira, 2011).

Fourth, this paper broadly falls in the literature on structural transformation and home production (Gottlieb et al., 2023; Ngai, Olivetti, and Petrongolo, 2022; Dinkelman and Ngai, 2021; Ngai and Petrongolo, 2017; Ngai and Pissarides, 2007). I contribute by studying a setting that fits into the initial stage of structural change, where labor moves out of the agricultural sector. More uniquely, agents must decide between being wage-employed in the manufacturing sector or self-employed in the service sector. This paper provides some insights on occupational choice along the structural transformation process.

The remainder of the paper follows. Section 2 presents two stylized facts regarding self-employment and wage employment across countries. Section 3 introduces a simple occupational choice model. Section 4 details the calibration where Tanzania is a case study. Section 5 shows the policy experiments. Finally, Section 6 concludes.

2 Stylized Facts

This section presents two stylized facts on wage employment and self-employment. The first fact regards whether wage-employed and self-employed are engaged in different professions in developing contexts. The second fact probes into what accounts for the major decline in the self-employment rate with economic development.

2.1 Occupational heterogeneity

First, I empirically test if a difference exists between the jobs taken by wage-employed and self-employed. The quantitative literature, which assumes two employment sectors produce the same goods, implies that wage-employed and self-employed have similar jobs. Hence, I examine the empirical occupational distribution to check if this assumption holds.

The International Labour Organization (ILO)'s dataset on employment by status in employment and occupation sheds light on this point. The dataset compiles labor force survey data from 136 countries from 2000 to 2022. The country coverage ranges from low-income countries whose GDP per capita (in 2023 US dollar) is lower than $500, such as Burundi, Cambodia, Congo, Ethiopia, and Somalia, to high-income countries whose GDP per capita is higher than $100,000, such as Luxembourg and Norway. Therefore, the dataset provides a comprehensive view of how occupational choices differ by status in employment for countries with different income levels.

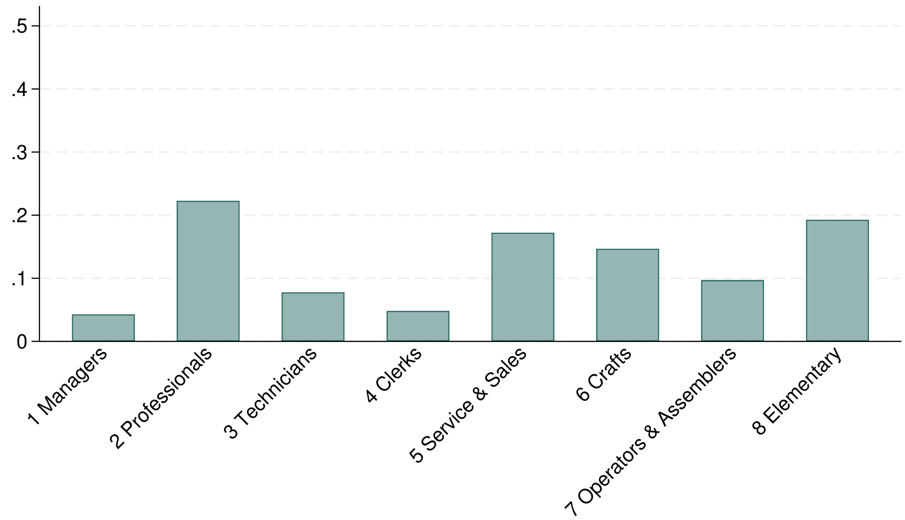
Classifications of employment status and occupations used in the ILO dataset follow international standards, which helps to harmonize labor force surveys in different countries. By status in employment, an employed person could be wage-employed or self-employed, where the self-employed category includes employers, own-account workers, members of producers' cooperatives, and contributing family members. The ILO follows the International Standard Classification of Occupation, 2008 (ISCO-08) and divides all occupations into ten major groups. I exclude agricultural and armed forces occupations because I focus on occupational choice in an urban setting, thus leaving me with eight major occupation groups.

Occupational distribution by employment status is quite diverse for countries at different stages of economic development. Following the standard of the World Bank, I classify countries into four groups based on GDP per capita in 2019: low income, lower-middle income, higher-middle income, and high income. This section focuses on presenting results for low-income countries because the occupational distribution of wage-employed and self-employed are more unlike. Moreover, since self-employment is more prevalent in low-income countries, it is essential to probe into what kind of occupation these self-employed do.

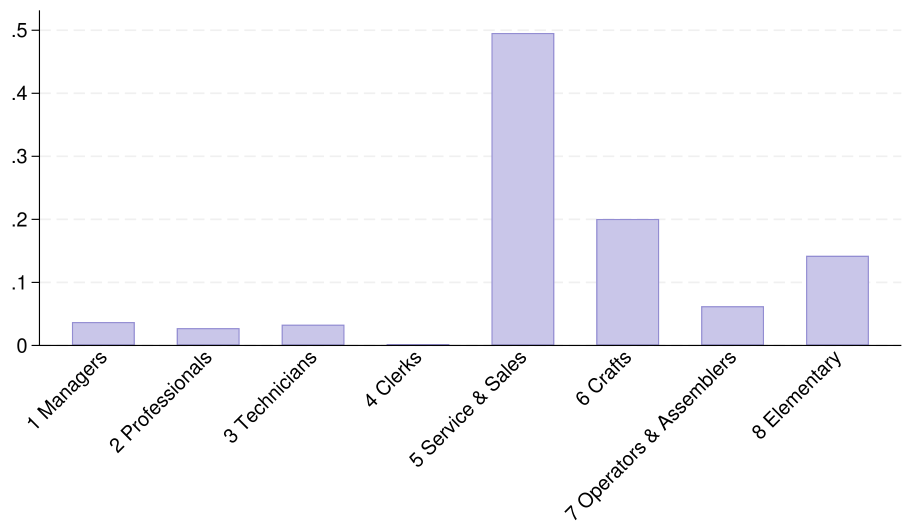
I use each country's most recent survey data to calculate the occupational distribution in eight major groups for wage-employed and self-employed separately. Within each employment status, the distribution among eight occupation groups sum up to 1. Then, I take the average for each occupation group across all countries in the same income-level category.

Figure 1 presents the occupational distribution by employment status in low-income countries. As you can see, the wage-employed and the self-employed take on quite distinguishable professions. Professional occupations, listed in categories 1 to 4, take up 39.14% of the labor force in the wage-employed, compared with barely 9.95% of the self-employed. Meanwhile, 24.36% of the wage-employed are doing jobs more likely to appear in manufacturing industries, such as crafts, operators, and assemblers (categories 6 and 7), while 26.28% of the self-employed are doing the same. The bulk (49.56%) of the self-employed lie in service-and-sales-related occupations; in the meantime, only 17.22% of the wage-employed are doing these occupations. Table 6 in the Appendix contains detailed distribution in each occupation group.

Since the service-and-sales-related occupations dominate the self-employed, I look into



(a) Wage-employed



(b) Self-employed

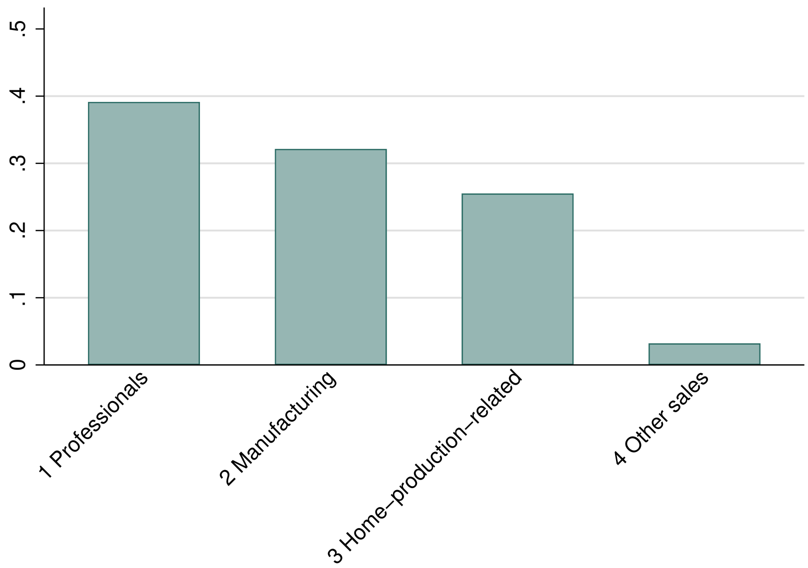
Figure 1: Occupational distribution by employment status in low-income countries

what kind of jobs this occupation group contains. It includes occupations such as personal service workers, personal care workers, protective service workers, street and related sales and service workers, etc. The goods and services these occupations provide, such as street foods, childcare, haircuts, and massages, are closely related to home-production substitutes. Therefore, I want to consolidate the occupation groups that provide insight into how two employment sectors distribute differently in home-production-related occupations.

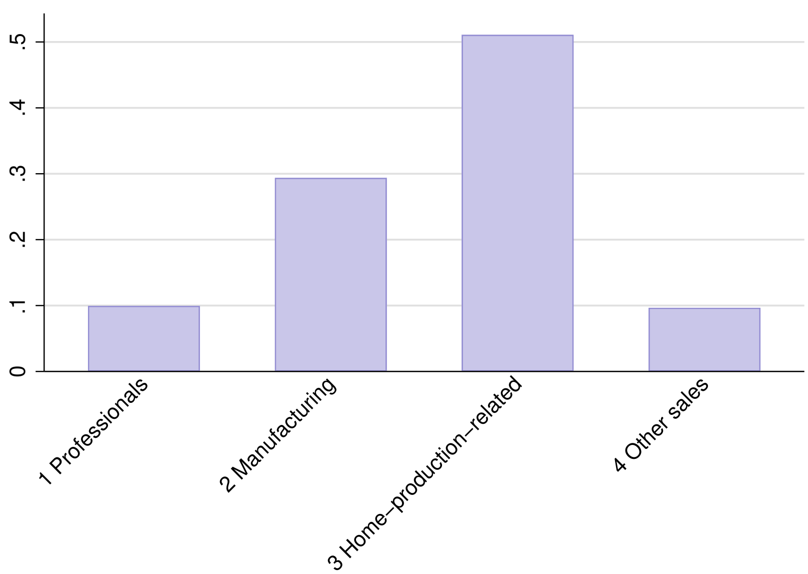
Home-production-related occupations include personal service workers, personal care workers, protective service workers, cleaners and helpers, food preparation assistants, street and related sales and service workers, refuse workers, and other elementary occupations. To single out home-production-related occupations, a challenge is that, according to ILO classification, some of the street food salespersons are in sales occupations, and it is not possible to further distinguish between street food sales and shop salespersons, which do not fall into home-production-related occupations. To solve this problem, I rely on Tanzania's data to determine what percentage of salespersons sell street foods. Tanzania Integrated Labour Force Survey 2020/2021 shows that, among the self-employed salespersons, 77.80% are stall and market sales, and 22.20% are shop salespersons and demonstrators, while among the wage-employed salespersons, the division is 34.35% and 65.55%, respectively. Therefore, for the sales occupations in ILO, I assign the corresponding proportion to stall and market salespersons and other sales based on the ratios I found in the Tanzania data.

A consolidation of occupational groups further illustrates that home-production-related occupations are the dominant self-employed form. The consolidated groups include (1) managers, professionals, technicians, associate professionals, and clerical support workers; (2) manufacturing occupations; (3) home-production-related occupations; (4) other sales. As Figure 2 shows, around half of the self-employed are doing home-production-related occupations in low-income countries. In the meantime, most of the wage-employed concentrate on professional and manufacturing occupations.

The first stylized fact is that self-employed and wage-employed have different occupation distributions in low-income countries. The self-employed concentrate on occupations in services and sales, which are closely related to providing home production substitute goods for a living. Most of the wage-employed work in professions that require higher levels of skill, such as managers, professionals, and technicians. Therefore, empirical evidence indicates that the self-employed and wage-employed are not producing the same goods.



(a) Wage-employed



(b) Self-employed

Figure 2: Consolidated occupational distribution by employment status in LICs

2.2 The decline of the self-employment with development

Drawing on the previous section, an interesting question is how the decline in self-employment with economic development differs by home-production-related self-employment and non-home-production-related self-employment.

ILO's data on employment by status in employment and occupation answers this question. I focus on non-agricultural occupations and divide them into home-production-related or non-home-production-related occupations. Then, I calculate the proportion of the self-employed in these two occupation groups out of all employed for each country using its latest survey data. A country's income level is measured by the log GDP per capita in 2019 (in 2023 US dollars).

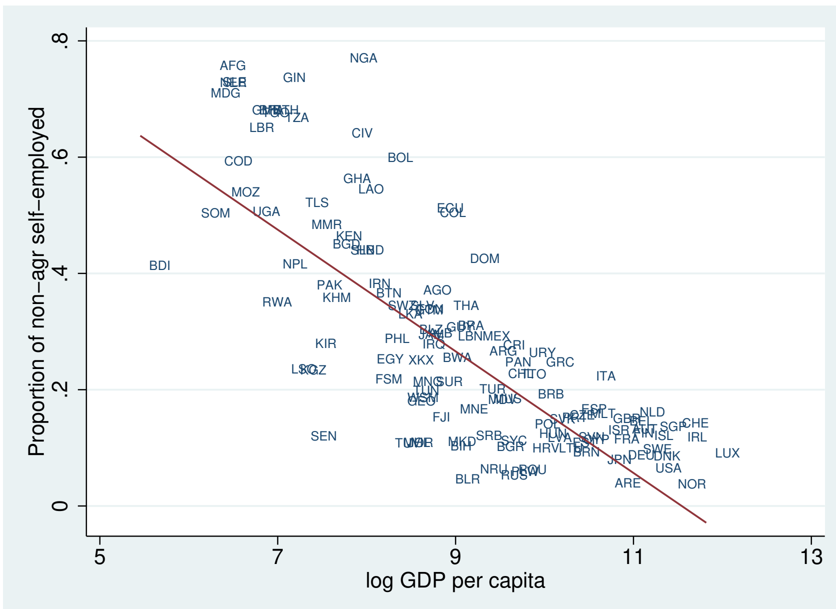
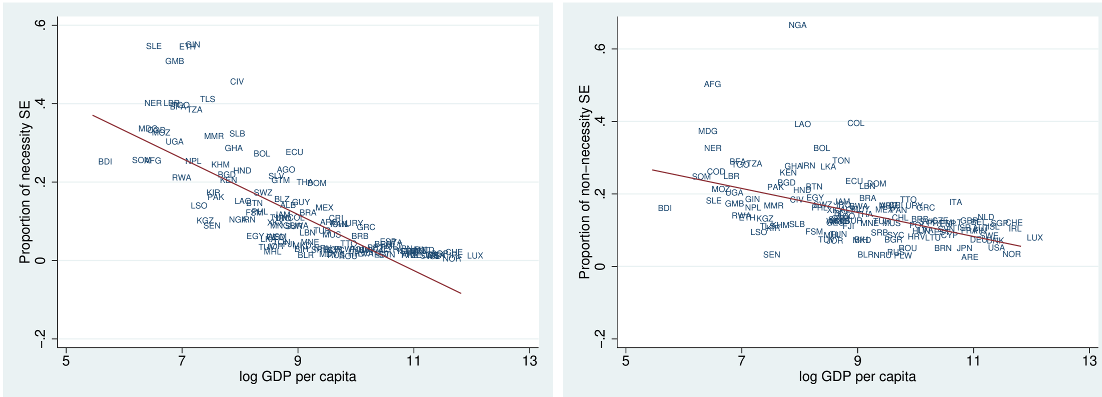


Figure 3: Proportion of (non-agr.) self-employed out of all (non-agr.) employed

Figure 3 replicates the empirical findings in (Poschke, 2023; Gollin, 2008) that the self-employment rate has a negative correlation with economic development. Self-employment is the dominant mode of employment in low-income countries. On average, 61% are self-employed in low-income countries where log GDP per capita is less than 7, compared to merely 12% of the self-employment rate in high-income countries whose log GDP per capita is more than 10.

Figure 4 indicates that the decline of self-employment in home-production-related occupations is the primary driver of the decrease in the self-employment rate with economic development. The average home-production-related self-employment rate plunges from 36% in low-income countries to 2% in high-income countries, while the average non-home-production-



(a) Home-production SE

(b) Non-home-production SE

Figure 4: Prop. of necessity and non-necessity SE out of all (non-agr.) employed

related self-employment rate decreases from 24% to 10% from low-income to high-income group. A back-of-envelope calculation shows that the sharp decrease in self-employment in home-production-related occupations accounts for around 70% of the decline in self-employment with GDP per capita. The slope of the fitted line in Figure 3 is -0.1046, while the slope of the fitted lines in Figure 4 are -0.0715 and -0.0331, respectively. The regression coefficients confirm that the big jump comes from home-production-related self-employment.

Since the decrease in home-production-related self-employment rate is the key to the decline in self-employment as a country develops, it is essential to understand how policies affect the size of this specific group of people.

3 A Simple Model

Since self-employment is the dominant form of employment in low-income countries, I need a model to quantitatively evaluate if policies can effectively lessen the self-employment rate and encourage wage employment in more productive firms. This section presents a standard occupational choice model (Gollin, 2008, 2006), incorporating the empirical evidence that the self-employed and the wage-employed produce different goods. My innovation is that I assume the self-employed produce home production substitute goods, while the wage-employed produce other distinguishable goods, which I will call *manufacturing goods* for simplification. Since there are marketable home production substitute goods in the economy, I also explicitly model home production goods and services the households make that are not tradeable, such as homemade meals, care for children, cleaning the houses, etc.

3.1 The model setup

Heterogeneous agents. A continuum of agents of measure 1 populates the economy. Agents are heterogeneous in two dimensions: earning ability as a worker, ν, and taste for self-employment, ι. I assume that ν follows log-normal distribution, ι follows normal distribution, and both distributions are independent, i.e., $\ln \nu \sim \mathbf{N}(\mu_\nu, \sigma_\nu^2)$, $\iota \sim \mathbf{N}(\mu_\iota, \sigma_\iota^2)$, and $\nu \perp \iota$. The probability distribution functions of ν and ι are $f(\nu)$ and $g(\iota)$, respectively.

Time use. Each agent has $\bar{T}$ amount of time endowment, which they can allocate in three activities: market work, n, home production, h, and leisure, l.

Preference. Agents value composite consumption goods, c_{com}, and leisure time, l. The utility function has the CRRA form:

$$u = \theta \frac{c_{com}^{1-\sigma}}{1-\sigma} + (1-\theta)\frac{l^{1-\sigma}}{1-\sigma}. \tag{1}$$

The composite consumption good c_{com} is a CES aggregate of manufacturing goods, c_m, and home goods, c_h, with elasticity of substitution being ϵ:

$$c_{com} = \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi) c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{2}$$

The corporate sector produces manufacturing goods, c_m. Home goods, c_h, is another CES aggregate of home production goods, c_{sh}, and home production substitute goods purchased on the market, c_{ph}, with elasticity of substitution being ζ:

$$c_h = \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi)\, c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}. \tag{3}$$

The corporate sector. A representative firm produces manufacturing goods, c_m, which is the numeraire in the economy. The wage-employed work in this firm. The corporate sector's production technology is $Y_m = zN^{1-\alpha}$, where z is the TFP term and N is the sum of the efficient amount of labor by all workers. For a worker with earning ability, ν, and spends n

amount of hour on market work, the efficient labor she supplies is νn. The firm is subject to the corporate tax at the rate τ. The corporate sector hires the optimal amount of labor by maximizing its profit:

$$\max_{N} \quad \Pi = (1-\tau)zN^{1-\alpha} - wN \tag{4}$$

FOC:

$$\frac{\partial \Pi}{\partial N} = (1-\tau)(1-\alpha)zN^{-\alpha} - w = 0 \Longrightarrow N^d = \frac{(1-\tau)(1-\alpha)Y_m}{w}. \tag{5}$$

The corporate sector earns positive profit:

$$\Pi^* = \alpha Y_m. \tag{6}$$

Home production technology. Two types of home goods exist in the economy. The first is home production goods, c_{sh}, that everyone makes and then consumes, which are not tradable on the market. The second is home production substitute goods, c_{ph}, that the self-employed make and are marketable. The two types of home goods are not identical. Due to customization and sophistication, comparing the productivity of the production of these two goods ex-ante is not apparent.

Home production goods. Agents do home production using their home production hours with a linear production technology: $y_{sh} = \rho h$. ρ measures the home production productivity. The inventions of more efficient home appliances, like washing machines and vacuum robots, greatly reduce the time required to perform a certain amount of home production, which can be captured by an increase in ρ. Home production goods are not tradeable, and agents consume all the home production goods they produce: $c_{sh} = y_{sh}$.

Home production substitute goods. Agents can purchase home production substitute goods on the market, c_{ph}. For example, you can buy your lunch from a food truck instead of cooking at home. The self-employed in the economy produce home production substitute goods using their market work time: $y_{ph} = \xi n$. ξ measures the productivity of the self-employed in their market work. The home production substitute goods are tradable at the equilibrium price p.

The government sector. The government collects taxes in the unit of manufacturing goods. The government spends all the tax revenue. I assume the agents do not value government spending for simplification, which is also a standard assumption in the literature.

Occupational choice. There are two occupational choices available in the economy. An agent could either be a wage worker, working in the corporate sector. Alternatively, an agent could be self-employed and produce home production substitute goods.

For the wage-employed, earning ability ν determines the efficient amount of labor a worker can supply for each working hour. Given equilibrium wage rate w, an agent's earning ability ν, and working hours n, the income for a worker is $w\nu n$.

For the self-employed, by assumption, they produce home production substitute goods, like restaurant meals, cleaning services, personal care services, etc. Home-production-related occupations take up a significant proportion of the self-employed in developing countries. Therefore, assuming all the self-employed are providing home-production substitute goods is a simplification to distinguish the nature of the self-employed is different from the wage-employed in developing countries. Given the equilibrium price of home production substitute goods p, the self-employed productivity ξ, and working hours n, the income for a self-employed is $p\xi n$.

Agents choose an occupation based on higher utility. Besides utility from composite consumption goods, c_{com}, and leisure time, l, agents receive an additional amount of relative utility, ι, from self-employment. ι follows normal distribution $\iota \sim N(\mu_\iota, \sigma_\iota^2)$; but for a given individual, his/her ι is fixed instead of random. The relative taste for self-employment differs across agents. Some agents might have a higher ι since they value the freedom from being self-employed; others may have a lower ι if they think providing home production substitute goods is less prestigious. The utility from each occupation comes from each agent's optimization problem by choosing the consumption bundle and time allocation. $u^*_{we}(c_m, c_h, l; \nu)$ is the optimized utility an agent can get from being wage-employed by solving the problem (8), which depends on the state variable, ν; while $u^*_{se}(c_m, c_h, l; \nu, \iota)$ is the optimized utility an agent can obtain by being self-employed from the problem (10), which depends on two state variables, ν and ι. By comparing optimized utility from two occupations, an agent chooses either wage-employed, $o(\nu, \iota) = we$, or self-employed, $o(\nu, \iota) = se$.

$$\max_{o \in \{we, se\}} \{u^*_{we}(c_m, c_h, l; \nu), u^*_{se}(c_m, c_h, l; \nu, \iota)\} \tag{7}$$

1. wage-employed:

$$\max_{h,l,c_m,c_{ph}} u_{we}(c_m, c_h, l; \nu) = \theta \frac{c_{com}^{1-\sigma}}{1-\sigma} + (1-\theta)\frac{l^{1-\sigma}}{1-\sigma} \tag{8}$$

$$s.t. \quad c_m + pc_{ph} = w\nu(\bar{T} - h - l) \tag{9}$$

2. self-employed with taste as ι:

$$\max_{h,l,c_m,c_{ph}} u_{se}(c_m, c_h, l; \nu, \iota) = \iota + \left[\theta \frac{c_{com}^{1-\sigma}}{1-\sigma} + (1-\theta)\frac{l^{1-\sigma}}{1-\sigma}\right] \tag{10}$$

$$s.t. \quad c_m + pc_{ph} = p\xi(\bar{T} - h - l) \tag{11}$$

   where c_{com} is a function of manufacturing goods, c_m, and home goods, c_h, as defined in equation (2); c_h is a function of home production goods, c_{sh}, and home production substitute goods, c_{ph}, as defined in equation (3); c_{sh} is a function of home production time h since $c_{sh} = \rho h$.

General equilibrium. The equilibrium consists of the wage rate w, relative price of home production substitute goods p, household's career decision $o(\nu, \iota) \in \{wk, se\}$, decision on time allocation in market work, home production, and leisure, $\{n, h, l\}$, and decision on consumption in c_m, c_{ph}, such that given prices, idiosyncratic earning ability ν, and preference for entrepreneurship ι, agents are maximizing their utility and the following markets clear:

1. Manufacturing goods market clears. The manufacturing goods the representative firm produces, Y_m, equals the total demand of goods $c_m(\nu, \iota)$ from everyone in the economy.

$$Y_m = \int \int c_m(\nu, \iota) f(\nu) g(\iota) d\nu d\iota \tag{12}$$

2. Home production substitute goods market clears. The home production substitute goods supplied by the agents who decide to become self-employed equals the total demand from the economy. Both the wage-employed and the self-employed can demand

home production substitute goods. The idea is that there are different kinds of home production substitute goods owing to specialization. For example, street food vendors can also pay someone to babysit their children when necessary.

$$\int\int_{o=se} \xi n(\nu,\iota) f(\nu) g(\iota) d\nu d\iota = \int\int c_{ph}(\nu,\iota) f(\nu) g(\iota) d\nu d\iota \tag{13}$$

3. The labor market clears. The efficient amount of labor supplied by the agents who opt to become wage-employed equals the labor demand from the corporate sector. Workers' decision on how much time to devote to market work, n_w, depends only on their idiosyncratic earning ability ν.

$$\int\int_{o=we} \nu n(\nu) f(\nu) g(\iota) d\nu d\iota = \frac{(1-\tau)(1-\alpha)Y_m}{w} \tag{14}$$

3.2 Equilibrium occupational choice

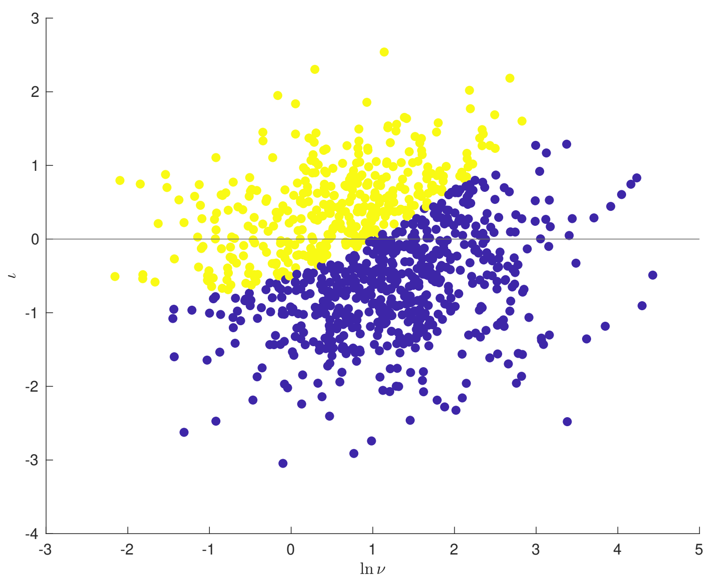
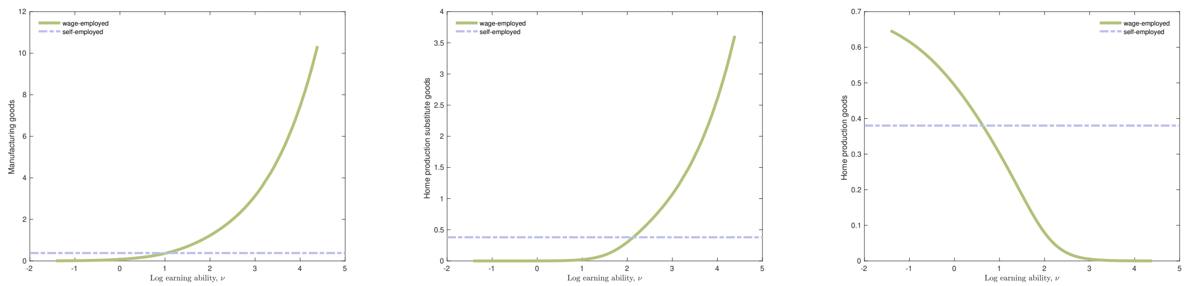


Figure 5: Occupational choice in ln ν - ι space

Using calibrated parameter values (details in Section 4), Figure 5 shows the equilibrium occupational choice in the $\ln\nu - \iota$ space. The blue dots represent the wage-employed, while the yellow dots represent the self-employed. Agents with higher earning ability ν provide more efficient labor in a given working hour as a worker, thus receiving higher income, are more likely to be wage-employed. Agents with higher ι have a stronger preference for self-employment, and are thus more likely to be self-employed.

3.3 Consumption choice



(a) c_m (b) c_{ph} (c) c_{sh}

Figure 6: Optimal consumption choice

Figure 6 displays the consumption choices by agents using calibrated parameters detailed in Section 4. Agents who decide to be self-employed have the same productivity in both market work and home production work. Therefore, all self-employed make the same consumption choices. Agents with higher earning ability, ν, have higher income and thus can afford more manufacturing goods, c_m, and home production substitute goods, c_{ph}. Meanwhile, higher-ability workers produce fewer home production goods due to less time devoted to it (see Figure 7).

3.3.1 Income allocation

Each agent spends their income between buying manufacturing goods and home production substitute goods. The optimal consumption bundle between these two goods, c_m and c_{ph}, is derived explicitly in the Appendix C but presented here:

(a) For the wage-employed:

$$\frac{c_m}{c_{ph}} = \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} \cdot \left[\psi + (1-\psi)\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{1-\zeta}\right]^{\frac{\zeta}{\zeta-1}} \tag{15}$$

(b) For the self-employed:

$$\frac{c_m}{c_{ph}} = \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} \cdot \left[\psi + (1-\psi)\left(\frac{\psi\xi}{\rho(1-\psi)}\right)^{1-\zeta}\right]^{\frac{\zeta}{\zeta-1}} \tag{16}$$

Holding other things constant, the $\frac{c_m}{c_{ph}}$ ratio is negatively correlated with p. The intuition is simple: when home production substitute goods become relatively more expensive, agents will respond by consuming more manufacturing goods compared to home production substitute goods. The $\frac{c_m}{c_{ph}}$ ratio is also positively correlated with ϵ, the elasticity of substitution between manufacturing goods and home goods. When two types of goods are easier to substitute, an increase in the price of c_{ph} will lead the agents to adjust consumption bundles by maintaining a higher $\frac{c_m}{c_{ph}}$ ratio.

3.3.2 Home goods allocation

Agents also have an optimal consumption bundle between home production goods, c_{sh}, and home production substitute goods, c_{ph}. The optimal ratio between two types of home goods:

(a) For the wage-employed:

$$\frac{c_{ph}}{c_{sh}} = \left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta} \tag{17}$$

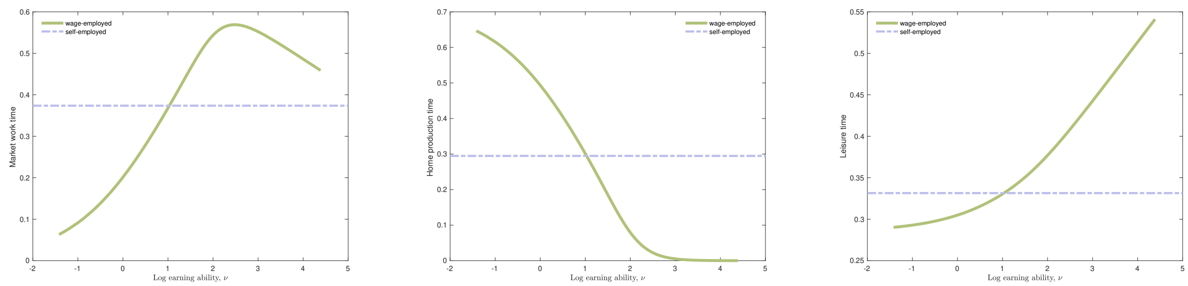
(b) For the self-employed:

$$\frac{c_{ph}}{c_{sh}} = \left(\frac{\psi\xi}{(1-\psi)\rho}\right)^{\zeta} \tag{18}$$

On the one hand, for the wage-employed, $\frac{c_{ph}}{c_{sh}}$ is negatively correlated with the relative price, p. Workers will demand less of c_{ph} and increase their home production time to produce more c_{sh} when home production substitute goods become more expensive. On the other hand, the self-employed people's $\frac{c_{ph}}{c_{sh}}$ is independent of the relative price p. When the price of goods they produce, p, increases, their incomes increase at the same rate, thus allowing them to maintain the same consumption bundle between c_{ph} and c_{sh}.

3.4 Time use choice

An agent allocates time between market work, home production, and leisure. The optimal time allocation for the three activities can be derived analytically, which you can find in the Appendix C. Figure 7 presented the optimal time use for agents with different earning abilities and occupations using calibrated parameter values detailed in Section 4.



(a) Market work (b) Home production (c) Leisure

Figure 7: Optimal time allocation

Agents who opt to be self-employed have the same productivity and preference, therefore, they have the same time use choice. Wage-employed with higher earning abilities spend more time on leisure and less time on work (market work plus home production), due to a stronger income effect. Among work, market work time has an inverse U-shape with workers' abilities, while home production time has a negative relationship with earning ability.

Market hour-to-home production hour ratio. In general, higher-ability workers maintain a higher market hour-to-home production hour ratio. Since wage-employed are heterogeneous in the productivity of market work while everyone has the same productivity in home production, high-ability workers will work more in the market, earn more income, and buy more home production substitute goods. Equation (19) confirms that $\frac{n}{h}$ has a positive correlation with earning ability ν.

$$\frac{n}{h} = \rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\left(\psi\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta-1}+(1-\psi)\right)^{\frac{\zeta-\epsilon}{\zeta-1}}+\left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{w\nu}{p\rho}\right)^{\zeta-1} \tag{19}$$

Meanwhile, an increase in the price of home production substitute goods p will bring down the $\frac{n}{h}$ ratio. The wage-employed will readjust their time allocation by spending more time on home production to make home goods. For example, when it becomes more expensive to eat outside, a rational worker will spend more time cooking at home and bringing her food.

4 Calibration: The Case of Tanzania

I calibrate the model to fit Tanzania's economy in 2020. Tanzania is a low-income country where around 60% of the self-employed are doing home-production-related occupations. The country's Development Vision 2050 highlights industrialization as one of the pillars to foster economic growth. Raising wage employment and reducing self-employment in a low-income country is generally a development goal of policymakers. As outlined in the United Nations' Millennium Development Goals, promoting inclusive and sustainable economic growth, employment, and decent work for all is paramount. Tanzania had some tax reforms to accommodate its development goal. For example, in 2018/2019, the government reduced a few industries' corporate income tax rates to encourage investment and increase employment opportunities for five years.

Tanzania also has excellent survey data that facilitates the calibration. The Tanzania Integrated Labour Force Survey 2020/2021 covers a wide range of topics and incorporates the Time Use Survey (TUS). It is very rare for a low-income country to have time-use data, which is critical for the calibration exercise. With TUS, I can target moments on household time allocation and gain better insights into how households spend time among market work, home production, and leisure. I set some parameters to common values from the literature and calibrate the rest internally.

4.1 Predetermined parameter values

To calibrate the model, I first predetermine some parameters using standard values in the literature or through normalization. α is $\frac{1}{3}$ so that $1-\alpha$, the labor income share, is $\frac{2}{3}$. I take σ, the relative risk aversion, from the estimated value in Fang and Zhu (2017). I normalize all the productivity parameters to 1. The benchmark corporate income tax rate, $\tau = 30\%$, is the tax rate in Tanzania before the tax reform.

ϵ, the elasticity of substitution between c_m and c_h, is an important parameter. When assuming the wage-employed and the self-employed produce homogeneous goods as in the previous literature (Gollin, 2008; Ihrig and Moe, 2004), it is equivalent to assume that the elasticity of substitution between manufacturing goods and home goods is infinity. Aguiar, Hurst, and Karabarbounis (2012) survey the literature that estimates the elasticity of substitution between market and home goods. The estimated parameter ϵ range from slightly less than 2 to 2.3, and I choose $\epsilon = 2$ as in Gottlieb et al. (2023). In the quantitative exercise, I also consider policy implications with a higher ϵ.

I set ζ, the elasticity of substitution between c_{ph} and c_{sh}, to be 4 because it is easier to substitute within home goods than between home goods and manufacturing goods ($\zeta > \epsilon$). Moro, Moslehi, and Tanaka (2017) estimate this parameter value in the online Appendix B. Their estimates range from 0.267 to 6.850, depending on different specifications. Therefore, setting $\zeta = 4$ is around the average of the estimates.

Table 1: Predetermined parameter values

Parameter	Value
α	$\frac{1}{3}$
σ, relative risk aversion	1.4
ϵ, elasticity of substitution between c_m and c_h	2
ζ, elasticity of substitution between c_{ph} and c_{sh}	4
z, TFP term	1
ρ, home production productivity	1
ξ, NE productivity	1
$\bar{T}$, total time endowment	1
μ_ν, mean of $\ln \nu$	1
τ, corporate income tax rate	0.3

4.2 Targeted moments

I use the following six moments to calibrate the six remaining unknown parameters: (1) standard deviation of log earning ability, σ_ν; (2) weight on consumption in the utility function, θ; (3) weight on c_m in the c_{com} composite, ϕ; (4) weight on c_{ph} in the c_h composite, ψ; (5) mean of taste for self-employment distribution, μ_ι; (6) standard deviation of taste for self-employment distribution, σ_ι.

Proportion of the self-employed. The data is from the ILO data on employment by status in employment and occupation for Tanzania in 2020. I exclude occupations in agriculture and armed forces and the self-employed who are not doing home-production-related occupations. Then, the remaining sample includes the wage-employed and the self-employed doing home-production-related occupations. The self-employment rate is 38%.

Average time use. The Time Use Survey within Tanzania's Integrated Labour Force Survey 2020/2021 provides a detailed 24-hour diary for each interviewee. I categorize each activity group into either market work, home production, or leisure. The market work includes time spent on employment and related activities and the production of goods for its own final use. Home production includes time spent on unpaid domestic services for household and family members, unpaid caregiving services for household and family members, unpaid volunteer, trainee, and other unpaid work. Leisure includes time spent on learning, socializing, community participation, and culture leisure mass-media and sports practices. I assume everyone spends 12 hours daily on self-care and maintenance; therefore, everybody allocates the remaining 12 hours daily to market work, home production, and leisure.

The working age population, those between 15 and 65 years old, spend, on average, 4.67 hours and 3.10 hours per day on market work and home production, respectively. For those who do multitasking, I consider only the primary activity and allocate all the time to it. Therefore, among the 12 hours of discretionary time, an average person allocates 39% to market work and 26% to home production, which become two targeted moments.

Standard deviation of log wage. The imputed hourly wage of the employees follows a log-normal distribution, whose standard deviation is one of the targets. I compute the hourly wage by dividing reported last week's total paid income by last week's working hours for the working-age employees. A normal distribution fits the imputed log hourly wage as seen in Figure 14 in the Appendix. The standard deviation of the log wage is 0.95.

Average income ratio of the wage-employed and the self-employed. The wage-employed have a higher average income than the self-employed, as revealed in the ILFS. I consider total paid income as income for the wage-employed and total self income as income for the self-employed doing home-production-related occupations. I exclude reported self income that is negative. On average, the income of the wage-employed is 2.77 that of the self-employed.

Non-home-production-related goods expenditure share. Tanzania's National Account sheds light on household expenditure between marketable home-production and non-home-production-related goods. I separate each non-agricultural activity into either group, which you can find detailed classification in the Appendix B.2. In 2020, around 71% of the non-agricultural GDP occurred in the non-home-production-related sectors, while the remaining 29% was in the home-production-related sector.

Table 2: Targeted moments

	Model	Data
Prop. of entrepreneur	0.38	0.38
Avg market work time	0.39	0.39
Avg household work time	0.27	0.26
Std log wage	0.95	0.95
Consumption goods expenditure share	0.71	0.71
Average income ratio: worker over NE	2.77	2.77

4.3 Calibrated parameter values

I calibrate the 6 unknown parameters jointly by minimizing the sum of squared distances between each moment in the data and that of the model. Table 3 presents the calibrated parameter values.

Some moments are more informative for calibrating specific parameters. Total working hours, market plus home production, help to pin down the weight of consumption θ. The weight of the manufacturing goods in the c_{com} composite, ϕ, determines household expenditure share. The weight of home production substitute goods in the total home goods composite, ψ, plays a role in how households allocate working time between market work and home production. The standard deviation of log ability, σ_ν, directly governs the workers' standard deviation of log hourly wage.

Both ability and taste for self-employment determine an agent's occupational choice, the distribution of taste for self-employment is essential to understand how the occupation choice is different from the one solely governed by the ability. The mean taste for self-employment, μ_ι, sheds light on the proportion of the self-employed in the economy. As you can see, on average, agents don't prefer the self-employment over wage-employment. A more dispersed taste distribution is more likely to lead to high-ability agent choose self-employment sector due to extremely higher preference, thus reducing the average income ratio between wage-employed and the self-employed.

Table 3: Calibrated parameter values

	Parameter value
σ_ν, std of $\ln \nu$	1.0175
θ, weight on consumption in the utility function	0.6471
ϕ, weight on c_m in the c_{com} composite	0.4348
ψ, weight on c_{ph} in the c_h composite	0.3462
μ_ι, mean of ι	-0.2568
σ_ι, std of ι	0.8238

5 Policy Experiments

Based on the calibrated model that fits Tanzania's economy in 2020, I conduct policy experiments to study the effectiveness of tax reforms on promoting wage employment. I consider two sets of policy reform: the first is to reduce the corporate income tax rate, and the second is to increase regulation against informal self-employment.

5.1 Corporate income tax cut

The rationale for using corporate income tax cuts to promote wage employment is this: a lower corporate income tax rate increases the firm's labor demand, thus elevating the equilibrium wage rate. As the corporate sector offers more competitive incomes, it persuades some self-employed to switch to being wage-employed. In 2018/2019, Tanzania's government reduced the corporate income tax rate for the pharmaceutical and leather industries, intending to promote employment in these two industries. No post-reform data is available yet to empirically examine this tax reform's effectiveness. Meanwhile, I will evaluate it quantitatively using the calibrated model.

5.1.1 Benchmark scenario

In this section, I assess quantitatively the effect of corporate income tax cuts on the share of self-employment. I reduce the tax rate by 10% at a time, from the original 30% to 0. Table 4 presents the occupational choice with tax reform. In the second column, you can find the proportion of the self-employed in the economy at a given tax rate indicated in the first column. The third column summarizes how many self-employed people have switched to the wage-employment sector compared to the original 30% tax rate case.

Table 4: Coporate income tax cut in the benchmark scenario

Tax rate	Prop. of SE	Change in prop. of WE
0.3	38.30%	
0.2	37.70%	+ 0.6%
0.1	36.90%	+ 1.4%
0	36.50%	+ 1.8%

The quantitative exercise shows that corporate income tax cuts have a limited effect on promoting wage employment, consistent with the empirical findings in Pham (2020) but contradicts the results in Gollin (2006). Figure 8 shows that few agents along the indifference curve switch from self-employment (yellow dots) to wage-employment (blue dots), which are highlighted in the green dots.

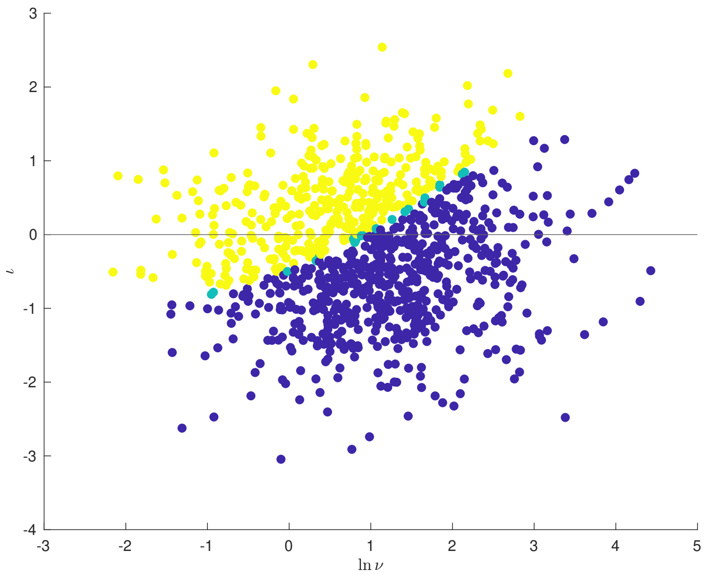


Figure 8: Transition of occupational choice with tax reform

The difference between my results and other optimistic quantitative results comes from the relative price of home production substitute goods that the self-employed produce. With tax cuts from 30% to 0, the wage rate in the corporate sector increases, as you can see on the left axis in Figure 9. Higher wage incentivizes the self-employed to switch to the wage-employment sector due to higher income. Now, less self-employed in the economy are producing home production substitute goods. Due to limited supply, the relative price of home-production-substitute goods increases, as the right axis of Figure 9 shows. The increase in the price of home production substitute goods makes self-employment more profitable, thus putting a break on the switch of occupations.

In the quantitative literature that assumes the self-employed and the wage-employed produce homogeneous goods, there is no relative price mechanism. Agents only respond to the wage rate. With higher wage rates in the corporate sector, more self-employed will switch occupations. Thus, the corporate income tax cut is more effective in promoting wage-employment in their scenarios.

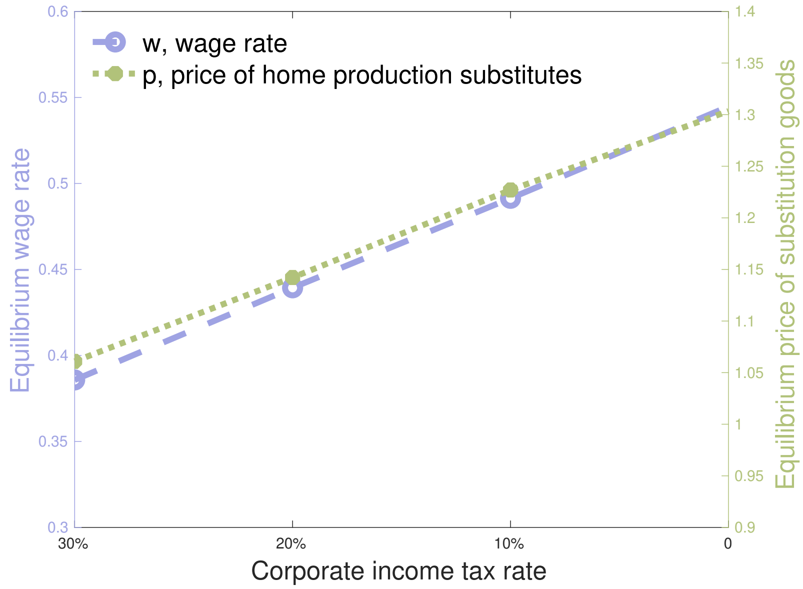


Figure 9: Equilibrium prices

5.1.2 A higher elasticity of substitution

Now, I increase the elasticity of substitution ϵ to a higher value 10. In the literature, where it assumes homogeneous goods across sectors, it is equivalent to think that the elasticity of substitution between goods produced by the wage-employed and the self-employed is infinity. A higher elasticity of substitution implies that two goods are more similar, thus bringing the analysis closer to the scenario studied in the previous quantitative literature. I recalibrate the model with the new ϵ, and you can find the details in the Appendix B.3.

Table 5: Coporate income tax cut with $\epsilon = 10$

Tax rate	Prop. of SE	Change in prop. of WE
0.3	38.30%	
0.2	36.80%	+ 1.5%
0.1	35.40%	+ 2.9%
0	33.40%	+ 4.9%

When the goods produced by the two sectors are more similar, corporate income tax cuts have a more powerful effect on promoting wage employment. Table 5 shows that at each level of the tax cut, almost three times as many self-employed would switch to wage-employment compared to the benchmark scenario with a lower elasticity of substitution presented in Table 4. As a result, we see more agents switch occupations along the indifference curve, shown in green dots in Figure 10.

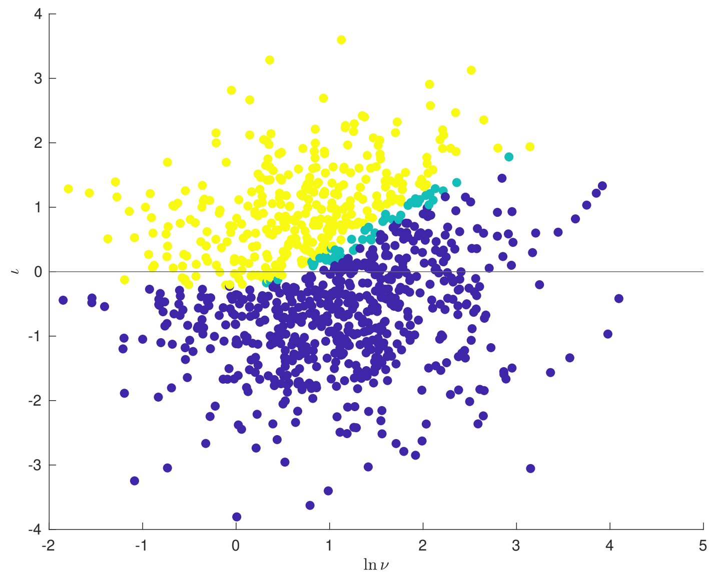


Figure 10: Transition of occupational choice with tax reform with $\epsilon = 10$

A less sharp increase in the relative price of home production substitute goods explains why tax cuts are more effective in promoting wage employment when the goods produced in two sectors are more substitutable. Each agent holds an optimal amount of consumption bundle between manufacturing goods and home-production-substitute goods as shown in equation (15) for wage-employed and equation (16) for self-employed. When manufacturing goods and home goods are easier to substitute (ϵ higher) and less self-employed producing home-production-substitute goods c_{ph} due to job switch, agents will replace more c_{ph} with manufacturing goods c_m. The reduced demand for c_{ph} translates into a slower-growing path of the price of home production substitute goods with tax cuts, as Figure 11 shows. Since the home production substitute goods' price increase does not keep up with the wage increase with the tax reform, working in the corporate sector has more comparative advantage for agents near the indifference curve. Therefore, more self-employed will switch to work in the corporate sector. As a result, the corporate tax cuts are more successful in promoting wage employment.

When $\epsilon = 10$, the trajectory of the relative price of home-production-substitute goods, p, is almost flat with tax cuts. Therefore, it closely resembles the one-good scenario commonly studied in the literature, where there doesn't exist a relative price effect.

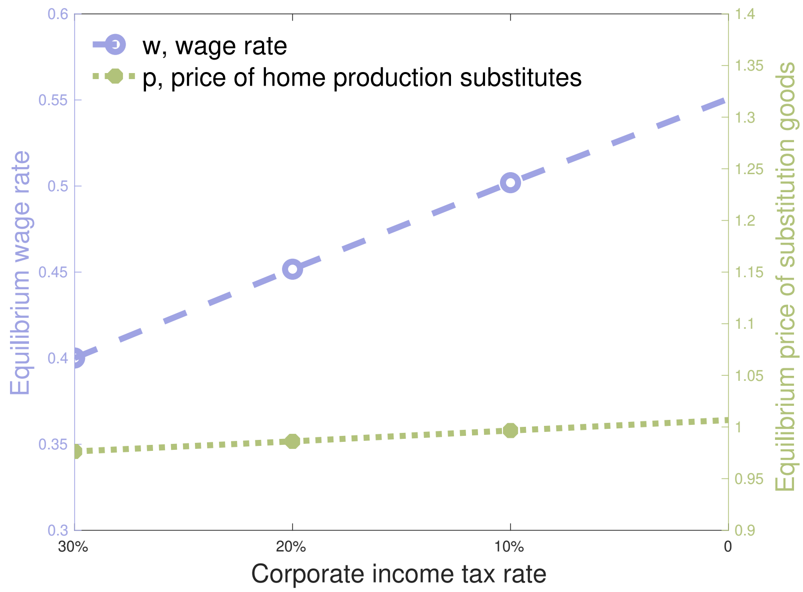


Figure 11: Equilibrium prices when $\epsilon = 10$

To summarize, when assessing the impact of corporate income tax cuts on promoting wage employment in a model with heterogeneous goods, the effect is minor, in line with some empirical findings. The model suggests that the relative price reacts to the occupation transition and brings unintended consequences. With tax cuts, the wage rate in the corporate sector will increase, attracting the self-employed to switch occupations. In the meantime, due to less supply of home-production-substitute goods as the self-employed left, the relative price of the goods provided by the self-employed will increase, thus attenuating the effect of the tax reform. If the goods produced by the self-employed and the wage-employed are easier to substitute, households demand fewer goods made by the self-employed. As a result, the relative price increase will be slight when the self-employed transit to the corporate sector, and the corporate tax cuts will be more powerful to promote wage employment in this case.

5.2 Increase regulation of the informal sector

Self-employment is generally connected with informal activities in low-income countries. Many low-skilled set up small businesses for subsistence and don't have any formal business registration. As a consequence, these informal self-employed escape tax obligations and impair the fiscal capacity of the country. Due to limited government revenues, developing countries often find themselves short of funding to finance projects that promote long-term growth, like infrastructure investments or education.

To eradicate the informal sector and encourage formalization, the governments usually take two measures. The first measure is to increase the surveillance of the informal sector. For example, the government might employ more police to supervise, or those caught in informal activities are subject to higher fines. The second measure is to reduce the entry cost of the formal sector. It includes policies such as minimizing the processes to set up the formal business, reducing the red tape, reimbursing formalization fees, providing registration assistance, etc. In a nutshell, the relative cost of staying in the informal sector is elevated with regulation of the informal sector.

I model the regulation of the informal sector by adding a fixed cost, s, to the operation of self-employment. In this case, the budget constraint of the self-employed becomes:

$$c_m + pc_{ph} = p\xi(\bar{T} - h - l) - s. \tag{20}$$

With a higher level of supervision, the informal activities have a chance of being caught and will pay a higher fine; thus s increases. Alternatively, policies that reduce the cost of formalization translate into a higher relative cost for the self-employed who stay in the informal sector ($s \uparrow$). The agents make the occupational choice based on higher utility as before, with a new budget constraint for the self-employed.

This section studies how increasing the regulation of the informal sector affects the share of wage employment in the economy. With a higher cost of staying self-employed, people switch to the corporate sector, thus bringing down the equilibrium wage rate. As fewer self-employed produce home-production-substitute goods, the relative price of the goods they make increases. Figure 12 shows the transition of equilibrium prices with more strict regulation,

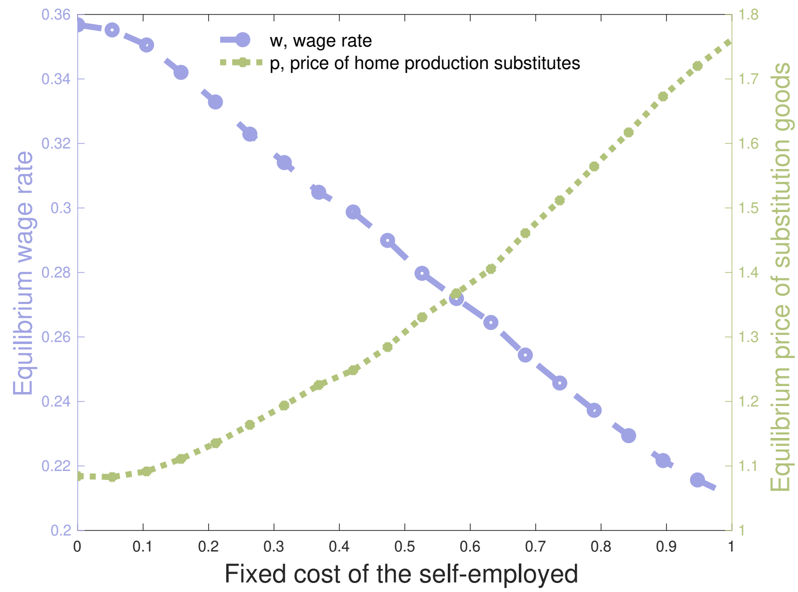


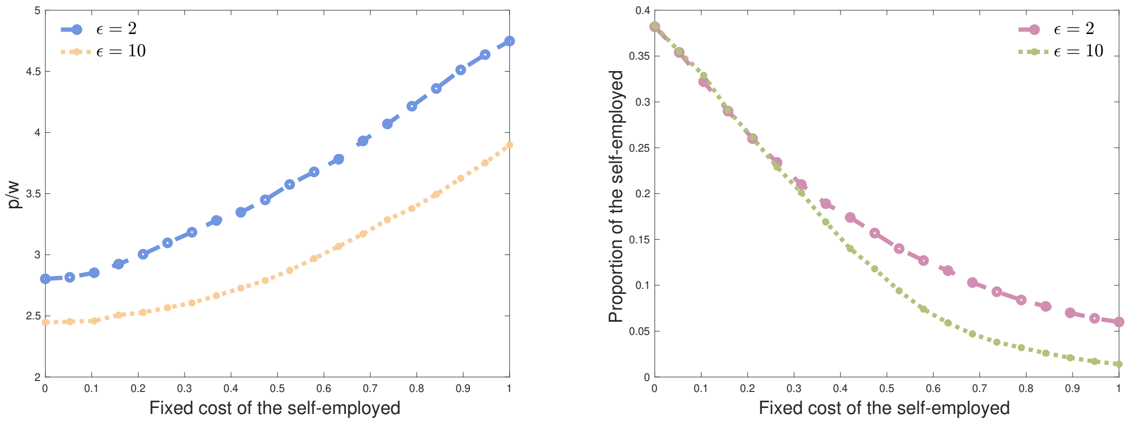
Figure 12: Equilibrium prices with regulation of the informal sector

The elasticity of substitution between goods produced in two sectors matters for the effectiveness of regulation on promoting wage employment. When goods are harder to substitute (lower ϵ), the home-production-substitute goods price over wage ratio rises faster with heavier regulation (see panel (a) of Figure 13) because consumers have a greater demand for the goods made by the self-employed. Hence, reducing self-employment is less significant, making regulation policies less efficient.

6 Conclusion

This paper explains why policy reforms, like corporate income tax cuts, may not successfully boost wage employment in some experiments. Empirically, I find that the self-employed and the wage-employed spread out in different occupations in developing countries, with the self-employed most likely taking jobs that provide home production substitute goods. Given this empirical evidence, I modify an assumption typically used in the quantitative literature that both employment sectors produce the same goods.

In an occupational choice model where the self-employed produce home production substitute goods and the wage-employed produce manufacturing goods, a relative price effect



(a) Price ratio (b) Prop. of self-employment

Figure 13: Regulation of the informal sector

attenuates any policy attempts to increase wage employment. With fewer self-employed in the economy, the reduced supply raises the price of the goods they provide. This unintended consequence weakens the policies that aim to lower the self-employment rate. The result is consistent with empirical literature studying the impact of corporate income tax cuts on wage employment in Vietnam, China, Brazil, etc.

The model also has some limitations. First, I am not considering the job search, which has more frictions in developing countries. Many people flow through self-employment while searching for more formal jobs. Second, I ignore some corporations that provide home production substitutes, like Starbucks and McDonald's, that are popular in Western countries. Therefore, the model is more suitable in a developing country setting, where corporatized home production substitute goods have a small presence. Third, I assume that all self-employed provide home production substitute goods and avoid paying taxes for simplification, which is not valid in reality. In low-income countries, street food vendors may be the most common form of self-employment; there are also self-employed who are not doing home-production-related occupations, follow the tax regulations, and are essential to the functioning of the economy. Future research may address these limitations.

This paper delivers several policy implications. First and foremost, while designing industrial policies that intend to increase employment share in specific sectors, governments should consider the previous sectors where the newly-attracted workers come from, how substitutable the goods/services in different sectors, and how policies affect the prices of goods in various sectors. As the model shows, the relative price change across sectors might dampen the effectiveness of policies promoting employment in specific industries. Second, given that most self-employed in developing countries produce home production substitutes, essential in everyday life and hard to replace, it is advisable to strengthen efforts to formalize these businesses and improve their productivity. Moreover, the emergence of more efficient home appliances, like vacuum robots and washing machines, will lessen household's demand for goods from the self-employed, thus lowering the self-employment rate.

References

Abras, Ana, Rita K Almeida, Pedro Carneiro, and Carlos Henrique L Corseuil. 2018. "Enforcement of labor regulations and job flows: evidence from Brazilian cities." *IZA Journal of Development and Migration* 8 (1):1–19.

Aguiar, Mark and Erik Hurst. 2007. "Measuring trends in leisure: The allocation of time over five decades." *The quarterly journal of economics* 122 (3):969–1006.

———. 2008. "The increase in leisure inequality." Tech. rep., National Bureau of Economic Research.

Aguiar, Mark, Erik Hurst, and Loukas Karabarbounis. 2012. "Recent developments in the economics of time use." *Annual review of Economics* 4 (1):373–397.

Almeida, Rita and Pedro Carneiro. 2012. "Enforcement of labor regulation and informality." *American Economic Journal: Applied Economics* 4 (3):64–89.

Bento, Pedro and Diego Restuccia. 2021. "On average establishment size across sectors and countries." *Journal of Monetary Economics* 117:220–242.

Bento, Pedro, Lin Shao, and Faisal Sohail. 2023. "Gender gaps in time use and entrepreneurship." *Working Paper* .

Bick, Alexander, Nicola Fuchs-Schündeln, and David Lagakos. 2018. "How do hours worked vary with income? Cross-country evidence and implications." *American Economic Review* 108 (1):170–99.

Bick, Alexander, Nicola Fuchs-Schündeln, David Lagakos, and Hitoshi Tsujiyama. 2022. "Structural change in labor supply and cross-country differences in hours worked." *Journal of Monetary Economics* 130:68–85.

Boppart, Timo and Per Krusell. 2020. "Labor supply in the past, present, and future: a balanced-growth perspective." *Journal of Political Economy* 128 (1):118–157.

Boppart, Timo and L Rachel Ngai. 2021. "Rising inequality and trends in leisure." *Journal of Economic Growth* 26 (2):153–185.

Bridgman, Benjamin, Georg Duernecker, and Berthold Herrendorf. 2018. "Structural transformation, marketization, and household production around the world." *Journal of Development Economics* 133:102–126.

Buera, Francisco J. 2009. "A dynamic model of entrepreneurship with borrowing constraints: theory and evidence." *Annals of finance* 5:443–464.

Burda, Michael C, Daniel S Hamermesh, and Philippe Weil. 2006. "The Distribution of Total Work in the EU and US." .

De Mel, Suresh, David McKenzie, and Christopher Woodruff. 2010. "Who are the microenterprise owners? Evidence from Sri Lanka on Tokman versus De Soto." In *International differences in entrepreneurship*. University of Chicago Press, 63–87.

———. 2013. "The demand for, and consequences of, formalization among informal firms in Sri Lanka." *American Economic Journal: Applied Economics* 5 (2):122–150.

Dinkelman, Taryn and L Rachel Ngai. 2021. "Home Production, Women's Market Work, and Structural Transformation." *STEG Pathfinding Paper* .

Duval-Hernández, Robert, Lei Fang, and L Rachel Ngai. 2021. "Marketization of Home Production and Gender Gaps in Working Hours." Tech. rep., Federal Reserve Bank of Atlanta.

Fairlie, Robert W and Frank M Fossen. 2018. "Opportunity versus necessity entrepreneurship: Two components of business creation." .

Fang, Lei and Guozhong Zhu. 2017. "Time allocation and home production technology." *Journal of Economic Dynamics and Control* 78:88–101.

Feng, Ying and Jie Ren. 2023. "Skill bias, financial frictions, and selection into entrepreneurship." *Journal of Development Economics* :103046.

Freeman, Richard B and Ronald Schettkat. 2005. "Marketization of household production and the EU–US gap in work." *Economic policy* 20 (41):6–50.

Garcia-Cabo, Joaquin and Rocio Madera. 2021. "Does Self-Employment Pay? The Role of Unemployment and Earnings Risk." .

Gindling, Thomas H and David Newhouse. 2014. "Self-employment in the developing world." *World development* 56:313–331.

Gollin, Douglas. 2006. "Do taxes on large firms impede growth? evidence from ghana." Tech. rep.

———. 2008. "Nobody's business but my own: Self-employment and small enterprise in economic development." *Journal of Monetary Economics* 55 (2):219–233.

Gottlieb, Charles, Douglas Gollin, Cheryl Doss, and Markus Poschke. 2023. "Gender, Work and Structural Transformation." .

Greenwood, Jeremy, Nezih Guner, and Ricardo Marto. 2023. "The great transition: Kuznets facts for family-economists." In *Handbook of the Economics of the Family*, vol. 1. Elsevier, 389–441.

Gu, Jiajia. 2021. "Financial intermediation and occupational choice." *Journal of Economic Dynamics and Control* 133:104238.

Guner, Nezih, Gustavo Ventura, and Yi Xu. 2008. "Macroeconomic implications of size-dependent policies." *Review of economic Dynamics* 11 (4):721–744.

Hasan, Rana, Yi Jiang, and Radine Michelle Rafols. 2021. "Place-based preferential tax policy and industrial development: Evidence from India's program on industrially backward districts." *Journal of Development Economics* 150:102621.

Hémous, David and Morten Olsen. 2022. "The rise of the machines: Automation, horizontal innovation, and income inequality." *American Economic Journal: Macroeconomics* 14 (1):179–223.

Herrendorf, Berthold, Richard Rogerson, and Akos Valentinyi. 2013. "Two perspectives on preferences and structural transformation." *American Economic Review* 103 (7):2752–2789.

Herreno, Juan and Sergio Ocampo. 2023. "The macroeconomic consequences of subsistence self-employment." *Journal of Monetary Economics* 136:91–106.

Ihrig, Jane and Karine S Moe. 2004. "Lurking in the shadows: the informal sector and government policy." *Journal of Development Economics* 73 (2):541–557.

International Labour Organization. 2022. "ILO modelled estimates database." ILOSTATA. https://ilostat.ilo.org/data/. Accessed November 20, 2022.

Kaplan, David S, Eduardo Piedra, and Enrique Seira. 2011. "Entry regulation and business start-ups: Evidence from Mexico." *Journal of Public Economics* 95 (11-12):1501–1515.

Li, Bing, Chang Liu, and Stephen Teng Sun. 2021. "Do corporate income tax cuts decrease labor share? Regression discontinuity evidence from China." *Journal of Development Economics* 150:102624.

McGrattan, Ellen R and Richard Rogerson. 2004. "Changes in hours worked, 1950-2000." *Federal Reserve Bank of Minneapolis Quarterly Review* 28 (1):14–33.

Morazzoni, Marta and Andrea Sy. 2022. "Female entrepreneurship, financial frictions and capital misallocation in the US." *Journal of Monetary Economics* 129:93–118.

Moro, Alessio, Solmaz Moslehi, and Satoshi Tanaka. 2017. "Does home production drive structural transformation?" *American Economic Journal Macroeconomics* :116–146.

National Bureau of Statistics (NBS) [Tanzania]. 2022. "Tanzania Integrated Labour Force Survey 2020/21, Dodoma, Tanzania: NBS." https://www.nbs.go.tz/tnada/index.php/catalog/34. Accessed January 4, 2023.

Ngai, L Rachel and Barbara Petrongolo. 2017. "Gender gaps and the rise of the service economy." *American Economic Journal: Macroeconomics* 9 (4):1–44.

Ngai, L Rachel and Christopher A Pissarides. 2007. "Structural change in a multisector model of growth." *American economic review* 97 (1):429–443.

Ngai, Rachel, Claudia Olivetti, and Barbara Petrongolo. 2022. "Structural Transformation over 150 years of Women's and Men's Work." *Unpublished Working Paper* .

Ordonez, Julio Cesar Leal. 2014. "Tax collection, the informal sector, and productivity." *Review of Economic Dynamics* 17 (2):262–286.

Pham, Anh. 2020. "Effects of temporary corporate income tax cuts: Evidence from Vietnam." *Journal of Development Economics* 146:102476.

Poschke, Markus. 2013a. "Who becomes an entrepreneur? Labor market prospects and occupational choice." *Journal of Economic Dynamics and Control* 37 (3):693–710.

———. 2013b. "'Entrepreneurs out of necessity': A snapshot." *Applied Economics Letters* 20 (7):658–663.

———. 2023. "Wage employment, unemployment and self-employment across countries." .

Quadrini, Vincenzo. 2000. "Entrepreneurship, saving, and social mobility." *Review of economic dynamics* 3 (1):1–40.

———. 2009. "Entrepreneurship in macroeconomics." *Annals of Finance* 5:295–311.

Ramey, Garey and Valerie A Ramey. 2009. "The rug rat race." Tech. rep., National Bureau of Economic Research.

Ramey, Valerie A and Neville Francis. 2009. "A century of work and leisure." *American Economic Journal: Macroeconomics* 1 (2):189–224.

Rocha, Rudi, Gabriel Ulyssea, and Laísa Rachter. 2018. "Do lower taxes reduce informality? Evidence from Brazil." *Journal of development economics* 134:28–49.

Salgado, Sergio. 2020. "Technical change and entrepreneurship." *Working paper* .

Ulyssea, Gabriel. 2018. "Firms, informality, and development: Theory and evidence from Brazil." *American Economic Review* 108 (8):2015–2047.

A Details on occupations

Table 6: Average proportion in each occupation for low-income countries (%)

	Self-employed	Wage-employed
Group 1: Managers	3.71	4.28
Group 2: Professionals	2.74	22.27
Group 3: Technicians and Associate Professionals	3.29	7.77
Group 4: Clerical Support Workers	0.21	4.82
Group 5: Service and Sales Workers	49.56	17.22
Group 6: Craft and Related Trades Workers	20.06	14.67
Group 7: Plant and Machine Operators, and Assemblers	6.22	9.69
Group 8: Elementary Occupations	14.21	19.27

B Calibration details

B.1 Log hourly wage

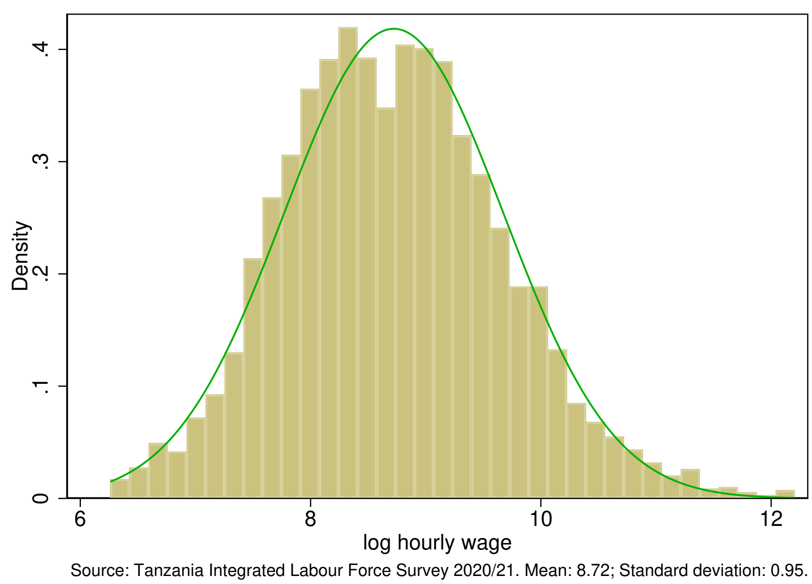


Figure 14: Distribution of log hourly wage for the working-age employees

B.2 Classification of National Account

For each non-agricultural activity in Tanzania's national account, I categorize it as either home-production-substitute goods or non-home-production-substitute goods to calculate the household's expenditure share in these two categories. The classification of each economic activity follows.

- Non-home-production-substitute goods
  - Mining and quarrying
  - Manufacturing
  - Electricity supply
  - Water supply; sewerage, waste management
  - Construction
  - Information and communication
  - Financial and insurance activities
  - Real estate
  - Professional, scientific and technical activities
  - Administrative and support service activities
  - Public administration and defence
  - Education
  - Arts, entertainment and recreation
- Home-production-substitute goods
  - Wholesale and retail trade; repairs
  - Transport and storage
  - Accommodation and Food Services
  - Human health and social work activities
  - Other service activities
  - Activities of households as employers

B.3 Calibration with a higher elasticity of substitution

B.3.1 Predetermined parameter values

Table 7: Predetermined parameter value

	Parameter value	Source
α	$\frac{1}{3}$	
ϵ, elasticity of substitution between c_m and c_h	10	A try
ζ, elasticity of substitution between c_{ph} and c_{sh}	4	Moro, Moslehi, and Tanaka (2017)
σ, relative risk aversion	1.4	Fang and Zhu (2017)
z, TFP term	1	normalization
ρ, home production productivity	1	normalization
ξ, NE productivity	1	normalization
$\bar{T}$, total time endowment	1	normalization
μ_ν, mean of $\ln \nu$	1	normalization
τ, corporate income tax rate	0.3	

B.3.2 Moments to match

Table 8: Moments to target

	Model	2020 Data
(A) Targeted moments		
Prop. of entrepreneur	0.3830	0.38
Avg market work time	0.3912	0.39
Avg household work time	0.2658	0.26
Std log wage	0.9535	0.95
Consumption goods expenditure share	0.7098	0.71
Average income ratio: worker over NE	2.7700	2.77

B.3.3 Calibrated parameter values

Table 9: Calibrated parameter values

	2020
σ_ν, std of $\ln \nu$	0.9328
θ, weight on consumption in the utility function	0.6398
ϕ, weight on c_m in the c_{com} composite	0.3384
ψ, weight on c_{ph} in the c_h composite	0.4618
μ_ι, mean of ι	-0.1032
σ_ι, std of ι	1.1207

C Analytical Results

C.1 Worker's maximization problem

$$\max_{h,l,c_m,c_{ph}} u_w(c_m, c_h, l) = \theta \frac{c_{com}^{1-\sigma}}{1-\sigma} + (1-\theta)\frac{l^{1-\sigma}}{1-\sigma} \tag{21}$$

$$s.t. \quad c_m + pc_{ph} = w\nu(\bar{T} - h - l) \tag{22}$$

where

$$c_{com} = \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{23}$$

$$c_h = \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}} \tag{24}$$

$$c_{sh} = \rho h \tag{25}$$

The Lagrangian function is:

$$\mathcal{L} = \theta \frac{c_{com}^{1-\sigma}}{1-\sigma} + (1-\theta)\frac{l^{1-\sigma}}{1-\sigma} + \lambda\left(w\nu(\bar{T} - h - l) - c_m - pc_{ph}\right) \tag{26}$$

1. FOC wrt c_m:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_m} &= \theta c_{com}^{-\sigma} \frac{\epsilon}{\epsilon-1}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} \phi \frac{\epsilon-1}{\epsilon} c_m^{-\frac{1}{\epsilon}} - \lambda = 0 \\ &\implies \theta\phi c_{com}^{-\sigma}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_m^{-\frac{1}{\epsilon}} = \lambda \end{aligned} \tag{27}$$

2. FOC wrt c_{ph}:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_{ph}} &= \theta c_{com}^{-\sigma} \frac{\epsilon}{\epsilon-1}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} (1-\phi)\frac{\epsilon-1}{\epsilon} c_h^{-\frac{1}{\epsilon}} \\ &\cdot \frac{\zeta}{\zeta-1}\left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} \psi\frac{\zeta-1}{\zeta} c_{ph}^{-\frac{1}{\zeta}} - \lambda p = 0 \\ &\implies \theta(1-\phi)c_{com}^{-\sigma}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_h^{-\frac{1}{\epsilon}} \cdot \psi\left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} c_{ph}^{-\frac{1}{\zeta}} = \lambda p \end{aligned} \tag{28}$$

3. FOC wrt h:

$$\frac{\partial \mathcal{L}}{\partial h} = \theta c_{com}^{-\sigma} \frac{\epsilon}{\epsilon-1} \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi) c_h^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{1}{\epsilon-1}} (1-\phi) \frac{\epsilon-1}{\epsilon} c_h^{-\frac{1}{\epsilon}}$$
$$\cdot \frac{\zeta}{\zeta-1} \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}} \right)^{\frac{1}{\zeta-1}} (1-\psi) \frac{\zeta-1}{\zeta} c_{sh}^{-\frac{1}{\zeta}} \rho - \lambda w \nu = 0$$
$$\Longrightarrow \theta(1-\phi) c_{com}^{-\sigma} \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi) c_h^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{1}{\epsilon-1}} c_h^{-\frac{1}{\epsilon}} \cdot (1-\psi)\rho \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}} \right)^{\frac{1}{\zeta-1}} c_{sh}^{-\frac{1}{\zeta}} = \lambda w \nu \tag{29}$$

4. FOC wrt l:

$$\frac{\partial \mathcal{L}}{\partial l} = (1-\theta) l^{-\sigma} - \lambda w \nu = 0$$
$$\Longrightarrow (1-\theta) l^{-\sigma} = \lambda w \nu \tag{30}$$

Combine FOCs:

1. Purchased substitute goods, c_{ph}, v.s. home production goods, c_{sh}

   Combine (28) and (29):

$$\frac{\psi}{(1-\psi)\rho} \left(\frac{c_{ph}}{c_{sh}} \right)^{-\frac{1}{\zeta}} = \frac{p}{w\nu} \tag{31}$$
$$\Longrightarrow \frac{c_{ph}}{c_{sh}} = \left(\frac{w\nu\psi}{p(1-\psi)\rho} \right)^{\zeta} \tag{32}$$

2. Manufactured goods, c_m, v.s. total home production goods, c_h

   Combine (27) and (28):

$$\frac{(1-\phi) c_h^{-\frac{1}{\epsilon}} \psi c_h^{\frac{1}{\zeta}} c_{ph}^{-\frac{1}{\zeta}}}{\phi c_m^{-\frac{1}{\epsilon}}} = p \tag{33}$$
$$\Longrightarrow \left(\frac{c_h}{c_m} \right)^{-\frac{1}{\epsilon}} \left(\frac{c_h}{c_{ph}} \right)^{\frac{1}{\zeta}} = \frac{p\phi}{\psi(1-\phi)} \tag{34}$$
$$\Longrightarrow \frac{c_h}{c_m} = \left[\frac{p\phi}{\psi(1-\phi)} \left(\frac{c_h}{c_{ph}} \right)^{-\frac{1}{\zeta}} \right]^{-\epsilon} \tag{35}$$
$$\Longrightarrow \frac{c_h}{c_m} = \left[\frac{\psi(1-\phi)}{p\phi} \left(\frac{c_h}{c_{ph}} \right)^{\frac{1}{\zeta}} \right]^{\epsilon} \tag{36}$$

3. Manufactured goods, c_m, v.s. self-made home production goods, c_{sh}

   Combine (27) and (29):

$$\frac{c_m}{c_h} = \left[\frac{w\nu\phi}{\rho(1-\psi)(1-\phi)} \left(\frac{c_h}{c_{sh}} \right)^{-\frac{1}{\zeta}} \right]^{\epsilon} \tag{37}$$

4. Amount of leisure l

   From (30):

$$l = \left(\frac{1-\theta}{\lambda w\nu} \right)^{\frac{1}{\sigma}} \tag{38}$$

Derive analytical solutions:

1. Since $\frac{c_{ph}}{c_{sh}} = \left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta}$ and $c_{sh} = \rho h$, then we have:

$$\begin{aligned} c_{ph} &= \left(\frac{w\nu\psi}{p\rho(1-\psi)} \right)^{\zeta} \cdot c_{sh} \\ &= \underbrace{\left(\frac{w\nu\psi}{p\rho(1-\psi)} \right)^{\zeta}}_{\mathbf{P}} \cdot \rho h \end{aligned} \tag{39}$$

2. By definition, $c_h = \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}$, then we have:

$$\begin{aligned} c_h &= \left(\psi(\mathbf{P}c_{sh})^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} \\ &= \left(\psi \mathbf{P}^{\frac{\zeta-1}{\zeta}} c_{sh}^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} \\ &= \left(\left(\psi \mathbf{P}^{\frac{\zeta-1}{\zeta}} + (1-\psi) \right) c_{sh}^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} \\ &= \underbrace{\left(\psi \mathbf{P}^{\frac{\zeta-1}{\zeta}} + (1-\psi) \right)^{\frac{\zeta}{\zeta-1}}}_{\mathbf{H}} c_{sh} \end{aligned} \tag{40}$$

   More specifically, we could write $\mathbf{H}$ as:

$$\begin{aligned} \mathbf{H} &= \left(\psi \mathbf{P}^{\frac{\zeta-1}{\zeta}} + (1-\psi) \right)^{\frac{\zeta}{\zeta-1}} \\ &= \left(\psi \left(\frac{w\nu\psi}{p\rho(1-\psi)} \right)^{\zeta-1} + (1-\psi) \right)^{\frac{\zeta}{\zeta-1}} \\ &= \left(\psi^{\zeta} \left(\frac{w\nu}{p\rho(1-\psi)} \right)^{\zeta-1} + (1-\psi) \right)^{\frac{\zeta}{\zeta-1}} \end{aligned} \tag{41}$$

3. Since $\frac{c_m}{c_h} = \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon}$, then

$$\begin{aligned} c_m &= \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} c_h \\ c_m &= \underbrace{\left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} \mathbf{H}}_{\mathbf{M}\equiv\mathbf{XH}} \cdot c_{sh} \end{aligned} \tag{42}$$

We could write $\mathbf{X}$ explicitly:

$$\mathbf{X} = \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon}\left(\psi\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\epsilon}{1-\zeta}} \tag{43}$$

Thus, $\mathbf{M} = \mathbf{XH}$:

$$\mathbf{M} = \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon}\left(\psi\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta-\epsilon}{\zeta-1}} \tag{44}$$

4. By definition, $c_{com} = \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$, which is also a linear function of c_{sh}.

$$\begin{aligned} c_{com} &= \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \left(\phi\left(\mathbf{M}c_{sh}\right)^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\left(\mathbf{H}c_{sh}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \underbrace{\left(\phi\mathbf{M}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\mathbf{H}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}}_{\mathbf{G}} c_{sh} \end{aligned} \tag{45}$$

To simplify $\mathbf{G}$:

$$\begin{aligned} \mathbf{G} &= \left(\phi\mathbf{M}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\mathbf{H}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \left(\phi(\mathbf{XH})^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\mathbf{H}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \left(\phi\mathbf{X}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\right)^{\frac{\epsilon}{\epsilon-1}} \mathbf{H} \end{aligned} \tag{46}$$

Since $\mathbf{X} = \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon}\left(\psi\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\epsilon}{1-\zeta}}$ and $\mathbf{H} = \left(\psi^{\zeta}\left(\frac{w\nu}{p\rho(1-\psi)}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}}$, then

$$\mathbf{G} = \left(\phi\left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon-1}\left(\psi\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta-1}+(1-\psi)\right)^{\frac{\epsilon-1}{1-\zeta}}+1-\phi\right)^{\frac{\epsilon}{\epsilon-1}} \cdot\left(\psi^{\zeta}\left(\frac{w\nu}{p\rho(1-\psi)}\right)^{\zeta-1}+(1-\psi)\right)^{\frac{\zeta}{\zeta-1}} \tag{47}$$

5. Because $\theta\phi c_{com}^{-\sigma}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}}+(1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_m^{-\frac{1}{\epsilon}} = \lambda$, now we simplify λ.

$$\lambda = \theta\phi\mathbf{G}^{-\sigma}\left(\frac{\mathbf{G}}{\mathbf{M}}\right)^{\frac{1}{\epsilon}} c_{sh}^{-\sigma} \tag{48}$$

6. Since $l = \left(\frac{1-\theta}{\lambda w\nu}\right)^{\frac{1}{\sigma}}$, then

$$l = \underbrace{\left(\frac{1-\theta}{w\nu\theta\phi}\right)^{\frac{1}{\sigma}}\mathbf{G}\left(\frac{\mathbf{G}}{\mathbf{M}}\right)^{-\frac{1}{\epsilon\sigma}}}_{\mathbf{L}} c_{sh} \tag{49}$$

7. In the budget constraint: $c_m + pc_{ph} = w\nu(\bar{T}-h-l)$, could solve for h, thus everything else.

$$\begin{aligned}\mathbf{M}\rho h + p\mathbf{P}\rho h &= w\nu\bar{T} - w\nu h - w\nu\mathbf{L}\rho h\\ \left(\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho\right)h &= w\nu\bar{T}\\ h &= \frac{w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho}\end{aligned} \tag{50}$$

The other variables could also be solved as:

$$c_{sh} = \rho h = \frac{\rho w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho} \tag{51}$$

$$c_{ph} = \mathbf{P}c_{sh} = \frac{\mathbf{P}\rho w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho} \tag{52}$$

$$c_{h} = \mathbf{H}c_{sh} = \frac{\mathbf{H}\rho w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho} \tag{53}$$

$$c_{m} = \mathbf{M}c_{sh} = \frac{\mathbf{M}\rho w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho} \tag{54}$$

$$c_{com} = \mathbf{G}c_{sh} = \frac{\mathbf{G}\rho w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho} \tag{55}$$

$$l = \mathbf{L}c_{sh} = \frac{\mathbf{L}\rho w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho} \tag{56}$$

8. The optimal consumption bundle, the ratio between c_m and c_{ph}:

$$
\begin{aligned}
\frac{c_m}{c_{ph}} &= \frac{\mathbf{M}}{\mathbf{P}} \\
&= \frac{\left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} \cdot \left(\psi\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}}}{\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{\zeta}} \\
&= \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} \cdot \left[\psi + (1-\psi)\left(\frac{w\nu\psi}{p\rho(1-\psi)}\right)^{1-\zeta}\right]^{\frac{\zeta}{\zeta-1}}
\end{aligned}
\tag{57}
$$

Derive the expressions for time use

1. Simplify $\rho\mathbf{L}$:

First of all, we know that

$$
\begin{aligned}
\mathbf{L} &= \left(\frac{1-\theta}{\theta\phi w\nu}\right)^{\frac{1}{\sigma}} \mathbf{G}\left(\frac{\mathbf{G}}{\mathbf{M}}\right)^{-\frac{1}{\epsilon\sigma}} \\
&= \left(\frac{1-\theta}{\theta\phi w\nu}\right)^{\frac{1}{\sigma}} \mathbf{H}\left(\phi\mathbf{X}^{\frac{\epsilon-1}{\epsilon}} + 1 - \phi\right)^{\frac{\epsilon}{\epsilon-1}} \left(\phi + (1-\phi)\mathbf{X}^{\frac{1-\epsilon}{\epsilon}}\right)^{\frac{1}{\sigma(1-\epsilon)}}
\end{aligned}
\tag{58}
$$

Plug in $\mathbf{X}$, which is

$$
\begin{aligned}
\mathbf{X} &= \left[\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} \\
&= \left(\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right)^{\epsilon} \mathbf{H}^{-\frac{\epsilon}{\zeta}}
\end{aligned}
\tag{59}
$$

Then we have

$$
\begin{aligned}
\mathbf{L} = {} & \frac{1-\theta}{\theta\phi w\nu}\mathbf{H}\left(\phi\left(\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right)^{\epsilon-1}\mathbf{H}^{\frac{1-\epsilon}{\zeta}} + 1 - \phi\right)^{\frac{\epsilon}{\epsilon-1}} \left(\phi + (1-\phi)\left(\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right)^{1-\epsilon}\mathbf{H}^{\frac{\epsilon-1}{\zeta}}\right)^{\frac{1}{1-\epsilon}} \\
& \cdot \underbrace{\left(\frac{1-\theta}{\theta\phi w\nu}\right)^{\frac{1-\sigma}{\sigma}} \left(\phi + (1-\phi)\left(\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right)^{1-\epsilon}\mathbf{H}^{\frac{\epsilon-1}{\zeta}}\right)^{\frac{1-\sigma}{\sigma(1-\epsilon)}}}_{\equiv\mathbf{I}}
\end{aligned}
\tag{60}
$$

Then,

$$\begin{aligned}
\rho\mathbf{L} &= \frac{1-\theta}{\theta\phi w\nu}\mathbf{H}\left(\rho\phi\left(\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right)^{\epsilon-1}\mathbf{H}^{\frac{1-\epsilon}{\zeta}}+\rho(1-\phi)\right)^{\frac{\epsilon}{\epsilon-1}} \\
&\cdot\left(\rho\phi+\rho(1-\phi)\left(\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right)^{1-\epsilon}\mathbf{H}^{\frac{\epsilon-1}{\zeta}}\right)^{\frac{1}{1-\epsilon}}\cdot\mathbf{I} \\
&= \frac{1-\theta}{\theta\phi w\nu}\mathbf{H}(1-\phi)^{\frac{\epsilon}{\epsilon-1}}\phi^{\frac{1}{1-\epsilon}}\left(\rho^{2-\epsilon}\underbrace{(w\nu)^{\epsilon-1}\left(\frac{\phi}{1-\phi}\right)^{\epsilon}(1-\psi)^{1-\epsilon}\mathbf{H}^{\frac{1-\epsilon}{\zeta}}}_{\mathcal{R}}+\rho\right)^{\frac{\epsilon}{\epsilon-1}} \\
&\cdot\left(\rho+\rho^{\epsilon}\underbrace{(w\nu)^{1-\epsilon}\left(\frac{\phi}{1-\phi}\right)^{-\epsilon}(1-\psi)^{\epsilon-1}\mathbf{H}^{\frac{\epsilon-1}{\zeta}}}_{\mathcal{R}^{-1}}\right)^{\frac{1}{1-\epsilon}}\cdot\mathbf{I} \\
&= \frac{1-\theta}{\theta w\nu}\left(\frac{1-\phi}{\phi}\right)^{\frac{\epsilon}{\epsilon-1}}\mathbf{H}\left(\rho^{2-\epsilon}\mathcal{R}+\rho\right)^{\frac{\epsilon}{\epsilon-1}}\left(\rho+\rho^{\epsilon}\mathcal{R}^{-1}\right)^{\frac{1}{1-\epsilon}}\cdot\mathbf{I} \\
&= \frac{1-\theta}{\theta w\nu}\left(\frac{1-\phi}{\phi}\right)^{\frac{\epsilon}{\epsilon-1}}\mathbf{H}(\mathcal{R}\rho^{1-\epsilon}+1)\mathcal{R}^{\frac{1}{\epsilon-1}}\cdot\mathbf{I}
\end{aligned} \tag{61}$$

Plug in $\mathcal{R}$:

$$\begin{aligned}
\rho\mathbf{L} &= \frac{1-\theta}{\theta(1-\psi)}\mathbf{H}^{1-\frac{1}{\zeta}}\left(\rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{1-\phi}\right)^{\epsilon}(1-\psi)^{1-\epsilon}\mathbf{H}^{\frac{1-\epsilon}{\zeta}}+1\right)\cdot\mathbf{I} \\
&= \frac{1-\theta}{\theta}\left[\rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{\frac{\zeta-\epsilon}{\zeta}}+\frac{1}{1-\psi}\mathbf{H}^{1-\frac{1}{\zeta}}\right]\cdot\mathbf{I} \\
&= \frac{1-\theta}{\theta}\left[\rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{\frac{\zeta-\epsilon}{\zeta}}+\left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{w\nu}{p\rho}\right)^{\zeta-1}+1\right]\cdot\mathbf{I}
\end{aligned} \tag{62}$$

2. Simplify $p\mathbf{P}\rho$:

$$p\mathbf{P}\rho = (w\nu)^{\zeta}\left(\frac{\psi}{1-\psi}\right)^{\zeta}(p\rho)^{1-\zeta} \tag{63}$$

3. Simplify $\mathbf{M}\rho$:

$$\begin{aligned}
\mathbf{M}\rho &= \mathbf{X}\mathbf{H}\rho \\
&= \left(\frac{w\nu\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{1-\frac{\epsilon}{\zeta}}\rho^{1-\epsilon}
\end{aligned} \tag{64}$$

4. Derive h:

$$\begin{aligned}
h &= \frac{w\nu\bar{T}}{\mathbf{M}\rho + p\mathbf{P}\rho + w\nu + w\nu\mathbf{L}\rho} \\
&= \frac{\bar{T}}{\frac{\mathbf{M}\rho}{w\nu} + \frac{p\mathbf{P}\rho}{w\nu} + 1 + \mathbf{L}\rho} \\
&= \frac{\bar{T}}{(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{1-\frac{\epsilon}{\zeta}}\rho^{1-\epsilon} + (w\nu)^{\zeta-1}\left(\frac{\psi}{1-\psi}\right)^{\zeta}(p\rho)^{1-\zeta} + 1 + \rho\mathbf{L}} \\
&= \frac{\theta\bar{T}}{\rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{w\nu}{p\rho}\right)^{\zeta-1} + 1} \cdot \underbrace{\frac{1}{\theta + (1-\theta)\mathbf{I}}}_{\text{new term}}
\end{aligned} \tag{65}$$

5. Derive l:

$$\begin{aligned}
l = \rho\mathbf{L}\cdot h &= (1-\theta)\bar{T}\cdot\frac{\mathbf{I}}{\theta + (1-\theta)\mathbf{I}} \\
&= (1-\theta)\bar{T}\cdot\underbrace{\frac{1}{\frac{\theta}{\mathbf{I}} + 1 - \theta}}_{\text{new term}}
\end{aligned} \tag{66}$$

$$\mathbf{I} = \underbrace{\left(\frac{1-\theta}{\theta\phi w\nu}\right)^{\frac{1-\sigma}{\sigma}}}_{(1)}\left(\phi + (1-\phi)\underbrace{\left(\frac{w\nu\phi}{\rho(1-\phi)(1-\psi)}\right)^{1-\epsilon}}_{(2)}\underbrace{\mathbf{H}^{\frac{\epsilon-1}{\zeta}}}_{(3)}\right)^{\frac{1-\sigma}{\sigma(1-\epsilon)}}$$

6. Derive n:

$$\begin{aligned}
n &= \bar{T} - l - h \\
&= \bar{T} - (1-\theta)\bar{T}\frac{\mathbf{I}}{\theta + (1-\theta)\mathbf{I}} - h \\
&= \underbrace{\theta\bar{T}\frac{\rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{w\nu}{p\rho}\right)^{\zeta-1}}{\rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{w\nu}{p\rho}\right)^{\zeta-1} + 1}}_{\text{old term}} \cdot \underbrace{\frac{1}{\theta + (1-\theta)\mathbf{I}}}_{\text{new term}}
\end{aligned} \tag{67}$$

7. Derive the work-to-home production time ratio, $\mathbf{N}$

$$\mathbf{N} = \frac{n}{h} = \rho^{1-\epsilon}(w\nu)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{H}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{w\nu}{p\rho}\right)^{\zeta-1} \tag{68}$$

C.2 Entrepreneur's maximization problem

$$\max_{h,l,c_m,c_{ph}} u_e(c_m, c_h, l) = \iota + \theta \frac{c_{com}^{1-\sigma}}{1-\sigma} + (1-\theta)\frac{l^{1-\sigma}}{1-\sigma} \tag{69}$$

$$s.t. \quad c_m + pc_{ph} = p\xi(\bar{T} - h - l) \tag{70}$$

where

$$c_{com} = \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{71}$$

$$c_h = \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}} \tag{72}$$

$$c_{sh} = \rho h \tag{73}$$

The Lagrangian function:

$$\mathcal{L} = \iota + \theta \frac{c_{com}^{1-\sigma}}{1-\sigma} + (1-\theta)\frac{l^{1-\sigma}}{1-\sigma} + \lambda\left(p\xi(\bar{T} - h - l) - c_m - pc_{ph}\right) \tag{74}$$

1. FOC wrt c_m:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_m} &= \theta c_{com}^{-\sigma} \frac{\epsilon}{\epsilon-1}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} \phi \frac{\epsilon-1}{\epsilon} c_m^{-\frac{1}{\epsilon}} - \lambda = 0 \\ &\Longrightarrow \theta\phi c_{com}^{-\sigma}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_m^{-\frac{1}{\epsilon}} = \lambda \end{aligned} \tag{75}$$

2. FOC wrt c_{ph}:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_{ph}} &= \theta c_{com}^{-\sigma} \frac{\epsilon}{\epsilon-1}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} \frac{\epsilon-1}{\epsilon}(1-\phi) c_h^{-\frac{1}{\epsilon}} \\ &\cdot \frac{\zeta}{\zeta-1}\left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} \psi \frac{\zeta-1}{\zeta} c_{ph}^{-\frac{1}{\zeta}} - \lambda p = 0 \\ &\Longrightarrow \theta(1-\phi) c_{com}^{-\sigma}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_h^{-\frac{1}{\epsilon}} \cdot \psi\left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} c_{ph}^{-\frac{1}{\zeta}} = \lambda p \end{aligned} \tag{76}$$

3. FOC wrt h:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial h} &= \theta c_{com}^{-\sigma} \frac{\epsilon}{\epsilon-1}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} (1-\phi)\frac{\epsilon-1}{\epsilon} c_h^{-\frac{1}{\epsilon}} \\ &\cdot \frac{\zeta}{\zeta-1}\left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} (1-\psi)\frac{\zeta-1}{\zeta} c_{sh}^{-\frac{1}{\zeta}} \rho - \lambda p\xi = 0 \\ &\Longrightarrow \theta(1-\phi) c_{com}^{-\sigma}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_h^{-\frac{1}{\epsilon}} \cdot (1-\psi)\rho\left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi)c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{1}{\zeta-1}} c_{sh}^{-\frac{1}{\zeta}} = \lambda p\xi \end{aligned} \tag{77}$$

4. FOC wrt l:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial l} &= (1-\theta) l^{-\sigma} - \lambda p \xi = 0 \\ \implies (1-\theta) l^{-\sigma} &= \lambda p \xi \end{aligned} \tag{78}$$

Combine FOCs

1. Purchased substitute goods, c_{ph}, v.s. home production goods, c_{sh}

   Combine (76) and (77):

$$\frac{\psi}{(1-\psi)\rho} \left(\frac{c_{ph}}{c_{sh}} \right)^{-\frac{1}{\zeta}} = \frac{1}{\xi} \tag{79}$$

$$\implies \frac{c_{ph}}{c_{sh}} = \left(\frac{\psi \xi}{(1-\psi)\rho} \right)^{\zeta} \tag{80}$$

2. Firm-manufactured goods, c_m, v.s. total home production goods, c_h

   Combine (75) and (76), get exactly the same results as in worker's maximization problem.

3. Firm-manufactured goods, c_m, v.s. self-made home production goods, c_{sh}

   Combine (75) and (77):

$$\frac{c_m}{c_h} = \left[\frac{p \xi \phi}{\rho(1-\phi)(1-\psi)} \left(\frac{c_h}{c_{sh}} \right)^{-\frac{1}{\zeta}} \right]^{\epsilon} \tag{81}$$

4. Amount of leisure, l

   From (78):

$$l = \left(\frac{1-\theta}{\lambda p \xi} \right)^{\frac{1}{\sigma}} \tag{82}$$

Derive analytical solutions

1. Since $\frac{c_{ph}}{c_{sh}} = \left(\frac{\psi \xi}{(1-\psi)\rho} \right)^{\zeta}$ and $c_{sh} = \rho h$, then we have:

$$\begin{aligned} c_{ph} &= \left(\frac{\psi \xi}{(1-\psi)\rho} \right)^{\zeta} \cdot c_{sh} \\ &= \underbrace{\left(\frac{\psi \xi}{(1-\psi)\rho} \right)^{\zeta}}_{\mathbf{Q}} \cdot \rho h \end{aligned} \tag{83}$$

2. By definition, $c_h = \left(\psi c_{ph}^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}}$, then we have:

$$\begin{aligned}
c_h &= \left(\psi (\mathbf{Q} c_{sh})^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}} \\
&= \left(\psi \mathbf{Q}^{\frac{\zeta-1}{\zeta}} c_{sh}^{\frac{\zeta-1}{\zeta}} + (1-\psi) c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}} \\
&= \left(\left(\psi \mathbf{Q}^{\frac{\zeta-1}{\zeta}} + (1-\psi)\right) c_{sh}^{\frac{\zeta-1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}} \\
&= \underbrace{\left(\psi \mathbf{Q}^{\frac{\zeta-1}{\zeta}} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}}}_{\mathbf{K}} c_{sh}
\end{aligned} \tag{84}$$

More specifically, we could write $\mathbf{K}$ as:

$$\begin{aligned}
\mathbf{K} &= \left(\psi \mathbf{Q}^{\frac{\zeta-1}{\zeta}} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}} \\
&= \left(\psi \left(\frac{\psi \xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}} \\
&= \left(\psi^{\zeta} \left(\frac{\xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}}
\end{aligned} \tag{85}$$

3. Since $\frac{c_m}{c_h} = \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)} \left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon}$, then

$$\begin{aligned}
c_m &= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)} \left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} c_h \\
&= \underbrace{\left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)} \left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} \mathbf{K}}_{\mathbf{N} \equiv \mathbf{Y} \cdot \mathbf{K}} \cdot c_{sh}
\end{aligned} \tag{86}$$

Simplify $\mathbf{Y}$:

$$\begin{aligned}
\mathbf{Y} &= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)} \left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} \\
&= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon} \mathbf{K}^{-\frac{\epsilon}{\zeta}} \\
&= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon} \left(\psi^{\zeta} \left(\frac{\xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\epsilon}{1-\zeta}}
\end{aligned} \tag{87}$$

Finally, $\mathbf{N} \equiv \mathbf{Y} \cdot \mathbf{K}$:

$$\begin{aligned}
\mathbf{N} &= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon} \mathbf{K}^{\frac{\zeta-\epsilon}{\zeta}} \\
&= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon} \left(\psi^{\zeta}\left(\frac{\xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta-\epsilon}{\zeta-1}}
\end{aligned} \tag{88}$$

4. By definition, $c_{com} = \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$, which is also a linear function of c_{sh}.

$$\begin{aligned}
c_{com} &= \left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\
&= \left(\phi\left(\mathbf{N}c_{sh}\right)^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\left(\mathbf{K}c_{sh}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\
&= \underbrace{\left(\phi\mathbf{N}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\mathbf{K}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}}_{\mathbf{F}} c_{sh}
\end{aligned} \tag{89}$$

To simplify $\mathbf{F}$:

$$\begin{aligned}
\mathbf{F} &= \left(\phi\mathbf{N}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\mathbf{K}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\
&= \left(\phi(\mathbf{YK})^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\mathbf{K}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\
&= \left(\phi\mathbf{Y}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)\right)^{\frac{\epsilon}{\epsilon-1}} \mathbf{K}
\end{aligned} \tag{90}$$

Since $\mathbf{Y} = \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon}\left(\psi^{\zeta}\left(\frac{\xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\epsilon}{1-\zeta}}$ and $\mathbf{K} = \left(\psi^{\zeta}\left(\frac{\xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}}$, then

$$\begin{aligned}
\mathbf{F} = &\left(\phi\left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right]^{\epsilon-1}\left(\psi^{\zeta}\left(\frac{\xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\epsilon-1}{1-\zeta}} + (1-\phi)\right)^{\frac{\epsilon}{\epsilon-1}} \\
&\cdot \left(\psi^{\zeta}\left(\frac{\xi}{(1-\psi)\rho}\right)^{\zeta-1} + (1-\psi)\right)^{\frac{\zeta}{\zeta-1}}
\end{aligned} \tag{91}$$

5. Because $\theta\phi c_{com}^{-\sigma}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_m^{-\frac{1}{\epsilon}} = \lambda$, now we simplify λ.

$$
\begin{aligned}
\lambda &= \theta\phi c_{com}^{-\sigma}\left(\phi c_m^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)c_h^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{1}{\epsilon-1}} c_m^{-\frac{1}{\epsilon}} \\
&= \theta\phi c_{com}^{-\sigma} c_{com}^{\frac{1}{\epsilon}} c_m^{-\frac{1}{\epsilon}} \\
&= \theta\phi \mathbf{F}^{-\sigma}\left(\frac{\mathbf{F}}{\mathbf{N}}\right)^{\frac{1}{\epsilon}} c_{sh}^{-\sigma}
\end{aligned}
\tag{92}
$$

6. Since $l = \left(\frac{1-\theta}{\lambda p\xi}\right)^{\frac{1}{\sigma}}$, l can be written as:

$$
l = \underbrace{\left(\frac{1-\theta}{p\xi\theta\phi}\right)^{\frac{1}{\sigma}} \mathbf{F}\left(\frac{\mathbf{F}}{\mathbf{N}}\right)^{-\frac{1}{\epsilon\sigma}}}_{\mathbf{L}} c_{sh} \tag{93}
$$

7. In the budget constraint: $c_m + pc_{ph} = p\xi(\bar{T} - h - l)$, could solve for h, thus everything else.

$$
\begin{aligned}
\mathbf{N}\rho h + p\mathbf{Q}\rho h &= p\xi\bar{T} - p\xi h - p\xi\mathbf{J}\rho h \\
\left(\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho\right) h &= p\xi\bar{T} \\
h &= \frac{p\xi\bar{T}}{\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho}
\end{aligned}
\tag{94}
$$

The other variables could also be solved as:

$$
c_{sh} = \rho h = \frac{\rho \cdot p\xi\bar{T}}{\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho} \tag{95}
$$

$$
c_{ph} = \mathbf{Q}c_{sh} = \frac{\mathbf{Q}\rho \cdot p\xi\bar{T}}{\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho} \tag{96}
$$

$$
c_h = \mathbf{K}c_{sh} = \frac{\mathbf{K}\rho \cdot p\xi\bar{T}}{\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho} \tag{97}
$$

$$
c_m = \mathbf{N}c_{sh} = \frac{\mathbf{N}\rho \cdot p\xi\bar{T}}{\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho} \tag{98}
$$

$$
c_{com} = \mathbf{F}c_{sh} = \frac{\mathbf{F}\rho \cdot p\xi\bar{T}}{\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho} \tag{99}
$$

$$
l = \mathbf{J}c_{sh} = \frac{\mathbf{J}\rho \cdot p\xi\bar{T}}{\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho} \tag{100}
$$

8. The optimal consumption bundle, the ratio between c_m and c_{ph}:

$$
\begin{aligned}
\frac{c_m}{c_{ph}} &= \frac{\mathbf{N}}{\mathbf{Q}} \\
&= \frac{\left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon}\cdot\left(\psi\left(\frac{\psi\xi}{\rho(1-\psi)}\right)^{\zeta-1}+(1-\psi)\right)^{\frac{\zeta}{\zeta-1}}}{\left(\frac{\psi\xi}{\rho(1-\psi)}\right)^{\zeta}} \\
&= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon}\cdot\left[\psi+(1-\psi)\left(\frac{\psi\xi}{\rho(1-\psi)}\right)^{1-\zeta}\right]^{\frac{\zeta}{\zeta-1}}
\end{aligned} \tag{101}
$$

Derive the expressions for time use

1. Simplify $\rho\mathbf{J}$:

   First of all, we know that

$$
\begin{aligned}
\mathbf{J} &= \left(\frac{1-\theta}{\theta\phi p\xi}\right)^{\frac{1}{\sigma}}\mathbf{F}\left(\frac{\mathbf{F}}{\mathbf{N}}\right)^{-\frac{1}{\epsilon\sigma}} \\
&= \left(\frac{1-\theta}{\theta\phi p\xi}\right)^{\frac{1}{\sigma}}\mathbf{K}\left(\phi\mathbf{Y}^{\frac{\epsilon-1}{\epsilon}}+1-\phi\right)^{\frac{\epsilon}{\epsilon-1}}\left(\phi+(1-\phi)\mathbf{Y}^{\frac{1-\epsilon}{\epsilon}}\right)^{\frac{1}{\sigma(1-\epsilon)}}
\end{aligned} \tag{102}
$$

   Plug in $\mathbf{Y}$, which is

$$
\begin{aligned}
\mathbf{Y} &= \left[\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\left(\frac{c_h}{c_{sh}}\right)^{-\frac{1}{\zeta}}\right]^{\epsilon} \\
&= \left(\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{-\frac{\epsilon}{\zeta}}
\end{aligned} \tag{103}
$$

   Then we have

$$
\begin{aligned}
\mathbf{J} = \frac{1-\theta}{\theta\phi p\xi}\mathbf{K}&\left(\phi\left(\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right)^{\epsilon-1}\mathbf{K}^{\frac{1-\epsilon}{\zeta}}+1-\phi\right)^{\frac{\epsilon}{\epsilon-1}} \\
&\cdot\left(\phi+(1-\phi)\left(\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right)^{1-\epsilon}\mathbf{K}^{\frac{\epsilon-1}{\zeta}}\right)^{\frac{1}{1-\epsilon}} \\
&\cdot\underbrace{\left(\frac{1-\theta}{\theta\phi p\xi}\right)^{\frac{1-\sigma}{\sigma}}\left(\phi+(1-\phi)\left(\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right)^{1-\epsilon}\mathbf{K}^{\frac{\epsilon-1}{\zeta}}\right)^{\frac{1-\sigma}{\sigma(1-\epsilon)}}}_{\equiv\mathbf{U}}
\end{aligned} \tag{104}
$$

Then,

$$
\begin{aligned}
\rho\mathbf{J} &= \frac{1-\theta}{\theta\phi p\xi}\mathbf{K}\left(\rho\phi\left(\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right)^{\epsilon-1}\mathbf{K}^{\frac{1-\epsilon}{\zeta}}+\rho(1-\phi)\right)^{\frac{\epsilon}{\epsilon-1}} \\
&\quad\cdot\left(\rho\phi+\rho(1-\phi)\left(\frac{p\xi\phi}{\rho(1-\phi)(1-\psi)}\right)^{1-\epsilon}\mathbf{K}^{\frac{\epsilon-1}{\zeta}}\right)^{\frac{1}{1-\epsilon}}\cdot\mathbf{U} \\
&= \frac{1-\theta}{\theta\phi p\xi}\mathbf{K}(1-\phi)^{\frac{\epsilon}{\epsilon-1}}\phi^{\frac{1}{1-\epsilon}}\left(\rho^{2-\epsilon}\underbrace{(p\xi)^{\epsilon-1}\left(\frac{\phi}{1-\phi}\right)^{\epsilon}(1-\psi)^{1-\epsilon}\mathbf{K}^{\frac{1-\epsilon}{\zeta}}}_{\mathcal{O}}+\rho\right)^{\frac{\epsilon}{\epsilon-1}} \\
&\quad\cdot\left(\rho+\rho^{\epsilon}\underbrace{(p\xi)^{1-\epsilon}\left(\frac{\phi}{1-\phi}\right)^{-\epsilon}(1-\psi)^{\epsilon-1}\mathbf{K}^{\frac{\epsilon-1}{\zeta}}}_{\mathcal{O}^{-1}}\right)^{\frac{1}{1-\epsilon}}\cdot\mathbf{U} \\
&= \frac{1-\theta}{\theta p\xi}\left(\frac{1-\phi}{\phi}\right)^{\frac{\epsilon}{\epsilon-1}}\mathbf{K}\left(\rho^{2-\epsilon}\mathcal{O}+\rho\right)^{\frac{\epsilon}{\epsilon-1}}\left(\rho+\rho^{\epsilon}\mathcal{O}^{-1}\right)^{\frac{1}{1-\epsilon}}\cdot\mathbf{U} \\
&= \frac{1-\theta}{\theta p\xi}\left(\frac{1-\phi}{\phi}\right)^{\frac{\epsilon}{\epsilon-1}}\mathbf{K}(\mathcal{O}\rho^{1-\epsilon}+1)\mathcal{O}^{\frac{1}{\epsilon-1}}\cdot\mathbf{U}
\end{aligned}
\tag{105}
$$

Plug in $\mathcal{O}$:

$$
\begin{aligned}
\rho\mathbf{J} &= \frac{1-\theta}{\theta(1-\psi)}\mathbf{K}^{1-\frac{1}{\zeta}}\left(\rho^{1-\epsilon}(p\xi)^{\epsilon-1}\left(\frac{\phi}{1-\phi}\right)^{\epsilon}(1-\psi)^{1-\epsilon}\mathbf{K}^{\frac{1-\epsilon}{\zeta}}+1\right)\cdot\mathbf{U} \\
&= \frac{1-\theta}{\theta}\left[\rho^{1-\epsilon}(p\xi)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{\frac{\zeta-\epsilon}{\zeta}}+\frac{1}{1-\psi}\mathbf{K}^{1-\frac{1}{\zeta}}\right]\cdot\mathbf{U} \\
&= \frac{1-\theta}{\theta}\left[\rho^{1-\epsilon}(p\xi)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{\frac{\zeta-\epsilon}{\zeta}}+\left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{p\xi}{p\rho}\right)^{\zeta-1}+1\right]\cdot\mathbf{U}
\end{aligned}
\tag{106}
$$

2. Simplify $p\mathbf{P}\rho$:

$$
p\mathbf{Q}\rho = (p\xi)^{\zeta}\left(\frac{\psi}{1-\psi}\right)^{\zeta}(p\rho)^{1-\zeta} \tag{107}
$$

3. Simplify $\mathbf{N}\rho$:

$$
\begin{aligned}
\mathbf{N}\rho &= \mathbf{YK}\rho \\
&= \left(\frac{p\xi\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{1-\frac{\epsilon}{\zeta}}\rho^{1-\epsilon}
\end{aligned}
\tag{108}
$$

4. Derive h:

$$
\begin{aligned}
h &= \frac{p\xi\bar{T}}{\mathbf{N}\rho + p\mathbf{Q}\rho + p\xi + p\xi\mathbf{J}\rho} \\
&= \frac{\bar{T}}{\frac{\mathbf{N}\rho}{p\xi} + \frac{p\mathbf{P}\rho}{p\xi} + 1 + \mathbf{J}\rho} \\
&= \frac{\bar{T}}{(p\xi)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{1-\frac{\epsilon}{\zeta}}\rho^{1-\epsilon} + (p\xi)^{\zeta-1}\left(\frac{\psi}{1-\psi}\right)^{\zeta}(p\rho)^{1-\zeta} + 1 + \rho\mathbf{J}} \\
&= \frac{\theta\bar{T}}{\rho^{1-\epsilon}(p\xi)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{p\xi}{p\rho}\right)^{\zeta-1} + 1} \cdot \underbrace{\frac{1}{\theta + (1-\theta)\mathbf{U}}}_{\text{new term}}
\end{aligned}
\tag{109}
$$

5. Derive l:

$$
\begin{aligned}
l &= \rho\mathbf{J}\cdot h = (1-\theta)\bar{T}\cdot\frac{\mathbf{U}}{\theta + (1-\theta)\mathbf{U}} \\
&= (1-\theta)\bar{T}\cdot\underbrace{\frac{1}{\frac{\theta}{\mathbf{U}} + 1 - \theta}}_{\text{new term}}
\end{aligned}
\tag{110}
$$

6. Derive n:

$$
\begin{aligned}
n &= \bar{T} - l - h \\
&= \bar{T} - (1-\theta)\bar{T}\frac{\mathbf{U}}{\theta + (1-\theta)\mathbf{U}} - h \\
&= \theta\bar{T}\frac{\rho^{1-\epsilon}(p\xi)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{p\xi}{p\rho}\right)^{\zeta-1}}{\rho^{1-\epsilon}(p\xi)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{p\xi}{p\rho}\right)^{\zeta-1} + 1} \cdot \underbrace{\frac{1}{\theta + (1-\theta)\mathbf{U}}}_{\text{new term}}
\end{aligned}
\tag{111}
$$

7. Derive the work-to-home production time ratio

$$
\frac{n}{h} = \rho^{1-\epsilon}(p\xi)^{\epsilon-1}\left(\frac{\phi}{(1-\phi)(1-\psi)}\right)^{\epsilon}\mathbf{K}^{\frac{\zeta-\epsilon}{\zeta}} + \left(\frac{\psi}{1-\psi}\right)^{\zeta}\left(\frac{p\xi}{p\rho}\right)^{\zeta-1}
\tag{112}
$$