

Introduction to Machine Learning Course

Prof. Mingkui Tan

SCUT Machine Intelligence Laboratory (SMIL)



SMIL内部资料 请勿外泄

Contents

- 1 Course Outline
- 2 Machine Learning
- 3 Probability Theory
- 4 Bayes' Theorem
- 5 Information Theory

课程教学大纲

- 机器学习基础 (3)
- Linear Regression and Gradient Descent (3)
线性回归与梯度下降
- Linear Classification and Stochastic Gradient Descent (3)
线性分类、支持向量机、随机梯度算法
- Logistic Regression and Ensemble Methods (Decision Tree, Adaboost) (3)
逻辑回归与集成学习算法
- Overfitting, Underfitting, Regularization and Cross-Validation (3)
过拟合、欠拟合、正则化与交叉验证
- Multiclass Classification and Cross-entropy Loss (3)
多类分类和交叉熵损失函数

课程教学大纲

- ~~Clustering and Dimension Reduction (PCA, Feature Selection) (3)~~
— 聚类算法与维度约简
- ~~Recommendation Systems (3)~~ 推荐系统
- Neural Networks and Deep Learning (Basics) (3)
神经网络与深度学习
- Image Processing Basics and Convolutional Neural Networks (3)
神经网络与深度学习
- 序列模型(RNN)、Transformer、Bert (3)
- ~~Markov Decision Process, Reinforcement Learning and AlphaGO (3)~~
马尔可夫决策过程、强化学习及AlphaGo

实验教学大纲

■ 随堂实验

- Linear Regression and Gradient Descent (2)
线性回归与梯度下降
- Linear Classification with Stochastic Gradient Descent (2)
线性分类、支持向量机、随机梯度算法

■ 课程实验

- Classification with AdaBoost (4)
科技论文阅读、写作;
逻辑回归与集成学习算法
- Face Detection and Recognition (4)
人脸检测与识别基础
- 基于Transformer的中英文翻译 (4)

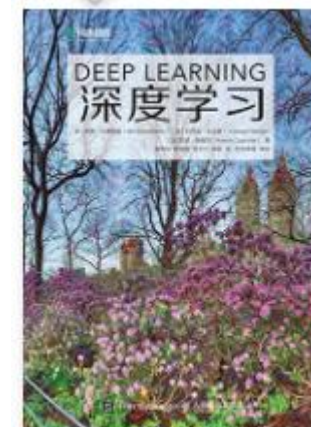
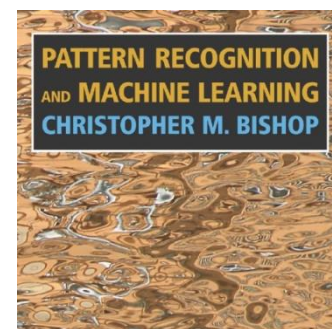
考核标准+参考书

■ 考核标准

考试 (50%) + 平时成绩 (25%) + 技术报告 (25%)

■ 参考书

- Pattern Recognition and Machine Learning **By Bishop**
- Understanding Machine Learning: From Theory to Algorithms **By Shai Shalev-Shwartz and Shai Ben-David**
- 深度学习 by Ian Goodfellow (伊恩·古德费洛)
- 《机器学习》 By 周志华



联系方式

■ 邮箱

mingkuitan@scut.edu.cn

■ QQ群

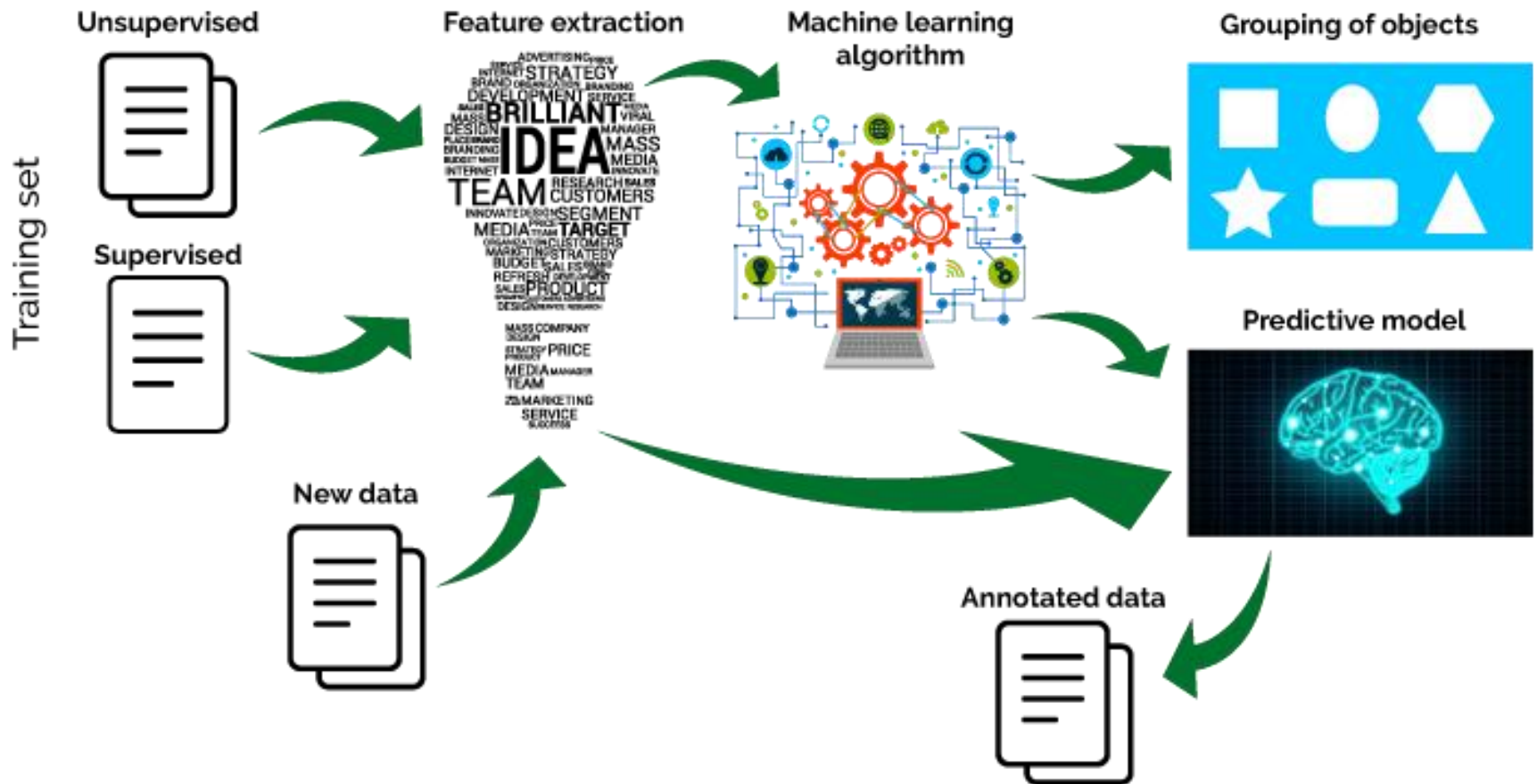


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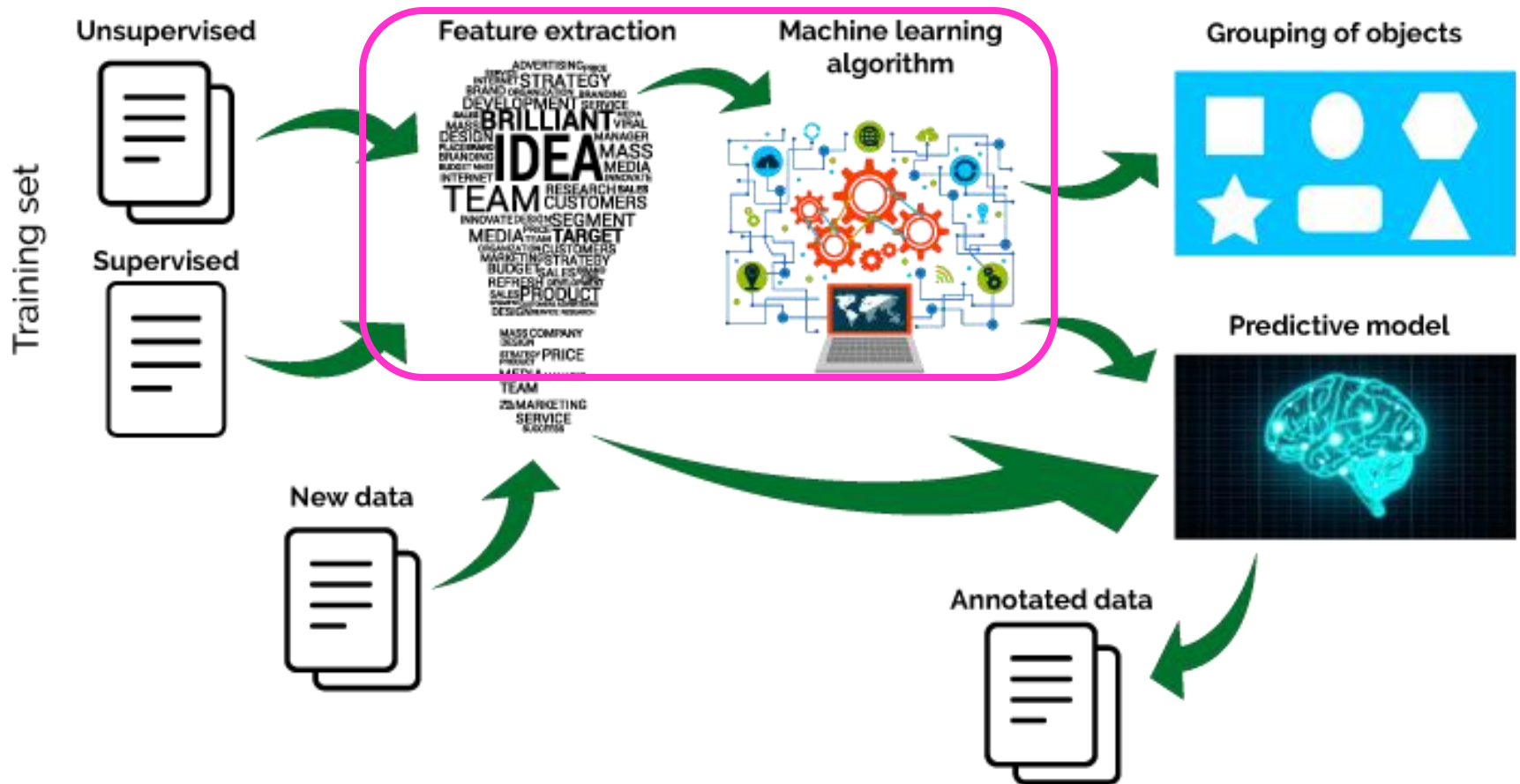
Big Picture: Machine Learning

Machine Learning



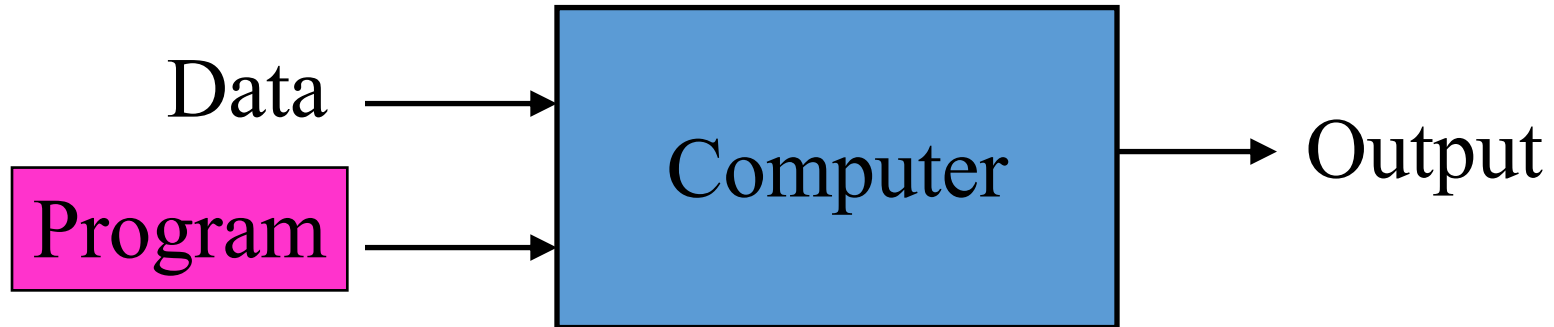
Big Picture: Machine Learning

Deep Learning

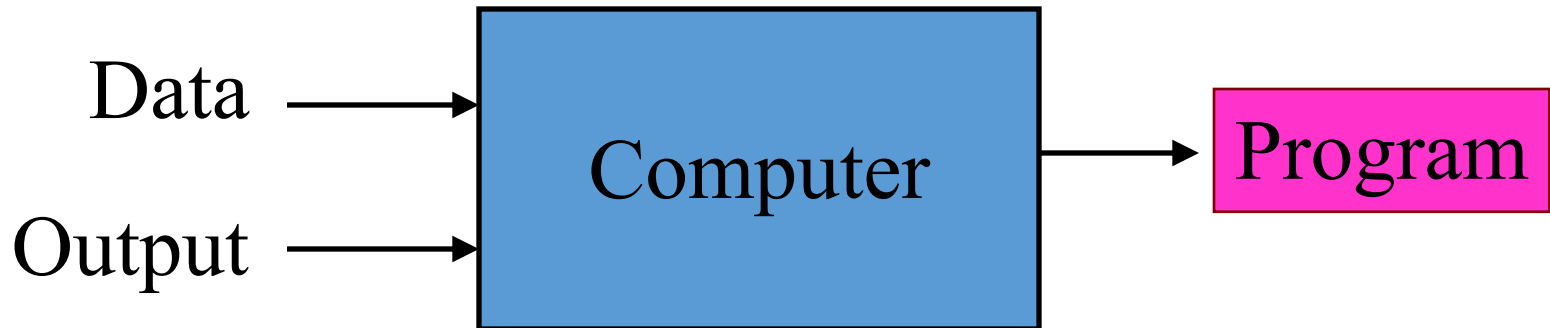


Traditional Programming and Machine Learning

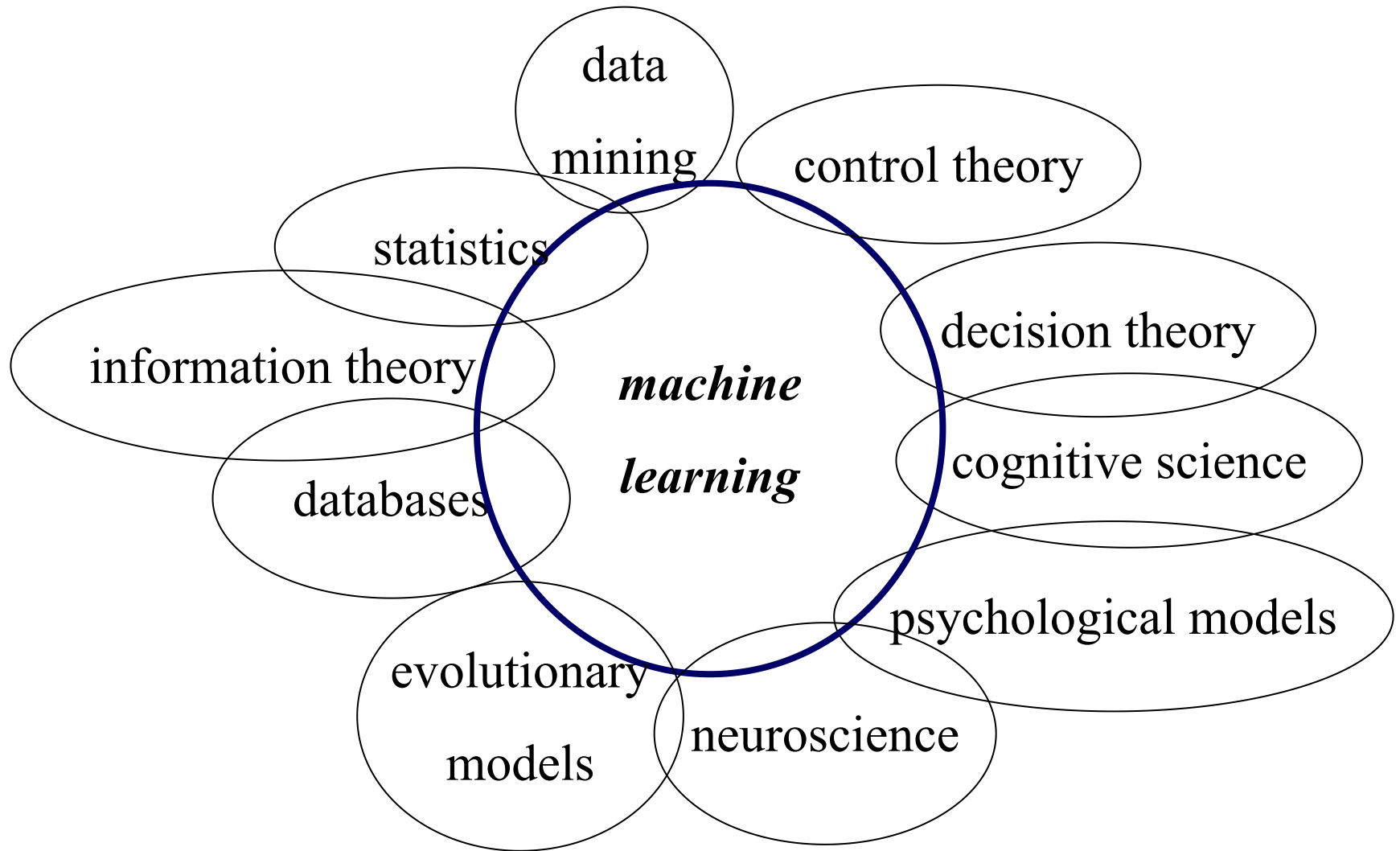
■ Traditional Programming



■ Machine Learning



Related Fields



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Probability Theory

■ Random Variables

$$P(A) = \frac{1}{6}, A$$



- Random variables describe the outcome of a random experiment in terms of a (real) number
- A random experiment is an experiment that can (in principle) be repeated several times under the same conditions
- Discrete or continuous random variables
- Independent and identically distributed (iid) experiment vs non-iid experiment

Probability Theory

■ Marginal Probability

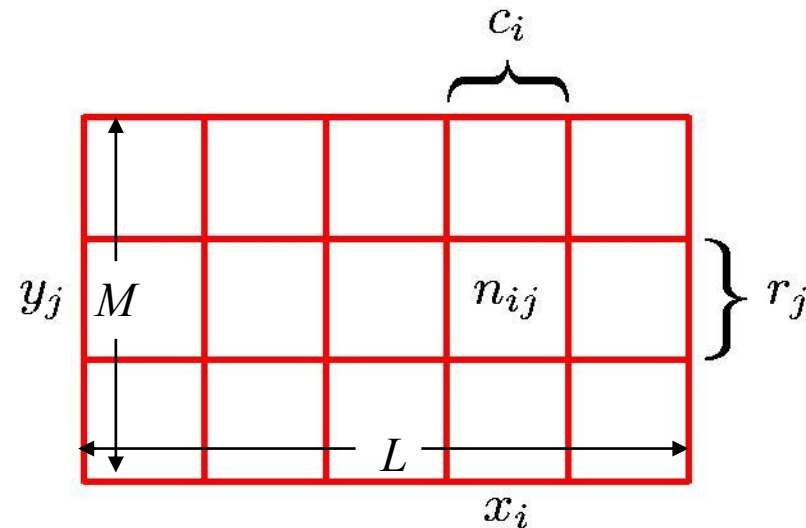
$$P(X = x_i) = \frac{c_i}{L}$$

■ Joint Probability

$$P(X = x_i, Y = y_j) = \frac{n_{ij}}{L \times M} = \frac{c_i \times r_j}{L \times M}$$

■ Conditional Probability

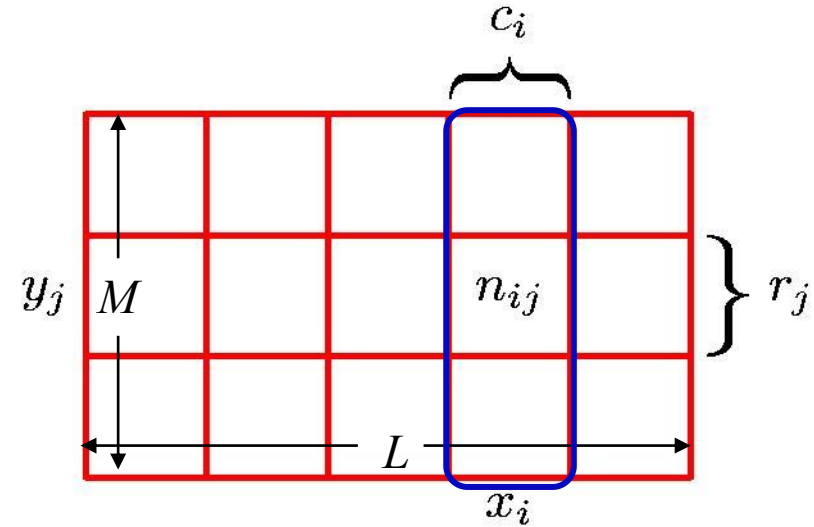
$$P(Y = y_j | X = x_i) = \frac{r_j}{M}$$



Probability Theory

■ Sum Rule

$$\begin{aligned} P(X = x_i) &= \frac{c_i}{L} = \frac{1}{L \times M} \sum_j n_{ij} \\ &= \sum_j P(X = x_i, Y = y_j) \end{aligned}$$



■ Product Rule

$$\begin{aligned} P(X = x_i, Y = y_j) &= \frac{n_{ij}}{L \times M} = \frac{r_j}{M} \cdot \frac{c_i}{L} \\ &= P(Y = y_j | X = x_i) P(X = x_i) \end{aligned}$$

Marginalization

Marginal Probability

Joint Probability

$$P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$$

$$= \sum_j P(X = x_i | Y = y_j) P(Y = y_j)$$

Conditional Probability

Marginal Probability

Y \ X	x ₁	x ₂	x ₃	x ₄	p _Y (Y)↓
y ₁	4/32	2/32	1/32	1/32	8/32
y ₂	2/32	4/32	1/32	1/32	8/32
y ₃	2/32	2/32	2/32	2/32	8/32
y ₄	8/32	0	0	0	8/32
p _X (X) →	16/32	8/32	4/32	4/32	32/32

Margin

This concept is called "marginal" because it can be found by summing values in a table along rows or columns, and writing the sum in the **margins** of the table

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Bayes' Theorem

The Rules of Probability

$$\text{Sum Rule: } P(X) = \sum_Y P(X, Y)$$

$$\text{Product Rule: } P(X, Y) = P(Y|X)P(X)$$

Bayes' Theorem

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)}$$

$$P(X) = \sum_Y P(X|Y)P(Y)$$

Bayes' Theorem

$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$

Posterior probability $P(Y|X)$: the likelihood of event Y occurring given that X is true, $P(Y|X)$ is a conditional probability

Posterior probability $P(X|Y)$: the likelihood of event X occurring given that Y is true, $P(X|Y)$ is a conditional probability

Prior probability $P(X)$ and $P(Y)$: the probabilities of observing X and Y independently of each other (the marginal probability)

Bayes' Theorem

$$P(\text{"taking a shower"}|\text{"wet"}) = P(\text{"wet"}|\text{"taking a shower"}) \frac{P(\text{"taking a shower"})}{P(\text{"wet"})}$$

$$P(\text{reason}|\text{observation}) = P(\text{observation}|\text{reason}) \frac{P(\text{reason})}{P(\text{observation})}$$

- Often useful in diagnosis situations, since $P(\text{observation}|\text{reason})$ might be easily determined
- Useful for reasoning
- Often delivers surprising results

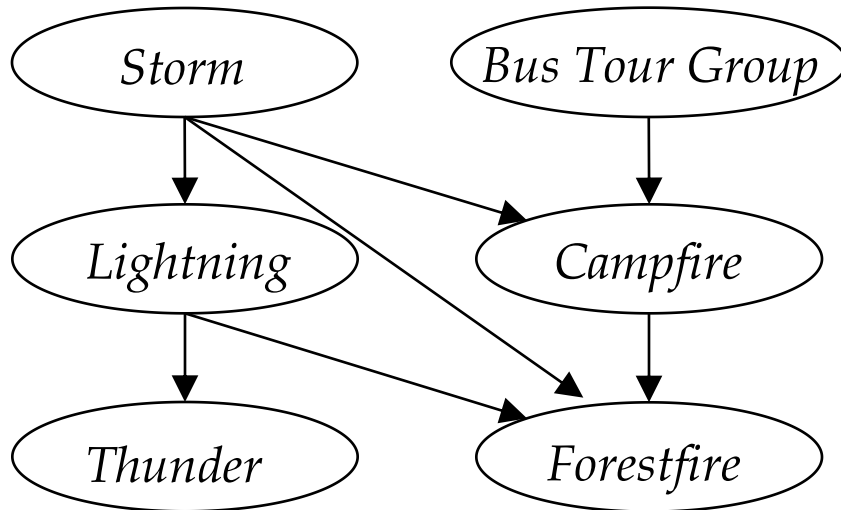
Bayes' Theorem in Bayesian Learning

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

- $P(h)$: prior probability of hypothesis h
- $P(D)$: prior probability of training data D
- $P(h|D)$: posterior probability of h given D
- $P(D|h)$: posterior probability of D given h

Bayesian Net

- Network represents conditional independence assertions
- Each node conditionally independent of its non-descendants, given its immediate predecessors (e.g. Campfire and Lightning are independence conditioned on Storm)



conditional probability tables (CPT)

	$S \wedge B$	$S \wedge \neg B$	$\neg S \wedge B$	$\neg S \wedge \neg B$
C	0.4	0.1	0.8	0.2
$\neg C$	0.6	0.9	0.2	0.8

C : Campfire

S : Storm

B : Bus Tour Group

Example

■ Random variables X and Y

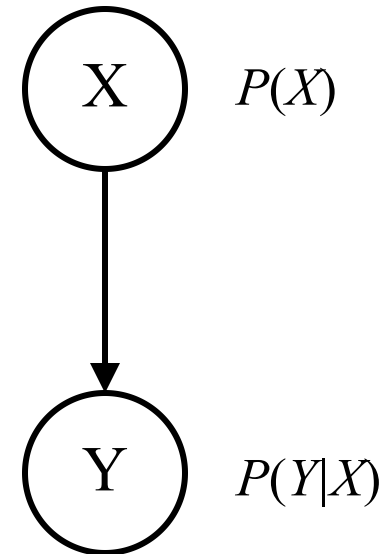
X : It is raining

Y : The grass is wet

■ X affects Y

Or, Y is a symptom of X

■ Draw two nodes and link them



■ Define the CPT(conditional probability tables) for each node

- $P(X)$ and $P(Y|X)$

■ Typical use: we observe Y and we want to query $P(X|Y)$

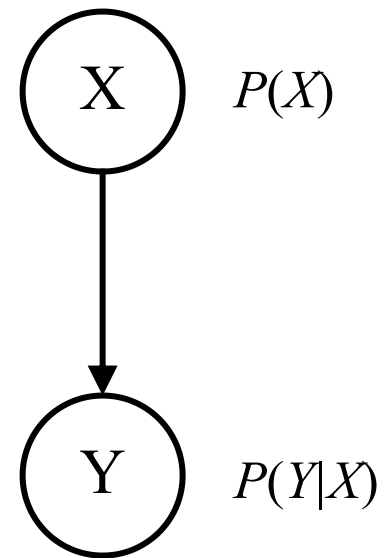
- - Y is an evidence variable
- X is a query variable

Example

■ What is $P(X|Y)$?

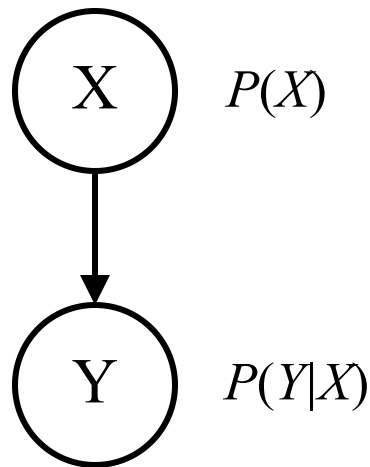
- Given that we know the CPTs of each node in the graph

$$\begin{aligned} P(X \mid Y) &= \frac{P(Y \mid X)P(X)}{P(Y)} \\ &= \frac{P(Y \mid X)P(X)}{\sum_X P(X, Y)} \\ &= \frac{P(Y \mid X)P(X)}{\sum_X P(Y \mid X)P(X)} \end{aligned}$$

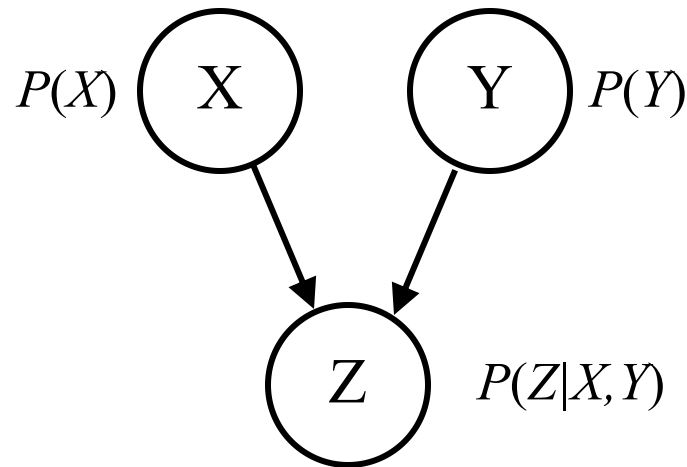


Belief Nets Represent Joint Probability

- The joint probability function can be calculated directly from the network
- It is the product of the CPTs of all the nodes
- $P(var_1, ..., var_n) = \prod_i P(var_i | Parents(var_i))$



$$P(X, Y) = P(X) P(Y|X)$$



$$P(X, Y, Z) = P(X) P(Y) P(Z|X, Y)$$

Probability Densities

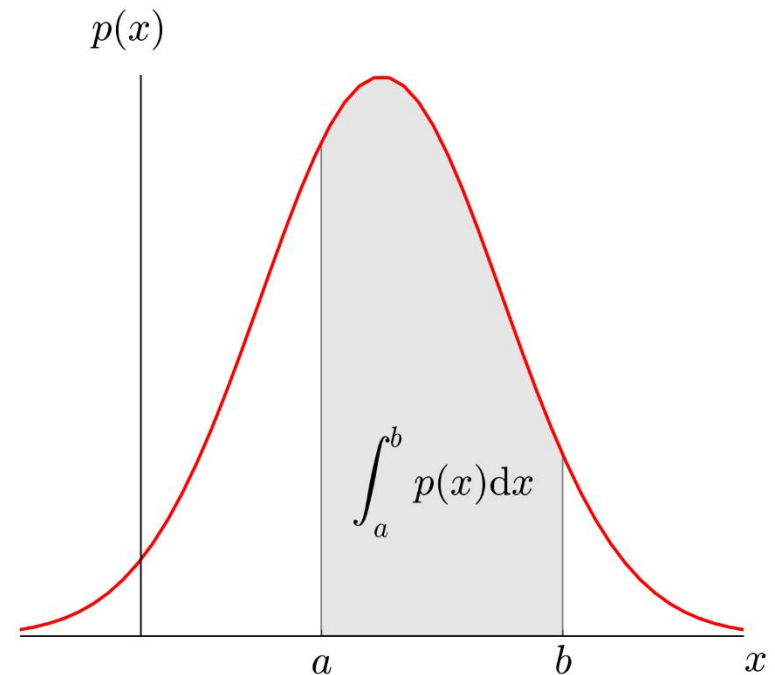
- The probability density function $p(x)$ has the following properties

$$p(x) \geq 0$$

$$p(x \in (a, b)) = \int_a^b p(x) dx$$

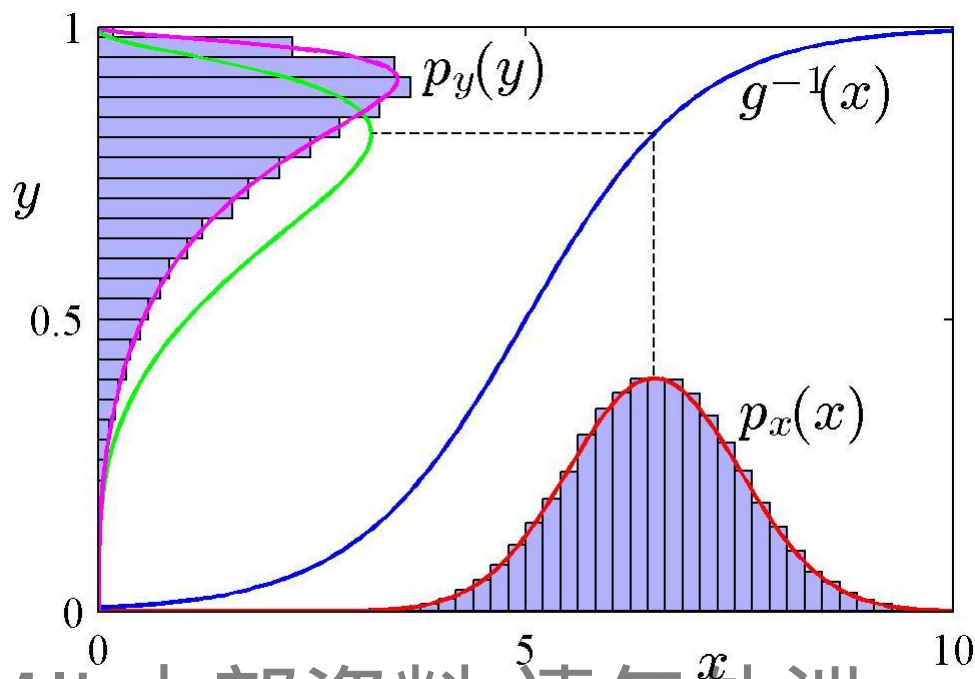
$$P(z) = \int_{-\infty}^z p(x) dx$$

$$\int_{-\infty}^{\infty} p(x) dx = 1$$



Transformed Densities

- x has a probability density $p_x(x)$
- $y = h(x)$ is some strictly monotonic continuous function
- Probability density $p_y(y)$ can be transformed from $p_x(x)$



$$y = h(x) = g^{-1}(x)$$

$$\begin{aligned} p_y(y) &= p_x(x) \left| \frac{dx}{dy} \right| \\ &= p_x(g(y)) |g'(y)| \end{aligned}$$

Maximum Likelihood Estimation

- A density f usually contains parameters $\theta \in \Omega$: $f(x|\theta)$
Parameters θ : a scalar or a vector

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Question: How to estimate θ given data $\mathcal{D} = \{x_i\}$?
- Likelihood function of θ given x :

$$L(\theta|x) = P(X = x|\theta)$$

- Likelihood function of θ given $\mathcal{D} = \{x_i\}$:

$$L_{\mathcal{D}}(\theta) = P(\mathcal{D}|\theta) = \prod_m P(x_i|\theta)$$

Maximum Likelihood Estimation

- Likelihood function of θ given $\mathcal{D} = \{x_i\}$ (iid x_i)

$$L_{\mathcal{D}}(\theta) = P(\mathcal{D}|\theta) = \prod_i P(x_i|\theta)$$

- Estimate θ by

$$\theta_* = \operatorname{argmax}_{\theta} \left(\prod_i P(x_i|\theta) \right)$$

- In practice, often use log likelihood function

$$\theta_* = \operatorname{argmax}_{\theta} \log \left(\prod_i P(x_i|\theta) \right)$$

- Then, we have

$$\theta_* = \operatorname{argmax}_{\theta} \left(\sum \log(P(x_i|\theta)) \right)$$

Maximum a Posteriori Estimation

- Replace the likelihood in the MLE formula with the posterior, and we get:

$$\theta_{MAP} = \operatorname{argmax}_{\theta} P(X|\theta)P(\theta)$$

$$= \operatorname{argmax}_{\theta} \log P(X|\theta) + \log P(\theta)$$

$$= \operatorname{argmax}_{\theta} \log \prod_i P(x_i|\theta) + \log P(\theta)$$

$$= \operatorname{argmax}_{\theta} \sum_i \log P(x_i|\theta) + \log P(\theta)$$

MLE vs MAP

- If we use uniform prior in MAP estimation, $P(\theta)$ is a const, so we have:

$$\begin{aligned}\theta_{MAP} &= \operatorname{argmax} \sum_i \log P(x_i|\theta) + \log P(\theta) \\ &= \operatorname{argmax} \sum_i \log P(x_i|\theta) + \text{const} \\ &= \operatorname{argmax} \sum_i \log P(x_i|\theta) = \theta_{MLE}\end{aligned}$$

- MLE is a special case of MAP, where the prior is uniform

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Probability and Information Theory

■ Information measure of an event A

$$I(A) = -\log_b P(A)$$

$I(A)$: self-information or information content, random variable

$P(A)$: probability of the event happening

b : base, usually $b=2$

base 2 = bits base 3 = trits

base 10 = Hartleys base e = nats

Information and Probability

■ Examples

The Chinese football team lost:

$$P(A)=1 \qquad I(A) = -\log_2 P(A) = 0$$

The Chinese table tennis team lost:

$$P(A)=0 \qquad I(A) = -\log_2 P(A) = +\infty$$

Probability $P(A)$: The degree of uncertainty of an event

Self-information $I(A)$: The elimination of uncertainty

Entropy

- Entropy is simply the average (expected) amount of the information from the event

$$H(A) = -E[\log_2 P(A)] = -\sum_A P(A) \log_2 P(A)$$

$H(A)$ is maximized when $P(A) = \frac{1}{n}$ for all A

- Joint Entropy

$$H(A, B) = -E[\log_2 P(A, B)] = -\sum_{A, B} P(A, B) \log_2 P(A, B)$$

- Conditional entropy of A given B

$$H(A|B) = -E[\log_2 P(A|B)] = -\sum_{A, B} P(A, B) \log_2 P(A|B)$$

Thank You