Lecture 7

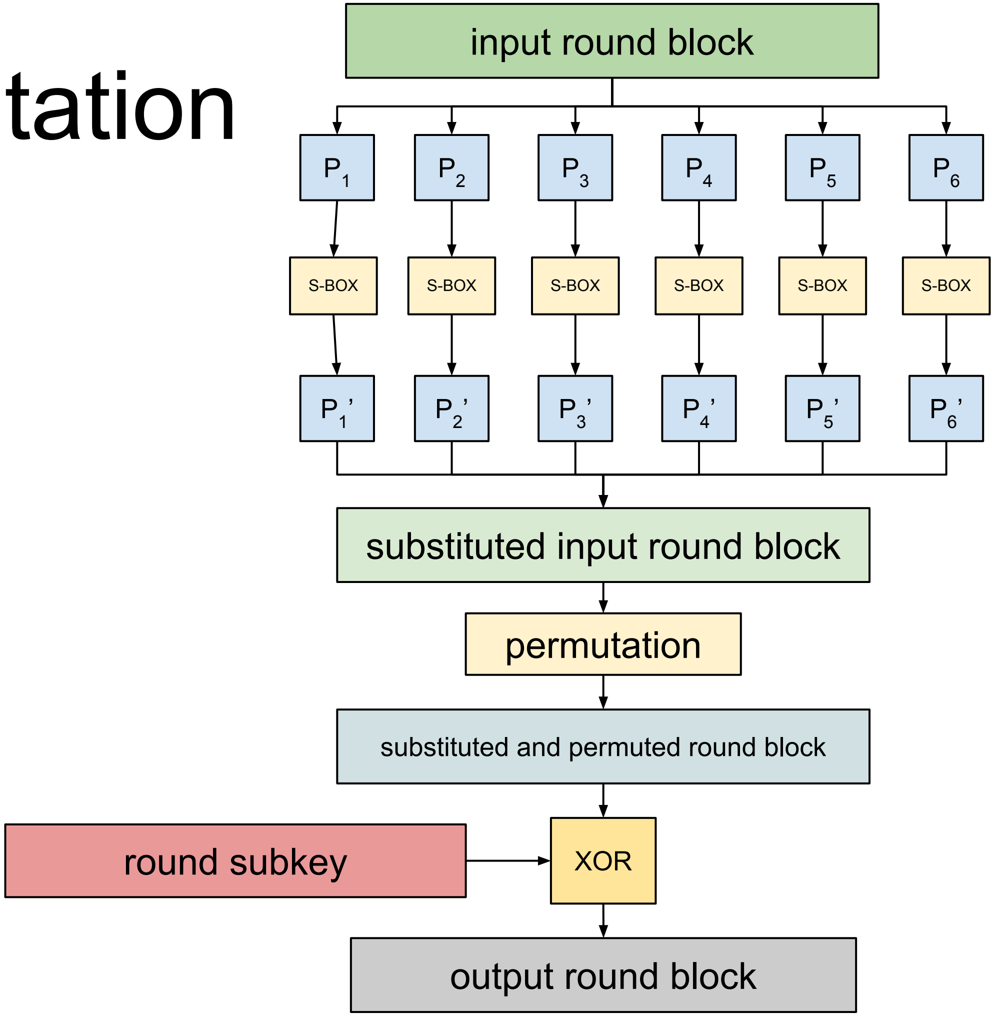
Cryptography 4

Substitution-permutation ciphers

- Square attack

Substitution-permutation ciphers

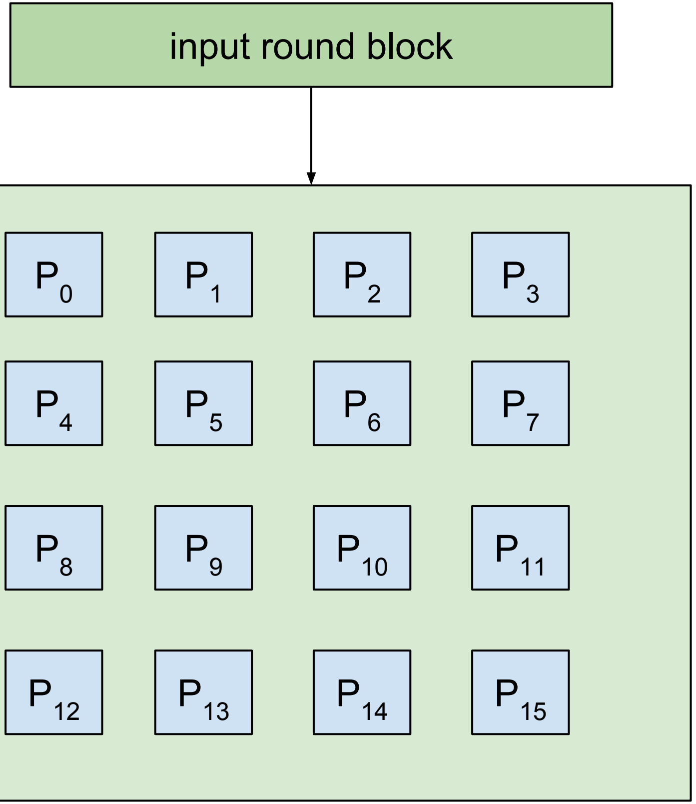
- S-Box
- P-Box
- Combine with key
- Confusion and diffusion



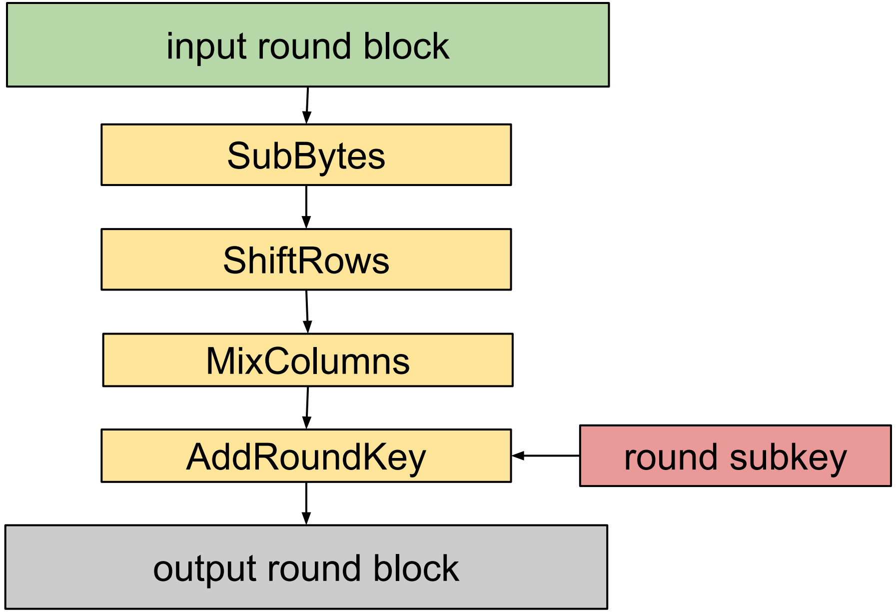
Square attacks

- Integral cryptanalysis
- AES (reduced to 4 rounds (from 10))
- Chosen plaintext

Square

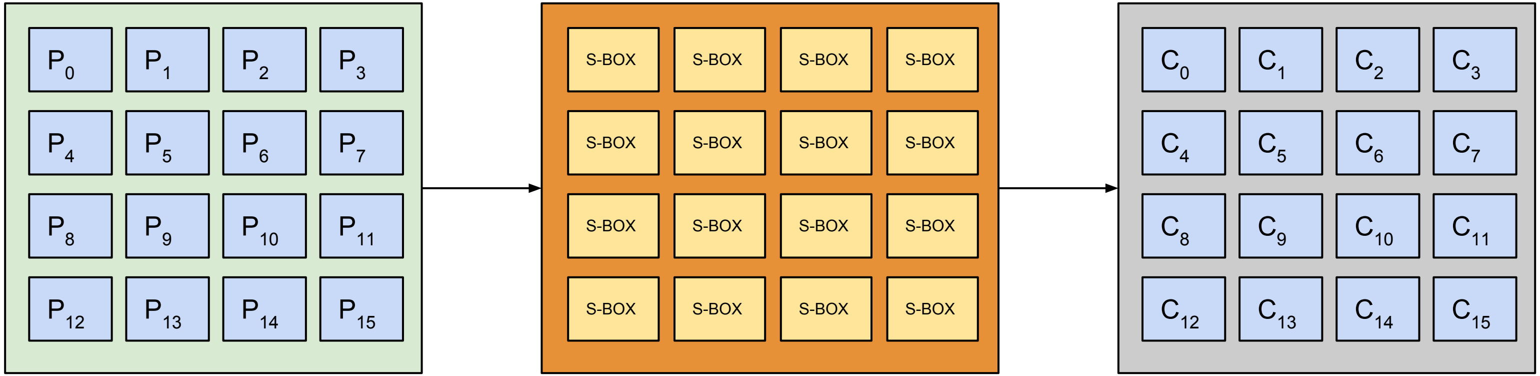

input round block
P_0
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9
P_{10}
P_{11}
P_{12}
P_{13}
P_{14}
P_{15}

AES



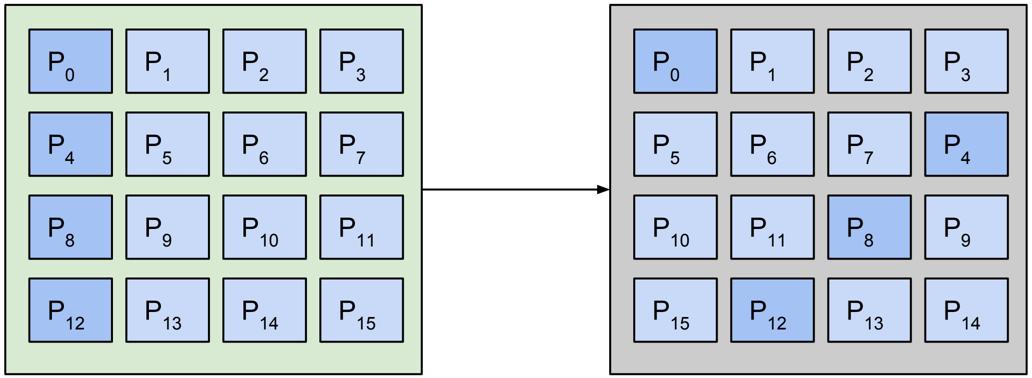
input round block
SubBytes
ShiftRows
MixColumns
AddRoundKey
round subkey
output round block

SubBytes



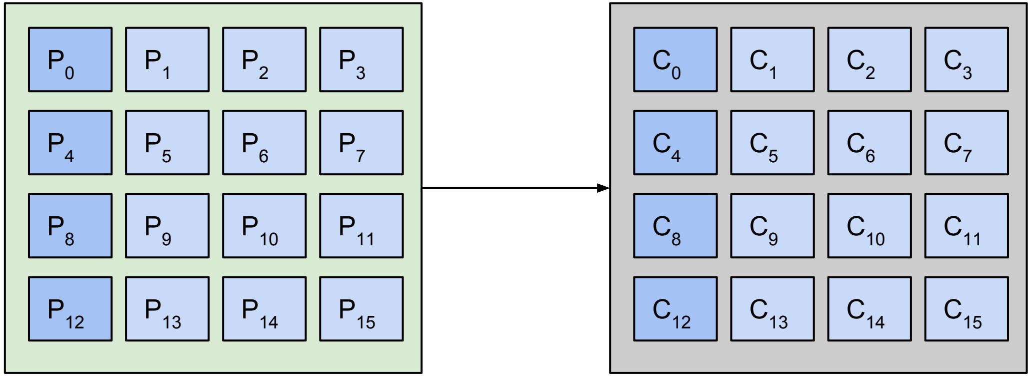
P_0
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9
P_{10}
P_{11}
P_{12}
P_{13}
P_{14}
P_{15}
S-BOX
C_0
C_1
C_2
C_3
C_4
C_5
C_6
C_7
C_8
C_9
C_{10}
C_{11}
C_{12}
C_{13}
C_{14}
C_{15}

ShiftRows



P_0 P_1 P_2 P_3
P_4 P_5 P_6 P_7
P_8 P_9 P_{10} P_{11}
P_{12} P_{13} P_{14} P_{15}
P_0 P_1 P_2 P_3
P_5 P_6 P_7 P_4
P_{10} P_{11} P_8 P_9
P_{15} P_{12} P_{13} P_{14}

MixColumns



$GF(2^8)$, $p(x) = P_{12}x^3+P_8x^2+P_4x+P_0$, $a(x) = 3x^3+x^2+x+2$, $p(x) * a(x)$, mod x^4+1

$C_0 = 2P_0 \oplus 3P_4 \oplus P_8 \oplus P_{12}$

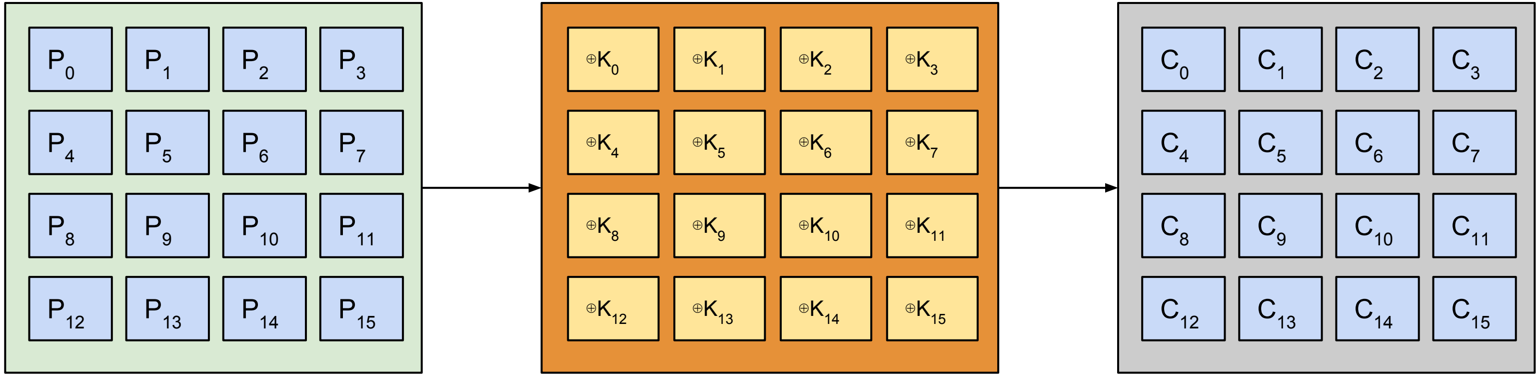
$C_4 = 2P_4 \oplus 3P_8 \oplus P_{12} \oplus P_0$

$C_8 = 2P_8 \oplus 3P_{12} \oplus P_0 \oplus P_4$

$C_{12} = 2P_{12} \oplus 3P_0 \oplus P_4 \oplus P_3$

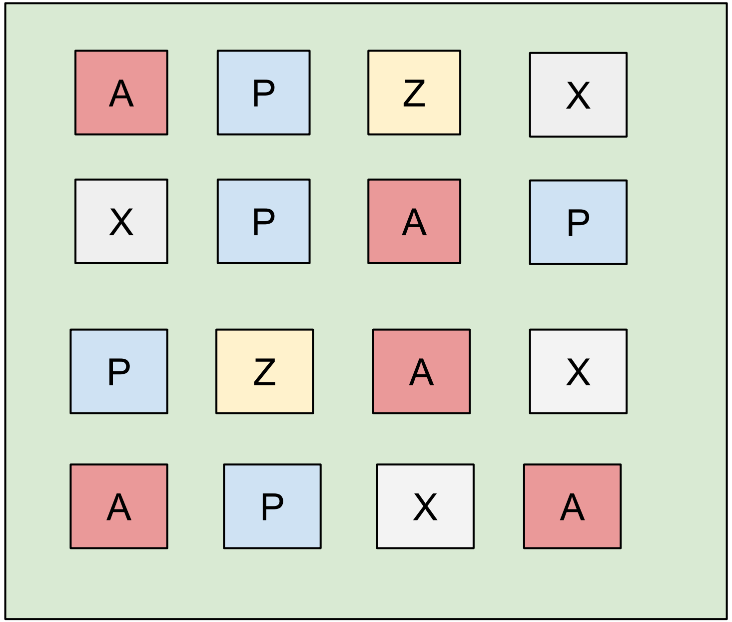
Not present in last round

AddRoundKey

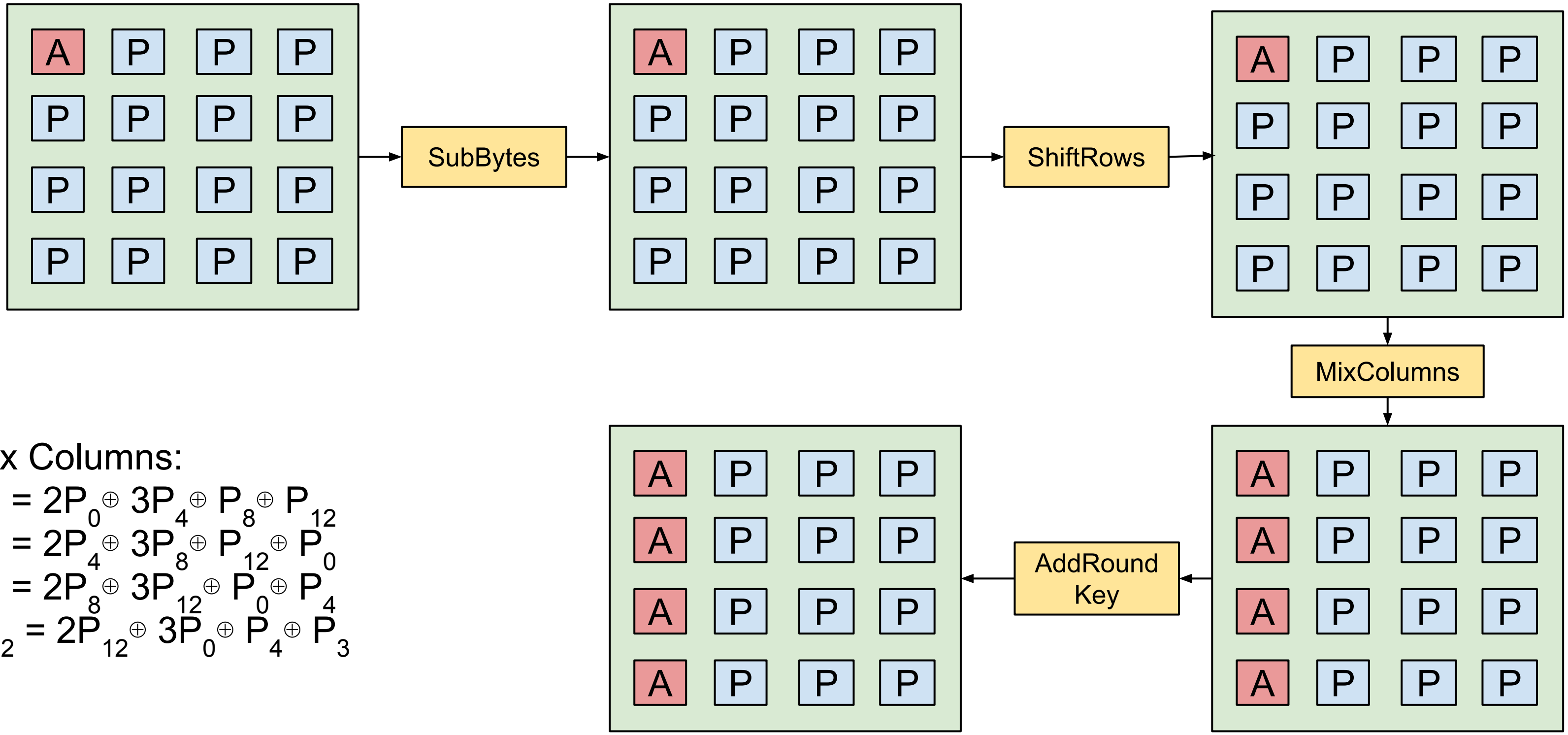


Active and passive byte states

- Set of plaintext
- Passive state: for each two plaintext the same square element
- Active state: for each two plaintext different square element
- Zero state: $\oplus$ elements from all plaintext = 0
    - $0 \oplus 1 \oplus ... \oplus 255 = 0$
- Example:
    - 256 plaintext, first byte: 0,1,2,..,255, other bytes: 0
    - Before first round: top left element is active, other elements passive



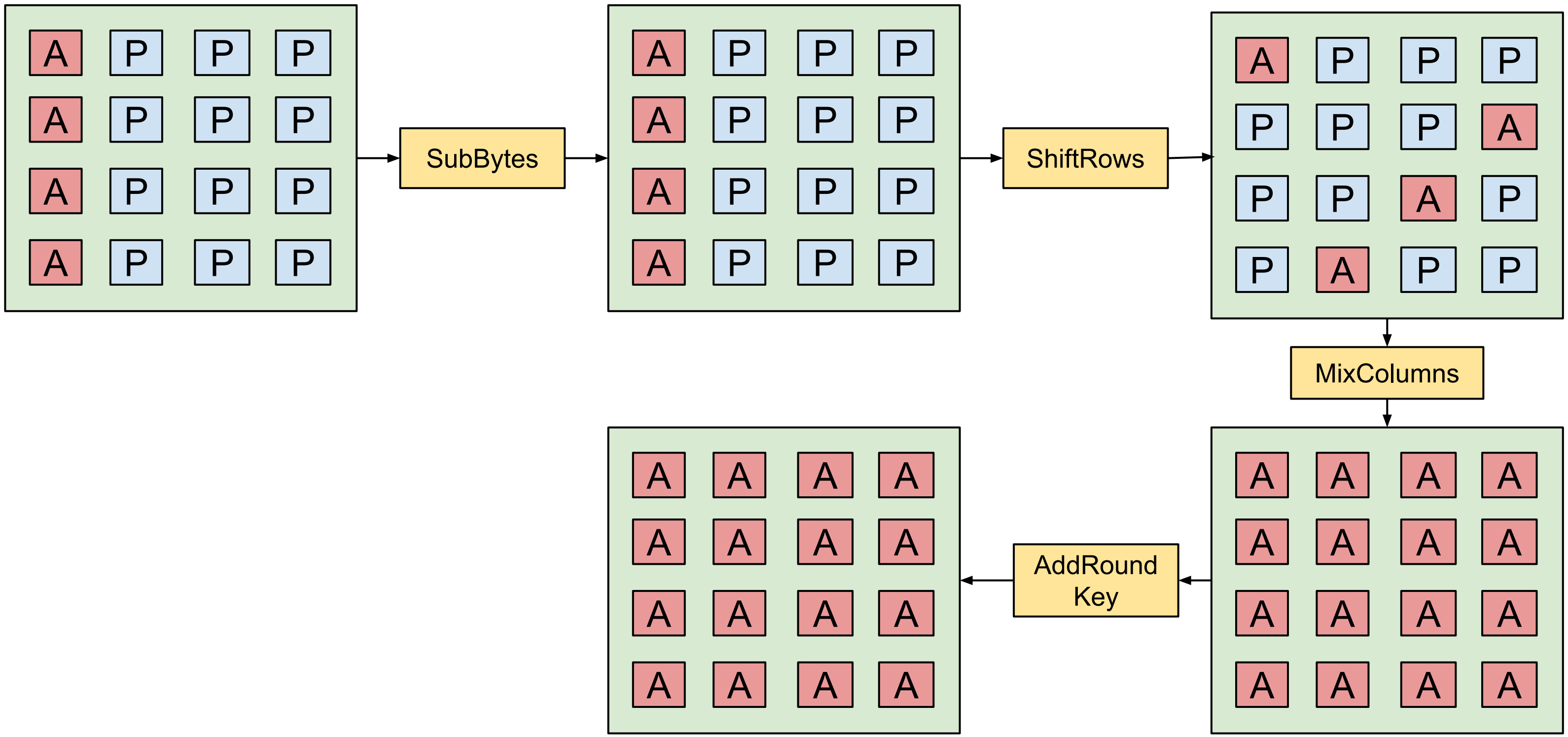
Round 1



Mix Columns:

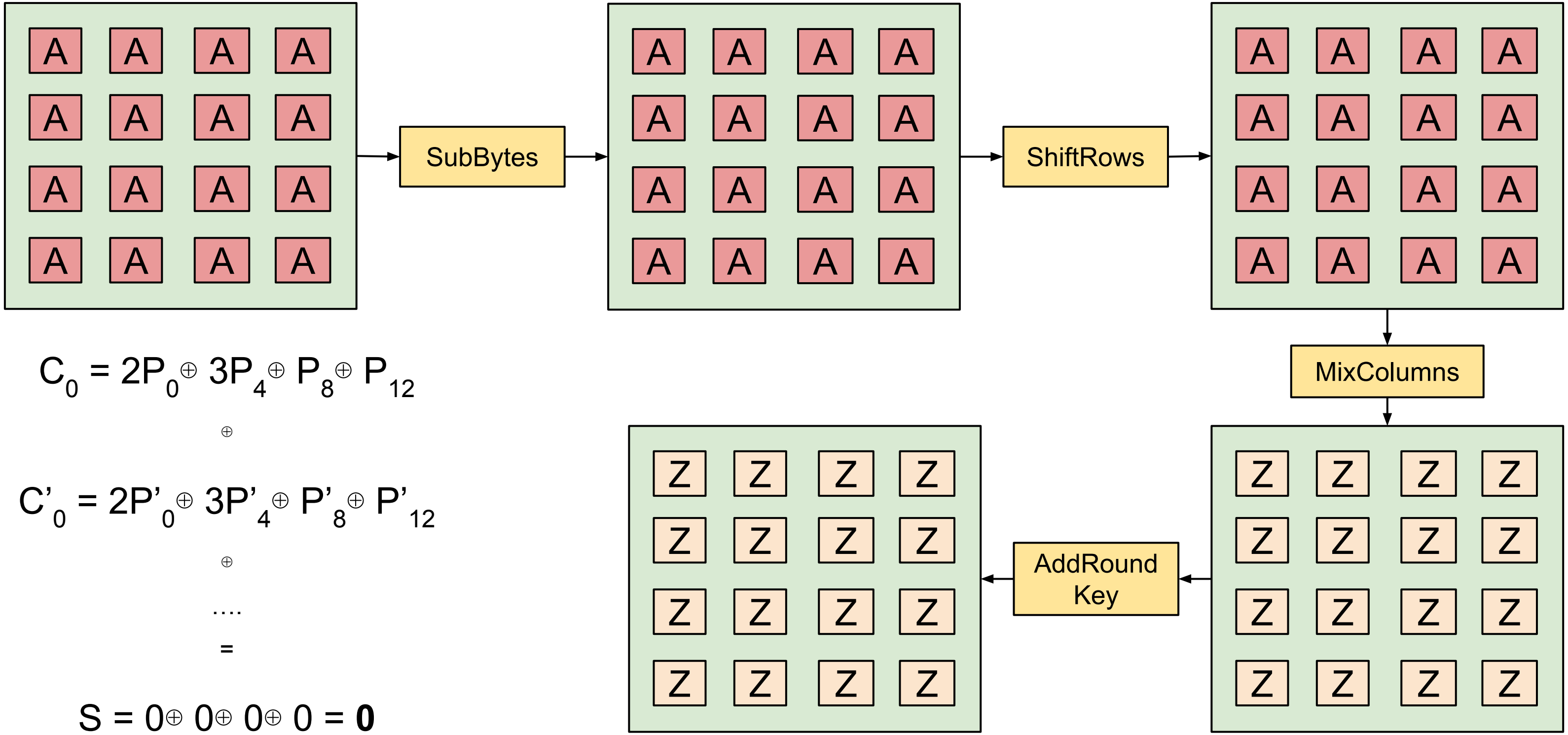
$C_0 = 2P_0 \oplus 3P_4 \oplus P_8 \oplus P_{12}$
$C_4 = 2P_4 \oplus 3P_8 \oplus P_{12} \oplus P_0$
$C_8 = 2P_8 \oplus 3P_{12} \oplus P_0 \oplus P_4$
$C_{12} = 2P_{12} \oplus 3P_0 \oplus P_4 \oplus P_3$

Round 2



A P P P
A P P P
A P P P
A P P P
SubBytes
A P P P
A P P P
A P P P
A P P P
ShiftRows
A P P P
P P P A
P P A P
P A P P
MixColumns
A A A A
A A A A
A A A A
A A A A
AddRound Key
A A A A
A A A A
A A A A
A A A A

Round 3



$$C_0 = 2P_0 \oplus 3P_4 \oplus P_8 \oplus P_{12}$$

$$\oplus$$

$$C'_0 = 2P'_0 \oplus 3P'_4 \oplus P'_8 \oplus P'_{12}$$

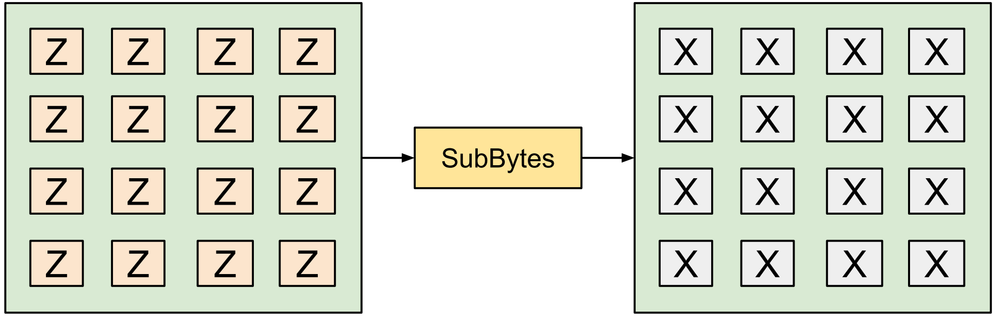
$$\oplus$$

....

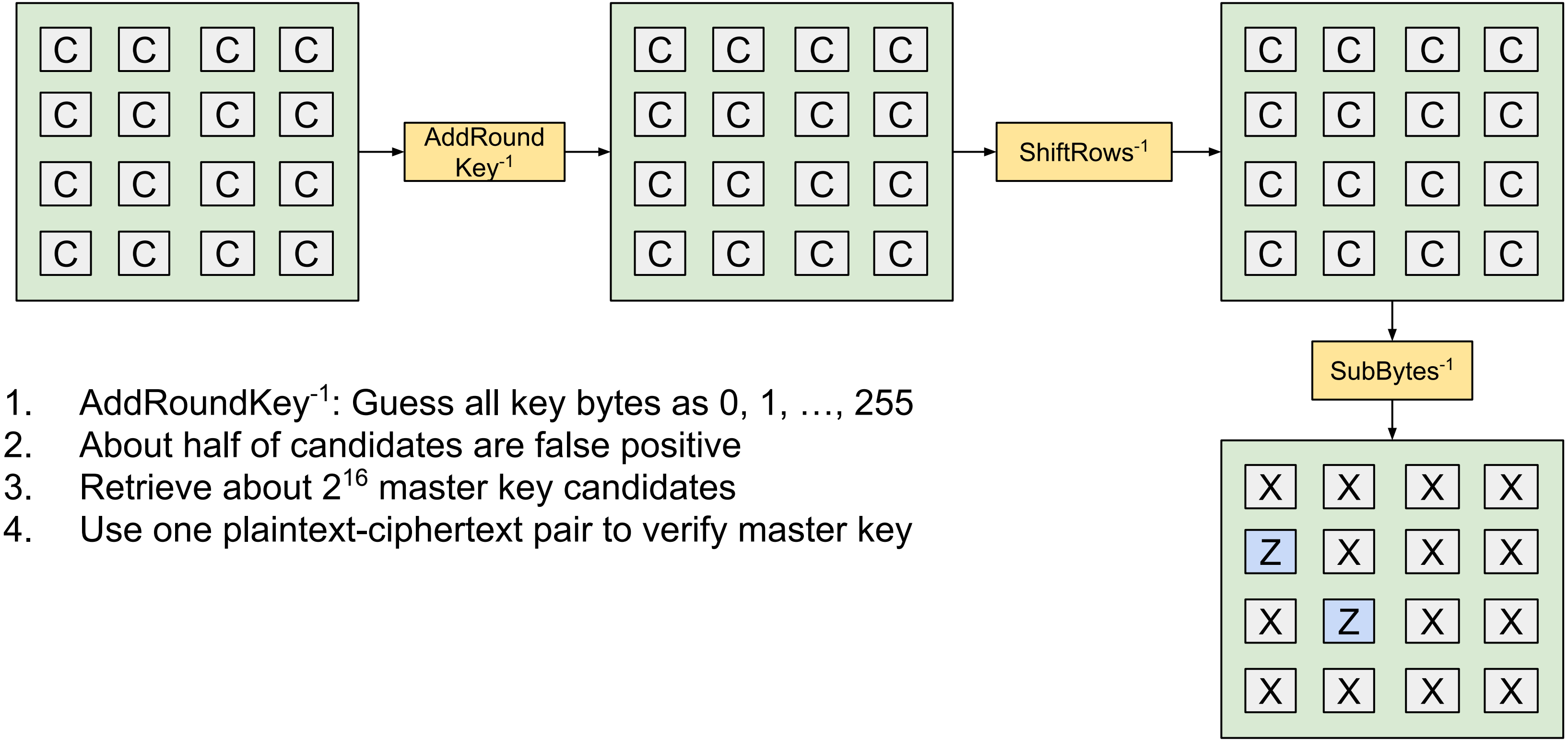
=

$$S = 0 \oplus 0 \oplus 0 \oplus 0 = \mathbf{0}$$

Round 4



Key byte guessing



1. AddRoundKey^{-1}: Guess all key bytes as 0, 1, …, 255
2. About half of candidates are false positive
3. Retrieve about 2^{16} master key candidates
4. Use one plaintext-ciphertext pair to verify master key

exercise

Drive: crypto4/square/

aes.py: aes implemntation, bonus method: master_from_round -> calculate master key from round key

client.py:

encrypt(data) -> returns encrypted data (one block)

get_encrypted_flag() -> returns encrypted flag

goal: **decrypt flag**

Pseudorandom generators

PRNG vs CSPRNG

Self explanatory:

Pseudo **R**andom **N**umber **G**enerator

Cryptographically **S**ecure **P**seudo **R**andom **N**umber **G**enerator

PRNG also known as a **D**eterministic **R**andom **B**it **G**enerator

PRNGs

Given

- P - a probability distribution on $(\mathbb{R}, \mathfrak{B})$ (where $\mathfrak{B}$ is the standard Borel field on the real line)
- $\mathfrak{F}$ - a non-empty collection of Borel sets $\mathfrak{F} \subseteq \mathfrak{B}$, e.g. $\mathfrak{F} = \{(-\infty, t] : t \in \mathbb{R}\}$. If $\mathfrak{F}$ is not specified, it may be either $\mathfrak{B}$ or $\{(-\infty, t] : t \in \mathbb{R}\}$, depending on context.
- $A \subseteq \mathbb{R}$ - a non-empty set (not necessarily a Borel set). Often A is a set between P's support and its interior; for instance, if P is the uniform distribution on the interval $(0, 1]$, A might be $(0, 1]$. If A is not specified, it is assumed to be some set contained in the support of P and containing its interior, depending on context.

We call a function $f : \mathbb{N}_1 \to \mathbb{R}$ (where $\mathbb{N}_1 = \{1, 2, 3, \ldots\}$ is the set of positive integers) a **pseudo-random number generator for P given $\mathfrak{F}$ taking values in A** iff

- $f(\mathbb{N}_1) \subseteq A$
- $\forall E \in \mathfrak{F} \quad \forall 0 < \varepsilon \in \mathbb{R} \quad \exists N \in \mathbb{N}_1 \quad \forall N \leq n \in \mathbb{N}_1, \quad \left| \frac{\# \{i \in \{1, 2, \ldots, n\} : f(i) \in E\}}{n} - P(E) \right| < \varepsilon$

($\#S$ denotes the number of elements in the finite set S.)

It can be shown that if f is a pseudo-random number generator for the uniform distribution on $(0, 1)$ and if F is the CDF of some given probability distribution P, then $F^* \circ f$ is a pseudo-random number generator for P, where $F^* : (0, 1) \to \mathbb{R}$ is the percentile of P, i.e. $F^*(x) := \inf\{t \in \mathbb{R} : x \leq F(t)\}$. Intuitively, an arbitrary distribution can be simulated from a simulation of the standard uniform distribution.

PRNGs

- *"Algorithm for generating a sequence of numbers whose properties **approximate** the properties of **sequences of random numbers**"*

- Perfect PRNG = indistinguishable from random sequence

- Trivia: with N bits of internal state, PRNG will repeat itself every 2**N steps.

- PRNG test: DieHard, DieHarder. For example:
    - The same number of 1s and 0s
    - Distribution of ngrams
    - Spacings between random points
    - Computing pi

PRNGs: general form

```
class prng_generic:
    def __init__(self, seed):
        self.state = self.initial_state(seed)

    def next(self):
        self.update_state()
        return self.current()
```

PRNGs: middle-square

- middle-square method: one of first methods of generating pseudorandom numbers
- Invented and used by John von Neumann

- **1234567890** **2
  1524**1578750190**52100

  next(**1234567890) = 1578750190**

PRNGs: middle-square

```
class prng_middle_square:
    def __init__(self, seed):
        self.state = seed

    def next(self):
        m = '0'*10 + str(self.state**2)
        self.state = int(m[-20:][5:15])
        return self.state
```

PRNGs: LCG

- **L**inear **C**ongruential **G**enerator
- One of oldest and most popular PRNGs
- Fast, very easy to implement
- Used by old Visual Studio C++, Visual Basic <= 6, Delphi, Pascal, glibc (sometimes), Borland C++, java.util.Random

- `state = (state*m + c) % n`

  next(**`1234567890) = 804752948`**
  (with m=1103515245, c=12345, n=2**31-1=2147483647)

PRNGs: LCG

```
class prng_lcg:
    def __init__(self, seed):
        self.state = seed

    def next(self):
        self.state = (self.state * m + c) % n
        return self.state
```

PRNGs: Mersenne Twister

- **The** most popular PRNG?
- Fast, rather simple
- Standard implementation is based on $2^{19937}-1$
- Used by Python, Ruby, PHP, Visual Studio C++, GMP, Common Lisp, Free Pascal, Sage, Excel...
- https://en.wikipedia.org/wiki/Mersenne_Twister#Python_implementation

Linear Feedback Shift Registers

```
state = (state << 13) ^ state
```

- Useful for pseudo-random numbers generation
- Reversible transformation
- Used in Mersenne Twister

PRNGs: Mersenne Twister

```
class prng_mt:
    def __init__(self, seed):
        self.index = 624
        self.mt = [0] * 624
        self.mt[0] = seed
        for i in range(1, 624):
            self.mt[i] = 0xffffffff & (1812433253 *
               (self.mt[i - 1] ^ self.mt[i - 1] >> 30) + i)
```

(code ~~stolen~~borrowed from wikipedia)

PRNGs: Mersenne Twister

```
def next(self):
    if self.index >= 624:
        self.twist()

    y = self.mt[self.index]
    y = y ^ y >> 11
    y = y ^ y << 7 & 2636928640
    y = y ^ y << 15 & 4022730752
    y = y ^ y >> 18
    self.index = self.index + 1
    return y & 0xFFFFFFFF
```

PRNGs: Mersenne Twister

```
def twist(self):
    for i in range(624):
        y = 0xFFFFFFFF & ((self.mt[i] & 0x80000000) +
                          (self.mt[(i + 1) % 624] & 0x7fffffff))
        self.mt[i] = self.mt[(i + 397) % 624] ^ y >> 1

        if y % 2 != 0:
            self.mt[i] = self.mt[i] ^ 0x9908b0df
    self.index = 0
```

PRNGs: Putting *pseudo* back in *P*RNG

https://uw2017.p4.team/lcg1-easy

(8+1 levels)

CSPRNGs

Like PRNGs... But **C**ryptographically **S**ecure

Next Bit Test:
Given k bits of output, adversary can't predict next bit with probability > 50%

State Compromise Extension Resistance:
After state exposed, impossible to predict previous numbers.

PRNG vs **CSPRNG**

	Next Bit Test	State Extension
LCG	✗	✗
Mersenne Twister	✗	✗

CSPRNG design

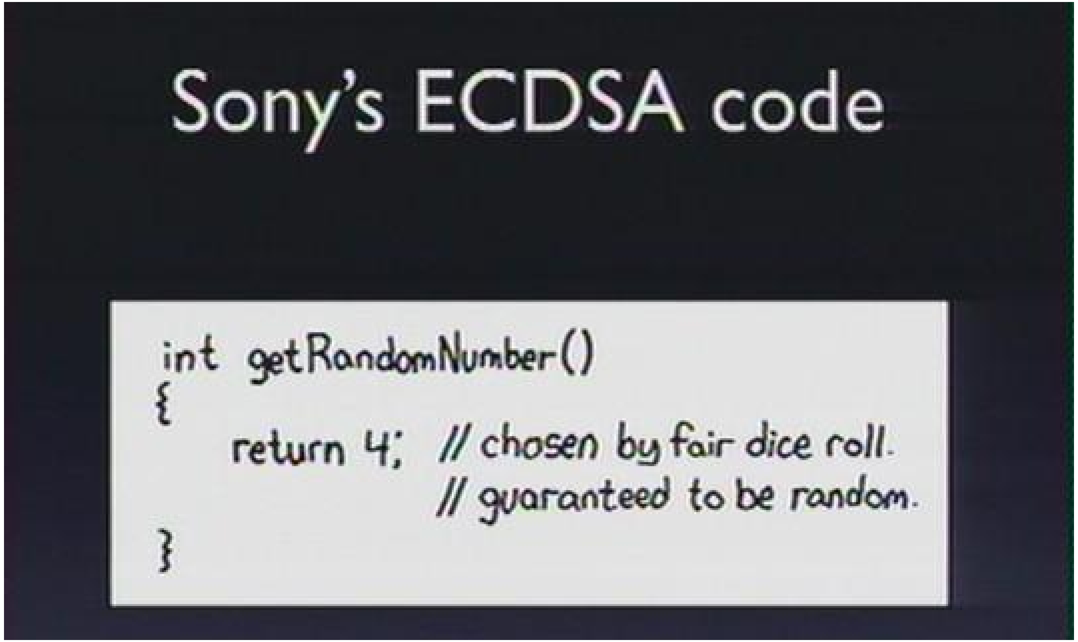
Designed "from the ground up":

- Blum Blum Shub (quadratic residuosity problem)
- Blum-Micali (discrete logarithm problem)

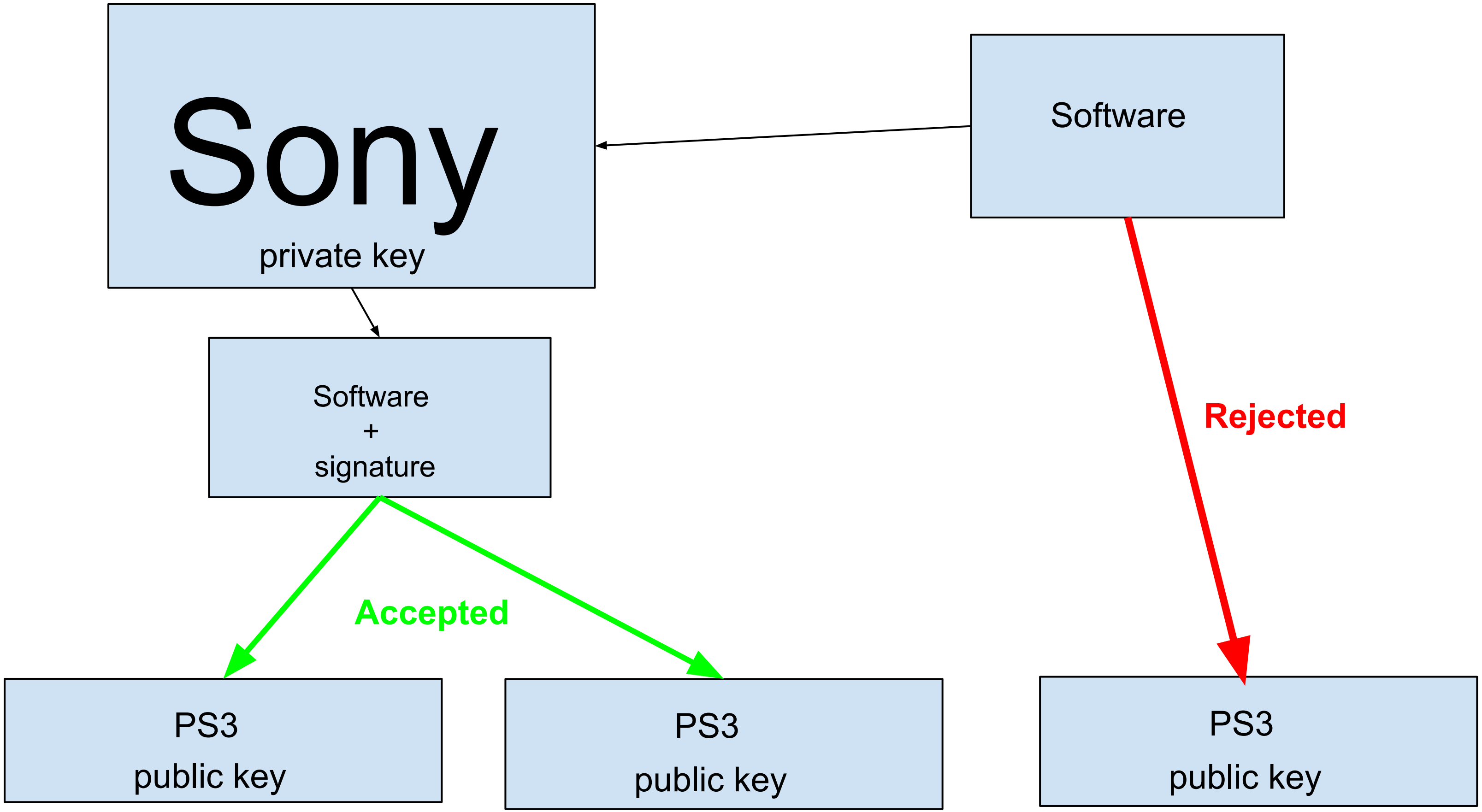
Based on crypto primitives:

- Block Cipher → CSPRNG
- Stream Cipher → CSPRNG
- Secure Hash → CSPRNG

Sony crypto fail



- Team fail0verflow
- Completely broken Playstation 3 console
- Recovered private key which wasn’t present on device
- Bug in Sony ECDSA signing implementation


Sony
private key
Software
Software
+
signature
Rejected
Accepted
PS3
public key
PS3
public key
PS3
public key

DSA algorithm

Parameters:
Numbers P, Q, G
Hash Function H

Private Key:
random X: $0 < X < Q$

Public Key:
Y: $Y = G^X \bmod P$

Singing: M - message
Choose random K: $0 < K < Q$
Calc: R: $R = (G^K \bmod P) \bmod Q$
Calc: S: $S = K^{-1}(H(M)+X*R) \bmod Q$
Signature: (R, S)

Verification: M - message, (R, S) - signature
$W = S^{-1} \bmod Q$
$V = (G^{H(M)*W \bmod Q} * Y^{R*W \bmod Q} \bmod P) \bmod Q$
Correct if $V = R$

Repeated K

Singing: M - message
Choose random K: 0 < K < Q
Calc: R: R = (G^K mod P) mod Q
Calc: S: S = K^{-1}(H(M)+X*R) mod Q
Signature: (R, S)

- Two signed messages: M, M’
- Signatures: (R, S), (R’, S’)
- K = K’
- R = R’
- S - S’ = (K^{-1}(H(M)+X*R) - K^{-1}(H(M’)+X*R)) mod Q = K^{-1} * (H(M) - H(M’)) mod Q
- K = (H(M) - H(M’)) * $(S - S’)^{-1}$ mod Q
- **X** = (S * K - H(M)) * R^{-1} mod Q

exercise

Drive: crypto4/dsa/

client.py:

verify(data, signature) - check if signature is correct for data

verify("flag", signature) - if signature is correct then flag is returned

sigs.txt:

msg, signature pairs for 50 messages generated by gen.py

Bibliography

Joan Daemen and Vincent Rijmen, "AES Proposal: Rijndael"

FIPS PUB 186-4: Digital Signature Standard (DSS), July 2013