STAT5204 Optimization for Data Science

Assignment 1

Due date & time: September 26, 2025 at 17:00

Instruction:

1. **File Format:** Your assignment must be submitted in PDF format. Only one PDF file should be submitted for the entire assignment.
2. **Handwritten Solutions:** All solutions must be handwritten. Please write clearly. Each solution must include all steps leading to your final answer.
3. **Numerical Answers:** All numerical answers must be presented in 3 significant figures.

Question 1

Points $\boldsymbol{x}_a = [2,3]^\top$, $\boldsymbol{x}_b = [4,0]^\top$, $\boldsymbol{x}_c = [4,3]^\top$ are possible minimizers of the problem

$$\begin{aligned}&\text{minimize } 2(4x_1 + 3x_2) + (x_1^2 + x_2^2 + 25)\\ &\text{subject to } x_1 \geq 0, x_2 \geq 0.\end{aligned}$$

(a) Find the feasible directions of $\boldsymbol{x}_a, \boldsymbol{x}_b$ and $\boldsymbol{x}_c$.

(b) For each point, check if the first-order necessary condition is satisfied.

Question 2 Consider the following optimization problem

$$\begin{aligned}&\text{minimize } 2x_1^3 + 3x_2^2 + 3x_1^2 x_2 - 24x_2\\ &\text{subject to } x_1 \in \mathbb{R}, x_2 \in \mathbb{R}.\end{aligned}$$

Find all points which satisfy FONC.

Question 3

An optimization algorithm has given a solution $\boldsymbol{x}_a = [0.6959, -1.3479]^\top$ for the problem

$$\text{minimize } x_1^4 + x_1 x_2 + (1 + x_2)^2.$$

(a) Is the Hessian of the objective function positive definite?

(b) Determine whether $\boldsymbol{x}_a$ is a minimizer.

Question 4

The 5th-order polynomial

$$f(x) = -5x^5 + 4x^4 - 12x^3 + 11x^2 - 2x + 1$$

is known to be a unimodal function on interval $[-0.5, 0.5]$.

(a) Use the golden section search to locate the minimizer to within a range of 0.5.

(b) Use the Fibonacci search to locate the minimizer to within a range of 0.5.

(c) Perform three iterations of the secant method to find minimizer with starting points $x^{(-1)} = 0.2$ and $x^{(0)} = 0.1$.

Question 5

Consider the optimization problem

$$\text{minimize } \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}\|^2$$

where $\boldsymbol{A} \in \mathbb{R}^{m\times n}$, $m \geq n$, and $\boldsymbol{b} \in \mathbb{R}^m$.

(a) Show that the objective function for this problem is a quadratic function (i.e., it can be expressed as $(1/2)\boldsymbol{x}^\top \boldsymbol{Q}\boldsymbol{x} - \boldsymbol{b}^\top \boldsymbol{x} + c$ where $\boldsymbol{Q} \in \mathbb{R}^{n\times n}$ is a symmetric positive definition matrix, $\boldsymbol{b}, \boldsymbol{x} \in \mathbb{R}^m$, and $c \in \mathbb{R}$.), and write down the gradient and Hessian of this quadratic.

(b) Suppose that

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \quad \text{and} \quad \boldsymbol{b} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}.$$

Find $\boldsymbol{x}^{(1)}$ by performing one iteration of the method of steepest descent with a starting point $\boldsymbol{x}^{(0)} = [0, 0]^\top$.

(c) Does $\boldsymbol{x}^{(1)}$ found in part (b) satisfy FONC?