Chong, Edwin Kah Pin., and Stanislaw H. Żak. *An Introduction to Optimization*. Fourth edition., John Wiley & Sons, Inc., 2013.

2 ONE-DIMENSIONAL SEARCH METHODS

7.2 Let $f(x) = x^2 + 4\cos x$, $x \in \mathbb{R}$. We wish to find the minimizer x^* of over the interval [1,2].

a. Plot $f(x)$ versus x over the interval $[1, 2]$.

b. Use the golden section method to locate x^* to within an uncertainty of 0.2. Display all intermediate steps using a table.

c. Repeat part b using the Fibonacci method, with $\varepsilon = 0.05$. Display all intermediate steps using a table.

d. Apply Newton's method, using the same number of iterations as in part b, with $x^{(0)} = 1$.

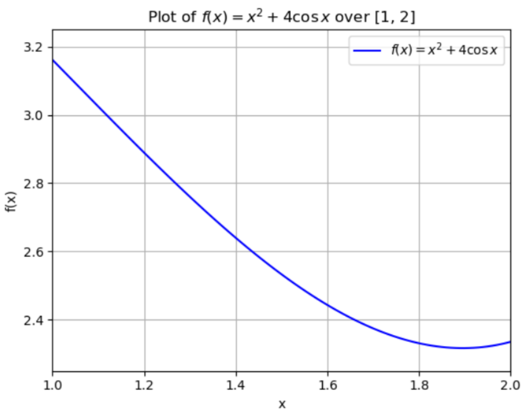
(a)

```python
import numpy as np
import matplotlib.pyplot as plt

# Define the function
def f(x):
    return x**2 + 4 * np.cos(x)

# Create an array of x values from 1 to 2
x = np.linspace(1, 2, 100)
y = f(x)

# Plotting the function
plt.plot(x, y, label=r'$f(x) = x^2 + 4 \cos x$', color='blue')
plt.title('Plot of $f(x) = x^2 + 4 \cos x$ over [1, 2]')
plt.xlabel('x')
plt.ylabel('f(x)')
plt.grid()
plt.xlim(1, 2)  # Set y-axis limits
plt.ylim(2.25, 3.25)  # Set y-axis limits
plt.legend()
plt.show()
```



(b) The number of steps needed for the Golden Section method is computed from the inequality:

$$0.61803^N \le 0.2/(2-1) \Rightarrow N \ge 3.34.$$

Therefore, the required number of steps is 4:

Iteration k	a_k	b_k	$f(a_k)$	$f(b_k)$	New uncertainty interval
1	1.3820	1.6180	2.6607	2.4292	[1.3820, 2]
2	1.6180	1.7639	2.4292	2.3437	[1.6180, 2]
3	1.7639	1.8541	2.3437	2.3196	[1.7639, 2]
4	1.8541	1.9098	2.3196	2.3171	[1.8541, 2]

The resulting interval is $[1.8541, 2]$ which has length 0.1459.

(c) The number of steps required is 4:

$$\frac{1+2\varepsilon}{F_{N+1}} \le \frac{0.2}{2-1}$$

$$F_{N+1} \ge \frac{1+2(0.05)}{0.2} = 5.5$$

Recall that $F_1 = 1, F_2 = 2, F_3 = 3, F_4 = 5, F_5 = 8$. Hence, $N + 1 \ge 5 \Rightarrow N \ge 4$.

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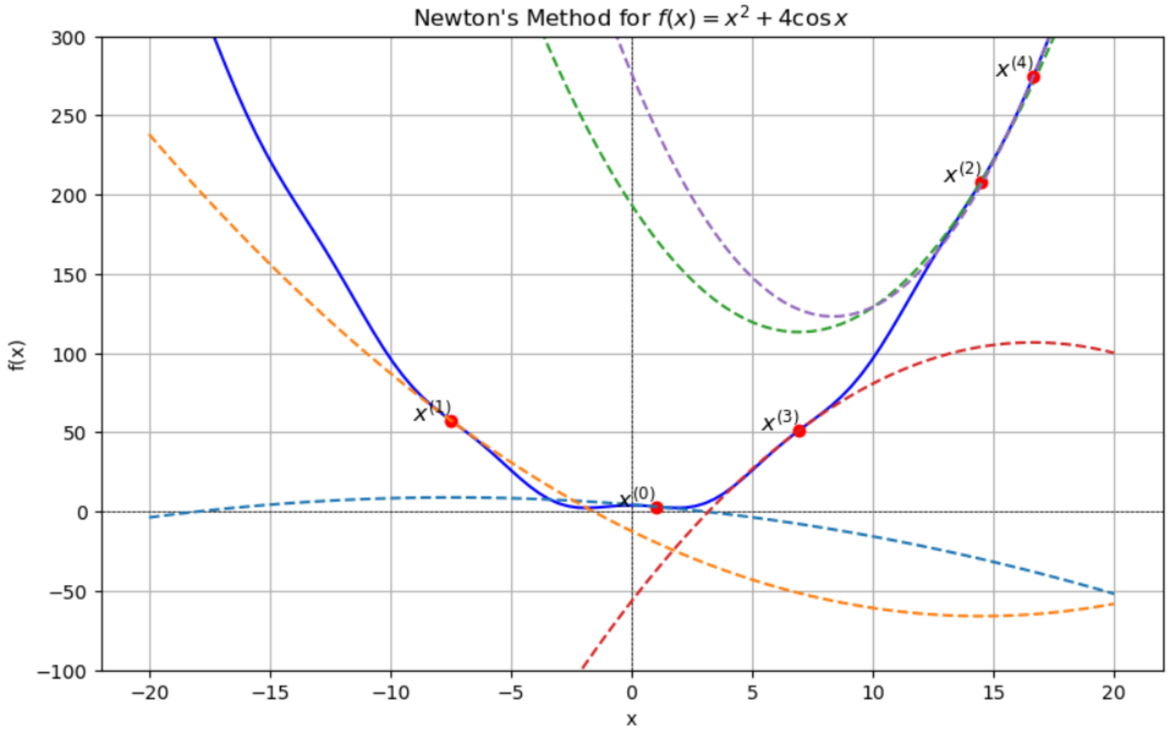
Iteration k	ρ_k	a_k	b_k	$f(a_k)$	$f(b_k)$	New uncertainty interval
1	0.3750	1.3750	1.6250	2.6688	2.4239	[1.3750, 2]
2	0.4000	1.6250	1.7500	2.4239	2.3495	[1.6250, 2]
3	0.3333	1.7500	1.8750	2.3495	2.3175	[1.7500, 2]
4	0.4500	1.8750	1.8875	2.3175	2.3169	[1.8750, 2]

The resulting interval is $[1.8750, 2]$ which has length 0.1250.

(d) We have $f'(x) = 2x - 4\sin x$, $f''(x) = 2 - 4\cos x$. Hence,

$$x^{(k+1)} = x^{(k)} - \frac{x^{(k)} - 2\sin x^{(k)}}{1 - 2\cos x^{(k)}}.$$

Starting with $x^{(0)} = 1$, we get $x^{(1)} = -7.4727$, $x^{(2)} = 14.4785$, $x^{(3)} = 6.9351$, $x^{(4)} = 16.6354$.



7.7 Suppose that we have an efficient way of calculating exponentials. Based on this, use Newton's method to devise a method to approximate log(2) [where "log" is the natural logarithm function]. Use an initial point of $x^{(0)} = 1$, and perform two iterations.

The number log(2) is the root of the equation $g(x) = 0$, where $g(x) = \exp(x) - 2$. The derivative of g is $g'(x) = \exp(x)$. Newton's method applied to this root finding problem is

$$x^{(k+1)} = x^{(k)} - \frac{g(x^{(k)})}{g'(x^{(k)})} = x^{(k)} - \frac{\exp(x^{(k)}) - 2}{\exp(x^{(k)})} = x^{(k)} - 1 + 2\exp(-x^{(k)}).$$

Performing two iterations, we get $x^{(1)} = 0.7358$ and $x^{(2)} = 0.6940$.