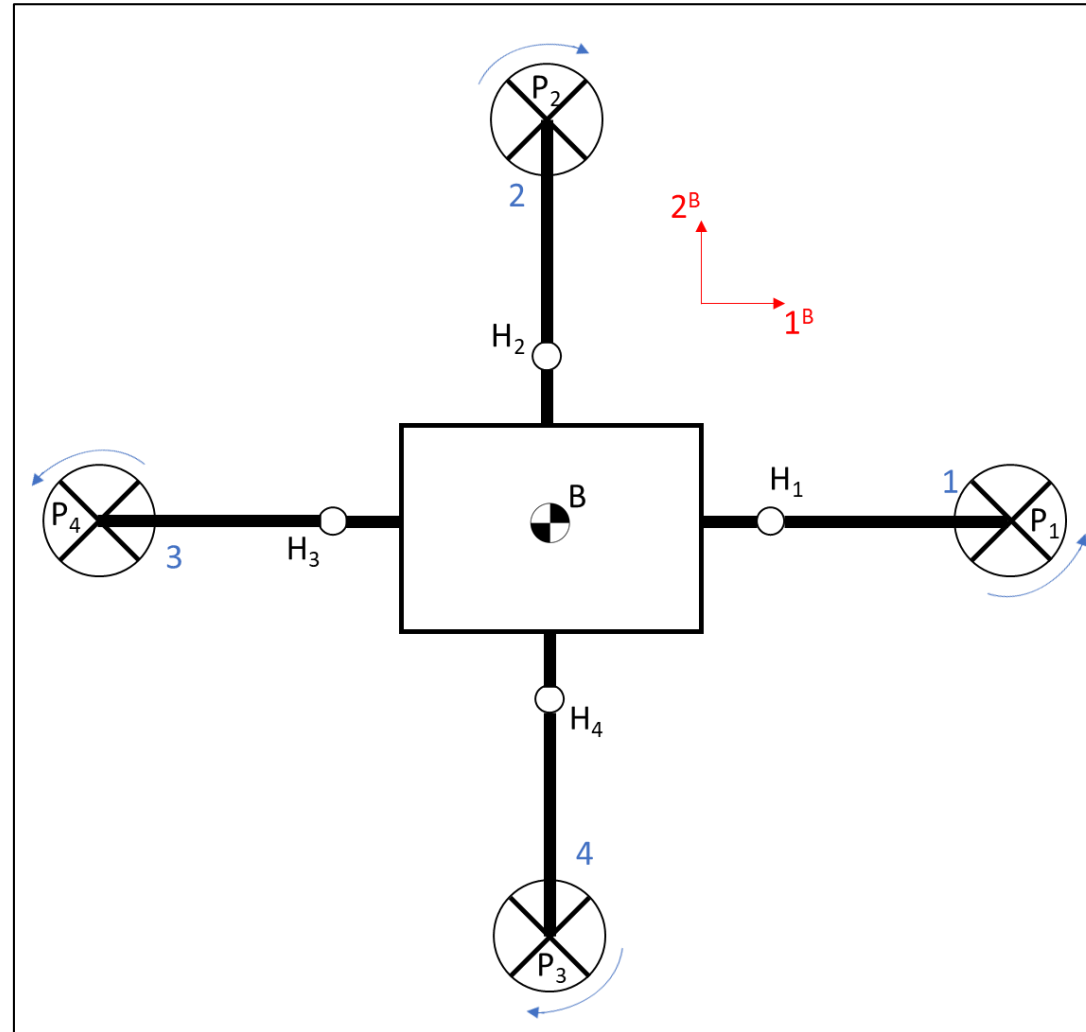


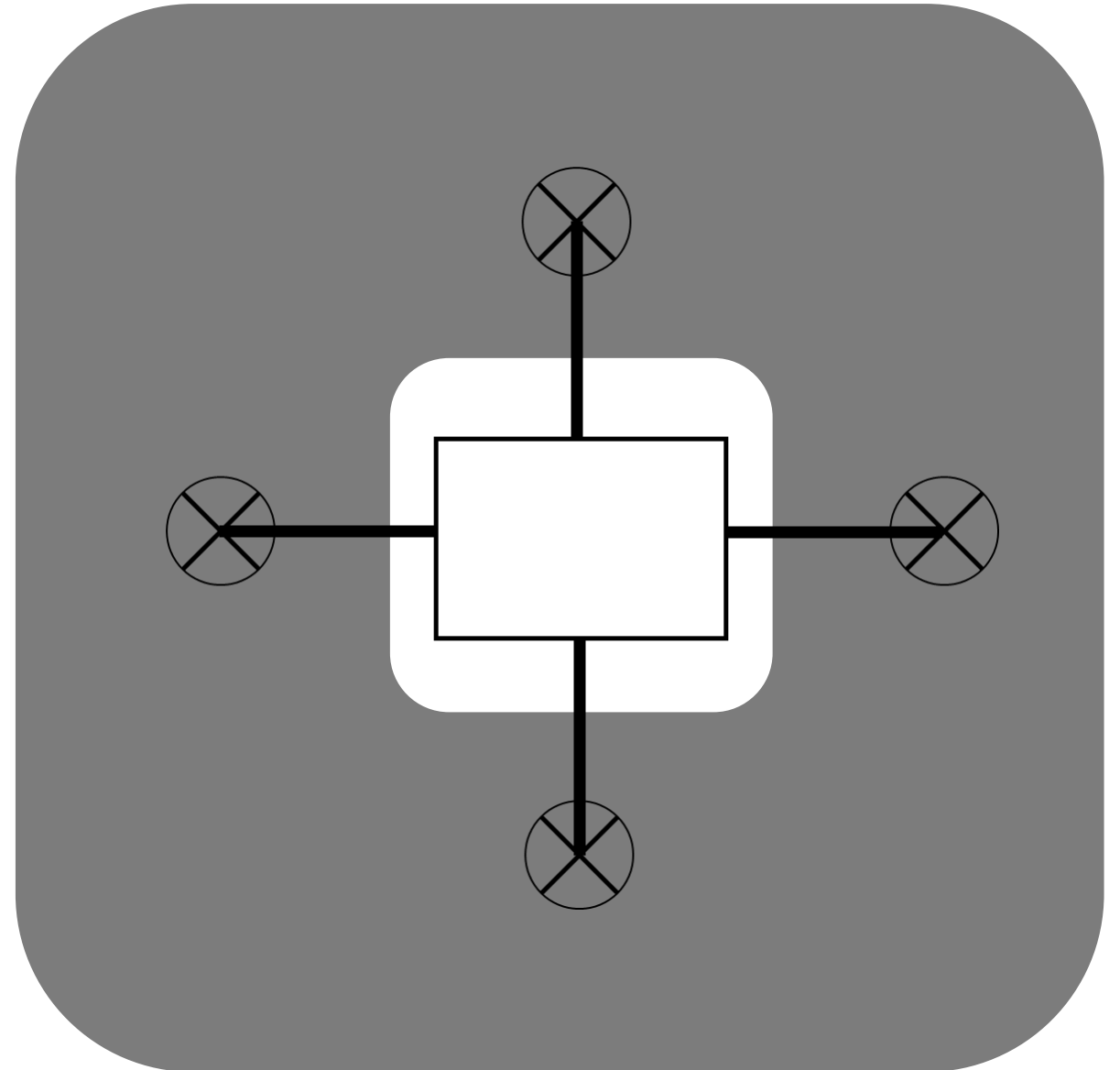
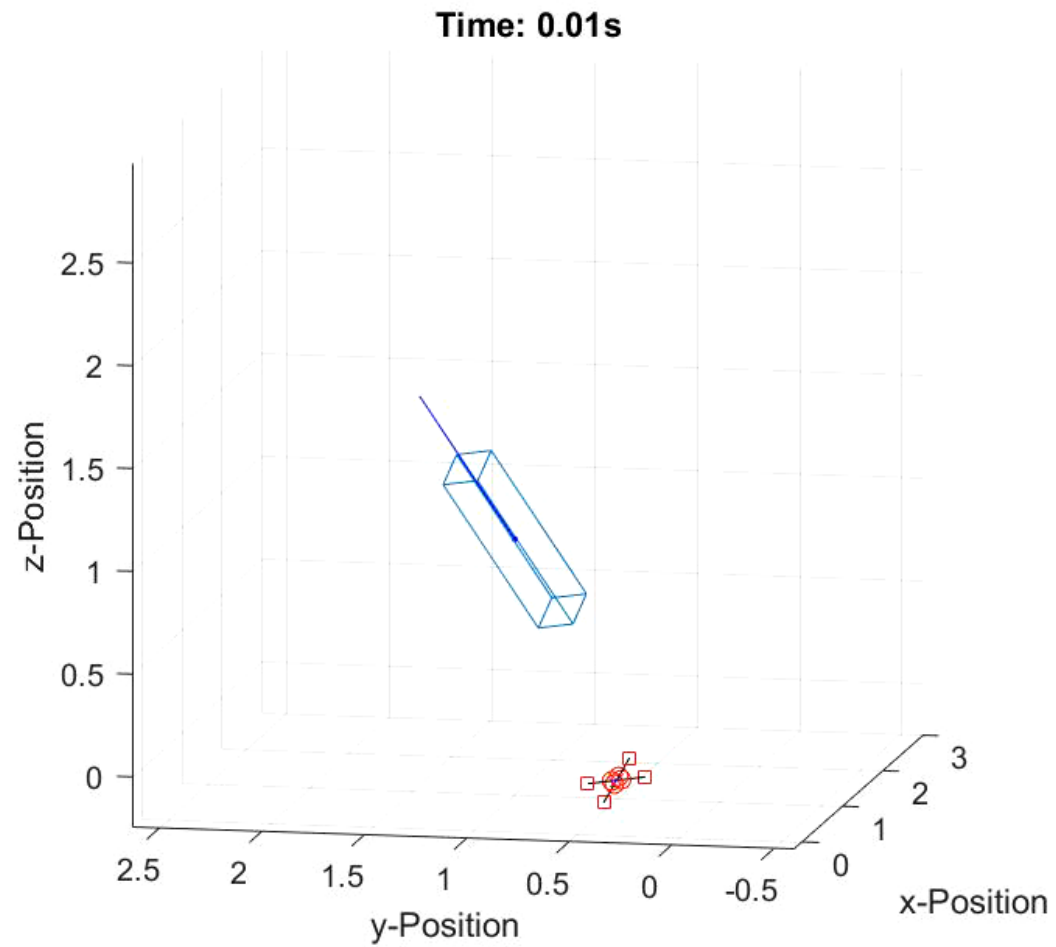
Simulation of a Quadcopter with Foldable Arms

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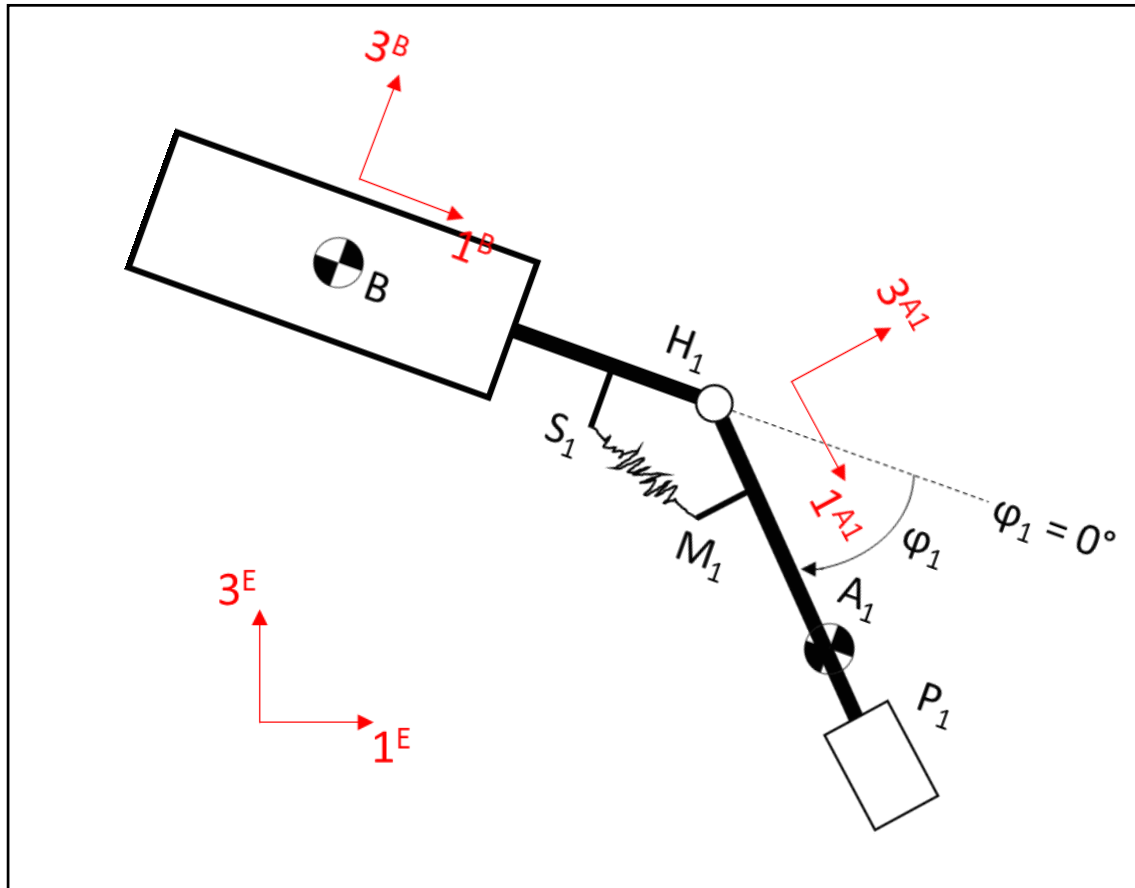
Introduction



Introduction

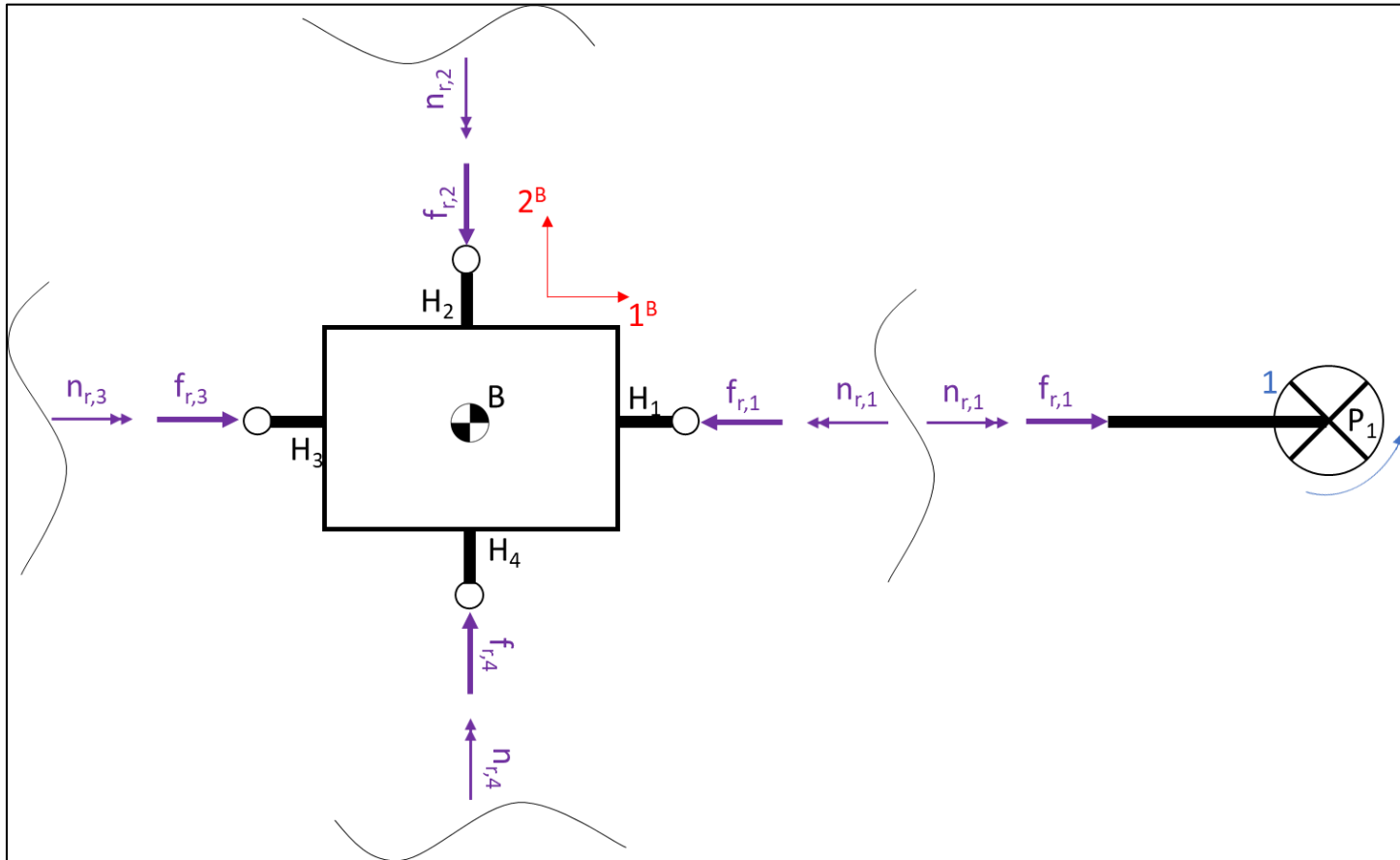


Overview of System



- Angle φ_1 being the only degree of freedom of the arm wrt. the body
- Mechanical stops at 0 and 90
- Constant force spring between body and arm
- **E**: earth-fixed frame (inertia)
- **B**: body-fixed frame
- **A_i**: arm-fixed frame

Derivation of dynamics



- Separate arms from body
- Introduce reaction torque and forces $n_{r,i}$, $f_{r,i}$
- Euler's and Newton's law for 1 arm and body

Derived Dynamics

$$\mathbf{I}_B^B D^B \omega^{BE} + \Omega^{BE} \mathbf{I}_B^B \omega^{BE} = \sum_{i=1}^4 (-\mathbf{n}_{r,i} - \mathbf{S}_{HB} \mathbf{f}_{r,i} - \mathbf{n}_{s,i})$$

$$m^B D^E (\mathbf{v}_B^E) = \sum_{i=1}^4 -\mathbf{f}_{r,i} + m^B \mathbf{g}$$

$$\begin{aligned} & (\mathbf{I}_A^A + \mathbf{I}_P^P) (D^A \omega^{AB} + D^B \omega^{BE}) \\ & + \mathbf{I}_P^P D^P \mathbf{w}^{PA} + \mathbf{I}_P^P ((\Omega^{BA} + \Omega^{AP}) \omega^{BE} + \Omega^{AP} \omega^{AB} \\ & + \mathbf{I}_A^A \Omega^{BA} \omega^{BE} + (\Omega^{AB} + \Omega^{BE}) \mathbf{I}_A^A (\omega^{AB} + \omega^{BE}) \\ & + (\Omega^{PA} + \Omega^{AB} + \Omega^{BE}) \mathbf{I}_A^A (\omega^{PA} + \omega^{AB} + \omega^{BE}) \\ & = \\ & \mathbf{n}_{r,1} + \mathbf{S}_{H_1 A_1} \mathbf{f}_{r,1} \\ & + \mathbf{n}_{p,1} + \mathbf{S}_{P_1 A_1} \mathbf{f}_{p,1} + \mathbf{n}_{s,1} \end{aligned}$$

$$\begin{aligned} & m^A [D^E \mathbf{v}_B^E - \mathbf{S}_{AH} D^A \omega^{AB} - (\mathbf{s}_{AH} + \mathbf{s}_{HB}) D^B \omega^{BE} \\ & (\Omega^{BE} \Omega^{BE} + (\Omega^{AB} + \Omega^{BE}) \Omega^{AB} + \Omega^{BE} \Omega^{AB}) \mathbf{s}_{AH} \\ & + \Omega^{BE} \Omega^{BE} \mathbf{s}_{HB}] \\ & = \\ & \mathbf{f}_{p,1} + \mathbf{f}_{r,1} + m^A \mathbf{g} \end{aligned}$$

$$[D^B \omega^{BE}]^B = [\dot{p} \quad \dot{q} \quad \dot{r}]'$$

$$[D^E v_B^E]^E = [a_1 \quad a_2 \quad a_3]'$$

$$[D^{A_i} \omega^{A_i B}]^{A_i} = [0 \quad \ddot{\varphi}_i \quad 0]'$$

$$[f_{r,i}]^{A_i} = [f_{1,i} \quad f_{2,i} \quad f_{3,i}]'$$

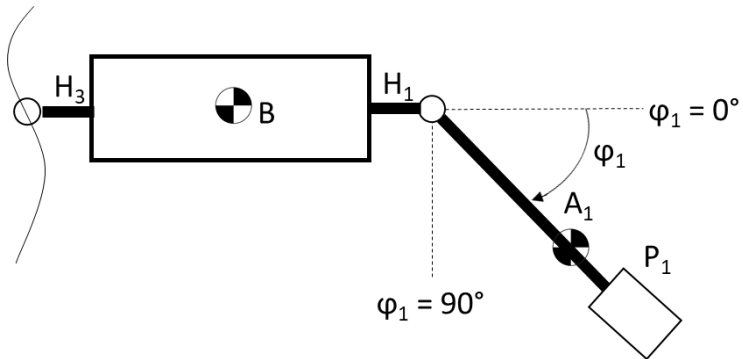
$$[n_{r,i}]^{A_i} = [n_{1,i} \quad n_{2,i} \quad n_{3,i}]'$$

x 4

x 4

Extra Constraint

Case 1:



$$[D^B \omega^{BE}]^B = [\dot{p} \quad \dot{q} \quad \dot{r}]'$$

$$[D^E v_B^E]^E = [a_1 \quad a_2 \quad a_3]'$$

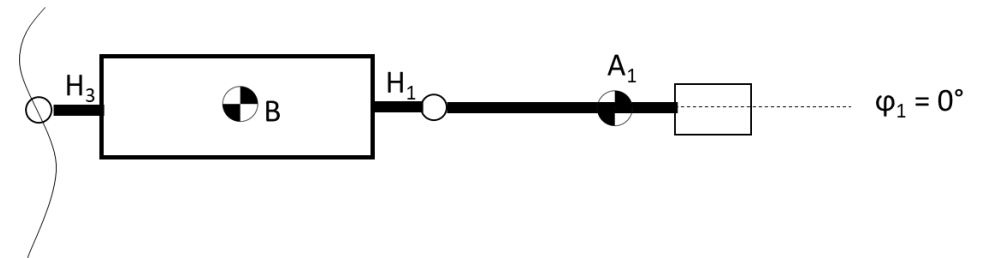
$$[D^{A_i} \omega^{A_i B}]^{A_i} = [0 \quad \ddot{\phi}_i \quad 0]'$$

$$[f_{r,i}]^{A_i} = [f_{1,i} \quad f_{2,i} \quad f_{3,i}]' \quad \times 4$$

$$[n_{r,i}]^{A_i} = [n_{1,i} \quad \underline{\underline{0}} \quad n_{3,i}]' \quad \times 4$$

30 unknowns
30 equations

Case 2:



$$[D^B \omega^{BE}]^B = [\dot{p} \quad \dot{q} \quad \dot{r}]'$$

$$[D^E v_B^E]^E = [a_1 \quad a_2 \quad a_3]'$$

$$[D^{A_i} \omega^{A_i B}]^{A_i} = [0 \quad \underline{\underline{0}} \quad 0]'$$

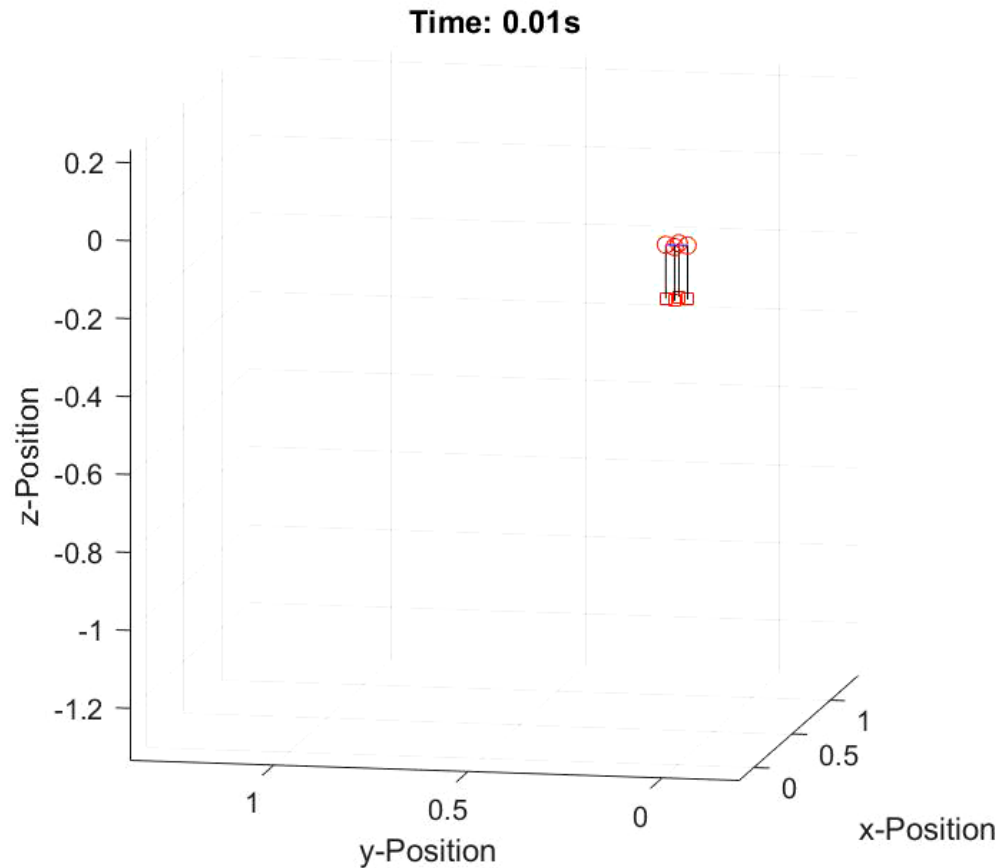
$$[f_{r,i}]^{A_i} = [f_{1,i} \quad f_{2,i} \quad f_{3,i}]' \quad \times 4$$

$$[n_{r,i}]^{A_i} = [n_{1,i} \quad n_{2,i} \quad n_{3,i}]'$$

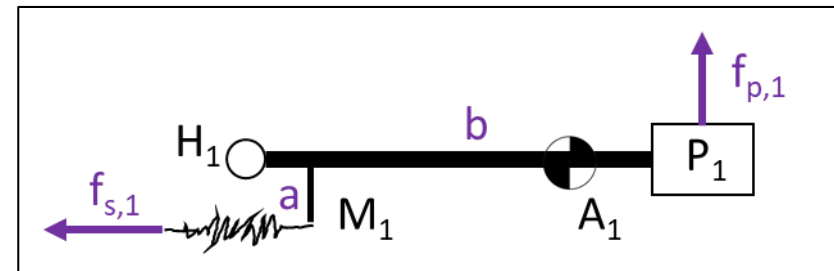
Summary Simulation

1. **Initialize** system at $k = 0$
2. **Set propeller speed** for each motor → calculate motor thrust/torque and angular acceleration of the propeller
3. **Calculate** accelerations, reaction forces and torques depending on current state of the arms by solving linear set of algebraic equations
4. Calculate angular/translational velocity, position and orientation through **Euler integration**
5. Check for **current state** of the arms (folded or stretched)
6. **Repeat** from step 2

Unfold/Hover/Fold

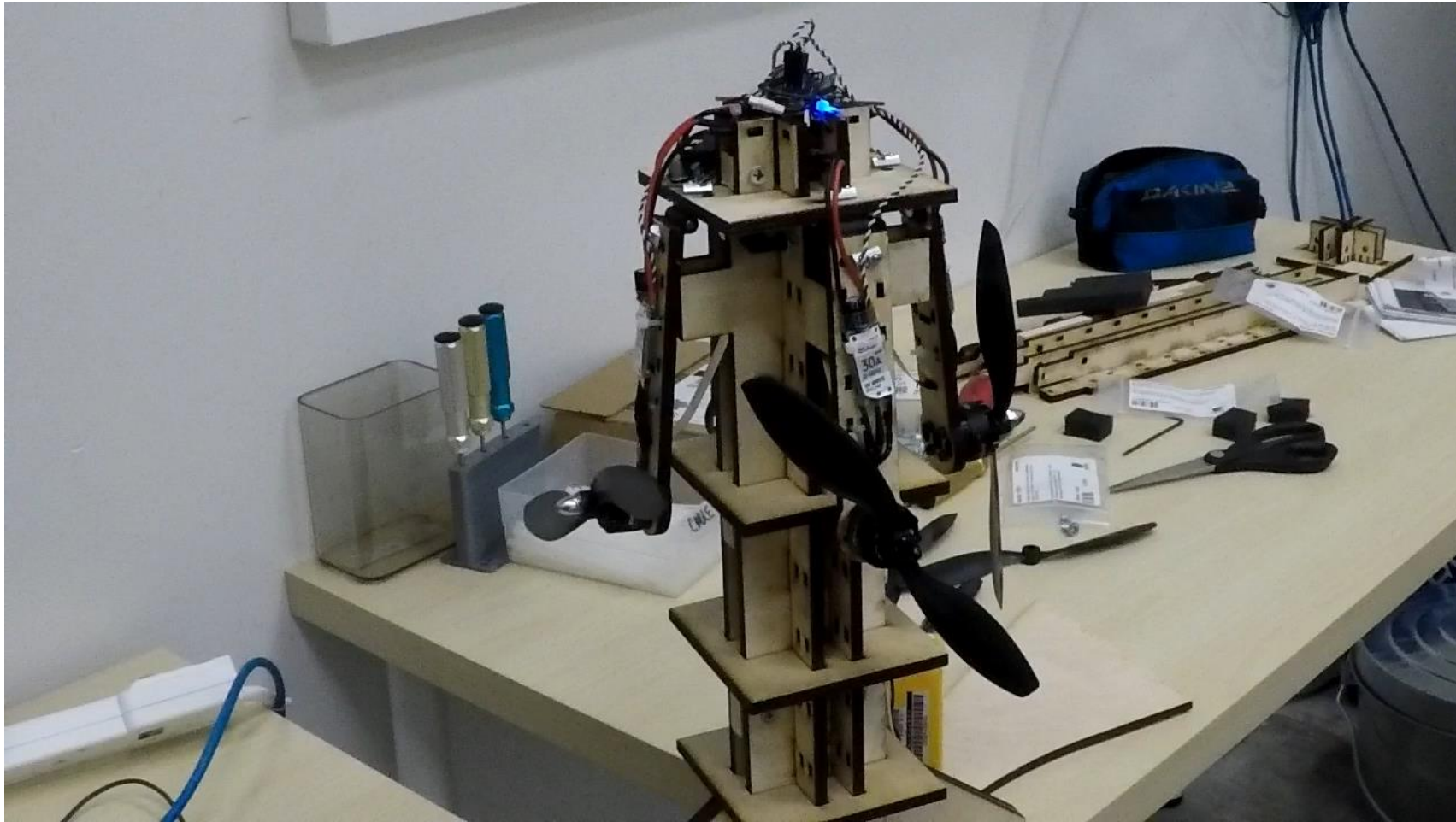


1. Unfold quadcopter by setting motor thrust to 100%
2. Use normal quadcopter controller to hover
3. Fold quadcopter by setting all motor thrust to zero

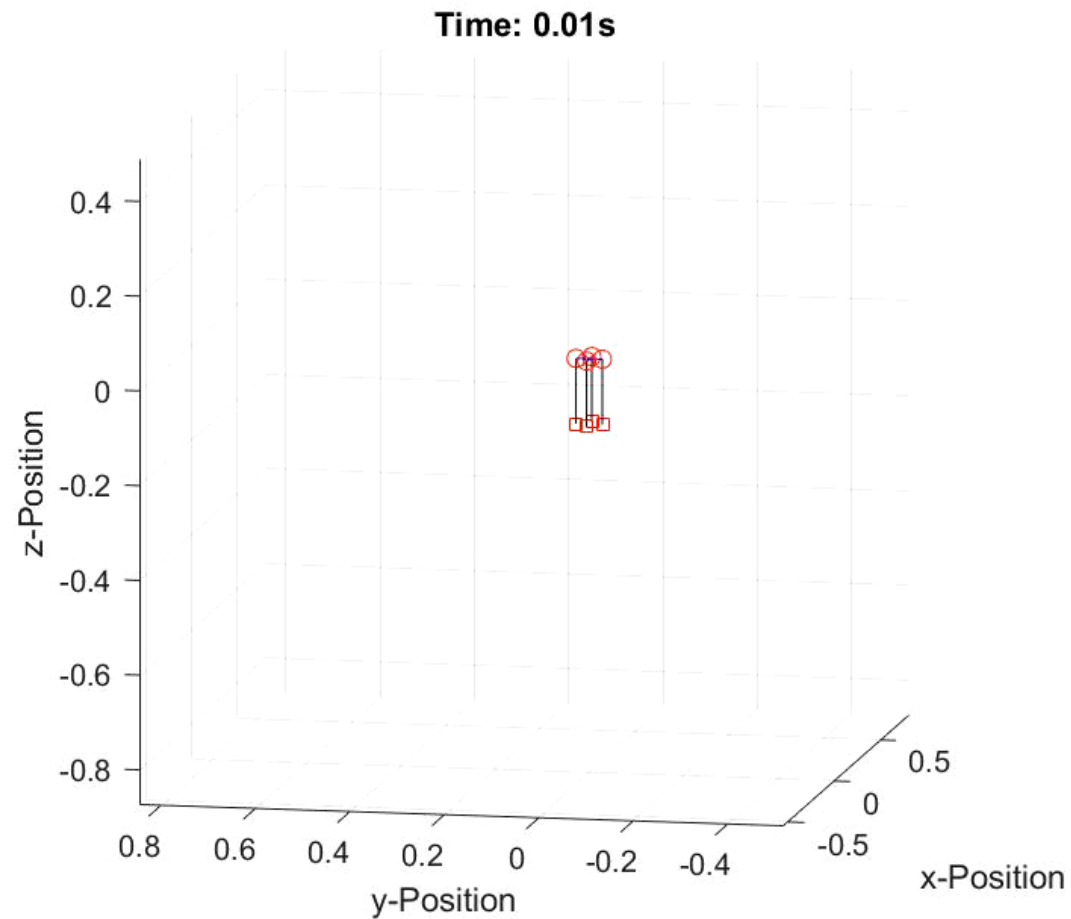


$$\text{minThrust} = f_{s,1} \cdot \frac{a}{b}$$

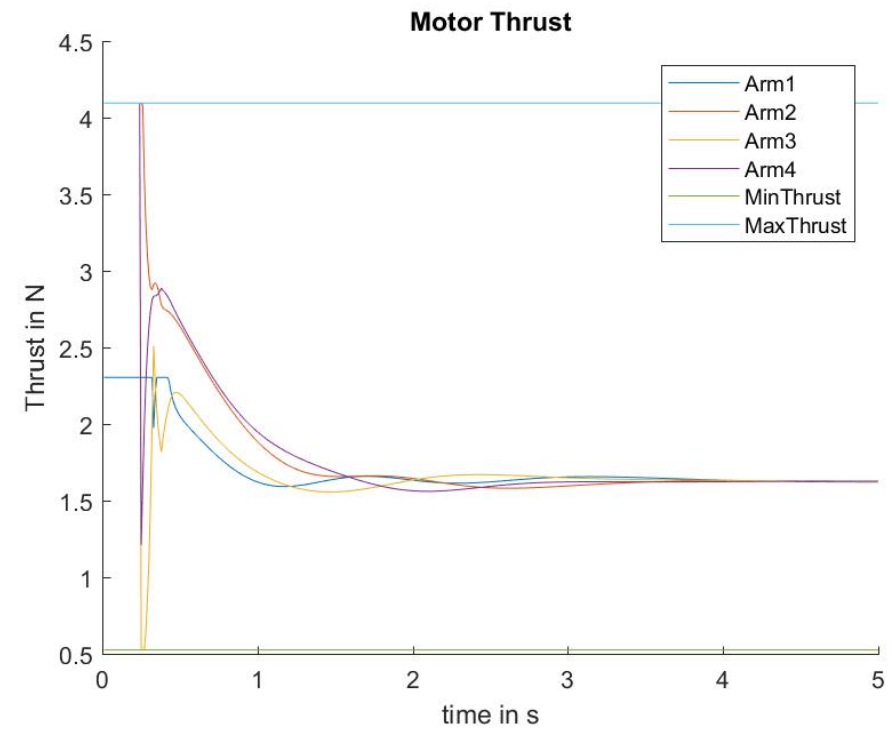
Physical Test



Asymmetric thrust

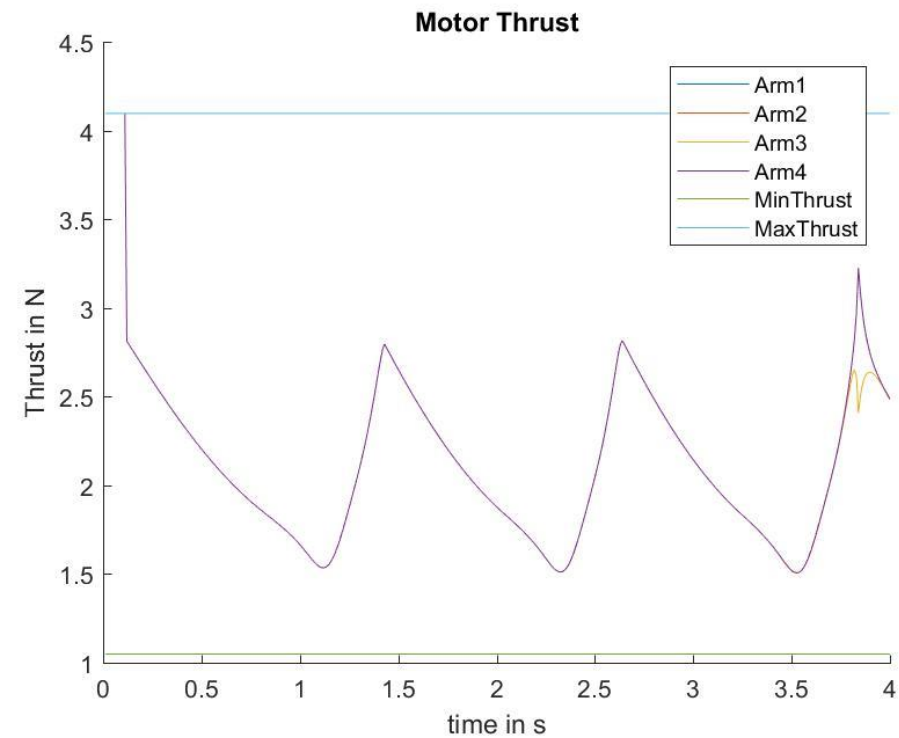
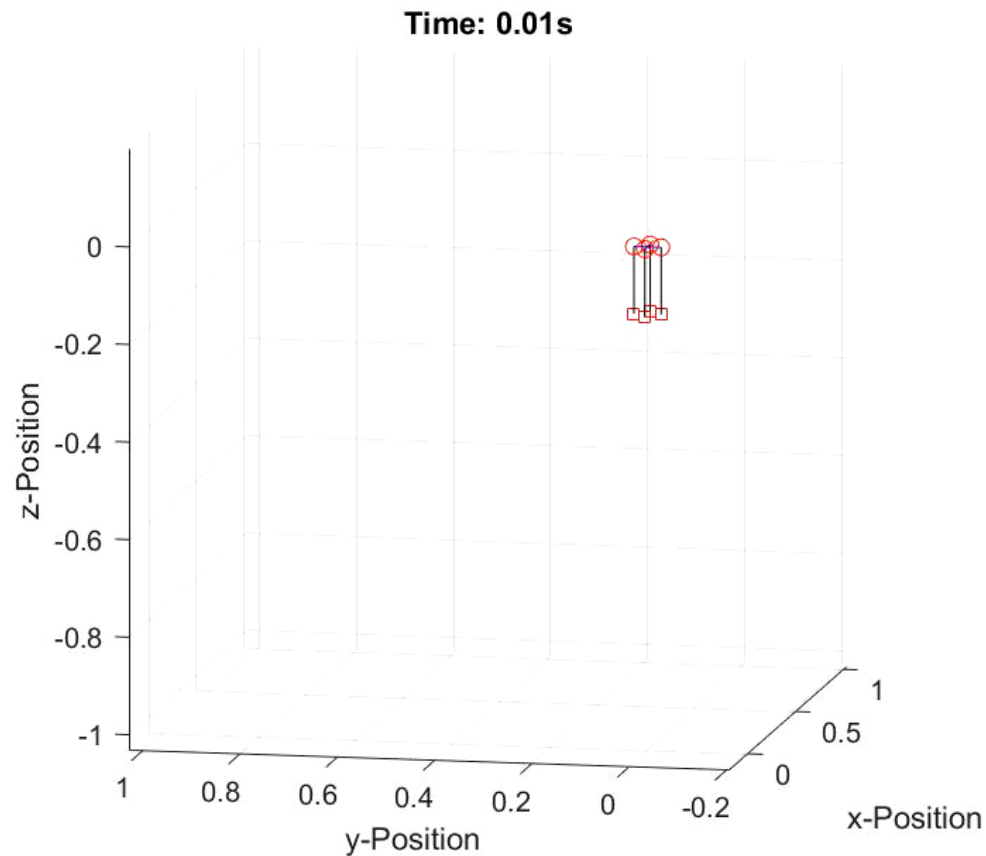


Disturbance in max. motor thrust
of Arm 1: 25% less



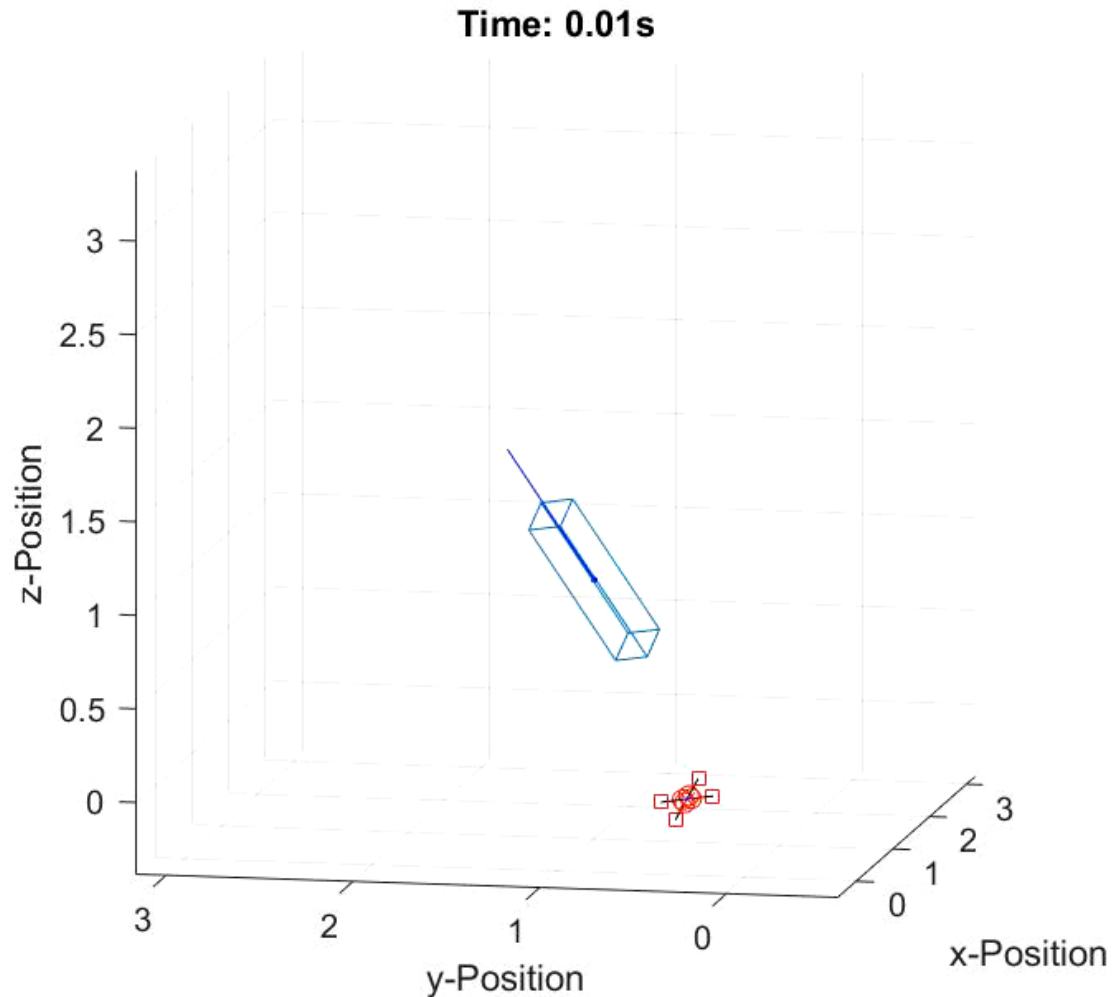
Minimum Threshold

springForce = 4
minThrust = 1.0483
hoverThrust = 1.6236



Gap with initial angular velocity

Set initial angular velocity
to $[0, 2, 0]$ rad/s, when
folding arms



Next Steps

- Import simulation in lab code (C++) and test with „real“ disturbances
- Improve algorithm for generating „Gap-trajectory“
- Finish and fabricate physical design and test control strategies on real system

Thank you for listening!

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