Given A, b

Prolongatn $= P$ $\}$ for symm matrix.

Restrictn $= P^T$

$A_c = P^T A P$ $\}$ Galerkin Project

Two-grid

$A \searrow A_c = P^T A P$

$A x = b$; $x = Py$

$P^T A x = P^T b$

$$\boxed{P^T A P y = P^T b}$$

$A_c y = P^T b \equiv b_c$

$\{ P \}^{(1)}$

Steps

① $b_c = P^T b$
$A_c = P^T A P$

② A_c

③ Solve $A_c y = b_c$

④ $x = Py$

MG has another component : Smoothing (P2)

$$M_{AMG}^{-1} = M_{smoother}^{-1} + M_{C}^{-1} - M_S A M_C^{-1}$$

$M_S = \begin{cases} \text{Jacobi} \\ \text{SSOR} \\ \text{block Jacobi} \\ \text{block SSOR} \\ \text{ILU} \end{cases}$

$A_{jacobi} \subset$ Jacobi (A)

$M_C^{-1} z = \left(P A_C^{-1} (P^T) z \right)$

$x = P_1 x^1$

$x^1 = P_2 x^2$

$x^2 = P_3 x^3$

From 2-Grid to MG

b A

P_1 $A^1 = P_1^T A P_1$

$b^1 = P_1^T b$

P_2 $A^2 = P_2^T A^1 P_2$

$b^2 = P_2^T b^1$

P_3 $A^3 = P_3^T A^2 P_3$

$b^3 = P_3^T b^2$

$$A^3 x^3 = b^3$$
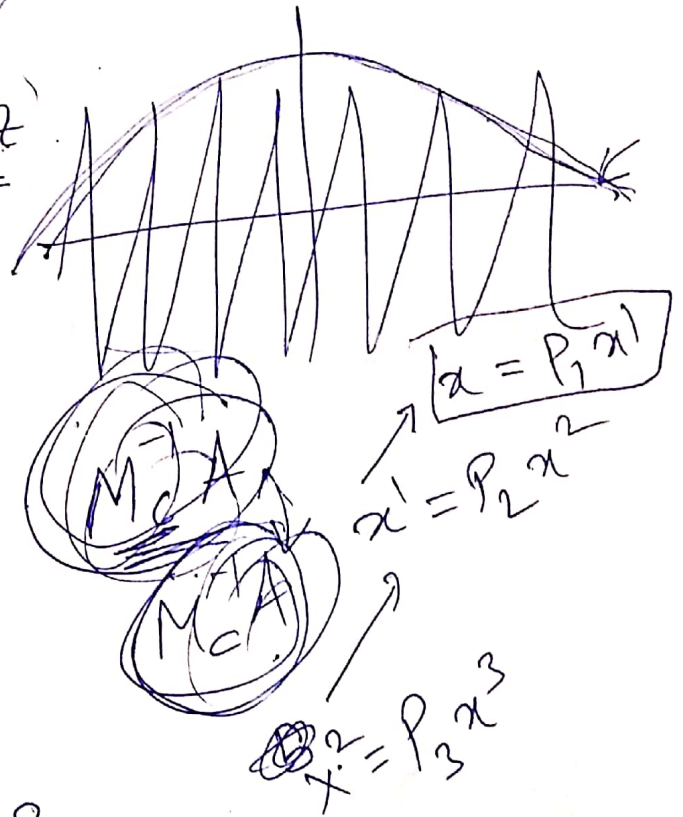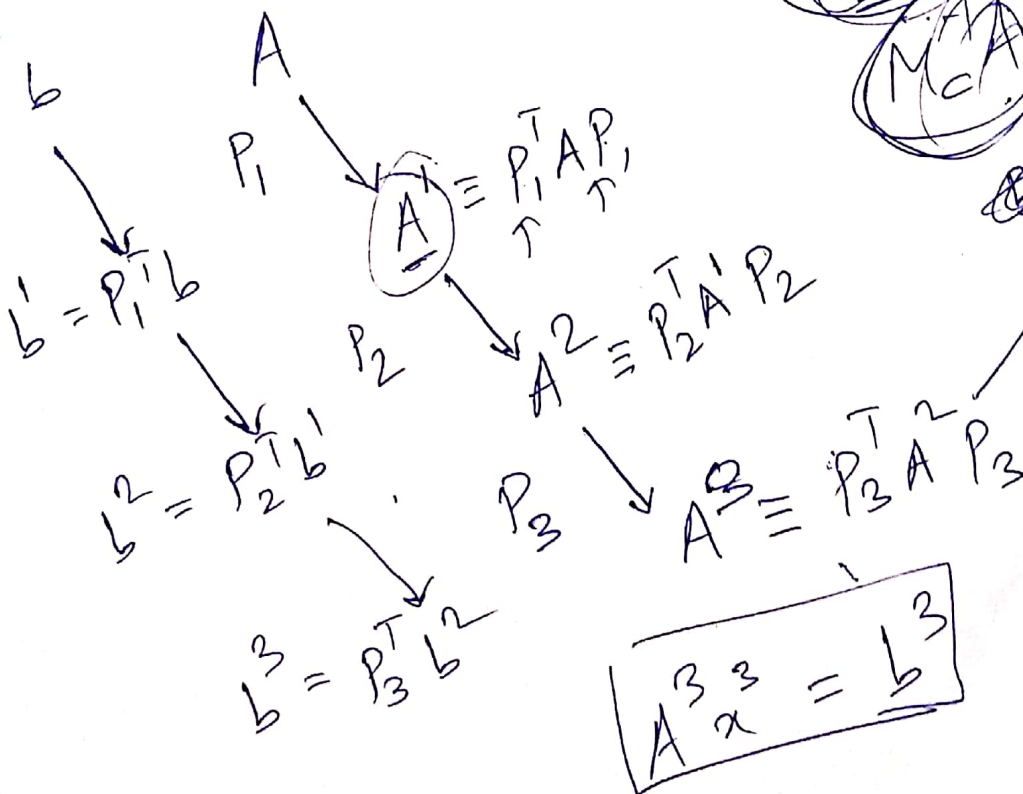
$$\text{AnySolu} \; (A, \; \text{SmootherData}, \; \text{Coar} \; \text{(3)} \quad P\{1\} = P_1$$
$$P\{2\} = P_2$$

$$y = \underline{\text{AnySolu}} \; \left(A, \; \overset{\downarrow}{J}, \; \underline{P = [P_1, P_2, \ldots, P_\ell]}, \; A_c, \; b, c \right)$$
$$\text{Smoothing}$$

$$z \leftarrow \text{JacobiSolu} \; (J, \; b)$$

$$t \leftarrow \text{CoarseGridSolu} \; \left(A, \; P, \; A_c, \; b_c \right)$$

$$\bullet \quad y = A_c^{-1} \, b_c$$

$$\bullet \quad t \leftarrow P_1^{\textcircled{4}} \; P_2^{\textcircled{8}} \cdots P_\ell^{\textcircled{5}} \; y$$

$$t \leftarrow At$$

$$y = z + \text{JacobiSolu} \; (J, \; t)$$