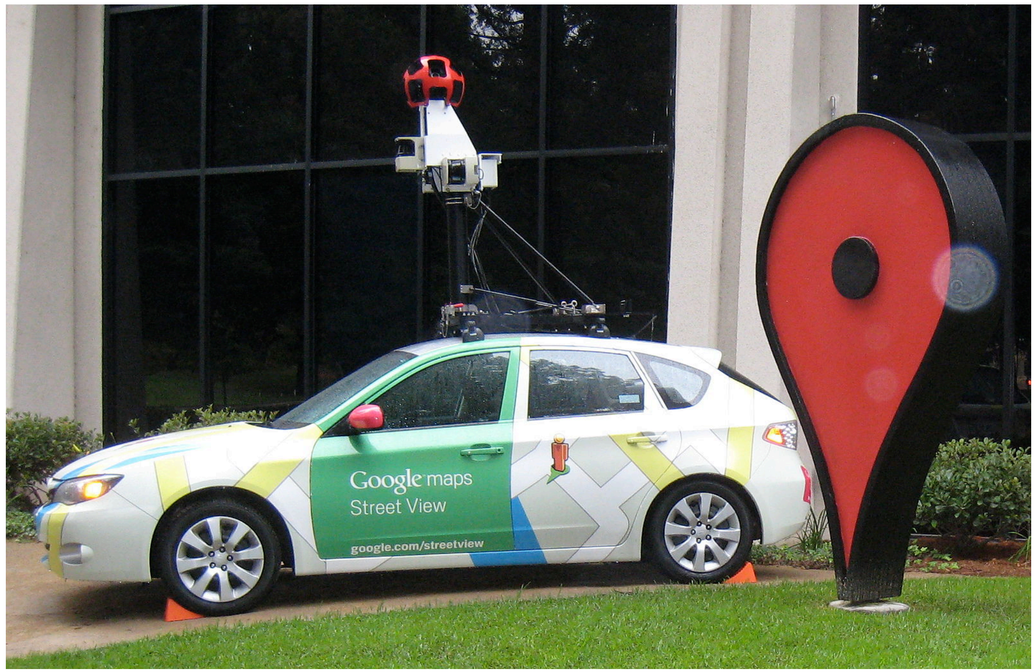
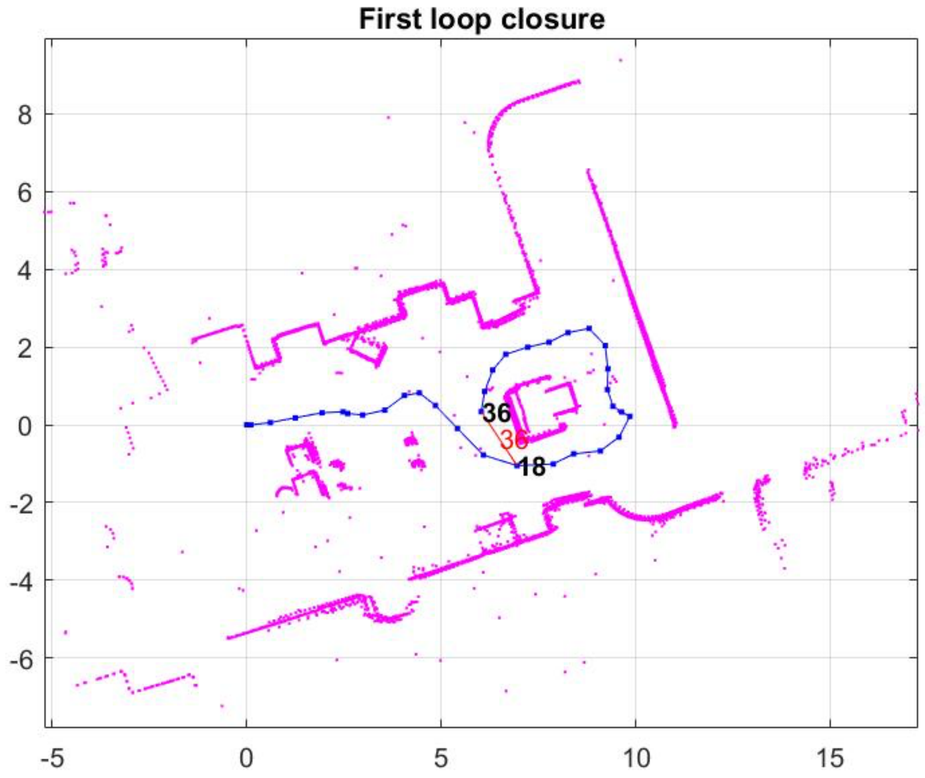
Multigrid for Bundle Adjustment

Tristan Konolige and Jed Brown
CU Boulder

Bundle Adjustment:

Nonlinear optimization problem from Computer Vision used in 3D reconstruction.





Outline

1. What is Bundle Adjustment?

2. How do we solve a Bundle Adjustment problem?

3. How do existing Multigrid solvers perform?

Outline

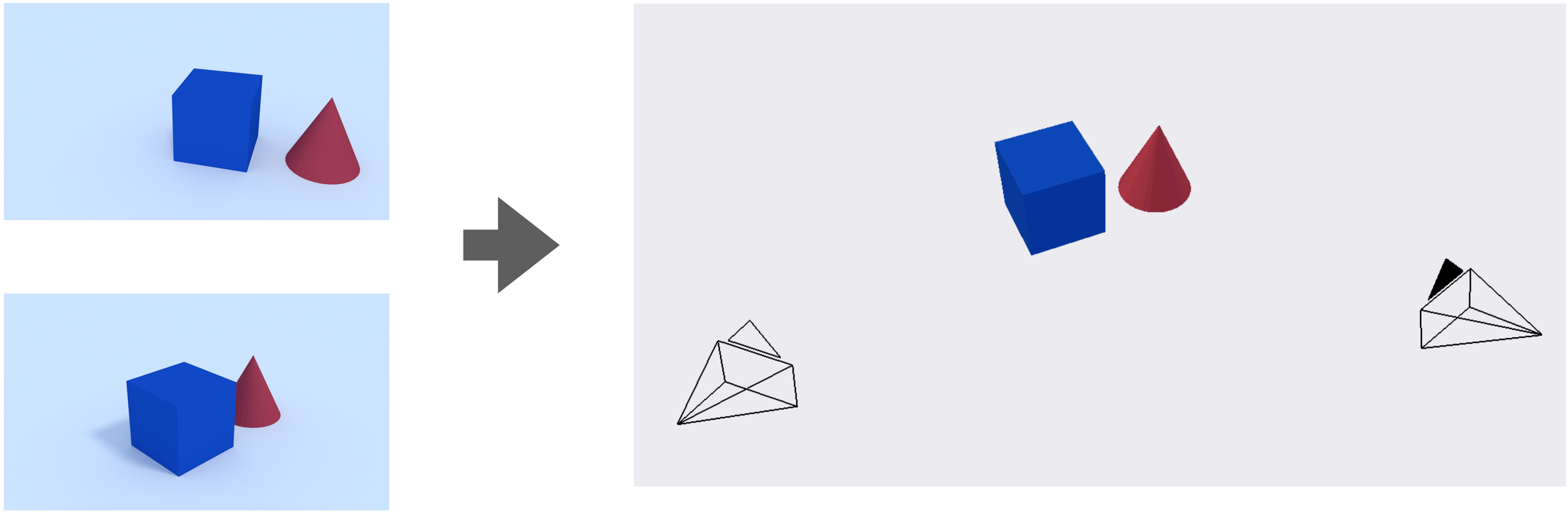
1. What is Bundle Adjustment?

2. How do we solve a Bundle Adjustment problem?

3. How do existing Multigrid solvers perform?

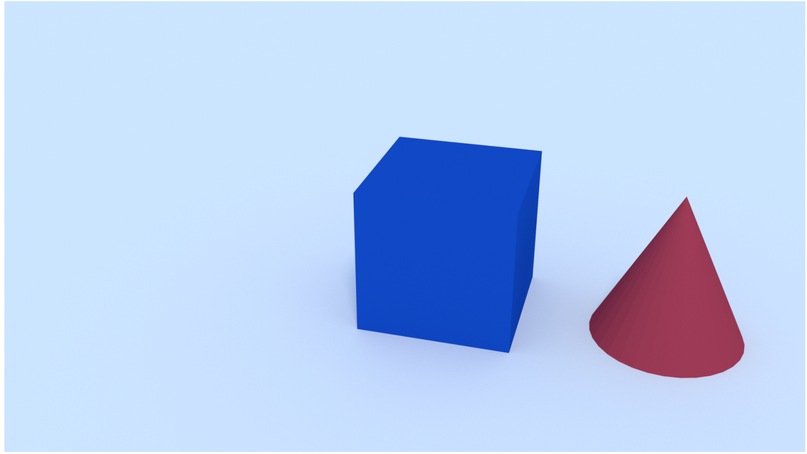
3D Reconstruction

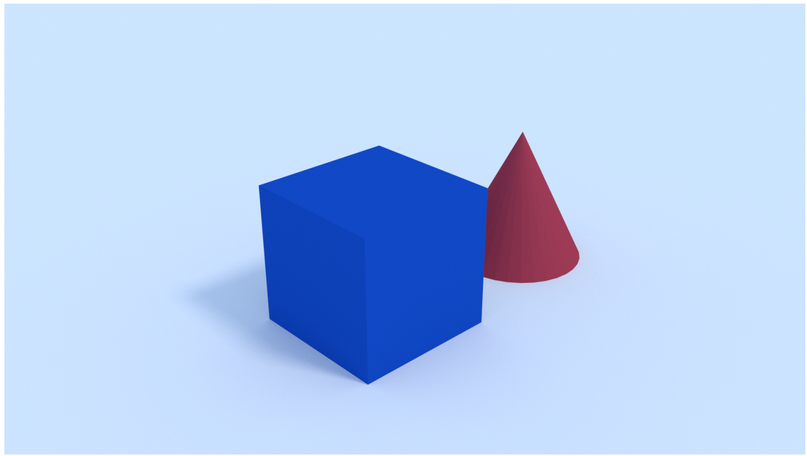
Reconstruct 3D geometry from a set of images



3D Reconstruction

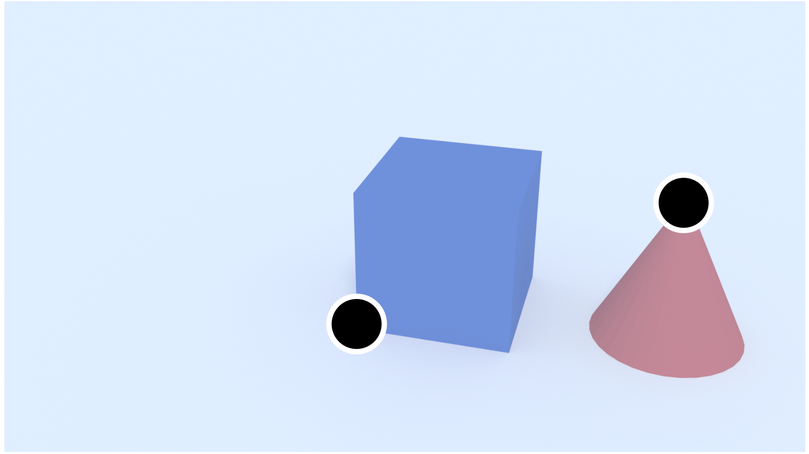
1. Find image features.
2. Match features.
3. Pose + point estimation.
4. Bundle Adjustment.
5. Dense Reconstruction.

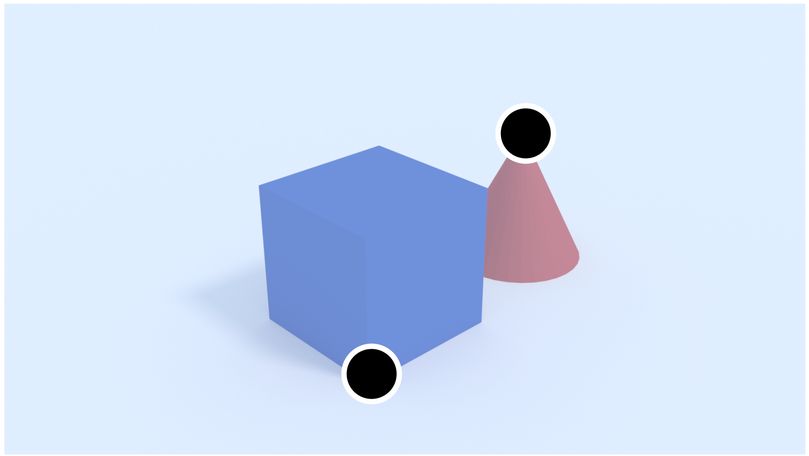




3D Reconstruction

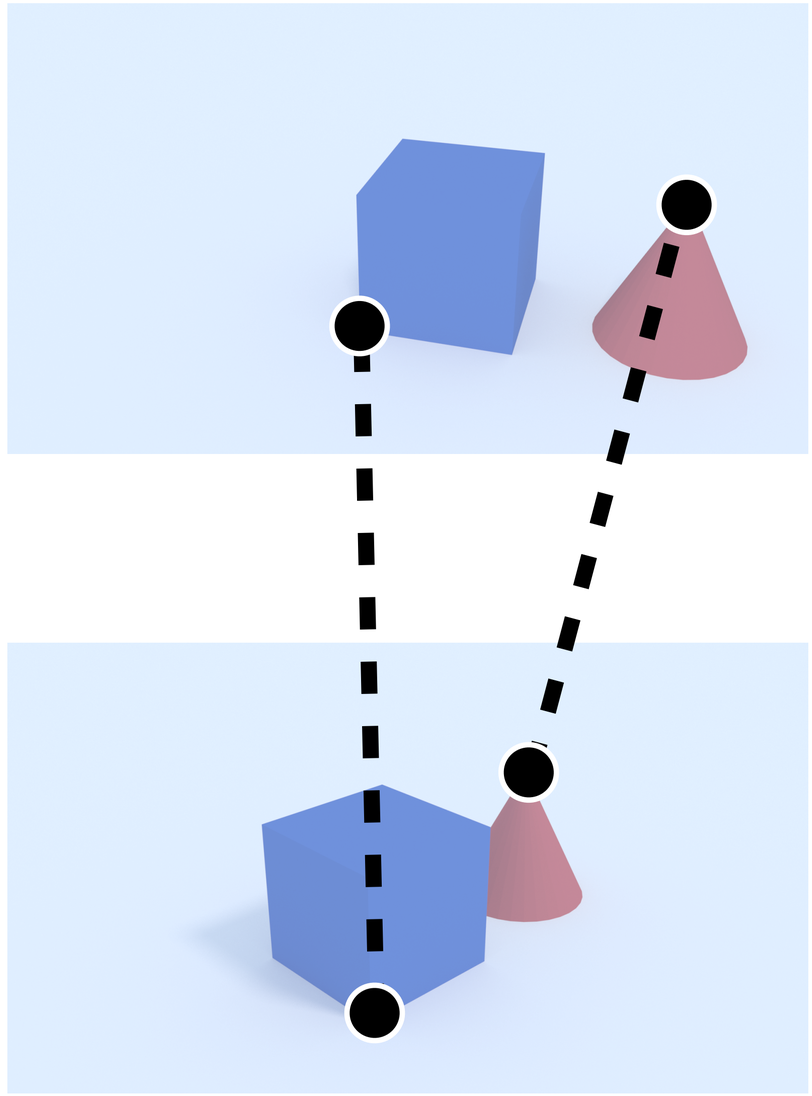
1. Find image features.
2. Match features.
3. Pose + point estimation.
4. Bundle Adjustment.
5. Dense Reconstruction.





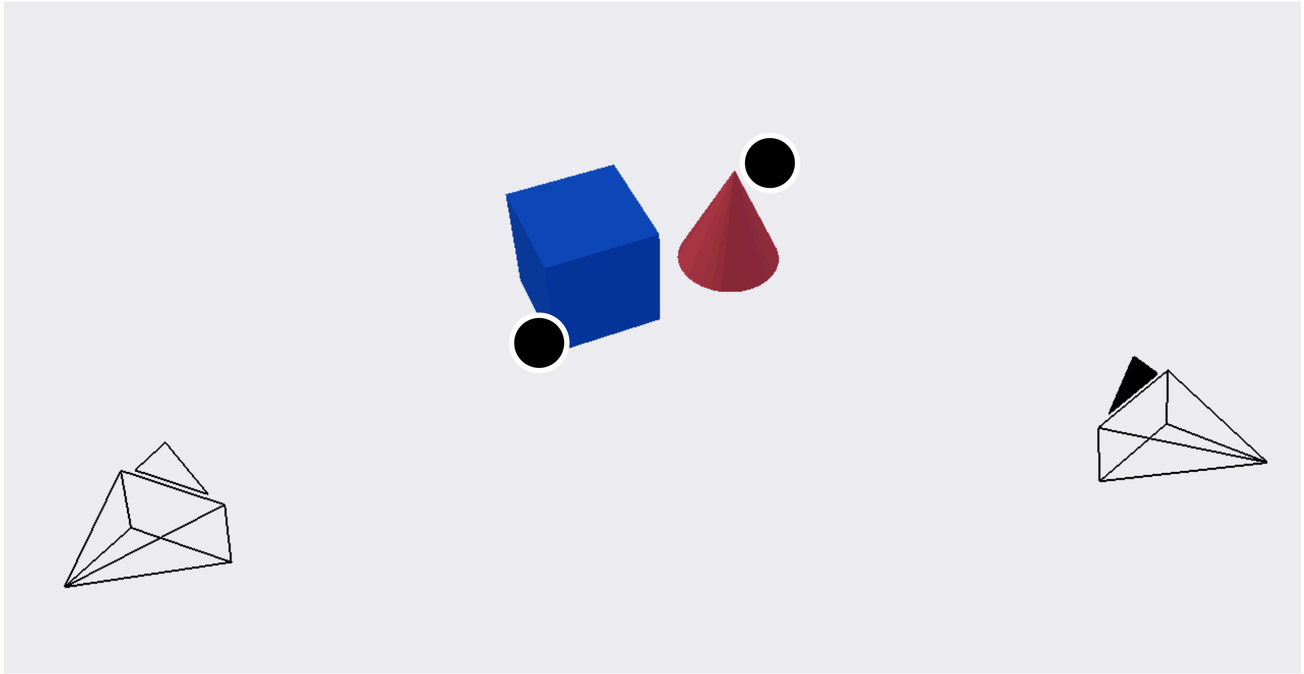
3D Reconstruction

1. Find image features.
2. Match features.
3. Pose + point estimation.
4. Bundle Adjustment.
5. Dense Reconstruction.



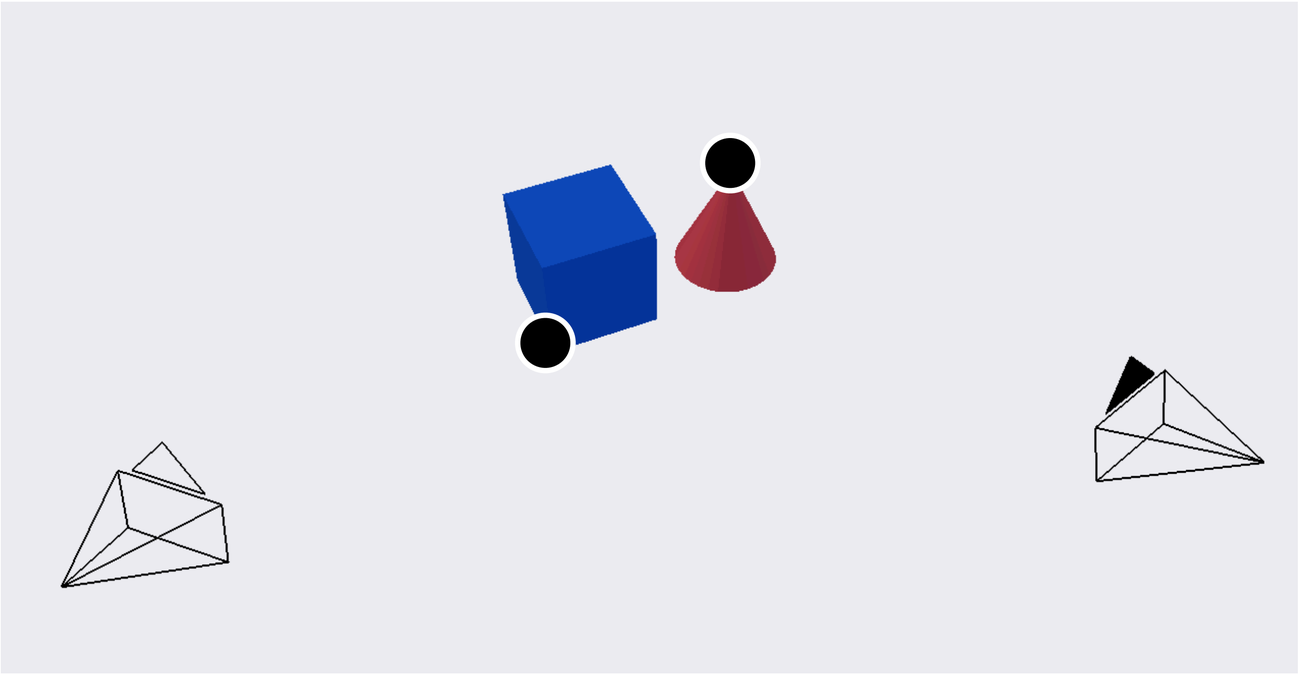
3D Reconstruction

1. Find image features.
2. Match features.
3. Pose + point estimation.
4. Bundle Adjustment.
5. Dense Reconstruction.



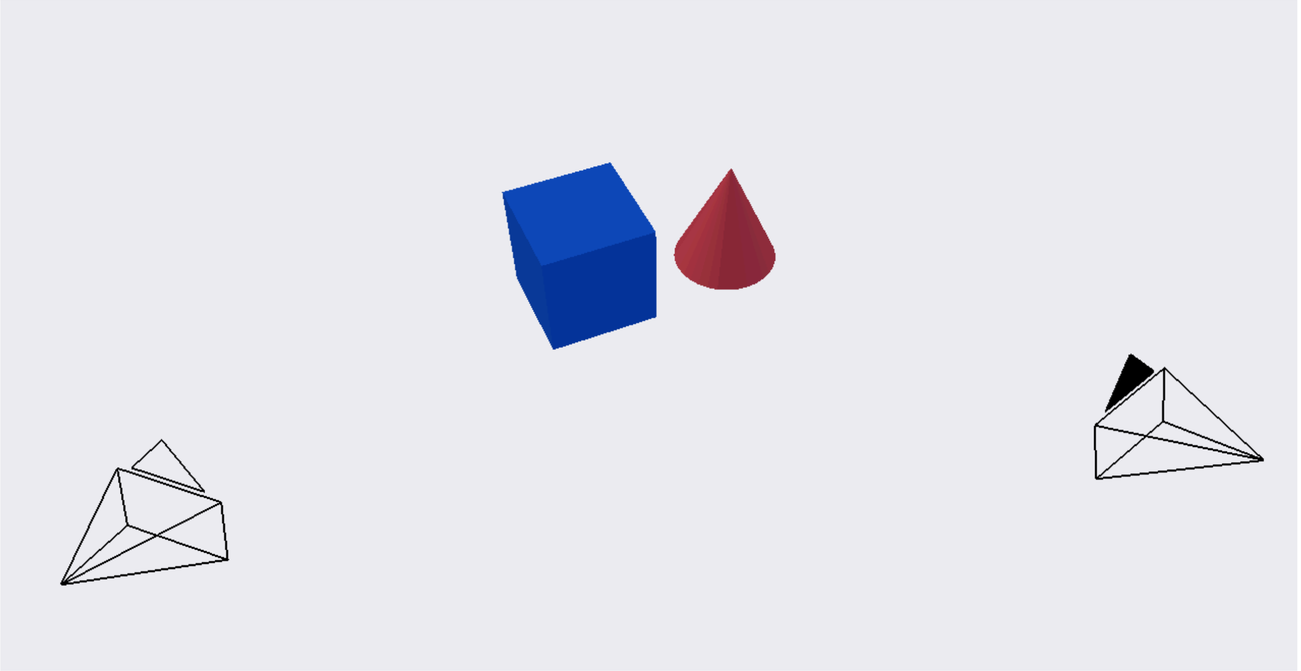
3D Reconstruction

1. Find image features.
2. Match features.
3. Pose + point estimation.
4. Bundle Adjustment.
5. Dense Reconstruction.



3D Reconstruction

1. Find image features.
2. Match features.
3. Pose + point estimation.
4. Bundle Adjustment.
5. Dense Reconstruction.



Bundle Adjustment

- Joint refinement of camera and point parameters.
  - Fixes issues with drift, noise, bias, and calibration.
- Nonlinear least squares optimization.
- Slow.

Camera and Point Parameters

Camera

R: rotation

3 element angle-axis

t: translation

x, y, z

K: intrinsics

focal length, 2x distortion

Point

t: translation

x, y, z

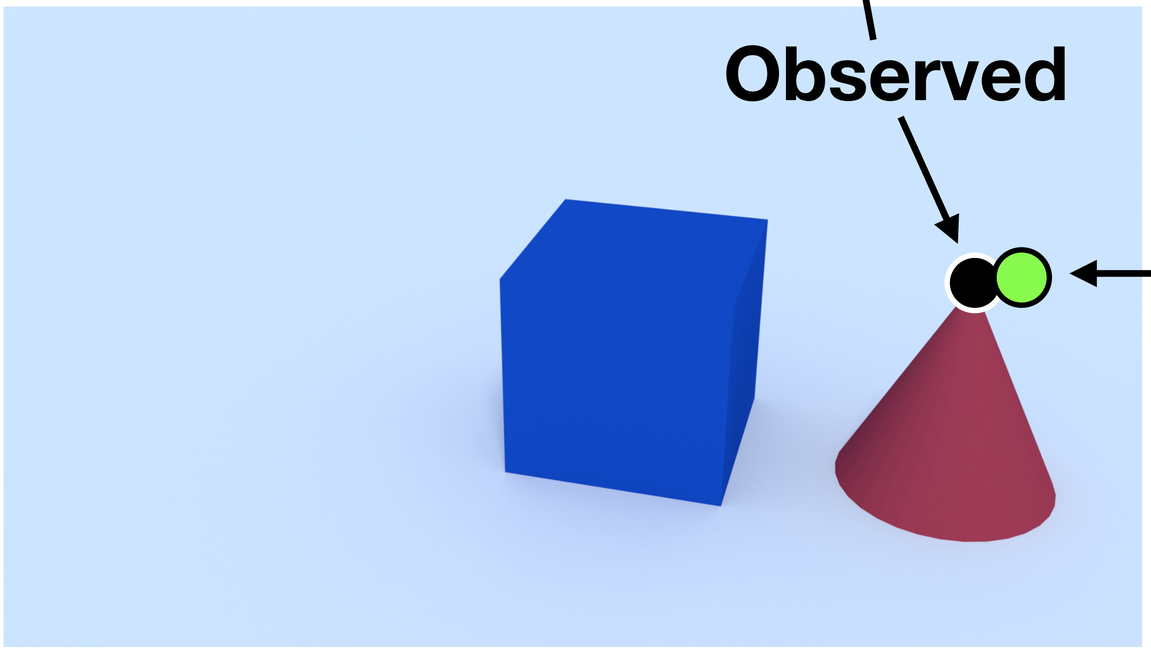
Reprojection Error

$$\text{Error} = \sum_{\text{observations}} \left\| \hat{x} - H\left(K \begin{pmatrix} R & t \\ 0 & 1 \end{pmatrix} x \right) \right\|^2$$

Point in world

Observed

Projected



Outline

1. What is Bundle Adjustment?

2. How do we solve a Bundle Adjustment problem?

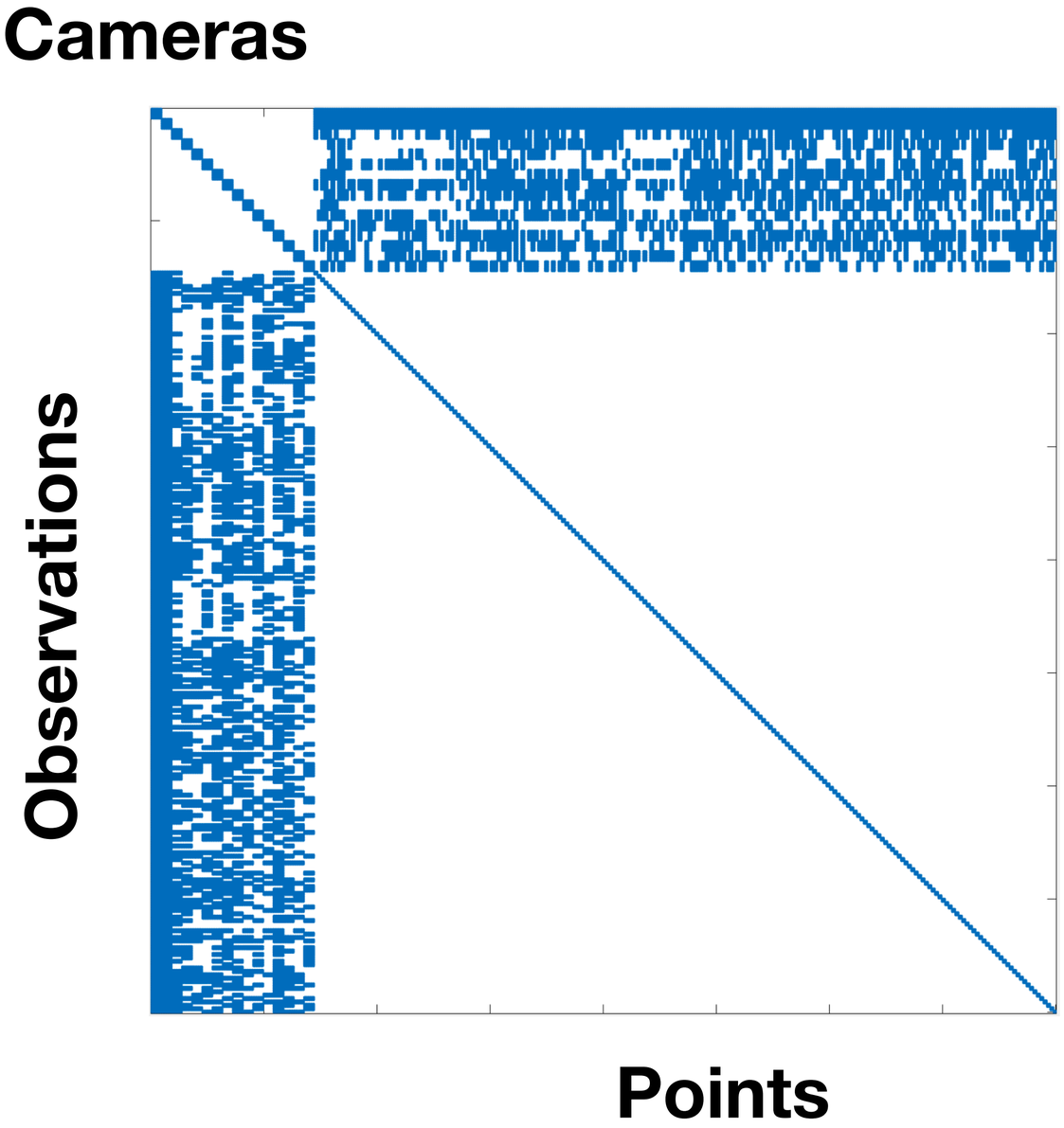
3. How do existing Multigrid solvers perform?

Nonlinear Least Squares

- Typically use Levenburg-Marquardt.
- Quasi-Newton method with $J^TJ \approx H$.
  - J = Jacobian
- Repeatedly solve $J^TJ + D$
  - D = diagonal matrix, determines step size.
- Expensive parts are constructing J^TJ and solving it.

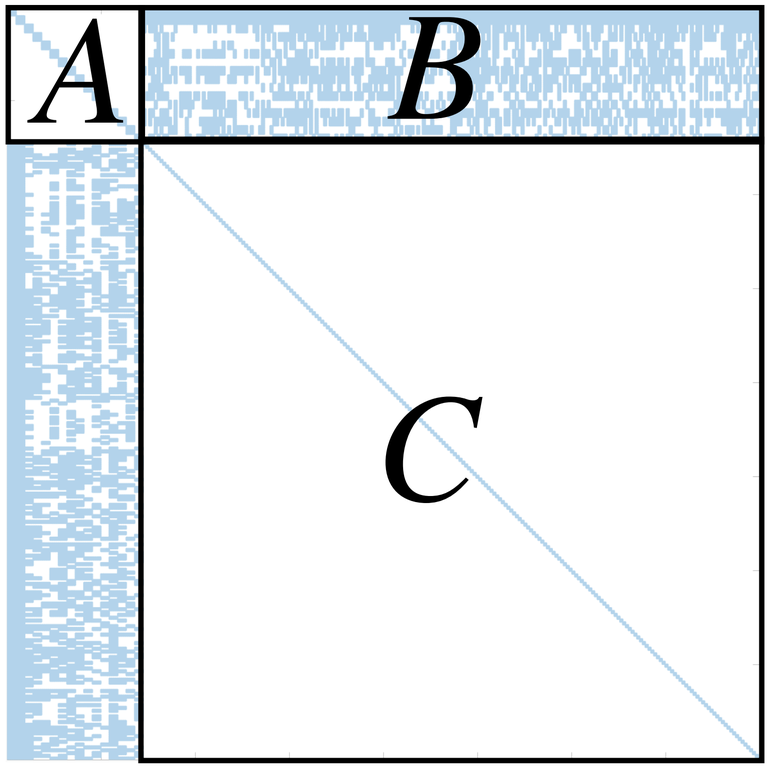
J^TJ Structure

- J^TJ has a special block structure
- J^TJ is symmetric, semipositive definite.
- $J^TJ + D$ is SPD.



Schur Complement

$$S = A - B^T C^{-1} B$$

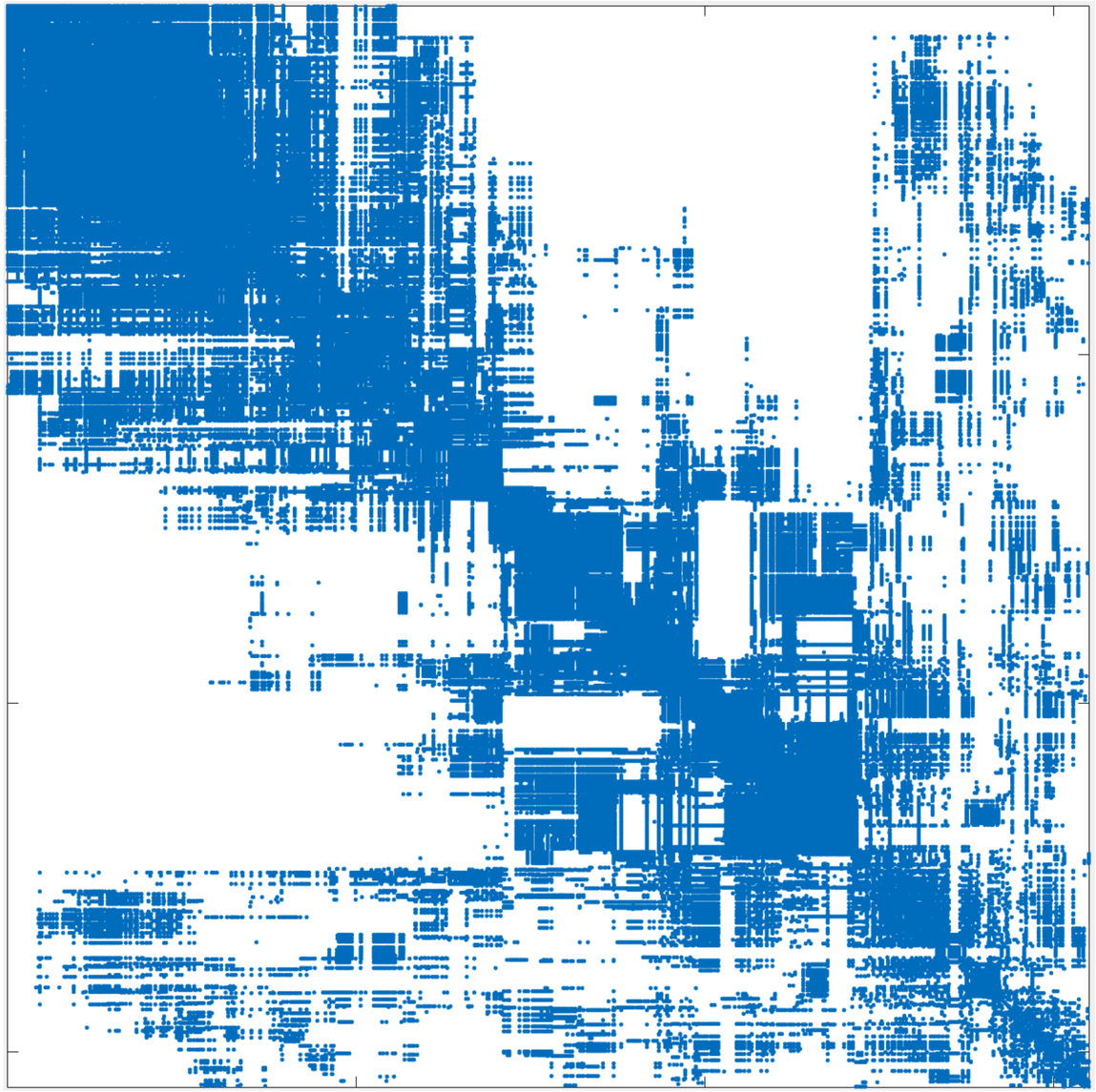


- Typically 1000x points than cameras.
- Greatly reduces problem size.
- C is easy to invert (block diagonal).

Structure of S

- Structure of S is determined by cameras observing the same point.
- If all the cameras are looking at the same thing, then S will be dense.
- If cameras don’t share many observations with other cameras, then S is sparse.
- Typical sparse situation: street view.

Example S



Flickr images of Dubrovnik, Croatia.

Outline

1. What is Bundle Adjustment?

2. How do we solve a Bundle Adjustment problem?

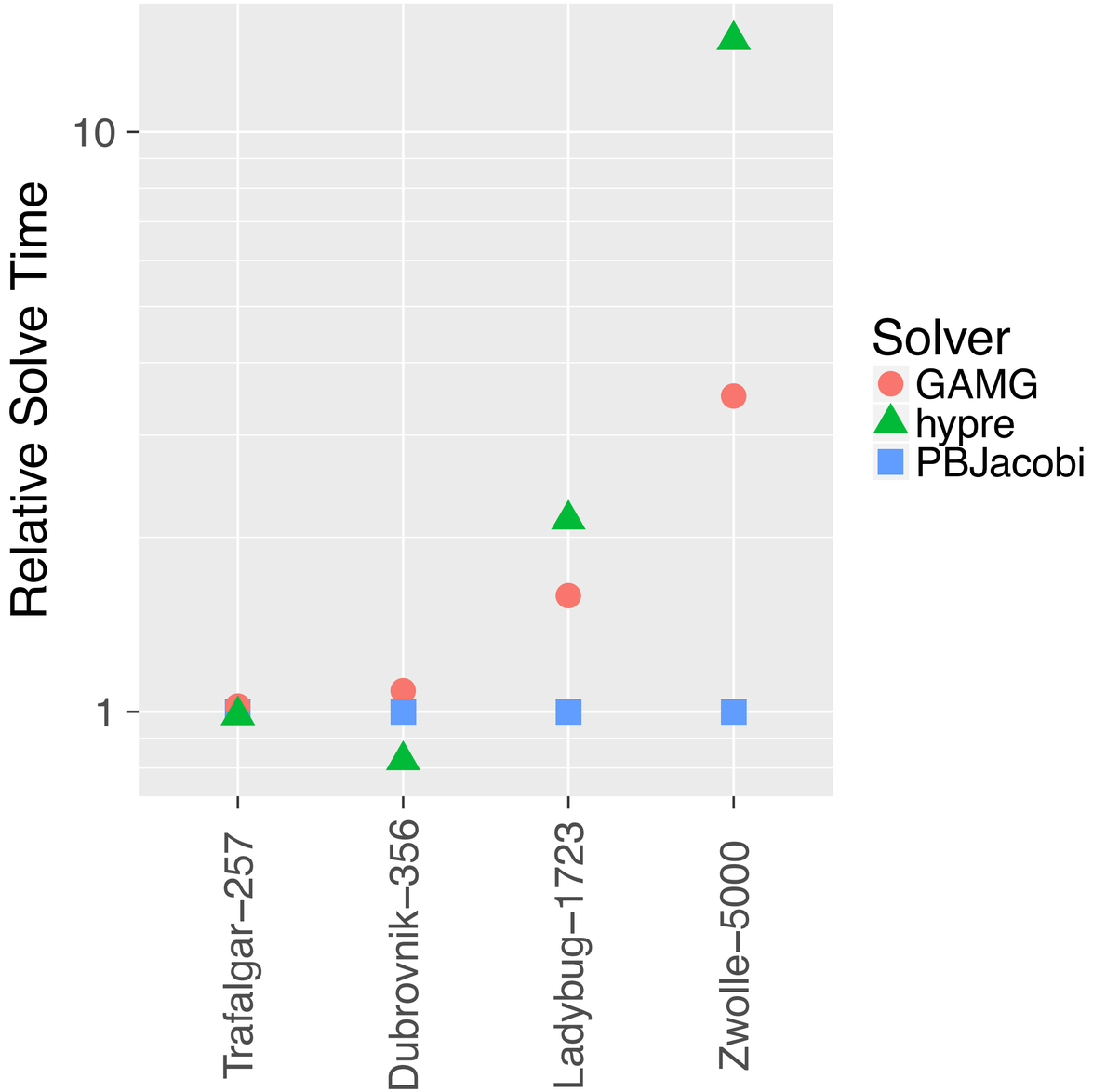
3. How do existing Multigrid solvers perform?

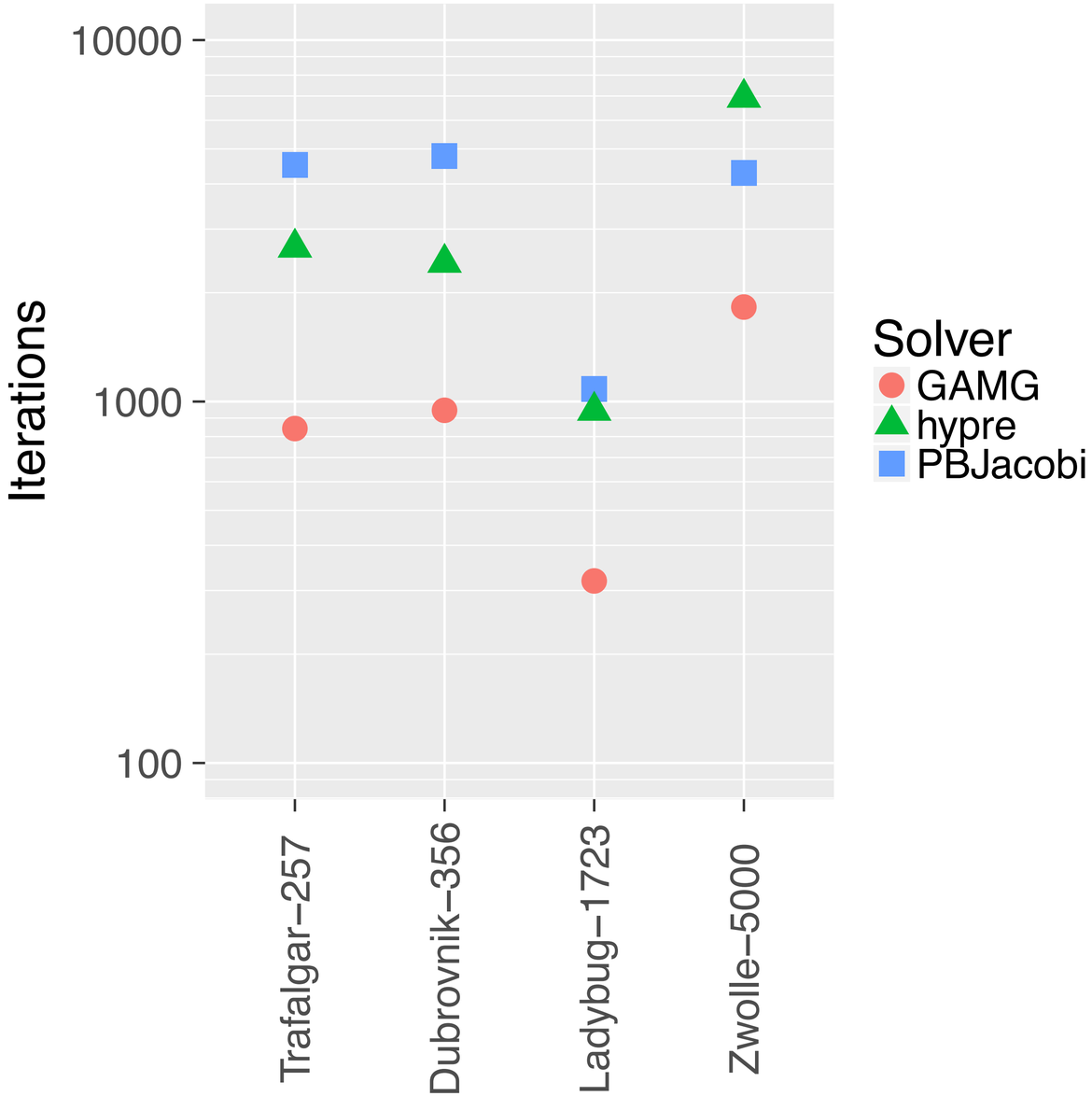
Why Multigrid

- Bundle Adjustment is usually a costly step in 3D reconstruction.
- Multigrid is usually a fast method for solving linear systems.
  - Hopefully O(nnz).
- Bundle Adjustment has underlying geometric information.

Performance

- Tested on 4 problems:
  - Dubrovnik — Flickr images of Dubrovnik, Croatia.
  - Trafalgar — Flickr images of Trafalgar Square.
  - Ladybug — images from a car mounted camera.
  - Zwolle — synthetic dataset from city model.
- Solved using Levenburg-Marquart in PETSc w/ Schur complement.
- Multigrid used as a preconditioned for CG.


Relative Solve Time
10
1
Trafalgar−257
Dubrovnik−356
Ladybug−1723
Zwolle−5000
Solver
GAMG
hypre
PBJacobi


Iterations
10000
1000
100
Trafalgar−257
Dubrovnik−356
Ladybug−1723
Zwolle−5000
Solver
GAMG
hypre
PBJacobi

Why Poor Performance?

- Irregular problem: no fixed number of nonzeros per row.
- Near nullspace not accurately represented.
  - Nonlinear problem can freely rotate, translate.
- Too aggressive coarsening.

Future Directions

- Determine near-nullspace of S.
- Better aggregation.
- Parallelism.
- How to handle dense Schur complement.

Conclusion

- Bundle Adjustment is essential in reconstructing 3D geometry.
- The slow part of bundle adjustment is solving S.
- Existing Multigrid methods for S are slow.

Thank You