A special case of general IQ modulation: amplitude modulation (AM)

Recall from the IQ signal notes that a general signal is written

$$s(t) = a(t)\cos(2\pi f_c t + \phi(t)) \;= \mathrm{Re}\{a(t)e^{j\phi(t)}e^{j2\pi f_c t}\}$$

The radio signal $s(t)$ is a cosine wave at frequency f_c with time-varying amplitude and phase $a(t), \phi(t)$ Thus the only way to encode the message $m(t)$ on the carrier wave is to vary $a(t)$ and/or $\phi(t)$ in step with the message $m(t)$. Thus $a(t), \phi(t)$ are specified as a function of the message $m(t)$ for some modulation types and as a constant for others.

For the case of amplitude modulation (AM), we assume a carrier frequency f_c on the order of MHz, whereas the message frequency f_m is typically on the order of KHz.

For AM, we set the amplitude

$$a(t) = A_c[1 + k_a m(t)]$$

with $\left|\mathrm{k}_a \mathrm{m}(\mathrm{t})\right| < 1$ or $\left[1 + \mathrm{k}_a \; \mathrm{m}(\mathrm{t})\right] > 0$ and $f_c >> w$
Where *w* is the bandwidth of *m(t).*

For the AM transmission type, we assume $\varphi(t)$ is a constant or zero. In many case we assume $\varphi(t)$ =0 for convenience. Thus the transmitted AM signal is of the form:

$$s(t) = a(t)\cos(2\pi f_c t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)\,; \qquad \varphi(t) = 0$$

For the special case where there is no message to send, $m(t) = 0$ and $s(t) = A_c\cos(2\pi f_c t)$ which is simply the carrier wave. For a message, $m(t) = A_m\cos(2\pi f_m t)$, the resulting AM wave is given by the following:

$$s(t) = A_c[1 + k_a A_m\cos(2\pi f_m t)]\cos(2\pi f_c t)$$

Recall that a message frequency is actually time varying in the practical case so one must keep in mind that the AM signal above is valid for one frame where the message contains only one frequency f_m during that frame. For our purposes here, we will assume the message is of constant frequency.

Practically, $m(t)$ could be a square wave or any type of signal.

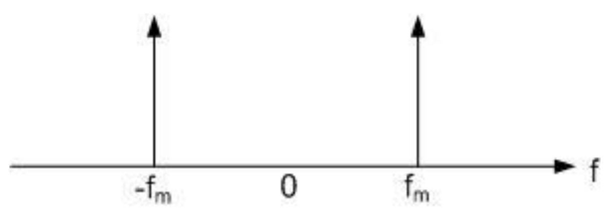
The AM wave generated from a square wave may be used for binary transmissions where the high amplitude would be read as the logical "one" and the low as the logical "zero".

For an analog message signal of single tone, the AM wave appears in the time domain to be the carrier function whose maxima replicate the form of the modulating tone (the oscillating values of the maxima form the "envelope"; see below).

$$m(t) = A_m \cos 2\pi f_m t \qquad f_m = \text{message frequency}$$
$$s(t) = A_c(1 + k_a m(t)]\cos 2\pi f_c t$$
$$s(t) = A_c(1 + \mu \cos 2\pi f_m t]\cos 2\pi f_c t$$
$$\mu = k_a A_m < 1 \text{ is called modulation index}$$

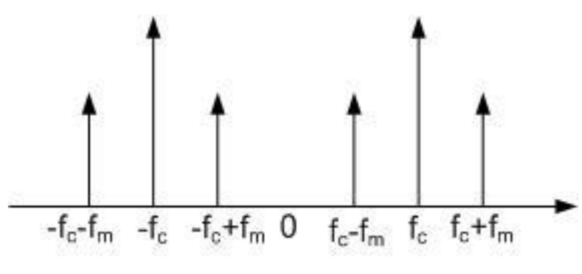
What does the AM wave look like in the frequency domain?

$$m(t) \xrightarrow{Fourier} M(f)$$
$$A_m \cos(2\pi f_m t) \xrightarrow{Fourier} \frac{1}{2} A_m[\partial(f - f_m) + \partial(f + f_m)]$$



Frequency Spectrum of a Single Tone

$$s(t) \xrightarrow{Fourier} S(f)$$



Frequency Spectrum of AM Wave

Demonstrating this algebraically is done by simplifying $s(t)$ in the time domain and then converting the resulting collection of individual sinusoidal terms to their frequency domain representation.

Note: The trigonometric identity $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$ is used to produce line three from line two below:

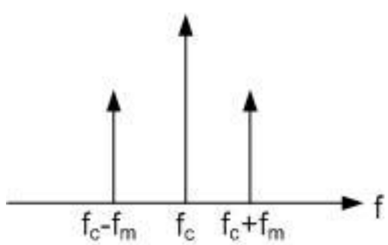
$$s(t) = A_c[1 + k_a A_m \cos(2\pi f_m t)]\cos(2\pi f_c t)$$
$$s(t) = A_c \cos(2\pi f_c t) + A_c \mu \cos(2\pi f_m t)\cos(2\pi f_c t)$$
$$s(t) = A_c \cos(2\pi f_c t) + (A_c \mu / 2)[\cos(2\pi[fc + f_m]t) + \cos(2\pi[f_c - f_m]t)]$$

The second and third cosine terms represent the sidebands as seen in the frequency domain. The frequency of the sidebands relative to the carrier frequency holds the useful information describing the message. We use the character $\mu = A_m k_a$ for simplification and to serve as the modulation index.

$$\mu = \frac{A_{cMAX} - A_{cMIN}}{A_{cMAX} + A_{cMIN}} = A_m k_a,$$

This index can be given as a percentage and A_{cMAX} and A_{cMIN} are respectively the highest and lowest positive amplitude obtained by $s(t)$. This oscillation of maxima holds all the information specified by $m(t)$ and is the effect of multiplying the carrier by the amplitude function, $a(t)$. Another name for $a(t)$ is the "envelope".

Note: $A_{cMAX} = A_c(1+\mu)$; $A_{cMIN} = A_c(1-\mu)$, $\frac{A_{c\max}}{A_{c\min}} = \frac{A_c(1+\mu)}{A_c(1-\mu)}$



Spectral Representation of AM signal

Observe that even though we seem to only modify amplitude, sidebands are observed in the frequency domain representation. The reason is that we are observing the signal over a long time (at least one cycle of the message *m(t)*). If we observe the signal over a small fraction of a cycle of *m(t)*, then we see only the carrier wave with amplitude $a(t) = A_c[1 + k_a m(t)]$

AM signal in time and frequency domain for a general message *m(t)*

$$s(t) = A_c[1 + k_a m(t)]\cos 2\pi f_c t$$
$$s(t) = A_c[1 + k_a m(t)]\frac{(e^{j2\pi f_c t} + e^{-j2\pi f_c t})}{2}$$
$$s(t) = \frac{A_c}{2}e^{j2\pi f_c t} + \frac{A_c}{2}e^{-j2\pi f_c t} + \frac{A_c k_a}{2}m(t)e^{j2\pi f_c t} + \frac{A_c k_a}{2}m(t)e^{-j2\pi f_c t}$$

To find the frequency domain expression *S(f)* use the Fourier transform properties

$$m(t) \leftrightarrow M(f)$$
$$e^{j2\pi f_c t} \leftrightarrow \delta(f - f_c)$$
$$e^{-j2\pi f_c t} \leftrightarrow \delta(f + f_c)$$
$$\exp(j2\pi f_c t)m(t) \leftrightarrow M(f - f_c)$$
$$\exp(-j2\pi f_c t)m(t) \leftrightarrow M(f + f_c)$$

The result is

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c k_a}{2}[M(f - f_c) + M(f + f_c)]$$

This expression shows both positive and negative frequencies. We often draw a picture of the positive frequencies only, and we can write

$$S(f > 0) = \frac{A_c}{2}\delta(f - f_c) + \frac{A_c k_a}{2}M(f - f_c)$$

The modulation spectrum $M(f)$ is shifted up so it is centered around f_c

In general, $M(f)$ will change shape with each frame. For a particular frame, and for illustration purposes, we draw it as it would be for a voice signal: an asymmetrical shape that is zero for $f < 300$Hz , peaks near f=1000 Hz, and is zero for $f > 2{,}700$ Hz. (Think of the green line in sndpeek). The negative frequencies will be the mirror image of this shape.

***Spectrum Analyzer**: How it works

In the lab section, a spectrum analyzer is used to view waveforms in the frequency domain. A signal is fed into the spectrum analyzer and the corresponding power spectrum is displayed on the screen. It is noteworthy to understand that the resulting frequency domain representation is one not of signal amplitude but signal power, displayed on a logarithmic scale. See the description of lab 1 for more detail.

Power is a quantity of interest and one may wish to calculate the power of the carrier or sidebands specifically. The power is proportional to the square of the cosine coefficients.

The power into one ohm $[s(t)]^2 = |S(f)|^2$

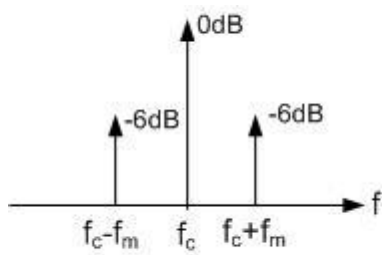
For example, we have an AM signal with single tone modulation $m(t) = A_m \cos 2\pi f_m t$

$$s(t) = A_c[1 + \mu \cos(2\pi f_m t)]\cos(2\pi f_c t)$$

which, in the frequency domain is:

$$s(t) = A_c \cos(2\pi f_c t) + (A_c \mu / 2)\cos(2\pi[f_c + f_m t)]t) + (A_c \mu / 2)\cos(2\pi[f_c - f_m]t)$$

Power at carrier at f_c: $A_c^2 / 2$
Power of sideband at $f_c + f_m$: $(1/2)(A_c \mu / 2)^2 = (1/8)A_c^2 \mu^2$
Power of sideband at $f_c - f_m$: $(1/2)(A_c \mu / 2)^2 = (1/8)A_c^2 \mu^2$



AM Spectrum: Sidebands With Half Carrier Amplitude

$$CarrierPower\, 2\left(\frac{A_c}{2}\right)^2 = \frac{A_c^2}{2}\, PositiveandNegativeFrequencies$$

$$UpperSidebandPower\, 2\left(\frac{A_c \mu}{4}\right)^2 = \frac{A_c^2 \mu^2}{8}$$

$$LowerSidebandpower\, 2\left(\frac{A_c \mu}{4}\right)^2 = \frac{A_c^2 \mu^2}{8}$$

$$\frac{USB + LSB\ Power}{Total\ Power} = \frac{\frac{A_c^2 \mu^2}{8} + \frac{A_c^2 \mu^2}{8}}{\frac{A_c^2}{2} + \frac{A_c^2 \mu^2}{8} + \frac{A_c^2 \mu^2}{8}} = \frac{\mu^2}{2 + \mu^2}$$

Question: If the power at $f_c = 0$ dBm, what is the power at $f_c + f_m$?

Answer: Given: $A_c^2/2 = 1mW$; $(1/8)A_c^2\mu^2 = (1mW)\mu^2/4$

If $\mu = 1$ then the power at $f_c + f_m$ is $(1mW)/4 = 0.25mW$

A drop in power by half is -3dB, thus one quarter of the power is -6dB

In the special case where $\mu = 1$, we have 100% modulation (minimum envelope value is zero).

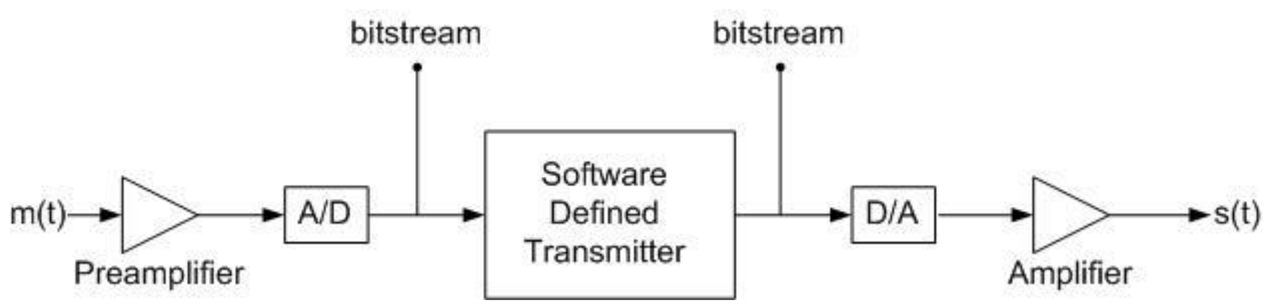
At modulation $\mu > 1$ (greater than 100%) the envelope, $[1 + \mu\cos(2\pi f_m t)]$ becomes less than zero and no longer looks like the message being sent. This case is called "overmodulation". The phase of the carrier wave is shifted by 180 degrees when $[1 + \mu\cos(2\pi f_m t)]$ is less than zero.

Building an AM transmitter

Next we look at how to build an AM transmitter. We have a message that we wish to send and we need a way to send and receive it.

Conceptual Modulation and Demodulation of the Signal m(t)

There are many choices and factors to consider when implementing this system. It can be done digitally using Software Defined Radio, as will be done in this lab.



Digital Transmitter Configuration