**Link Budget**

For any communications system designed to send a message from point A to point B linked via some channel (fiber, coax, radio), the available resources are transmit power and channel bandwidth, and the obstacles to be overcome are noise and interference. Both power and bandwidth cost money; how much will depend on the details of the communications system.

To design a working link with acceptable quality at the lowest possible cost we need to first specify the customer performance requirements, and then compute the necessary power and bandwidth. These performance requirements are usually expressed for digital signals as the data rate at a given error rate, or for analog signals as the fidelity (bandwidth) and signal-to-noise ratio. Another performance requirement is the distance to be covered. Alternately, we may be given the power and/or bandwidth that we can afford, and then need to compute the maximum data rate and distance that can be achieved.

The power obtained over a link for a given distance is given by

$$P_{r,o} = P_t G_t G_r / L_0 \tag{1}$$

where
$P_t$ is the transmit power in watts
$G_t$ is the transmit antenna gain (dimensionless)
$G_r$ is the receiver antenna gain (dimensionless)
$L_0$ is the path loss between points A and B

Usually, the quantities are expressed in dB, so by taking $log_{10}$ of both sides, we write

$$P_{r,o} = P_t + G_t + G_r - L_0 \tag{2}$$

The antenna gains depend on the details of the antenna design. An estimate of the antenna gain in terms of it's size is given by

$$G = \frac{4\pi A_e}{\lambda^2} \tag{3}$$

where $A_e$ is the effective antenna area, which is close to the physical area, and $\lambda = c/f_c$ is the wavelength used. The approximate beamwidth of the antenna is given by $\lambda/\sqrt{A}$.

The path loss $L_0$ is a function of the distance $d$ and the carrier frequency $f_c$ or wavelength $\lambda = c/f_c$ and is usually defined to be greater than 1 (0 dB). The function depends on the particular channel model between point A and point B. If the channel is a cable (fiber of coax), then we set $G_t = G_r = 1$, and find the loss from the manufacturer loss specifications in terms of dB/meter, and multiply by the path length in meters. If the channel is radio, then for free space propagation (e.g.

between point A on the earth and point B in a satellite or aircraft) the signal spreads out over a sphere in all directions, resulting in

$$L_0 = \left(\frac{4\pi d}{\lambda}\right)^2 > 1 \tag{4}$$

The path loss increases as the square of the distance. We can show (exercise) that for $d$ in meters, $f_c$ in GHz,

$$L_0 = 32.4 + 20log\ f_c + 20log\ d > 0 \tag{5}$$

where $log$ is to the base 10. The free space formula also applies for propagation along a flat earth between antennas at heights $h_1, h_2$, provided that $d$ is less than a breakpoint distance $d_{brk} = 4\pi h_1 h_2/\lambda = 4\pi h_1 h_2 f_c/c$. However, when $d > d_{brk}$, the ground-reflected ray is almost the same length as the direct ray. Since the reflection coefficient is close to -1, the two rays almost cancel. In this case, we find that the path loss increases as the fourth power of the distance

$$L_0 = \frac{d^4}{h_1^2 h_2^2} > 1 \tag{6}$$

or in dB

$$L_0 = 40logd - 20log\ h_1 - 20log\ h_2 > 0 \tag{7}$$

independent of $f_c$. We can show(exercise) that both path loss formulas give the same result when $d = d_{brk}$. On plot of $L_0$ versus $d$ on a log scale, the slope changes from 2 to 4 at $d = d_{brk}$. The free space and flat earth path loss formulas can be combined into one formula

$$L_0 = \left(\frac{4\pi d}{\lambda}\right)^2 \left(1 + \frac{d}{d_{brk}}\right)^2 \tag{8}$$

or in dB

$$L_0 = 32.4 + 20log\ f_c - 20log\ d_{brk} + 40log\ d \tag{9}$$

which is equivalent to free space loss for $d \ll d_{brk}$ and to flat earth loss for $d \gg d_{brk}$ (exercise).

In practice, the path loss depends on the details of the environment, and cannot be precisely determined by these formulas. Thus, in general, we measure the path loss over a range of distances, and write

$$L_0 = L_m + 10n_1 log\ d \tag{10}$$

for $d \ll d_{brk}$, where $L_m$ is the measured path loss at a specified distance (usually 1 m or 1 km) and $n_1 \simeq 2$ is the measured slope. For any value of $d$, we write

$$L_0 = L_m + 10(n_1 - n_2)log\ d_{brk} + 10n_2 log\ d \tag{11}$$

where $d_{brk}$ is a measured breakpoint distance, and $n_2 \simeq 4$ is the measured slope for $d > d_{brk}$.

Other propagation models depend on the details of the terrain, and take into account diffraction around edges, Fresnel zones where two paths differ by $\lambda/2$, atmospheric effects, earth curvature K factor, and refractivity gradients.

For example, for a line of sight path to have free space path loss, any obstructions or obstacles must be outside the "first Fresnel zone", i.e. the distance from the obstacle to the line of sight path

$$F_1 = \sqrt{\frac{\lambda d_1 d_2}{d_1 + d_2}} \tag{12}$$

where $d_1$ is the distance from the obstacle to one end of the link, and $d_2$ is the distance from the obstacle to the other end of the link.

The power required at the receiver to achieve acceptable performance (also called the receiver sensitivity) is given by

$$P_{r,n} = kT_0(S/N)WF \tag{13}$$

where

| | |
|---|---|
| $k$ | $= 10^{-22.86}$ watts/Hz is Boltzmann's constant |
| $T_0$ | is the noise temperature (normally 290K on earth), and |
| $F$ | is the (dimensionless) noise figure of the system representing the imperfect amplifiers and/or interference. |

Usually, the quantities are expressed in dB, so by taking $log_{10}$ of both sides, we write

$$P_{r,n} = k + T_0 + (S/N) + W + F \tag{14}$$

where $k = -228.6$. Thus $P_{r,n}$ increases with $(S/N), W$ and $F$.

The link budget parameters are sometimes combined as follows (linear not in dB) Effective Isotropic Radiated Power (EIRP) $EIRP = P_T G_T$, receiver figure of merit $G_R/T$, signal-to-noise power density also called carrier-to-noise ratio $S/N_0 = P_R/(kT)$ where $N = N_0 W$, $N_0 = kT$, $F = 1 + T/T_0$, where $T$ is the system noise temperature that combines the noise figure with all other noise effects.

The link will work as long as the power obtained is greater than the power needed, i.e. as long as the link margin $M$

$$M = \frac{P_{r,o}}{P_{r,n}} > 1 \tag{15}$$

or in dB

$$M = P_{r,o} - P_{r,n} > 0dB \tag{16}$$

To do a link budget calculation, we are given all link parameters except one, and then compute that one. For example, given $S/N, W, F, P_t, G_t, G_r, d, f_c$ and free space propagation, we can compute $M$. In many cases, it is necessary to make reasonable assumptions about some of these link parameters. We can combine the link budget equations in various ways.e.g $(S/N) + M = EIRP + (G/T_0) - L_0 - k - W$.

For digital systems, the modem performance is usually specified as bit error rate versus $E_b/N_0$, where

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|---|---|
| $E_b$ | is energy per bit (joules/bit), |
| $N_0$ | is the noise power spectral density (watts/Hz) |

$N_0$ is the noise power per unit bandwidth. The signal power $S$ in watts

$$S = E_b R \tag{17}$$

(units of joules/bit times bits/sec = joules/sec = watts). and the noise power $N$ in watts

$$N = N_0 W \tag{18}$$

(units of watts/Hz times Hz = watts). Thus we can write

$$\frac{S}{N} = \frac{E_b}{N_0}\frac{R}{W} \tag{19}$$

The maximum possible data rate $R_{max} = C$ is related to the power and bandwidth via the Shannon capacity formula

$$C = Wlog_2(1 + \frac{S}{N}) \tag{20}$$

where:

| | |
|---|---|
| $C$ | is the capacity in bits per second, |
| $W$ | is the bandwidth in Hz |
| $S$ | is the signal power |
| $N$ | is the noise power |

The Shannon capacity represents a theoretical upper limit to the data rate as a function of bandwidth and signal-to-noise ratio. If we use a modulation technique so that $C = W$, then we need $S/N = 1$ (0 dB). However, this requires complex and expensive circuity. For simple low cost systems with $C = W$, we need $S/N = 10$ dB.