ELEC 350 fall 2013

THEORY OF SOFTROCK ANALOG I-Q RECEIVER

I. Objective

Learn theory related to the Softrock analog I-Q receiver.

II. Theory

1. I-Q signals

Review the class notes posted on moodle.

In summary, any signal $s(t)$ can be written as a carrier wave at frequency f_c with time-varying amplitude and phase, i.e.

$$
\begin{aligned}
s(t) &= a(t)\cos[2\pi f_c t + \phi(t)] \\
&= \mathrm{Re}\{a(t)e^{j\phi(t)}e^{j2\pi f_c t}\} \\
&= \mathrm{Re}\{[I(t) + jQ(t)][\cos 2\pi f_c t + jsin2\pi f_c t]\} \\
&= I(t)\cos 2\pi f_c t - Q(t)sin2\pi f_c t \\
&= a(t)\cos[2\pi f_c t + \phi(t)]
\end{aligned}
$$

where

$$I(t) = a(t)\cos\phi(t) = \mathrm{Re}\{a(t)e^{j\phi(t)}\},$$

$$Q(t) = a(t)sin\phi(t) = \mathrm{Im}\{a(t)e^{j\phi(t)}\}$$

and

$$\tilde{s}(t) = I(t) + jQ(t) = a(t)e^{j\phi(t)}$$

is called the complex envelope of the signal. The complex envelope contains two real waveforms.

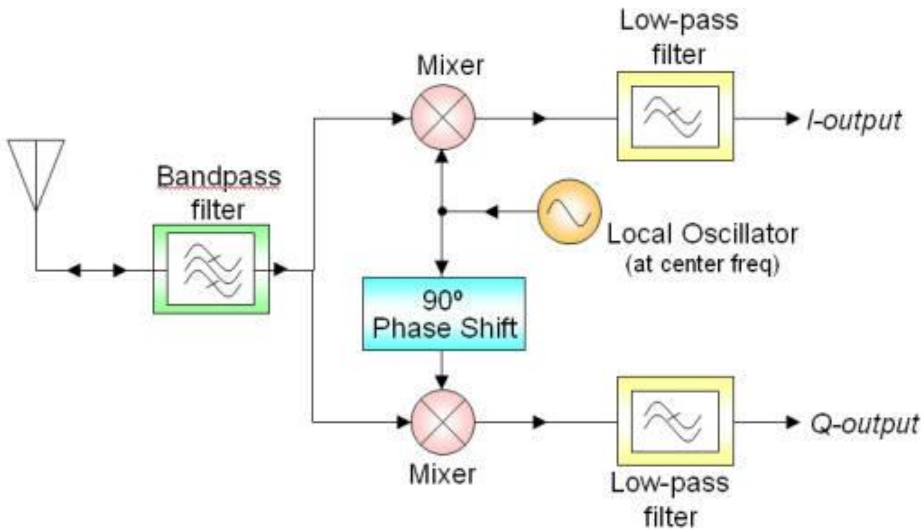
The complex envelope (or the two real waveforms) contain the information or message.

The real signal $s(t)$ is obtained by multiplying the complex envelope $\tilde{s}(t)$ with the complex carrier wave $e^{j2\pi f_c t} = cos2\pi f_c t + jsin2\pi f_c t$ and taking the real part to yield $s(t) = \mathrm{Re}\{\tilde{s}(t)e^{j2\pi f_c t}\}$

2. Softrock analog I-Q receiver

The Softrock receiver is designed to receive radio frequency (RF) signals at frequencies f_c in the range $f_{LO} \pm f_s / 2$ MHz, where f_{LO} is the local oscillator (LO) frequency set to about 14.047 MHz and f_s is the sampling rate of the soundcard. If $f_s = 48$ KHz, then the frequency range is 14.023-14.071 MHz. This frequency range is in the shortwave (HF) radio band, located in between the AM radio broadcast band 0.5-1.6 MHz and the FM radio broadcast band 88-108 MHz.

The Softrock receiver operates by generating two local oscillator signals at f_{LO} and mixing (multiplying) it with a desired radio frequency (RF) signal at f_c (picked up by the antenna or fed in by a signal generator) to yield a signal at the difference frequency $f_b = f_c - f_{LO}$



Mathematical proof:
In what follows, we need some trigonometric identities

$$\cos\alpha\cos\beta = [\cos(\alpha-\beta)+\cos(\alpha+\beta)]/2$$
$$\sin\alpha\cos\beta = [\sin(\alpha-\beta)+\sin(\alpha+\beta)]/2$$
$$\sin\alpha\sin\beta = [\cos(\alpha-\beta)-\cos(\alpha+\beta)]/2$$

One of the local oscillator signals is written $\cos 2\pi f_{LO} t$,
and the desired RF signal is written $a(t)\cos[2\pi f_c t + \phi(t)]$.
We assume $a(t)=1, \phi(t)=0$ for the moment, so the desired RF signal is simply an unmodulated carrier wave $\cos 2\pi f_c t$.
The *cos* mixer function multiplies these two signals to yield

$$\cos 2\pi f_c t \cdot \cos 2\pi f_{LO} t \ .$$

Using one of the trigonometric identities , we can write

$$\cos 2\pi f_{LO} t \cdot \cos 2\pi f_c t = 0.5\cos 2\pi(f_c + f_{LO})t + 0.5\cos 2\pi(f_c - f_{LO})t$$

Thus multiplying two sine (or cosine) waves at frequencies f_{LO} and f_c results in two new sine waves, one at the sum frequency $f_c + f_{LO}$ and one at the difference frequency $f_c - f_{LO}$.

The signal at the sum frequency $f_c + f_{LO}$ is filtered out by analog low pass filters in the Softrock receiver. The signal at the difference frequency $f_c - f_{LO}$ is written

$$I(t) = 0.5\cos 2\pi(f_c - f_{LO})t = 0.5\cos 2\pi f_b t \ .$$

$I(t)$ can be sampled by the sound card and processed by the computer, provided that f_c is close enough to f_{LO}, i.e. the difference is less than half the sampling rate, $|f_c - f_{LO}| < f_s / 2$ or

$$f_{LO} - f_s / 2 < f_c < f_{LO} + f_s / 2$$

The difference frequency $f_b = f_c - f_{LO}$, where $|f_b| < f_s / 2$

The Softrock receiver effectively has two local oscillators operating 90 degrees out of phase, $\cos 2\pi f_{LO} t$ and $-\sin 2\pi f_{LO} t$ and two mixers. Thus there are two outputs that we call $I(t)$ and $Q(t)$ (see figure above) where $I(t) = 0.5\cos 2\pi(f_c - f_{LO})t$ was calculated above. To calculate

$$\cos 2\pi f_c t \cdot (-\sin 2\pi f_{LO} t)$$

we apply a trigonometric identity with $\alpha = 2\pi f_{LO} t, \beta = 2\pi f_c t$ and write

$$-\sin 2\pi f_{LO} t \cdot \cos 2\pi f_c t = -0.5\sin 2\pi(f_c + f_{LO})t + 0.5\sin 2\pi(f_c - f_{LO})t$$

The signal at the difference frequency is written

$$Q(t) = 0.5\sin 2\pi(f_c - f_{LO})t = 0.5\sin 2\pi f_b t$$

We can write these two signals $I(t)$ and $Q(t)$ as one complex signal $\tilde{s}(t) = I(t) + jQ(t) = a(t)e^{j\phi(t)}$ that has time varying amplitude and phase $a(t), \phi(t)$

Thus the Softrock receiver function can also be described using complex signals as follows.

The RF signal $\hat{r}(t) = a(t)e^{j\phi(t)}e^{j2\pi f_c t}$ is multiplied by the complex local oscillator

$$e^{-j2\pi f_{LO} t|} = \cos 2\pi f_{LO} t - jsin 2\pi f_{LO} t$$

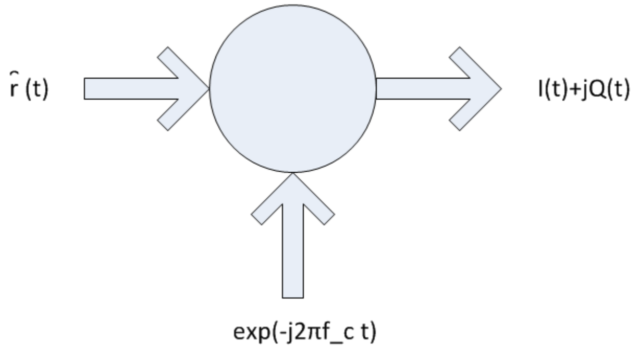
to yield

$$\hat{r}(t)e^{-j2\pi f_{LO} t} = [a(t)e^{j\phi(t)}e^{j2\pi f_c t}]e^{-j2\pi f_{LO} t} = a(t)e^{j\phi(t)}e^{j2\pi f_b t} = I(t) + jQ(t)$$

where the received complex baseband signal is

$$\tilde{r}(t) = I(t) + jQ(t)$$

The diagram below using complex signals performs the same function as the previous diagram above using real signals. Note that the complex signal diagram does not use the low pass filters.



Softrock analog IQ receiver in complex notation, in this diagram f_c = f_{LO}

For the case $a(t)=1, \phi(t)=0$ where the RF input signal is a simple carrier wave $\hat{r}(t)=e^{j2\pi f_c t}$, and $I(t)=\cos 2\pi f_b t$, $Q(t)=\sin 2\pi f_b t$, we can write

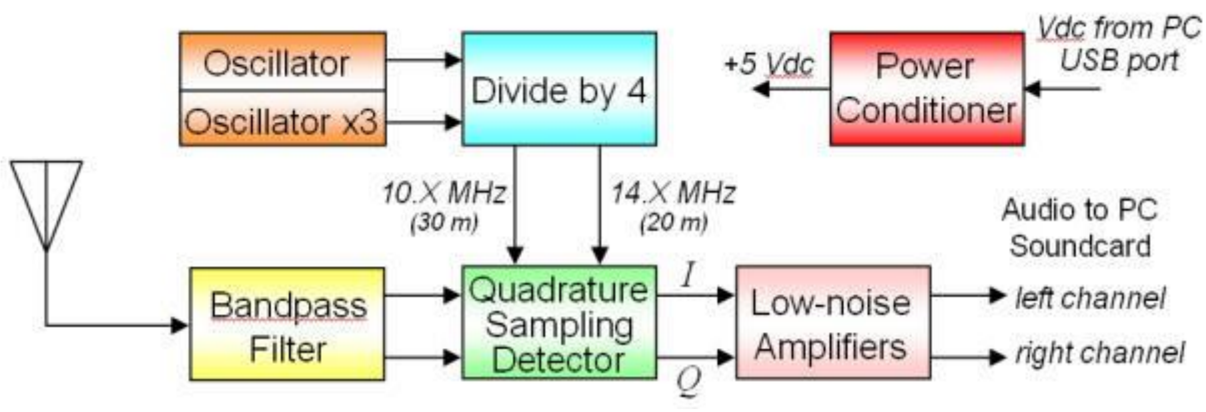
$\tilde{r}(t)=I(t)+jQ(t)=\cos 2\pi f_b t + j\sin 2\pi f_b t = e^{j2\pi f_b t}$

the same result as above, apart from a factor 0.5 arising from the complex notation.

If $I(t)$ and $Q(t)$ are displayed on a *x-y* scope, a circle is displayed. If $f_b < 5$ Hz or so, then the dot on the scope can be seen tracing out the circle.

Softrock implementation in analog circuitry:

The two local oscillators (*cos* and *sin*) are implemented using a single oscillator at $4 f_{LO} = 40.5$ MHz and a divide by 4 circuit (to get 4 phases 90 degrees apart, *cos, sin, -cos, -sin*, we use only the *cos* and -*sin*). The 40.5 MHz LO signal is generated using a 13.5 MHz crystal oscillator and taking the 3rd harmonic .

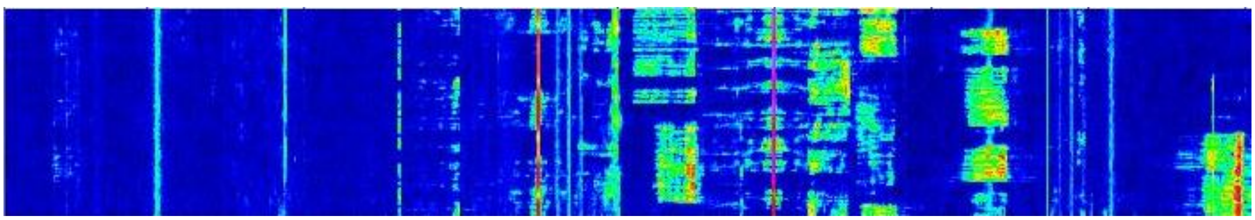


The mixing occurs in the quadrature sampling detector (QSD). Details of the QSD are posted on the moodle site under “SDR practical introduction 2002”, page 6.

3. Spectrogram

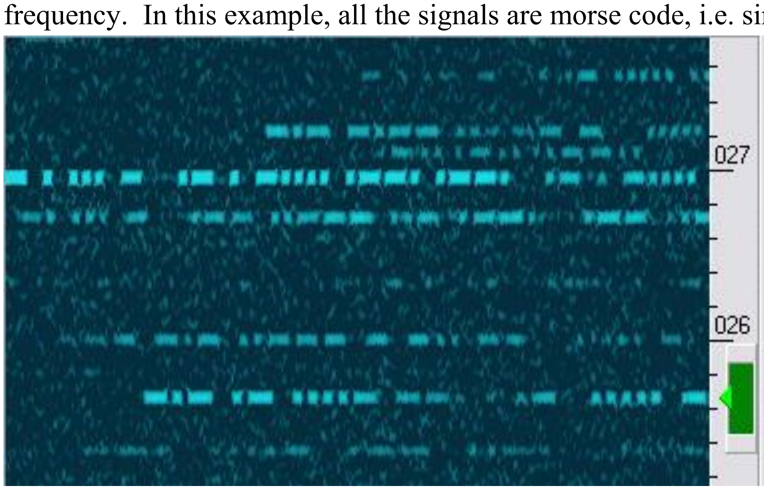
Radio signals (and other types of one dimensional signals) can be displayed with a *spectrogram* that represents the signal simultaneously in the time and frequency domains. A spectrogram has 3 dimensions: time, frequency and amplitude. Often the amplitude is represented by colour, just like a topographical map. A *waterfall* display is a spectrogram where the time axis is moving, showing the most recent signal first.

The figure below shows an example spectrogram (equivalent to a snapshot in time of a moving waterfall display). The *x* axis is frequency, the *y* axis is time (most recent at the top, time increasing downwards). On the left of the display, the blue line represents a sine wave at a fixed frequency. The green blocks represent radio signal modulated with information, and thus occupy a finite non-zero bandwidth.



Waterfall spectrogram with frequency on *x* axis, time on *y* axis, amplitude color coded to display *z* axis.

Another example of a waterfall spectrogram is shown below, where the *x* axis is time and the *y*-axis is frequency. In this example, all the signals are morse code, i.e. sine waves switched on and off.



Waterfall spectrogram with time on *x* axis, frequency on *y* axis, amplitude grey scale coded to display *z* axis, tunable filter shown in green to select desired signal

The spectrogram at a particular time is found by computing $\tilde{S}(f) = FFT\{\tilde{s}(t)\}$. Since $\tilde{s}(t)$ is complex, the spectrum $\tilde{S}(f)$ is not symmetrical around zero. The frequency axis zero point is often labeled f_{LO} instead of zero, since an RF signal at exactly $f_c = f_{LO}$ will be mixed by the IQ receiver down to $f_c - f_{LO} = f_b = 0$ Hz exactly. In this way, the frequency axis displays signals at their actual RF frequencies within a bandwidth f_s from $f_{LO} - f_s/2$ to $f_{LO} + f_s/2$. Thus the maximum bandwidth of a complex signal sampled at f_s is f_s, whereas the maximum bandwidth of a real signal sampled at f_s is $f_s/2$.

Live waterfall displays with tunable filter and audio output can be found at http://www.websdr.org/

4. IQ imbalance

In practice, the two local oscillators of the Softrock analog IQ receiver are such that $Q(t)$ is not exactly the same amplitude as $I(t)$ and not exactly 90 degrees out of phase with $I(t)$. As a result, the amplitude and phase of $Q(t)$ is shifted relative to what it should be by a complex factor $(1+\alpha)e^{j\theta}$ for small values of α, θ. We write the complex output $\tilde{r}'(t)$ of the imperfect receiver with IQ imbalance as

$$\tilde{r}'(t) = I'(t) + jQ'(t) = I(t) + j(1+\alpha)e^{j\theta}Q(t)$$

In this analysis we have assumed that the complex factor is independent of frequency f_b.

Thus we can write the output of an imperfect receiver

$$I'(t) = I(t) - (1+\alpha)\sin\theta Q(t)$$
$$Q'(t) = (1+\alpha)cos\theta Q(t)$$

These expressions can be approximated by

- Re{out} = Re{in} * (1 + Magnitude)
- Im{out} = Im{in} + Re{in} * Phase

Where Phase is defined such that $\theta = \pi/2 - \text{Phase}$ and using

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

In the frequency domain, IQ imbalance appears in the form of an image, i.e. every signal at frequency f_b will have a mirror image at $-f_b$. For a signal at f_b, the desired complex baseband signal is $\tilde{r}(t) = \tilde{s}(t) = e^{j2\pi f_b t}$, but with IQ imbalance, the actual observed complex baseband signal is

$$\tilde{r}'(t) = \mu e^{j2\pi f_b t} + \nu e^{-j2\pi f_b t}$$

with complex constants μ, ν , thus explicitly showing the desired signal at f_b and the mirror image at $-f_b$. The complex constants μ, ν are functions of α, θ . When $\alpha = \theta = 0, \nu = 0$

In general, we can write the received complex signal obtained at the output of an imperfect IQ receiver as

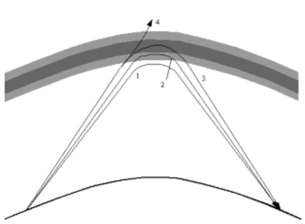
$$\tilde{r}'(t) = \mu\tilde{r}(t) + \nu\tilde{r}^*(t)$$

where $\tilde{r}'(t)$ is the receiver output, $\tilde{r}(t) = \tilde{s}(t)$ is the desired complex signal, and $\tilde{r}^*(t)$ is the image The image rejection is $20\log|\nu/\mu|$ dB.

The mirror image can be nulled out by multiplying $Q'(t)$ by the inverse of this complex factor $(1+\alpha)e^{j\theta}$, i.e. $\frac{1}{1+\alpha}e^{-j\theta} = \beta e^{j\psi}$. The values of the magnitude β and phase ψ can be found manually (with hardware or software controls) or by an adaptive algorithm.

5. Shortwave (HF) radio propagation

The Softrock receiver operates in the 10 MHz shortwave (HF) radio band, located in between the AM radio broadcast band 0.5-1.6 MHz and the FM radio broadcast band 88-108 MHz. Shortwave radio signals can propagate worldwide, because they are bent as they travel through the ionosphere. Shortwave radio signals are used for voice and data transmissions where satellites may not be practical or economic An excellent non-mathematical introduction to HF radio propagation may be found at http://www.ips.gov.au/Educational/5/2 "Introduction to HF Radio Propagation"



When testing radio propagation paths between two locations on earth, it is useful to identify the location of the transmitter and receiver using the Maidenhead grid locator system http://en.wikipedia.org/wiki/Maidenhead_Locator_System. UVic is located in grid square CN88il.

III. Preparation

Read the theory above and also the abovementioned I-Q notes (posted on moodle)
TASK FOR LAB WRITEUP:

1. For an complex baseband signal $\tilde{s}(t) = I(t) + jQ(t) = a(t)e^{j\phi(t)}$, find an expression for $a(t)$ as a function of $I(t), Q(t)$
2. Given a Softrock receiver with $f_{LO} = 14.047000$ MHz and an incoming RF signal $f_c = 14.060000$ MHz, find an expression for
    a. the two real output signals from the Softrock receiver $I(t), Q(t)$, and
    b. the complex output signal $I(t) + jQ(t)$ written in polar form $a(t)e^{j\phi(t)}$
3. Verify the expressions for the imperfect receiver outputs $I'(t)$ and $Q'(t)$ given in section 4 above.
4. Verify the approximate expressions for IQ imbalance correction given in Section 4 above.

IV. Procedure details (expanded version of instructions on web, not required but may be helpful)

Softrock I-Q receiver testing with oscilloscope

1. Familiarize yourself with the setup at each lab station.
   a. The computer is running Linux (Scientific Linux 6) with GNU Radio software installed. GNURadio has a GUI that enables the construction of a software radio. The username and password are both "radiolab" (without quotes). The GNU Radio software is launched by the icon at the top left, or by going to Applications > Programs> GRC.
   b. The grey box is a software defined radio that will not be used in this lab 0.

2. Identify the BNC cabled labeled "HF antenna" that is connected to shortwave antennas on the roof of ELW as well as to the signal generator at the back of the lab. The signal generator is set up to transmit a carrier wave signal at f_c =14.060 MHz at power level -30 dBm. The HF antenna lead will contain the sum of the signals received by the rooftop antennas and the signal generator output.
3. Identify the Softrock antenna lead that is a black coaxial cable with center conductor and a concentric braided shield that surrounds the center conductor.
4. Connect the Softrock antenna lead to the HF antenna lead at the bench. Use a cable that converts a BNC connector to two clip leads. Clip the red lead to the center conductor and the black lead to the braid.
5. Identify the Softrock audio output lead that has a standard 1/8 inch stereo audio connector and identify the 1/8 inch stereo jack provided at each bench that mates with the stereo audio connector.
6. Connect the Softrock audio output to the oscilloscope channel 1 and channel 2 input. Plug the connector into the jack and use the scope clip leads to connect the jack to the oscilloscope.
7. If you see two sine waves on the scope that are 90 degrees out of phase, then your Softrock receiver is correctly receiving the carrier wave from the signal generator. If you see only one sine wave or no sine waves, then there is a fault in the Softrock receiver. The sine waves contain a DC offset. The scope will display a DC-shifted circle in XY mode.
8. Determine the frequency f_b of the sine waves. This frequency represents the difference between the received RF signal and the local oscillator, i.e. $f_b = f_c - f_{LO}$.
9. DELIVERABLE: Find and record the value of f_{LO}. Since $f_c = 14.060$ MHz (step 5),
   $f_{LO} = f_c - f_b = 14.060 - f_b$
   f_{LO} is nominally 14.047 MHz but will vary from one receiver to another.

<u>Softrock I-Q receiver testing with computer and GNURadio software</u>

10. We now use GNU Radio software on the computer to observe the Softrock output. Connect the Softrock audio output to the microphone input on the front of the computer.
11. Log into the computer and double click the file **IQ_Balance.grc** on the desktop. The GNURadio

GUI will open and display the software block diagram (flowgraph). The blocks include an audio block that takes two channels of samples from the soundcard $I(t), Q(t)$, a block to convert the two channels to a complex signal $I(t)+jQ(t)$, an oscilloscope and a spectrum analyzer and some blocks used to correct IQ imbalance (see step 15).

12. Execute the flowgraph and observe the signals on the scope and spectrum analyzer. The scope will display the two sine waves at f_b 90 degrees out of phase. Note that since f_b is over half of the Nyquist frequency $f_s/2$ the sine waves have only about 3 to 4 samples per cycle and will appear ragged. Recall that a sine wave at exactly $f_s/2$ Hz will have 2 samples per cycle (plus and minus)
13. In XY mode, the scope will display a circle, i.e. a phasor rotating at f_b Hz.
14. The spectrum analyzer will display the wave at f_b and its mirror image at $-f_b$ at a different amplitude . This mirror image will not be present if the two Softrock oscillators were identical in amplitude and exactly 90 degrees out of phase. In practice, the two oscillators have an IQ imbalance that can be corrected in software by multiplying the Q branch by a complex constant.
15. Correct the IQ imbalance by adjusting the amplitude and phase blocks in the flowgraph.
16. Record the scope and spectrum analyzer outputs without IQ imbalance correction and again after IQ correction.

Listening to on-air signals

17. The receiver can also be used to listen to signals using one or more of many SDR software packages. The SDR software will display a waterfall of the spectrum along with a filter with adjustable bandwidth and center frequency that is used as a "tuning knob" to select individual signals of interest, and audio output for listening or routing to decoding software (step 23). An example of SDR software that uses a Softrock kit and runs in a web browser over the internet is at http://www.websdr.org/
18. SDR software can be built using GNU Radio Companion by adapting the receiver in Tutorial 4.
19. Ready-built software is available for all platforms:

    Windows:

    http://sdrsharp.com/
    http://www.dxatlas.com/rocky/ includes PSK31 decoder
    http://dxatlas.com/cwskimmer/ decodes Morse code CW signals
    http://www.hdsdr.de/
    http://www.winrad.org/
    http://www.sdr-radio.com/
    http://code.google.com/p/powersdr-iq/
    http://james.ahlstrom.name/quisk/

    Linux

    http://gqrx.dk/ based on GNURadio
    http://www.sm5bsz.com/linuxdsp/linrad.htm
    http://napan.ca/ghpsdr3/index.php/Main_Page

http://napan.ca/ghpsdr3/index.php/QtRadio_Installation
http://openhpsdr.org/download.php
http://openhpsdr.org/wiki/index.php?title=HPSDR_related_software

Mac OS

http://mac.softpedia.com/get/Utilities/CuteSDR.shtml
http://www.dxzone.com/cgi-bin/dir/jump2.cgi?ID=26327

20. The receiver can be used to decode digital signals such as phase-shift keying (PSK) and frequency shift keying (FSK) and variations thereof. Some of the SDR software packages above have digital decoding built-in. For others, the audio output from the SDR software package can be routed to decoder software using

    Virtual Audio Cable (For Windows 7) http://en.wikipedia.org/wiki/Virtual_Audio_Cable
    OSX http://www.jackosx.com/ or http://cycling74.com/soundflower-landing-page/
    Linux http://jackaudio.org/

21. Digital data decoding software

    Linux Windows http://www.w1hkj.com/index.htm
    http://www.w1hkj.com/FldigiHelp-3.21/Modes/index.htm
    Mac OS http://www.w7ay.net/site/Applications/cocoaModem/index.html

22. For more background information, see

    http://www.complextoreal.com/tutorial.htm
    http://f4dan.free.fr/sdr_eng.html
    http://www.scoop.it/t/low-cost-software-defined-radio-sdr-panorama

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