FM Transmitter and Receiver

I. Objective:

In this lab we are targeting the models for demonstrating the features of Frequency Modulation (FM) transmission and reception. This model enables to select

1. Sinusoidal and square message waveforms
2. The FM parameters (Modulation index and carrier frequency)

II. Theory:

A: FM with General Message

To derive the equation for an FM wave, we have 3 starting points:

1. The idea of frequency modulation is that the instantaneous frequency $f_i(t)$ is varied linearly with the baseband signal $m(t)$. Thus for a general message $m(t)$ the idea of FM is:

$$f_i(t) = f_c + k_f m(t) \tag{1}$$

where the constant k_f represents the frequency sensitivity of the modulator, expressed in hertz per volt.

2. Recall that a general signal is written:

$$s(t) = a(t)\cos\theta(t) = a(t)cos[2\pi f_c t + \phi(t)] \tag{2}$$

where $\theta(t) = 2\pi f_c t + \phi(t)$ has a linear variation at rate f_c and a time varying part $\phi(t)$

3. Recall the instantaneous frequency of a signal is:

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} = f_c + \frac{1}{2\pi}\frac{d\phi(t)}{dt} \tag{3}$$

Combining these 3 starting points, we can write:

$$\theta(t) = 2\pi\int_0^t f_i(\alpha)d\alpha = 2\pi\int_0^t \left[f_c + k_f m(\alpha)\right]d\alpha = 2\pi f_c t + 2\pi k_f \int_0^t m(\alpha)d\alpha \tag{4}$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\alpha)d\alpha\right] \tag{5}$$

The FM signal $s(t)$ can be written in standard IQ format:

$$s(t) = i(t)\cos 2\pi f_c t - q(t) sin 2\pi f_c t \tag{6}$$

$$
\begin{aligned}
s(t) &= \mathrm{Re}\{a(t)e^{j\phi(t)}e^{j2\pi f_c t}\} = \\
&\mathrm{Re}\{[I(t) + jQ(t)][\cos 2\pi f_c t + j sin 2\pi f_c t]\} \\
&= I(t)\cos 2\pi f_c t - Q(t) sin 2\pi f_c t \\
&= a(t)\cos[2\pi f_c t + \phi(t)]
\end{aligned}
$$

Thus for FM:

$$
\begin{aligned}
a(t) &= A_c \\
\phi(t) &= 2\pi k_f \int_0^t m(\alpha)d\alpha \\
I(t) &= A_c \cos 2\pi k_f \int_0^t m(\alpha)d\alpha \\
Q(t) &= A_c \sin 2\pi k_f \int_0^t m(\alpha)d\alpha
\end{aligned}
\tag{7}
$$

B: FM with Sinusoidal Message

Now consider a sinusoidal modulating wave defined by

$$m(t) = A_m \cos(2\pi f_m t) \tag{8}$$

We re-derive the equation for the FM wave with the same 3 starting points.

1. The idea of FM is that the instantaneous frequency of the resulting FM wave equals:

$$
\begin{aligned}
f_i(t) &= f_c + k_f A_m \cos(2\pi f_m t) \\
&= f_c + \Delta f \cos(2\pi f_m t)
\end{aligned}
\tag{9}
$$

   where $\Delta f = k_f A_m$. Thus the message causes the frequency to vary above and below the carrier frequency f_c. The quantity Δf is called the <u>frequency deviation</u>.

2. A general signal:

$$s(t) = a(t)\cos\theta(t) \tag{10}$$

3. The instantaneous frequency of a signal is:

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \tag{11}$$

Combining these 3 starting points, we can write:

$$
\begin{aligned}
\theta(t) &= 2\pi f_c t + \frac{\Delta f}{f_m}\sin(2\pi f_m t) \\
&= 2\pi f_c t + \beta \sin(2\pi f_m t)
\end{aligned}
\tag{12}
$$

Where $\beta = \frac{k_f A_m}{f_m} = \frac{\Delta f}{f_m}$ is called the modulation index of the FM wave.

Thus the FM wave itself is given In terms of βby:

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right] \tag{13}$$

If β is small we have narrowband FM (NBFM) and if β is large (compared to one radian) we have wideband FM.

Thus for a sinusoidal modulating wave $m(t) = A_m \cos(2\pi f_m t)$ the FM wave can be written:

$$s(t) = I(t)\cos 2\pi f_c t - Q(t) sin 2\pi f_c t = a(t)\cos[2\pi f_c t + \phi(t)] \tag{14}$$

where:

$$\begin{aligned} a(t) &= A_c \\ \phi(t) &= \beta sin 2\pi f_m t \\ i(t) &= A_c \cos \beta sin 2\pi f_m t \\ q(t) &= A_c \sin \beta sin 2\pi f_m t \end{aligned} \tag{15}$$

C: Power Spectrum and Bandwidth of an FM Signal

Expanding $s(t)$ in the form of a Fourier series, we get:

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t] \tag{16}$$

where $J_n(\beta)$ is the Bessel function of the first kind of order *n*.

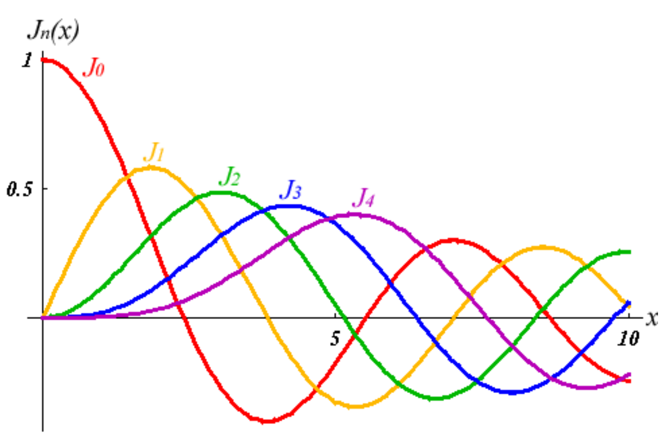
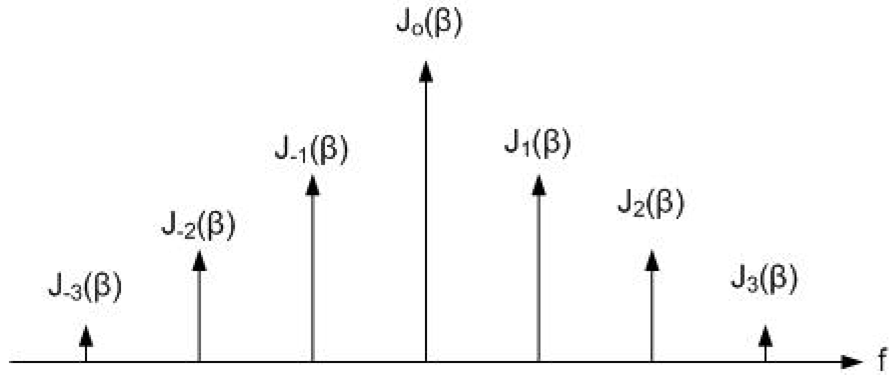


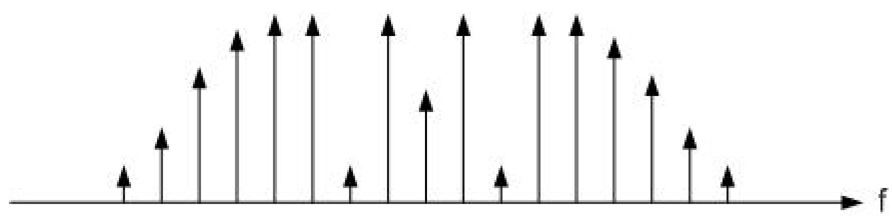
Figure 1 Bessel Functions

The discrete spectrum of the FM wave is obtained as

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)\right] \tag{17}$$



Bessel Function value [$J_n(\beta)$] defining the FM spectrum depends upon β



Example of FM spectrum with relatively large β

To visualize the spectrum $S(f)$, we note that:

$$J_n \beta = (-1)^n J_{-n}(\beta) \tag{18}$$

and:

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1 \tag{19}$$

Also, for small values of β, we have:

$$J_0(\beta) \simeq 1,\ J_1(\beta) \simeq \frac{\beta}{2} \tag{20}$$

and:

$$J_n(\beta) \simeq 0, \quad n > 1 \tag{21}$$

The average power of an FM wave developed across a 1 ohm resistor is given by:

$$P = 1/2 \ \ A_c^2 \cdot \sum_{n=-\infty}^{\infty} J_n^2(\beta) \tag{22}$$

For practical purposes, the <u>bandwidth of the FM wave</u> corresponds to the bandwidth containing 98% of the signal power.

The effective bandwidth of the FM signal is approximately given by Carson's formula:

$$B = 2(1+\beta) f_m \tag{23}$$

D: Frequency Demodulation

Frequency demodulation extracts the original message wave from the frequency-modulated wave. We describe a digital FM demodulator.
A digital FM demodulator starts with the I and Q outputs of a general IQ receiver. Recall for an FM signal:

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\alpha) d\alpha\right] \tag{24}$$

$$I(t) = A_c \cos 2\pi k_f \int_0^t m(\alpha) d\alpha$$

$$Q(t) = A_c \sin 2\pi k_f \int_0^t m(\alpha) d\alpha \tag{25}$$

To extract $m(t)$ from $I(t), Q(t)$ we show consider *I(t)* and *Q(t)* as a complex signal.

$$s(t) = \mathrm{Re}\{a(t) e^{j\phi(t)} e^{j2\pi f_c t}\} = \mathrm{Re}\{[I(t) + jQ(t)] e^{j2\pi f_c t}\} = \mathrm{Re}\{\tilde{s}(t) e^{j2\pi f_c t}\} \tag{26}$$

It can be shown that *m(t)* is obtained from the following formula:

$$m(t) = \arg[\tilde{s}(t-1)\tilde{s}^*(t)] \tag{27}$$

Where:

$$(t-1) \rightarrow z^{-1} \tag{28}$$

represents one sample delay

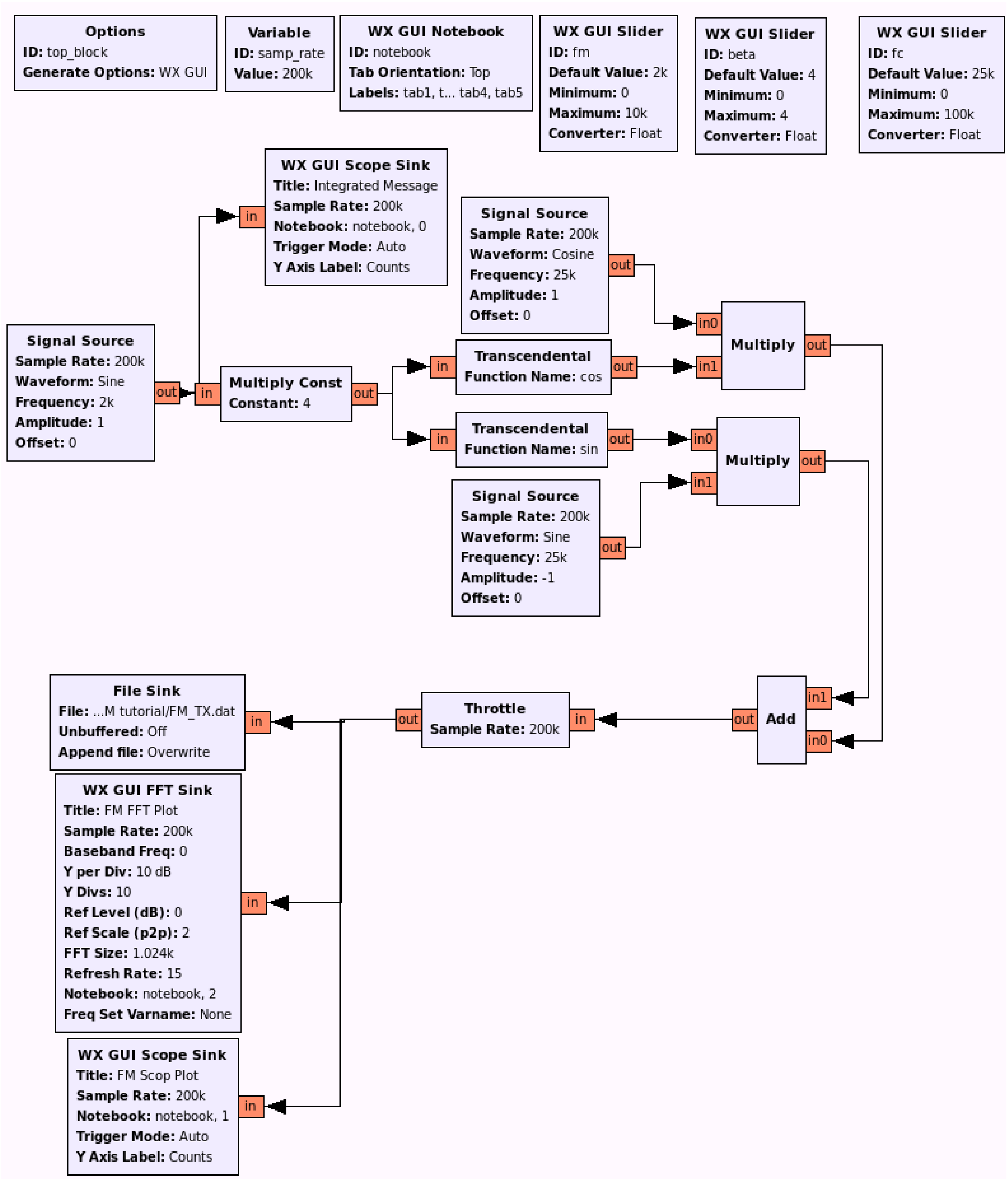
Proof:

$$\arg[s(t-1)s^*(t)] = \arg[a(t-1)e^{j\phi(t-1)}a(t)e^{-j\phi(t)}]$$

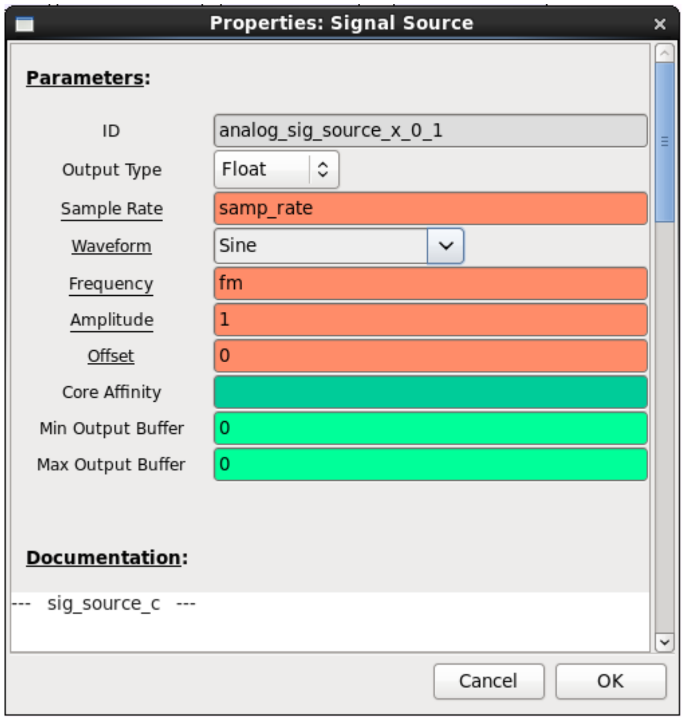
$$= \phi(t-1) - \phi(t) \approx \frac{d\phi}{dt} = 2\pi k_f m(t)$$

III. FM Implementation with Sinusoidal Message

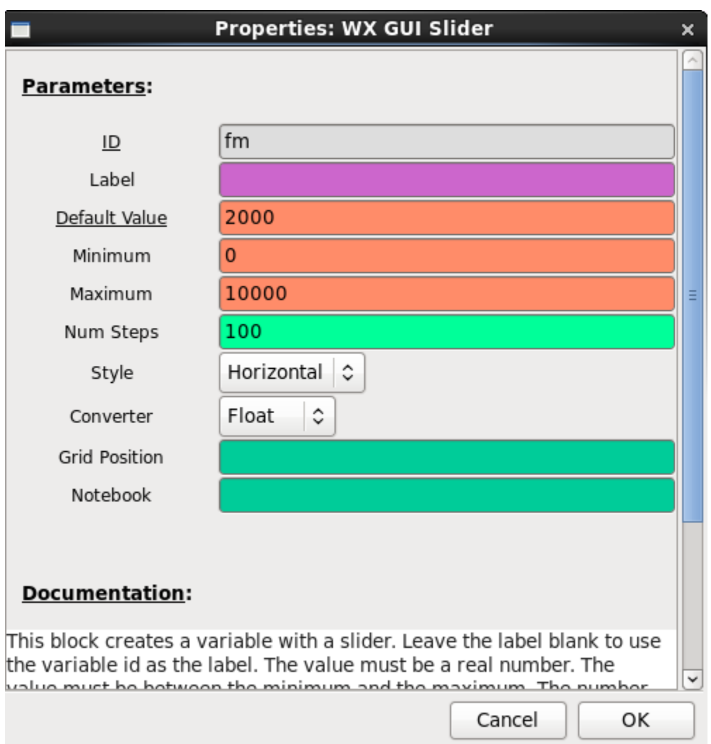
1- Construct the flow graph shown below which is based on equations (14) and (15).



2- The two first blocks are used for creating the $\phi(t)$ in equation (15). The signal source is $sin(2\pi fmt)$which is parameterized as below:

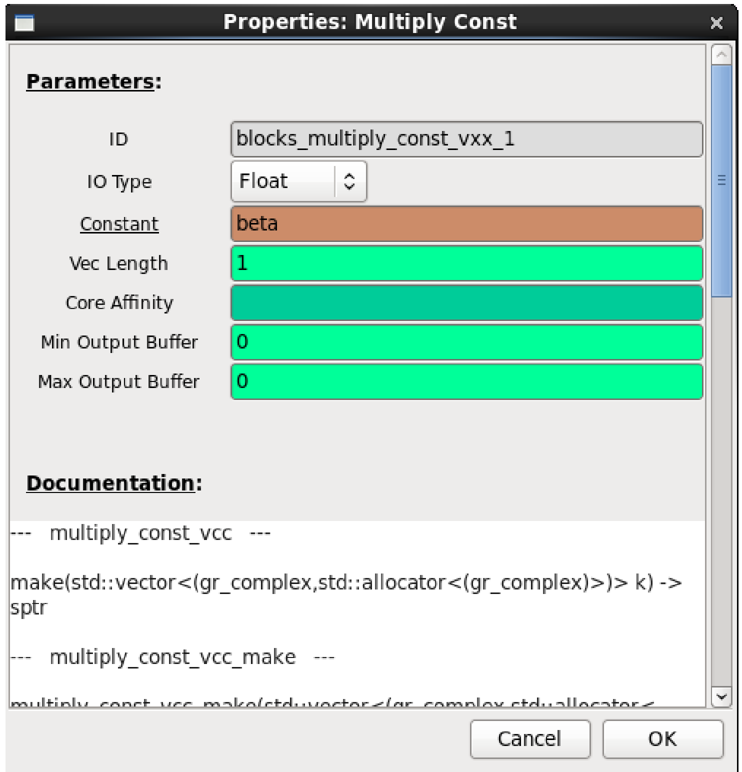


2- *fm* is the message frequency which is controlled by a WX GUI Slider as following

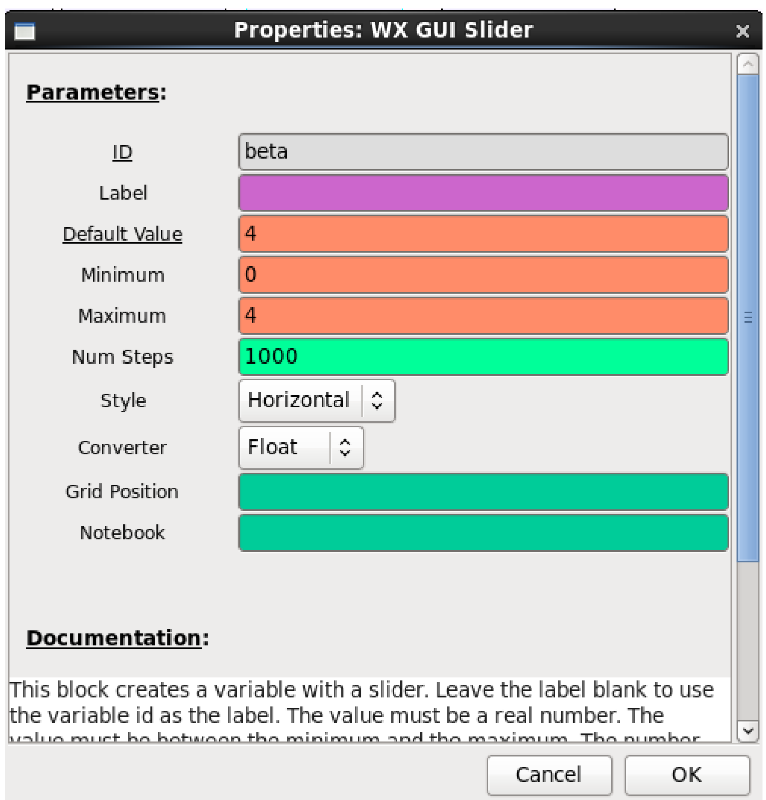


:

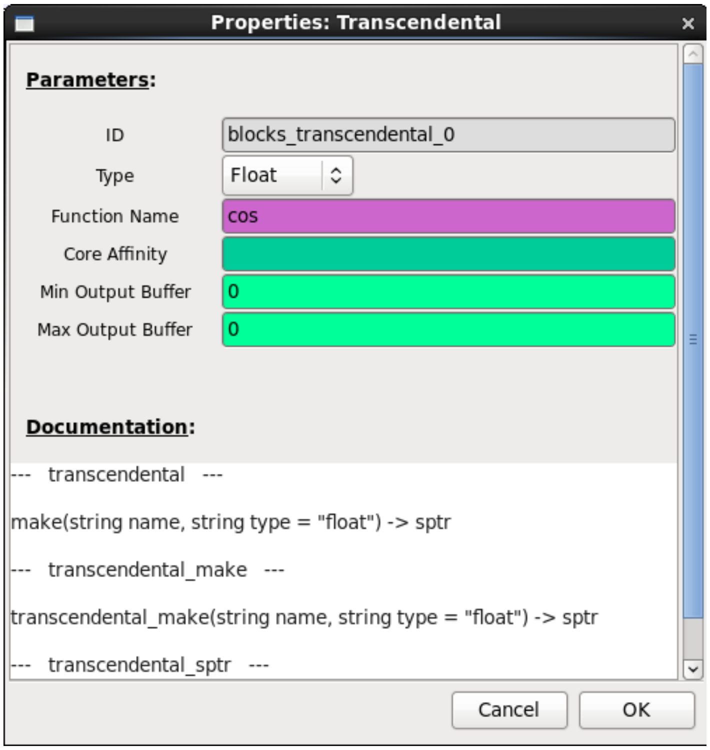
3- Double click on Multiply Const block. This block controls the modulation index β.

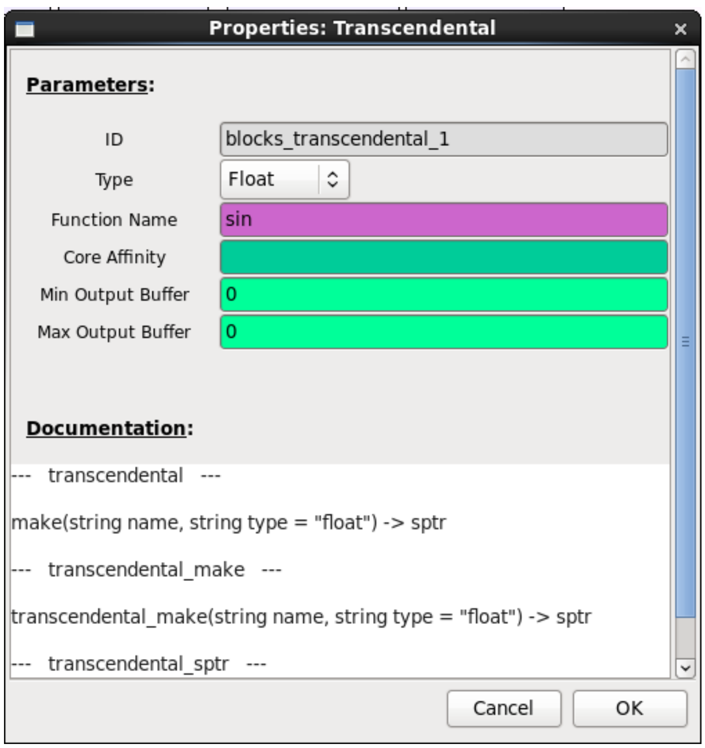


4- You can change the value of *beta* from zero to four using the following WX GUI Slide block:

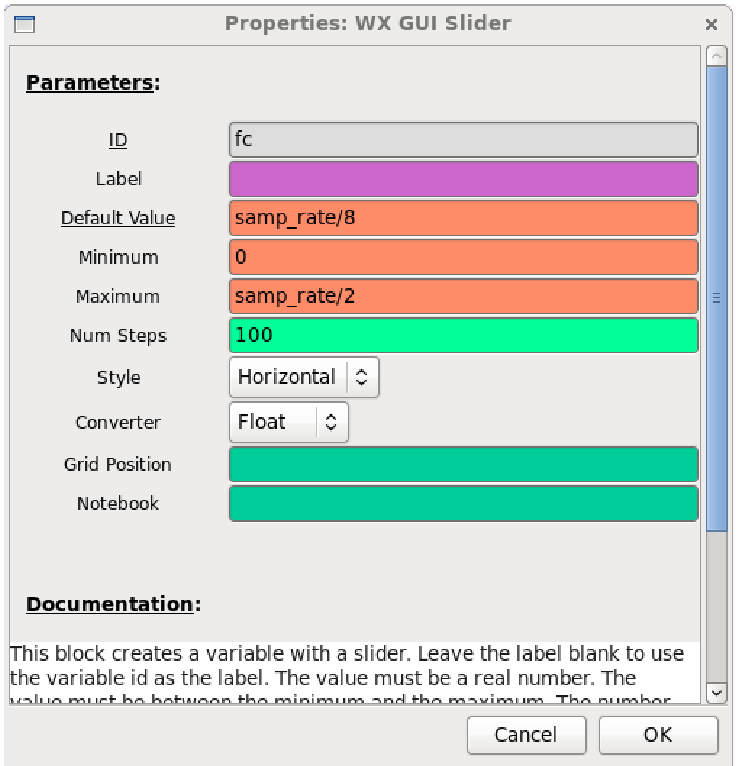


5- Transcendental Blocks are used to obtain the sine and cosine of $\phi(t)$ and create $I(t)$ and $Q(t)$ in equation (15). Double click on the Transcendental Blocks and set their parameters as below:





5- $I(t)$ and $Q(t)$ are multiplied by $cos(2\pi f_c t)$ and $-sin(2\pi f_c t)$ using two signal source blocks. fc is the carrier frequency and its value is controlled using a WX GUI Slider with following parameters:

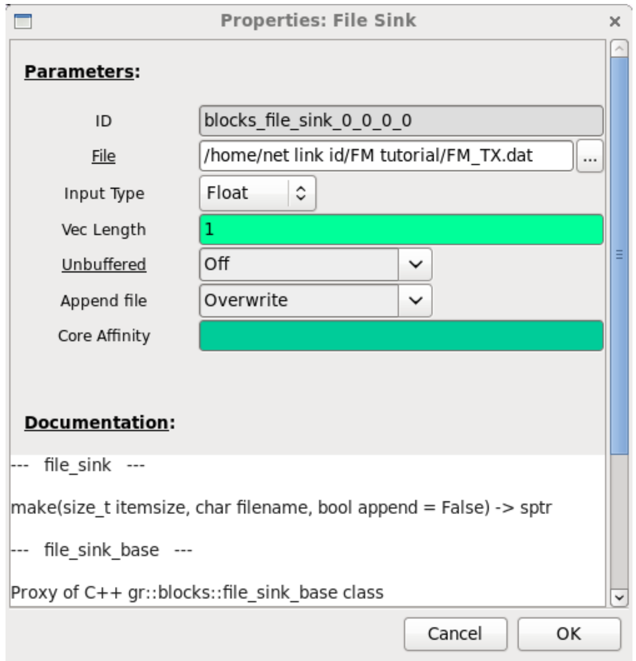


6- Set samp_rate variable to 200000Hz.

7- Add the result of step 5 using the Add block with two inputs.

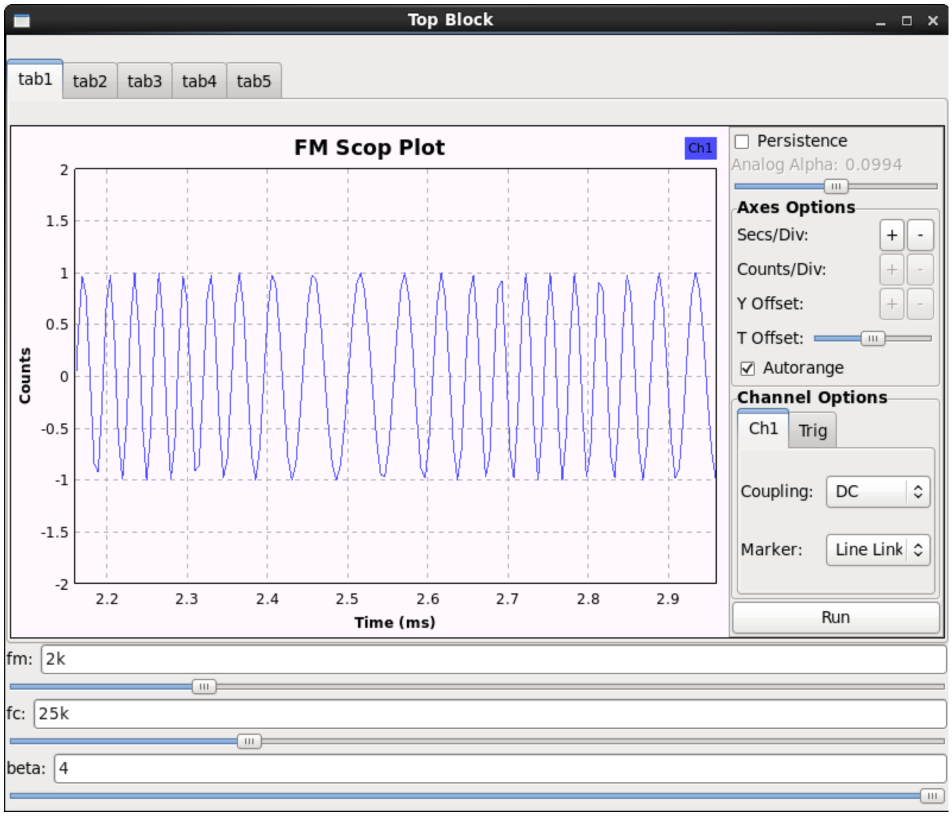
8- Set the Throttle sample rate to samp_rate.

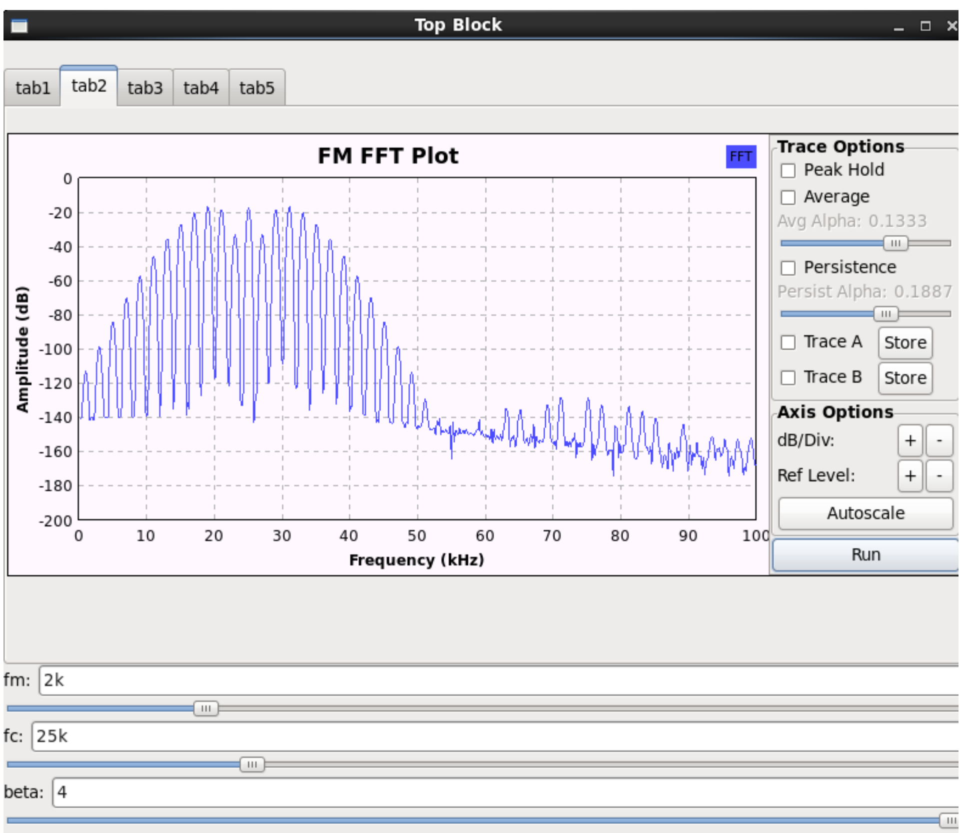
9- You can save the generated waveform using a File Sink. Double click on this block. Click on the ellipsis (…) next to the File parameter and choose the place in which you want to save your file.



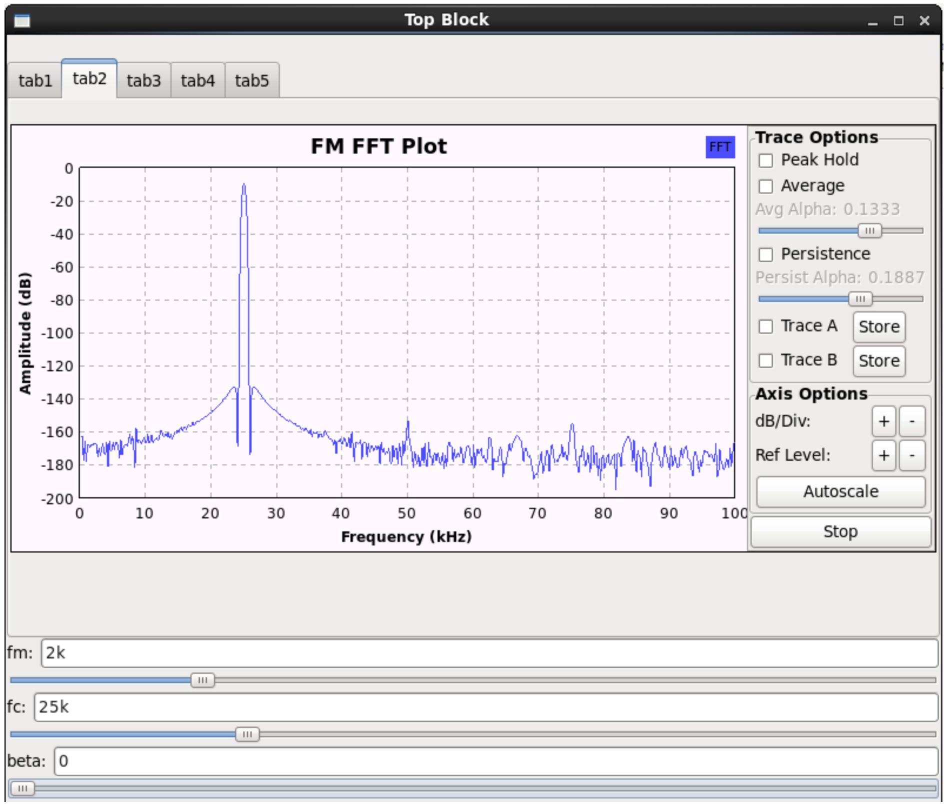
10-Use WX GUI Scope Sink and WX GUI FFT Sink to see the properties of FM signal with Sinusoidal message. You can change the modulation index as well as carrier frequency and message frequency and see the changes both in time domain and frequency domain. Following figures shows the results for the default parameters and also for some other sets of parameters.

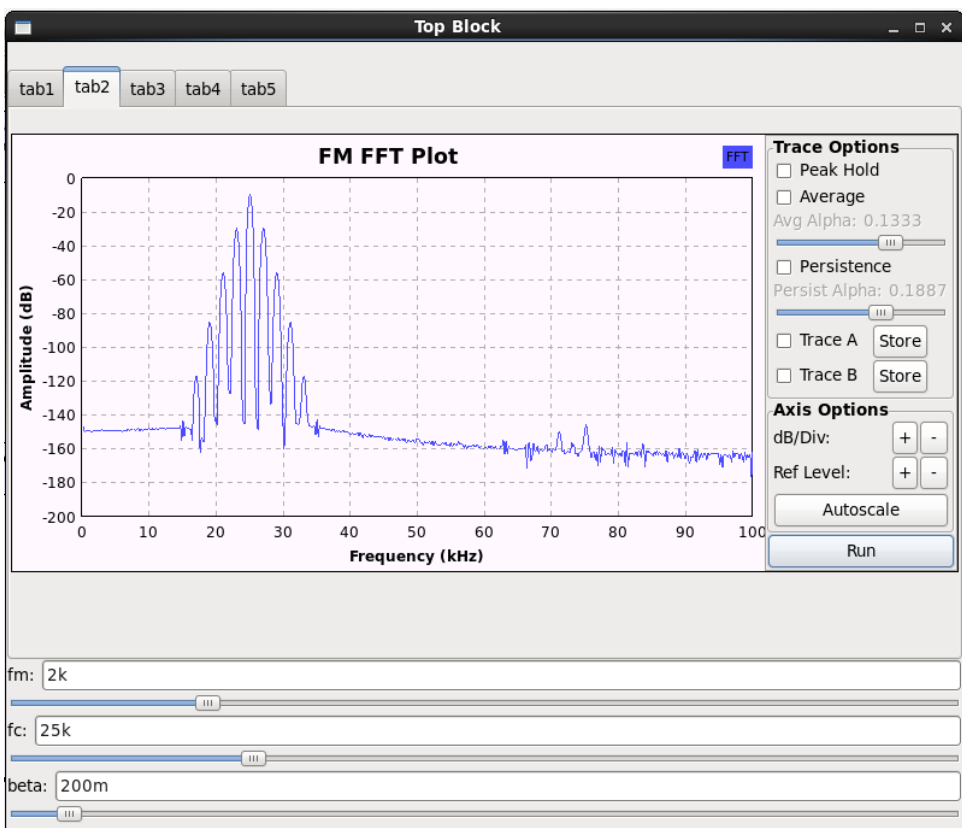
“Default Parameters”



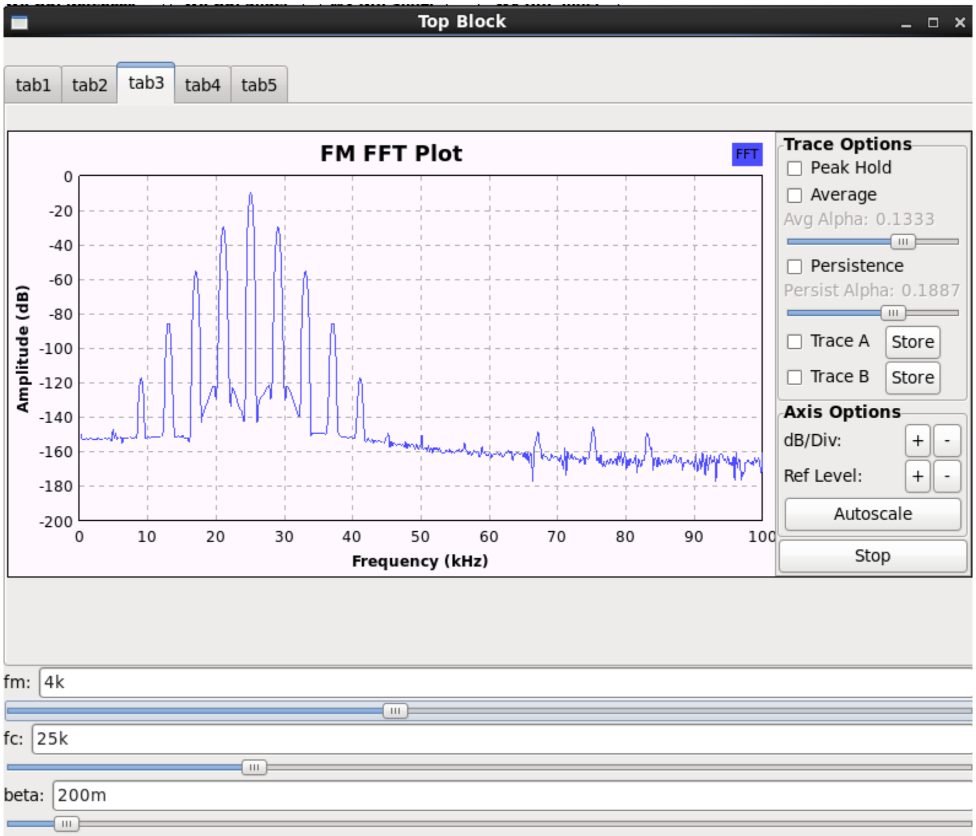


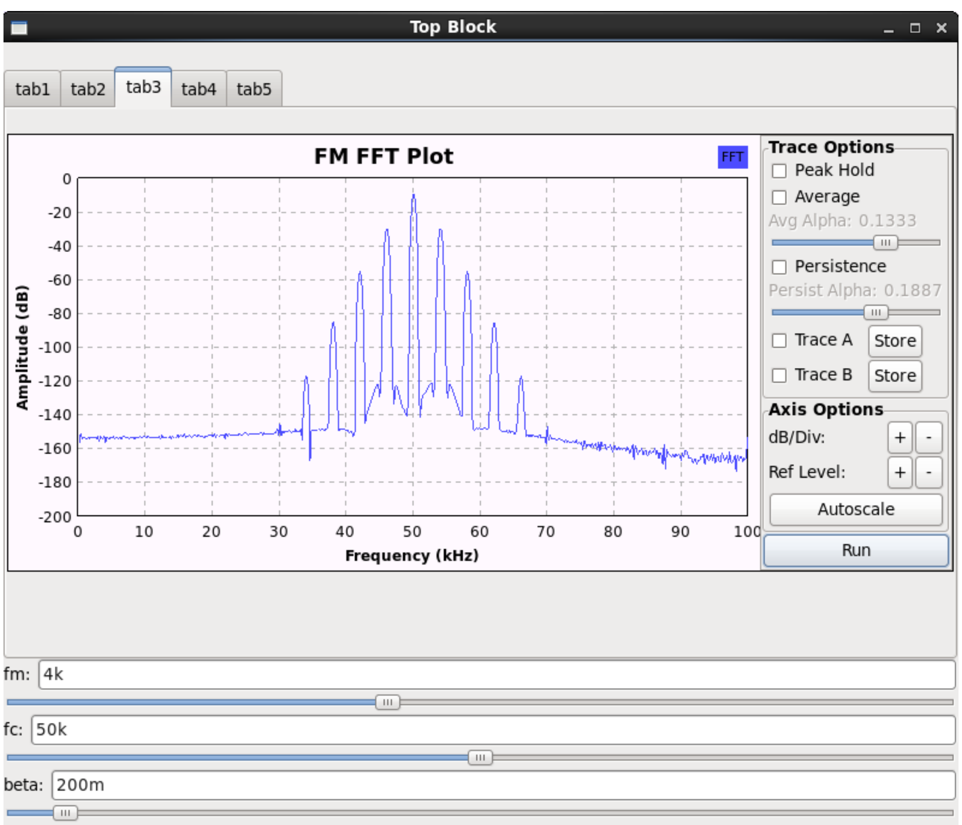
“Increasing Beta”





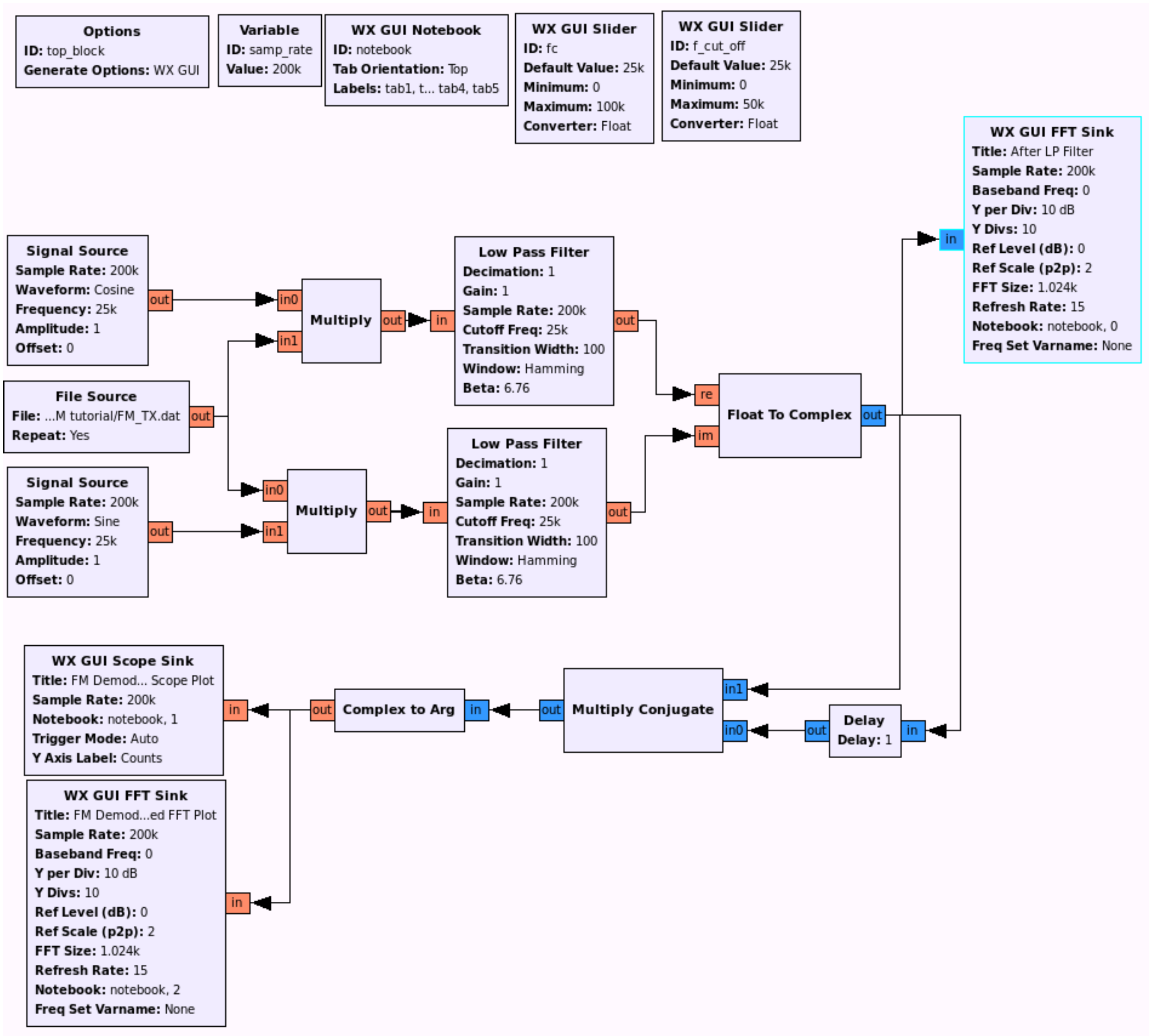
“Changing *fm* and *fc*”





VI. Frequency Demodulation

1- The following flow graph is the implementation of frequency demodulator based on equations (24-28).



2- The first part of the graph is for generating the base band signal $\tilde{S}(t) = I(t) + jQ(t)$ from the real band pass source signal *S(t).*

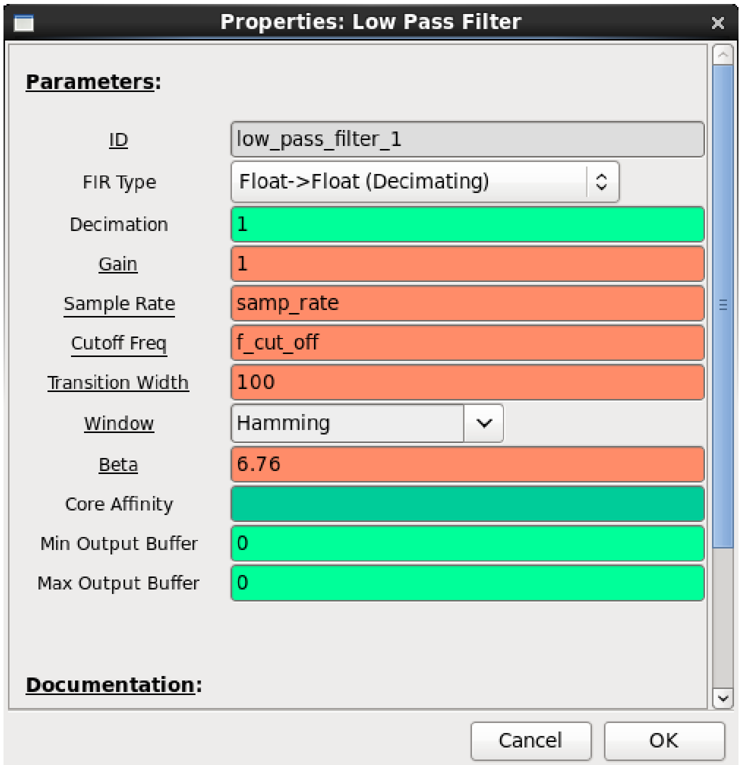
3- Recall the default FM signal of the previous part using the File Source block. Click on the ellipsis (…) next to the File parameter and choose the place where you saved your FM signal. Set repeat to yes to have continuously playing signal.

4- Multiply the File Source (FM Signal) by the cosine and sine signals with the same carrier frequency as the transmitter. Each multiplication will result in two low and high frequency components.

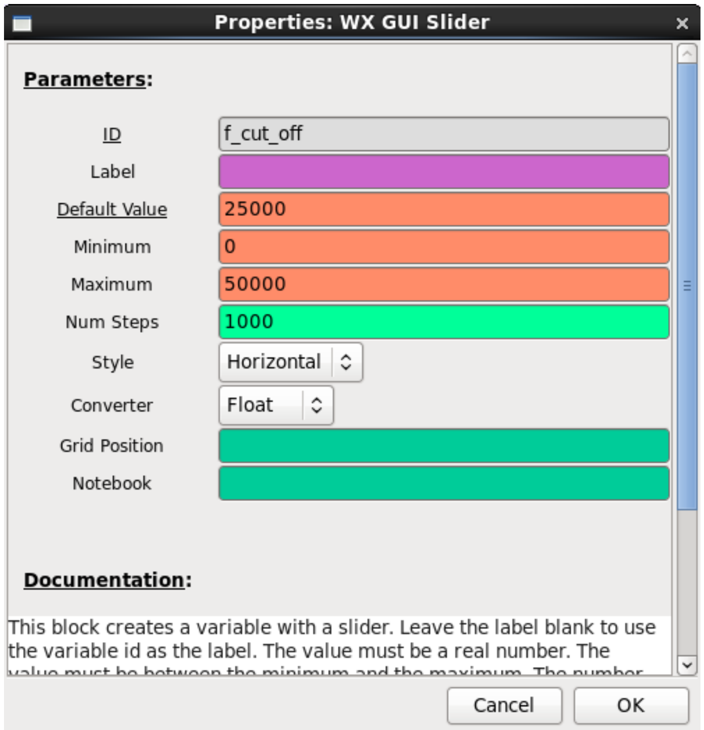
(Note: $\cos(\alpha)\cos(\beta)=(\cos(\alpha+\beta)+\cos(\alpha-\beta))/2$ & $\sin(\alpha)\cos(\beta)=(\sin(\alpha+\beta)+\sin(\alpha-\beta))/2$)

5- Lowpass filter the result of multiplication to save the baseband signal and omit the higher frequencies.

Set the parameters of the lowpass filter as below.

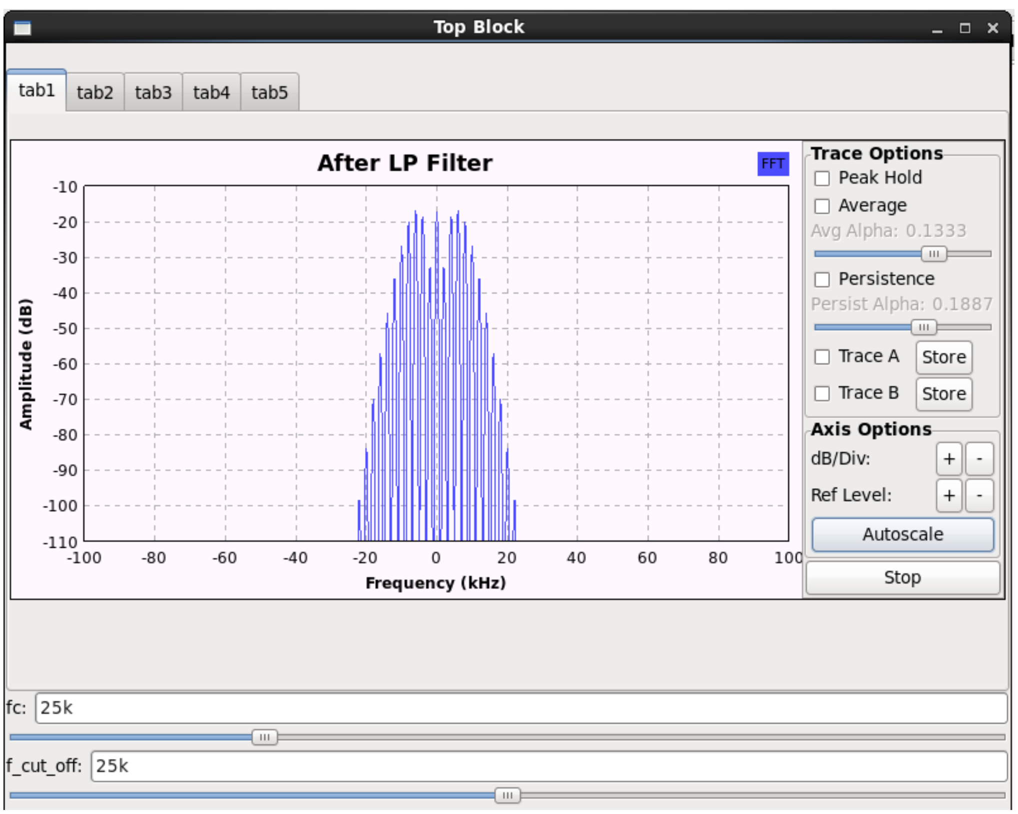


6- *f_cut_off* is the cut off frequency of the filter which is controlled by the following WX GUI Slider. The default value of the *f_cut_off* is set to 25000 based on the bandwidth of the FM signal which is observed in step 10 of previous part (for default parameters). You can also estimate the cut off frequency using equation (23).

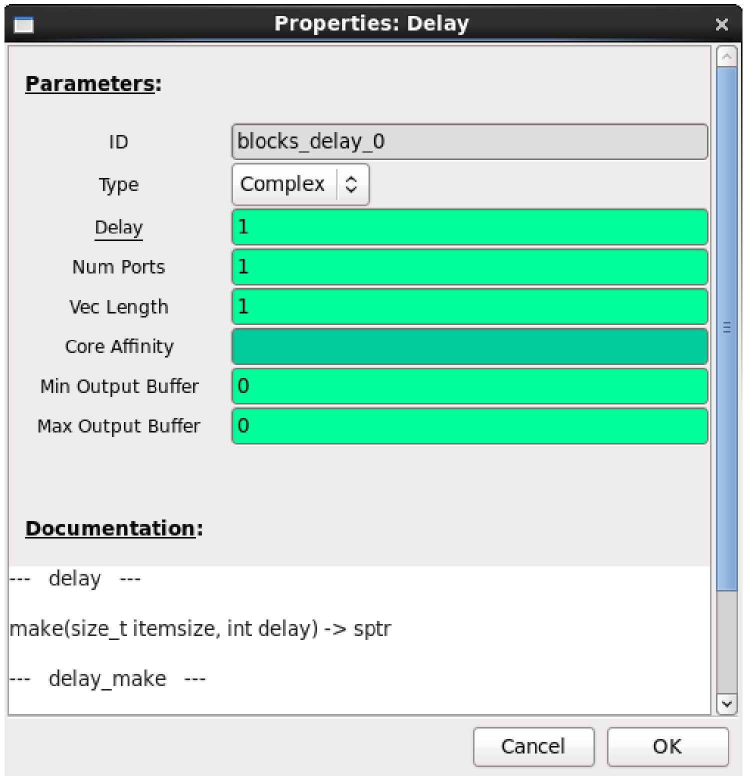


7- Use the Float to Complex block to combine *I(t)* and *Q(t)* and produce the complex baseband signal $\tilde{S}(t)$.

8- The following figure shows the spectrum of the baseband FM signal after lowpass filtering.



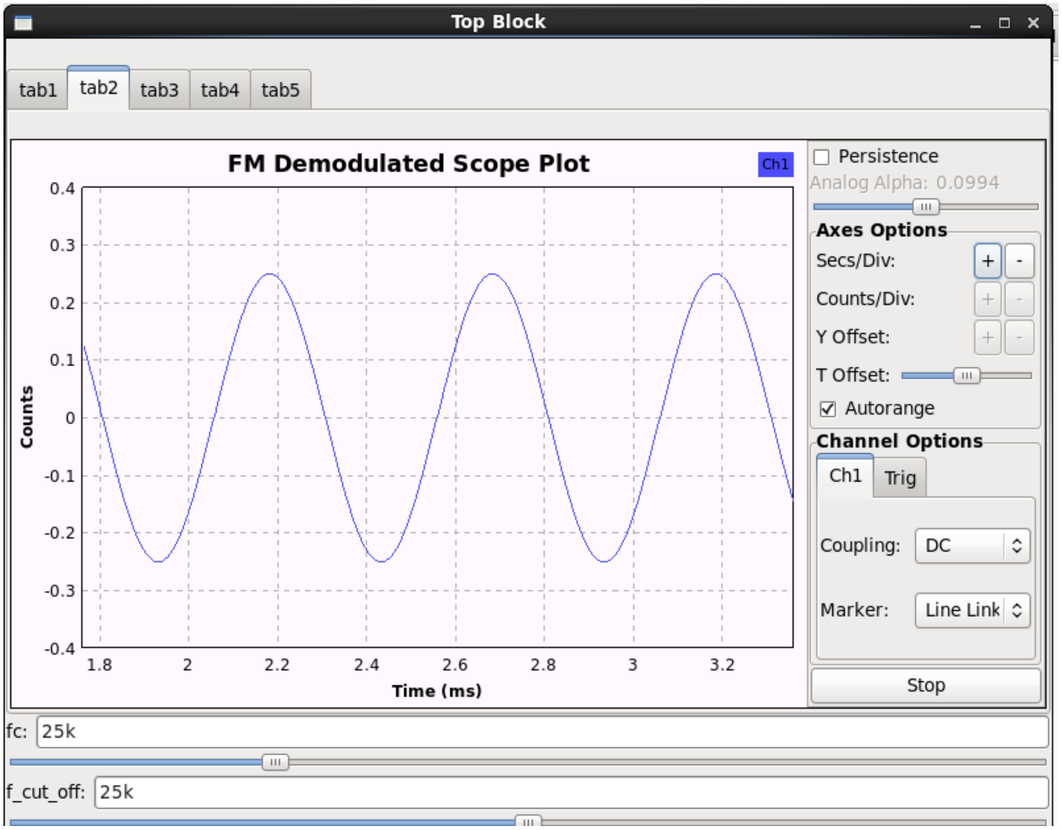
9- Now you should use equation (27) to recover the message signal. For doing that, you need a Delay block with the following properties:

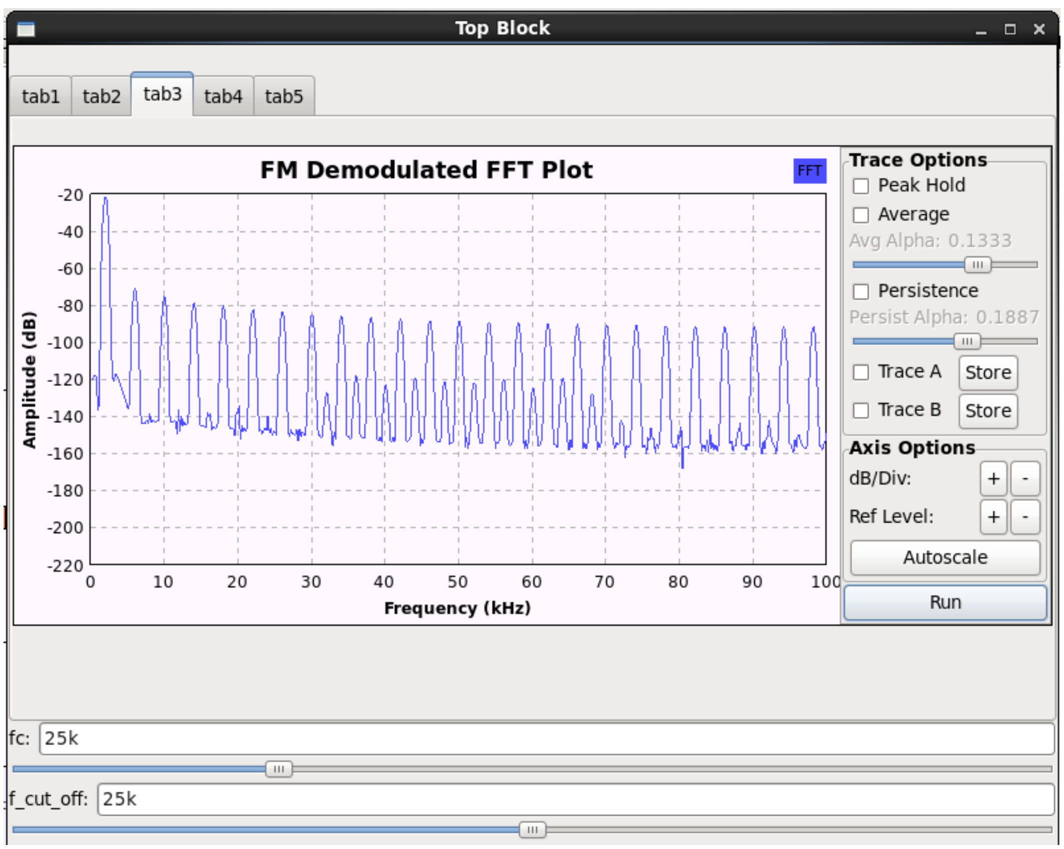


10- Use the Multiply Conjugate Block. This block multiplies in0 ($\tilde{S}(t-1)$) by the complex conjugate of in1 ($\tilde{S}(t)$).

11- Use the Complex to Arg block to recover *m(t).*

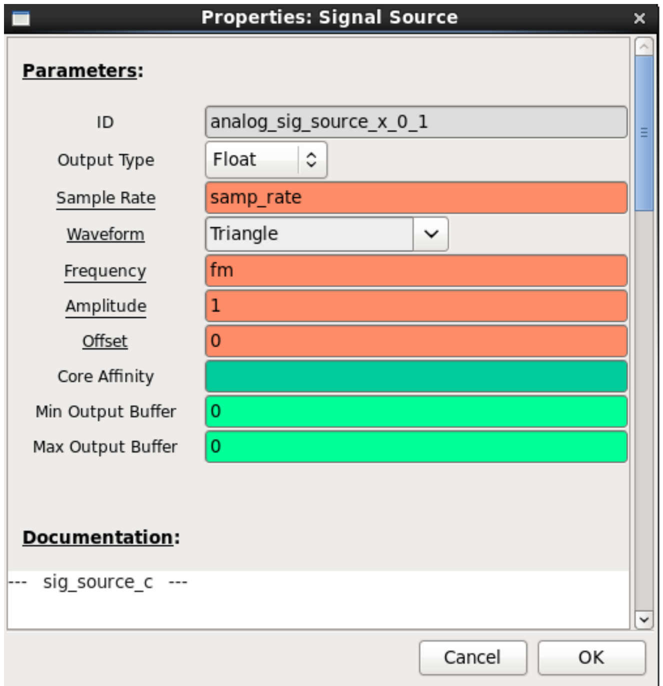
12- The following figures show the demodulated signal in both time and frequency domains. It is clear that the sinusoidal message is recovered successfully.

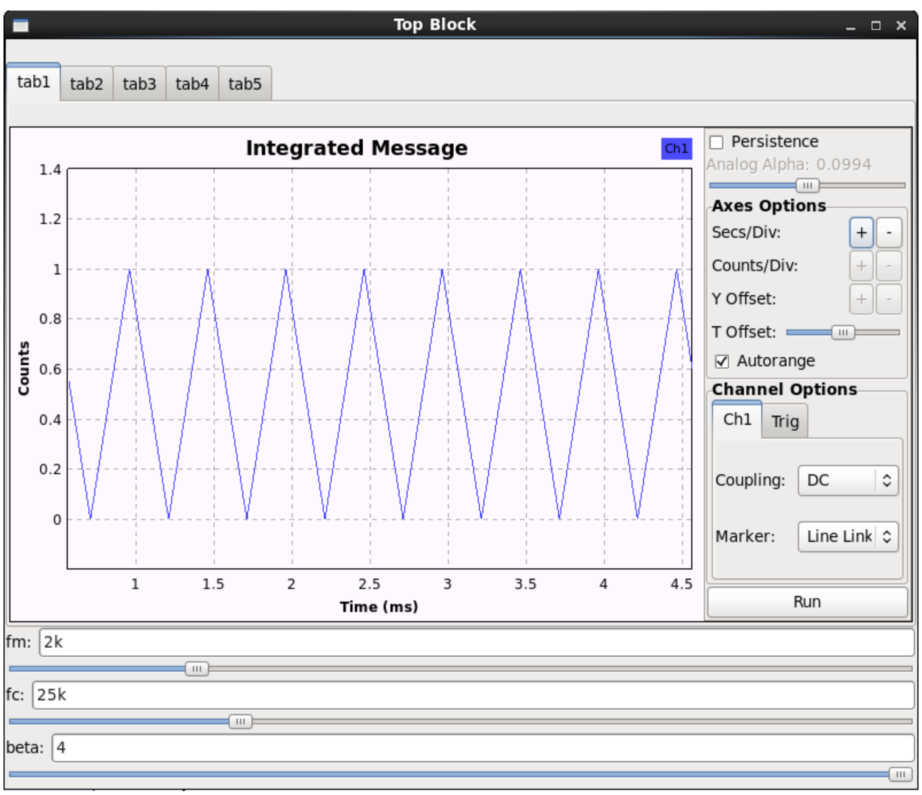

Top Block
tab1
tab2
tab3
tab4
tab5
FM Demodulated Scope Plot
Ch1
Counts
Time (ms)
Persistence
Analog Alpha: 0.0994
Axes Options
Secs/Div:
Counts/Div:
Y Offset:
T Offset:
Autorange
Channel Options
Ch1
Trig
Coupling:
DC
Marker:
Line Link
Stop
fc:
25k
f_cut_off:
25k


Top Block
tab1
tab2
tab3
tab4
tab5
FM Demodulated FFT Plot
FFT
Amplitude (dB)
Frequency (kHz)
Trace Options
Peak Hold
Average
Avg Alpha: 0.1333
Persistence
Persist Alpha: 0.1887
Trace A
Store
Trace B
Store
Axis Options
dB/Div:
Ref Level:
Autoscale
Run
fc:
25k
f_cut_off:
25k

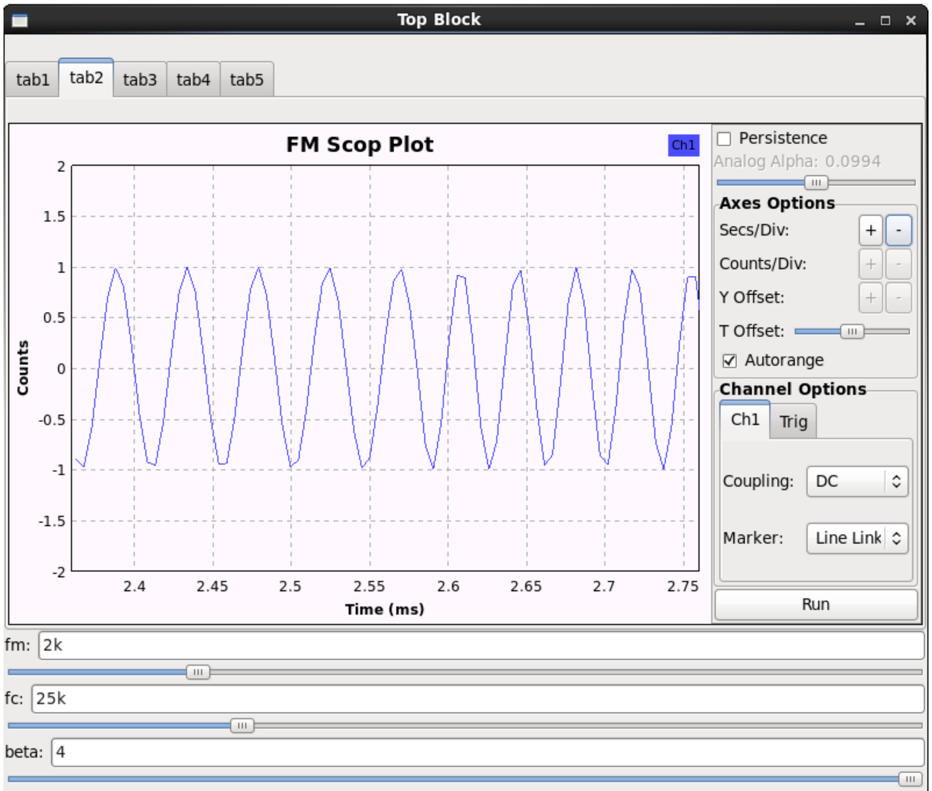
V. FM Implementation with Square Wave Message

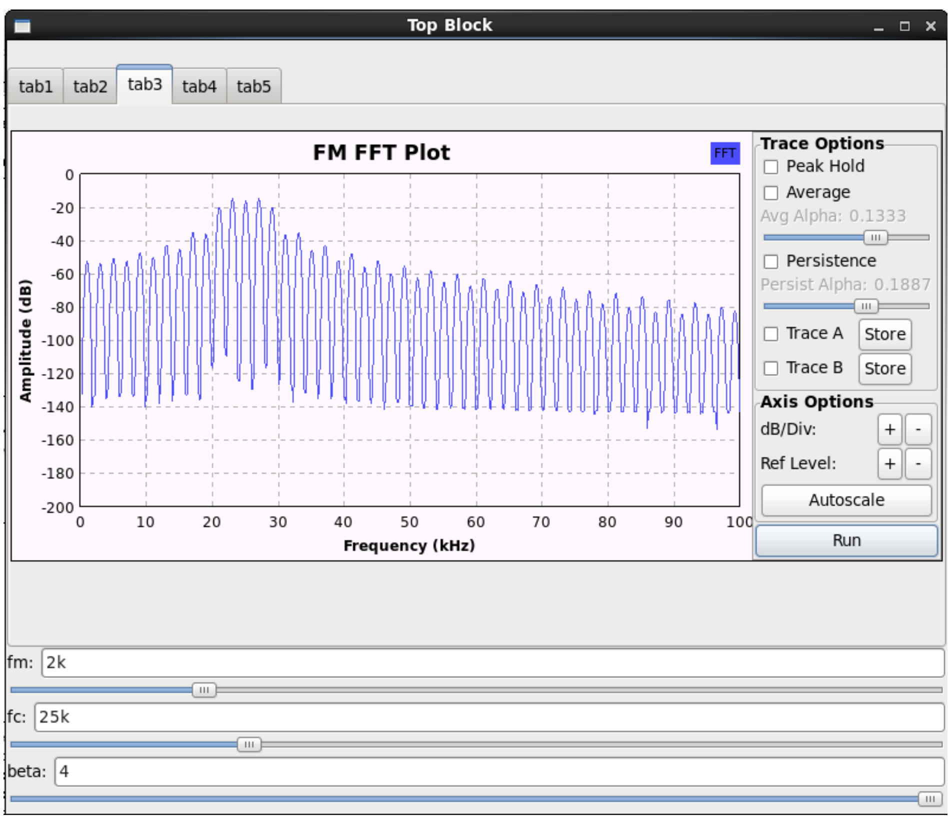
1- In this part, the frequency modulation and demodulation is implemented for square waveform signal. The integral of a square waveform is a triangular waveform with the same frequency as the square waveform. So for frequency modulation, it is enough to replace the sinusoidal signal source in the flowgraph of section III with a triangular signal source with following parameters:





2- The following figures show the FM signal with square waveform message in time and frequency domains for default parameters:





3- For demodulation we use the same flow graph as in section IV. You can see from the spectrum of the FM signal that the power of side-lobes is less than -60 dB for frequencies greater that 50K Hz. So the FM bandwidth can be considered as 50KHz-*fc*=25KHz and we can consider the default cut off frequency as before. You can change the cut off frequency and see its effect on the demodulated signal.

4- Following figures show the demodulated signal in time and frequency domain.