1. IQ Signals general overview
2. IQ receivers

IQ Signals general overview

Radio waves are used to carry a message over a distance determined by the link budget. The radio wave (called a carrier wave) is “modulated” (modified) by the message signal $m(t)$; in other words the amplitude and/or phase of the carrier wave is modified so as to include the information stored within the message.

The general form of a radio signal is as follows:

$$s(t) = \mathrm{Re}\{a(t)e^{i\theta(t)}\} = a(t)\cos\theta(t),$$

where $a(t)$ represents the amplitude of the signal after modulation and $\theta(t)$ is the phase of the carrier wave.

In the special case where there is no modulation applied to the carrier signal (ie: no message sent), a we write $s(t) = c(t)$ oscillating at a frequency of f_c and scaled by the carrier amplitude coefficient A_c is the result.

$$a(t) = A_c \text{ (constant independent of time)}$$
$$\theta(t) = 2\pi f_c t \text{ (linear increase of phase with time, } 2\pi \text{ radians every } 1/f_c \text{ seconds.}$$
$$\therefore$$
$$c(t) = A_c\cos(2\pi f_c t) = A_c\cos(\omega_c t)$$

This signal is called the carrier wave $c(t)$ and can also be visualized as a phasor with angular frequency $\omega_c = 2\pi f_c$ and with period $T = 1/f_c$.

In general the phase of the carrier $\theta(t)$ may have a time varying phase component $\varphi(t)$ that is added to the linear phase, thus in general:

$$s(t) = a(t)\cos(\theta(t))$$
$$\theta(t) = 2\pi f_c t + \varphi(t)$$
$$s(t) = a(t)\cos(2\pi f_c t + \phi(t)) = \mathrm{Re}\{a(t)e^{j\phi(t)}e^{j2\pi f_c t}\},$$

The radio signal $s(t)$ is a cosine wave at frequency f_c with time-varying amplitude and phase $a(t), \phi(t)$ Thus the only way to encode the message $m(t)$ on the carrier wave is to vary $a(t)$ and/or $\phi(t)$ in step with the message $m(t)$ Thus $a(t), \phi(t)$ are specified as a function of the message $m(t)$ for some modulation types and as a constant for others. These functions will be specified later when we discuss specific modulation types.

Using the identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$
the general radio signal $s(t) = a(t)\cos(2\pi f_c t + \phi(t))$ $= \mathrm{Re}\{a(t)e^{j\phi(t)}e^{j2\pi f_c t}\}$ may also be written

$$\begin{aligned} s(t) &= a(t)\cos 2\pi f_c t \cos\phi(t) - a(t)\sin 2\pi f_c t \sin\phi(t) \\ &= I(t)\cos 2\pi f_c t - Q(t)\sin 2\pi f_c t \end{aligned}$$

where

$$\begin{aligned} I(t) &= a(t)\cos\phi(t) = \mathrm{Re}\{a(t)e^{j\phi(t)}\}, \\ Q(t) &= a(t) sin\phi(t) = \mathrm{Im}\{a(t)e^{j\phi(t)}\} \end{aligned}$$

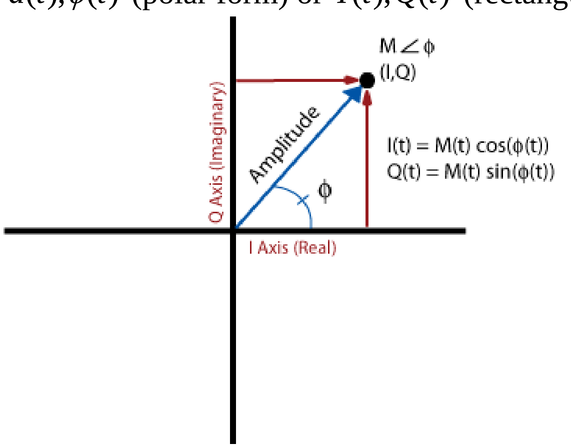
and thus $I(t) + jQ(t) = a(t)e^{j\phi(t)}$
Thus we can write

$$\begin{aligned} & s(t) = \mathrm{Re}\{a(t)e^{j\phi(t)}e^{j2\pi f_c t}\} = \\ & \mathrm{Re}\{[I(t) + jQ(t)][\cos 2\pi f_c t + j sin 2\pi f_c t]\} \\ & = I(t)\cos 2\pi f_c t - Q(t) sin 2\pi f_c t \\ & = a(t)\cos[2\pi f_c t + \phi(t)] \end{aligned}$$

The general radio signal $s(t)$ must be a real signal that we can view on an oscilloscope. We can write $s(t)$ as the real part of a complex signal. $s(t)$ can be described as either

- a cosine wave with amplitude $a(t) \geq 0$ and phase $\phi(t)$, or
- the sum of a cosine wave with amplitude $I(t)$ and a sine wave with amplitude $Q(t)$, where $I(t), Q(t)$ can be greater or less than zero.

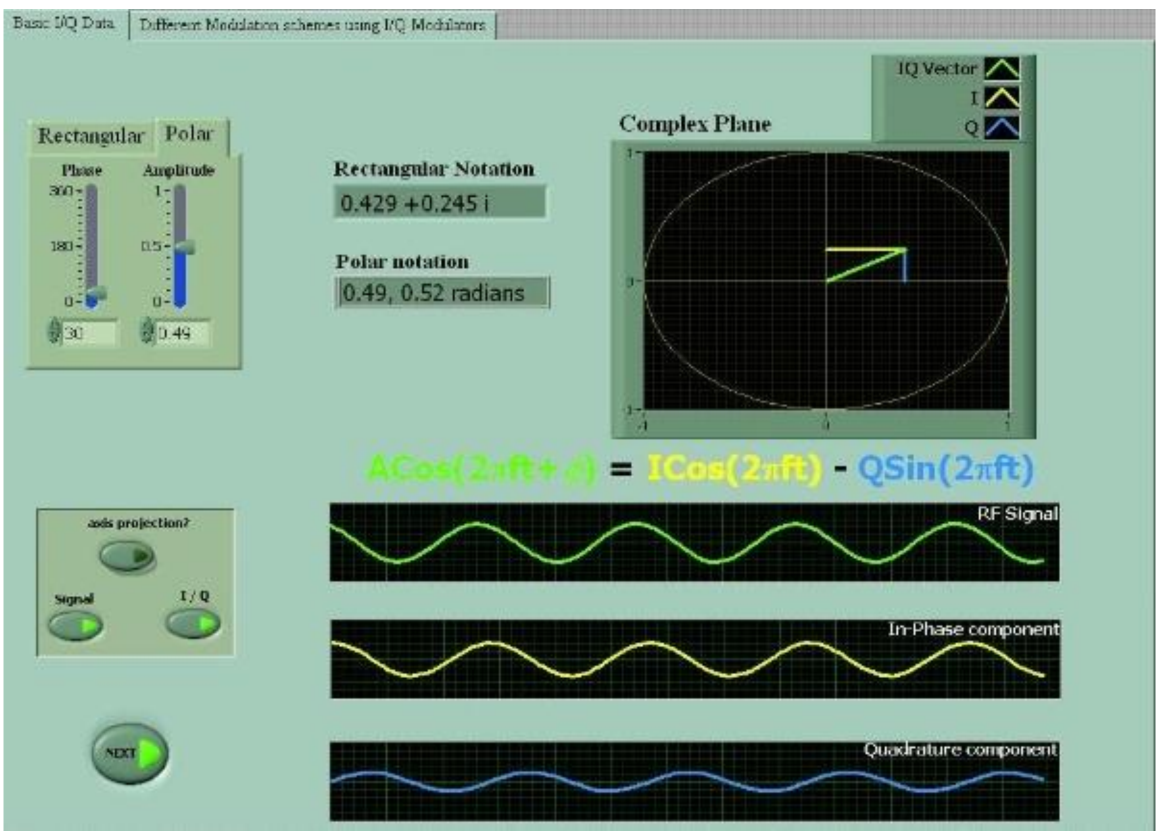
In both cases, the message is a two-dimensional (complex) signal represented using either $a(t), \phi(t)$ (polar form) or $I(t), Q(t)$ (rectangular form). In the figure, $M(t) = a(t)$.



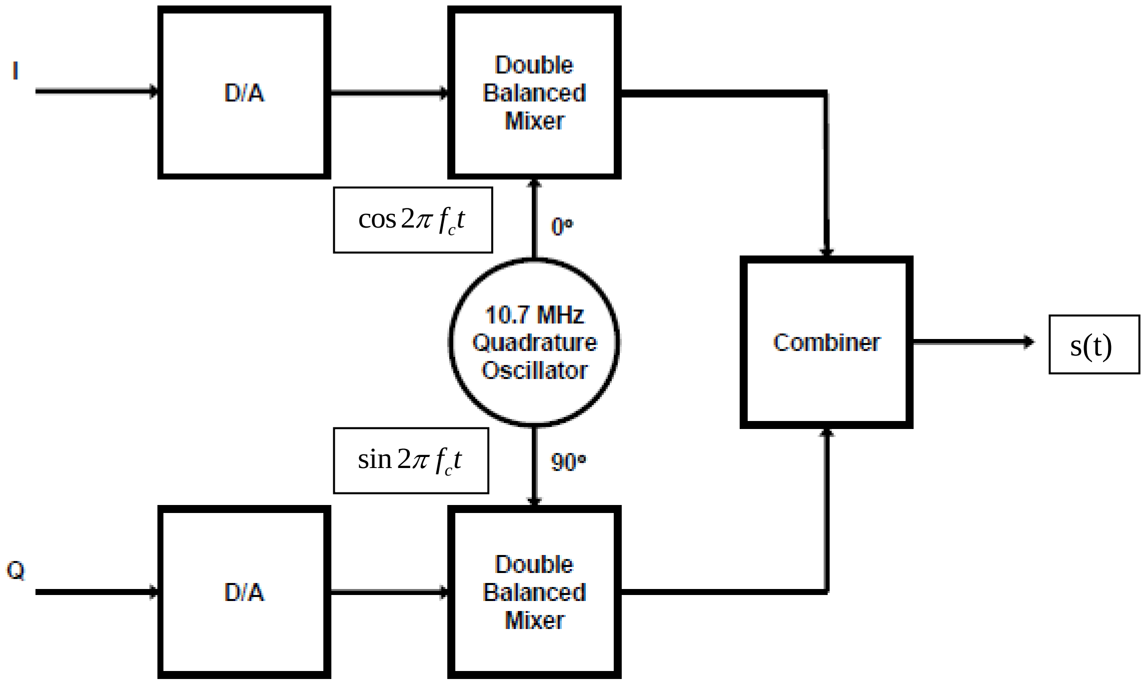
$I(t), Q(t)$ are functions of the message signal(s), where the exact function depends on the modulation type. A simple example is to consider the message signal to be a stereo (2-channel) music signal written as $m_L(t), m_R(t)$ and choose $m_L(t) = I(t), m_R(t) = Q(t)$

$I(t) + jQ(t)$ is the so-called *complex baseband* signal that is a function of the message. For a stereo music signal, the complex baseband signal is $m_L(t) + jm_R(t)$ .The radio signal $s(t)$ is the so-called *real passband* signal that contains the message modulated onto the carrier wave at frequency f_c

In the special case where $I(t), Q(t)$ are both constants I, Q, then $s(t)$ is a cosine wave with constant amplitude and phase. The figure below shows the radio frequency (RF) signal $s(t) = I\cos 2\pi f_c t - Q\sin 2\pi f_c t = a\cos 2\pi f_c t + \phi$ is a cosine wave at frequency f_c with constant amplitude and phase a, ϕ. The in-phase component $I\cos 2\pi f_c t$ and the quadrature component $Q\sin 2\pi f_c t$ are also cosine waves with constant amplitude and phase.

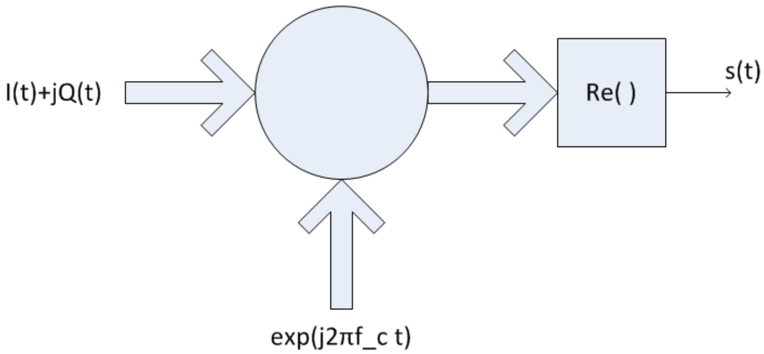


<u>Radio transmitter (modulator)</u>



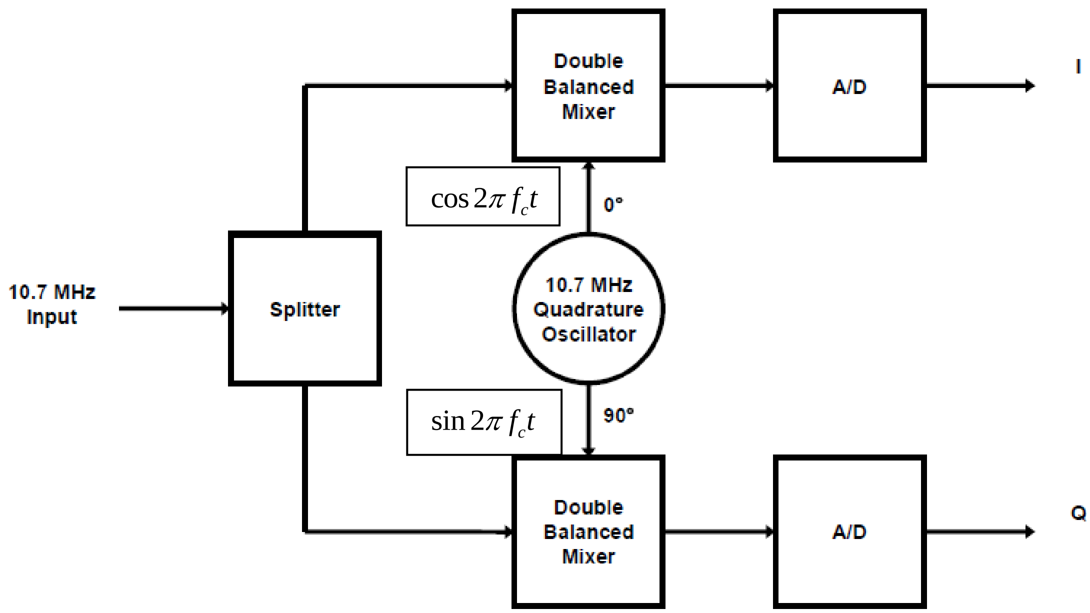
The figure shows a radio transmitter (or modulator) that produces the radio waveform $s(t) = I(t)\cos 2\pi f_c t - Q(t)\sin 2\pi f_c t$ from the message signals $I(t), Q(t)$, where in this example, $f_c = 10.7MHz$

The radio transmitter (modulator) may also be described in complex form with a complex multiplication of the complex baseband message
$I(t) + jQ(t)$ with $\cos 2\pi f_c t + jsin2\pi f_c t$ to yield a complex signal
$\hat{s}(t) = a(t)e^{j\phi(t)}e^{j2\pi f_c t} = [I(t) + jQ(t)][\cos 2\pi f_c t + jsin2\pi f_c t]$
whose real part is $s(t) = I(t)\cos 2\pi f_c t - Q(t)sin2\pi f_c t$
The complex message $I(t), Q(t)$ s multiplied by the complex carrier wave
$e^{j2\pi f_c t|} = \cos 2\pi f_c t - jsin2\pi f_c t$ The radio signal $s(t)$ is the real part of this complex multiplication $s(t) = I(t)\cos 2\pi f_c t - Q(t)\sin 2\pi f_c t$

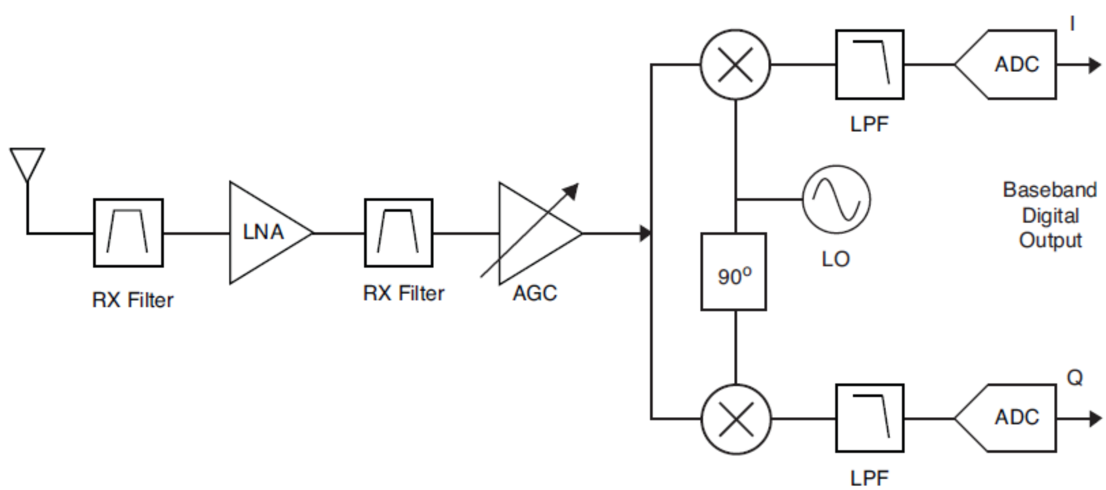


Radio receiver (demodulator)

The signal $s(t)$ is transmitted over a distance via some channel (wired or wireless), attenuated by the path loss L_0 and picked up by a receiver in the form $r(t) = s(t) / L_0$ The receiver's task is to recover the message signals $I(t), Q(t)$ from the signal $r(t)$. This can be done using the receiver shown below. In this receiver, $I(t), Q(t)$ are digitized by an analog-to-digital converter (A/D or ADC).



A more complete drawing of the receiver adds some practical components



In this figure, we have added: RX filters needed to filter out undesired signals on nearby frequencies, a Low Noise Amplifier (LNA) to amplify $r(t)$ that is typically the the microvolt range to a level in the volt range suitable for ADC, Automatic Gain Control (AGC) to adjust the gain to compensate for variations in the level $r(t)$ and low pass filters (LPF) before the ADC.

To see how this receiver works, we calculate the signals $x(t), y(t)$ at the two ADC inputs, and find that they are equal to $I(t), Q(t)$

Exercise: prove this. Several trigonometric identities will be needed:

$$\cos\alpha\cos\beta = [\cos(\alpha-\beta)+\cos(\alpha+\beta)]/2$$
$$\sin\alpha\cos\beta = [\sin(\alpha-\beta)+\sin(\alpha+\beta)]/2$$
$$\sin\alpha\sin\beta = [\cos(\alpha-\beta)-\cos(\alpha+\beta)]/2$$

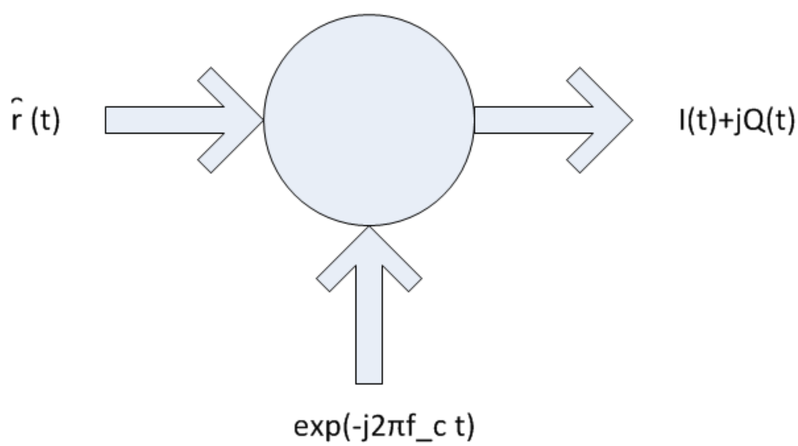
Also, the double frequency terms (cosine waves at $2f_c$) are filtered out by the low pass filter (LPF).

Alternately, we can prove this using the complex signals.
We write the received signal in complex form

$$\hat{r}(t) = L_0^{-1}a(t)e^{j\phi(t)}e^{j2\pi f_c t} =$$
$$L_0^{-1}[I(t)+jQ(t)][\cos 2\pi f_c t + jsin2\pi f_c t]$$

The receiver demodulates the complex radio signal by multiplying $\hat{r}(t)$ by the complex local oscillator $e^{-j2\pi f_c t|} = \cos 2\pi f_c t - jsin2\pi f_c t$ to yield

$$\hat{r}(t)e^{-j2\pi f_c t} = [L_0^{-1}a(t)e^{j\phi(t)}e^{j2\pi f_c t}]e^{-j2\pi f_c t} = L_0^{-1}a(t)e^{j\phi(t)} = L_0^{-1}[I(t)+jQ(t)]$$



Summary

We have created a communication system with message signals $I(t), Q(t)$ that are modulated onto a carrier wave at frequency f_c to create the radio signal $s(t) = I(t)\cos 2\pi f_c t - Q(t)\sin 2\pi f_c t$. $s(t)$ travels over a distance via a channel with path loss L_o. The receiver picks up the signal $r(t) = s(t)/L_0$ and recovers the messages $I(t), Q(t)$

IQ transmitter-receiver system

Messages

When we study modulation, we often choose a simple message, $m(t) = A_m \cos(2\pi f_m t)$, where f_m is the modulation/message frequency and is usually on the order of Hz or kHz, In practice, we wish to transmit a message with more than a single tone at a single frequency. A general analog (message would be represented by a sum of cosine waves with different amplitudes and phases A_i, ψ_i at each frequency f_i

$$m(t) = \sum_i A_i(t)\cos(2\pi f_i t + \psi_i(t))$$

We often assume that $m(t)$ is divided into frames of length T in the range 5-20 milliseconds. During each frame k at time $t = kT$, we assume $A_i(t) = A_{i,k}, \psi_i(t) = \psi_{i,k}$ are constant, so we can write $m(t = kT) = \sum_i A_{i,k} \cos(2\pi f_i t + \psi_{i,k})$. At the discrete times $t = kT$, the message is the sum of cosine waves with frequencies f_i amplitudes $A_{i,k}$ and phases $\psi_{i,k}$ that are different for each frame k. Frames are normally overlapped.
An analog message can also represent a digital symbol sequence a_k by writing

$$m(t) = \sum_k a_k p(t - kT)$$

where $p(t)$ is a pulse that spans a finite time period, a_k may be binary symbol ± 1 (to represent binary 1 or 0) or multilevel (e.g. $\pm 1, \pm 3$ to represent 00, 01, 10, 11) and T is the symbol time. We can have two such sequences

$$m_I(t) = \sum_k a_k p(t - kT) = I(t)$$
$$m_Q(t) = \sum_k b_k p(t - kT) = Q(t)$$

Thus we can write the complex baseband signal as
$I(t) + jQ(t) = \sum_k (a_k + jb_k) p(t - kT),$ where

$$c_k = a_k + jb_k = r_k e^{j\phi_k}$$

is a complex data symbol. If both a_k, b_k are binary, then c_k represents 2 bits of information at each time $t = kT$ If both a_k, b_k are or multilevel $\pm 1, \pm 3$, then c_k represents 4 bits of information at each time $t = kT$

2. IQ receivers

2.1. IQ signal review.

In summary, any signal $s(t)$ can be written as a carrier wave at frequency f_c with time-varying amplitude and phase, i.e.

$$
\begin{aligned}
& s(t) = a(t)\cos[2\pi f_c t + \phi(t)] \\
& = \mathrm{Re}\{a(t)e^{j\phi(t)}e^{j2\pi f_c t}\} \\
& = \mathrm{Re}\{[I(t) + jQ(t)][\cos 2\pi f_c t + jsin2\pi f_c t]\} \\
& = I(t)\cos 2\pi f_c t - Q(t)sin2\pi f_c t \\
& = a(t)\cos[2\pi f_c t + \phi(t)]
\end{aligned}
$$

where

$$I(t) = a(t)\cos\phi(t) = \mathrm{Re}\{a(t)e^{j\phi(t)}\},$$

$$Q(t) = a(t)sin\phi(t) = \mathrm{Im}\{a(t)e^{j\phi(t)}\}$$

and

$$\tilde{s}(t) = I(t) + jQ(t) = a(t)e^{j\phi(t)}$$

is called the complex envelope of the signal. The complex envelope contains two real waveforms. The complex envelope (or the two real waveforms) contain the information or message.

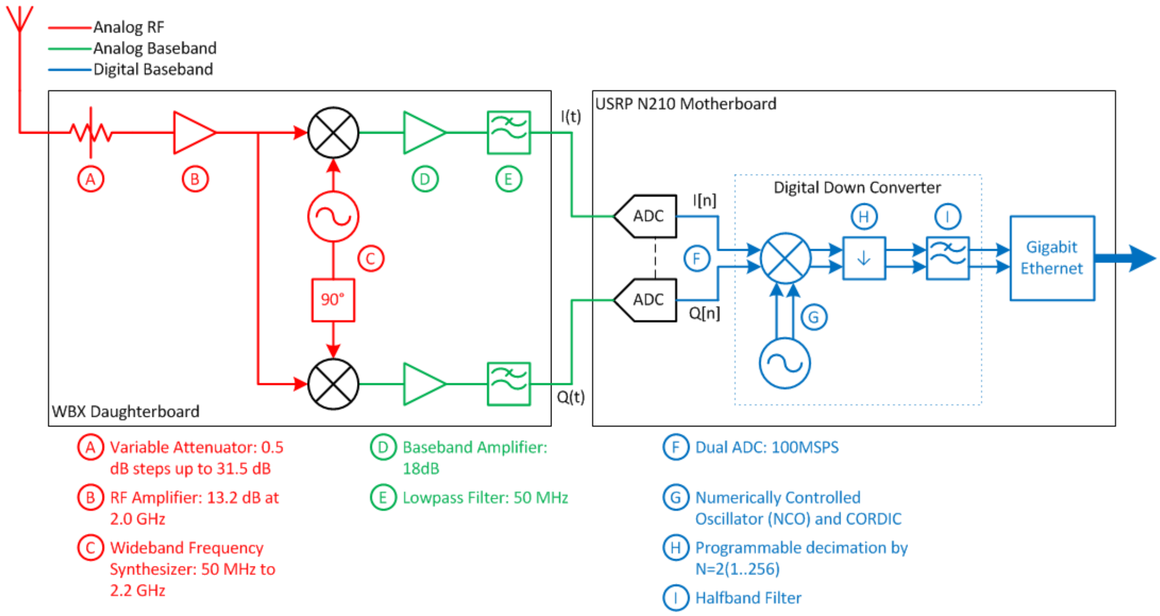
The real signal $s(t)$ is obtained by multiplying the complex envelope $\tilde{s}(t)$ with the complex carrier wave $e^{j2\pi f_c t} = cos2\pi f_c t + jsin2\pi f_c t$ and taking the real part to yield $s(t) = \mathrm{Re}\{\tilde{s}(t)e^{j2\pi f_c t}\}$.

2.2 USRP digital I-Q receiver

An IQ receiver's job is to extract the complex envelope $\tilde{s}(t) = I(t) + jQ(t) = a(t)e^{j\phi(t)}$ from the signal $s(t) = \mathrm{Re}\{\tilde{s}(t)e^{j2\pi f_c t}\}$

This is done by multiplying $s(t)$ by a complex carrier wave $e^{-j2\pi f_c t} = cos2\pi f_c t - jsin2\pi f_c t$ (note the minus sign).

This multiplication is called complex downconversion. It can be done mathematically using real signals or complex signals, as shown below.



The USRP receiver has two stages of of IQ downconversion, one analog IQ downconverter stage in the WBX daughterboard, and a digital IQ downconverter (DDC) stage on the motherboard. Both IQ downconverters operate by generating two local oscillator signals at f_{LO} and mixing (multiplying) it with a desired radio frequency (RF) signal at f_c (picked up by the antenna or fed in by a signal generator) to yield a signal at the difference frequency $f_b = f_c - f_{LO}$

Mathematical proof:
In what follows, we need some trigonometric identities

$$\cos\alpha\cos\beta = [\cos(\alpha-\beta)+\cos(\alpha+\beta)]/2$$
$$\sin\alpha\cos\beta = [\sin(\alpha-\beta)+\sin(\alpha+\beta)]/2$$
$$\sin\alpha\sin\beta = [\cos(\alpha-\beta)-\cos(\alpha+\beta)]/2$$

<u>Mathematics of complex downconversion in real notation</u>

One of the local oscillator signals is written $\cos 2\pi f_{LO} t$,
and the desired RF signal that we wish to receive is written $a(t)\cos[2\pi f_c t + \phi(t)]$.

We assume $a(t)=1, \phi(t)=0$ for the moment, so the desired RF signal is simply an unmodulated carrier wave $\cos 2\pi f_c t$. The RF signal is a real signal that can be seen on a scope.

The IQ receiver effectively has two local oscillators operating 90 degrees out of phase, $\cos 2\pi f_{LO}t$ and $-\sin 2\pi f_{LO}t$ and two mixers. Thus there are two outputs that we call $I(t)$ and $Q(t)$

The *cos* mixer function multiplies these two signals to yield

$$\cos 2\pi f_c t \cdot \cos 2\pi f_{LO}t \; .$$

we apply a trigonometric identity with $\alpha = 2\pi f_{LO}t, \beta = 2\pi f_c t$ and write

$$\cos 2\pi f_{LO}t \cdot \cos 2\pi f_c t = 0.5\cos 2\pi(f_c + f_{LO})t + 0.5\cos 2\pi(f_c - f_{LO})t$$

Thus multiplying two sine (or cosine) waves at frequencies f_{LO} and f_c results in two new sine waves, one at the sum frequency $f_c + f_{LO}$ and one at the difference frequency $f_c - f_{LO}$.

The signal at the difference frequency $f_c - f_{LO}$ is written

$$I(t) = 0.5\cos 2\pi(f_c - f_{LO})t = 0.5\cos 2\pi f_b t \; .$$

The *sin* mixer function multiplies these two signals to yield
$\cos 2\pi f_c t \cdot (-\sin 2\pi f_{LO}t)$
we apply a trigonometric identity with $\alpha = 2\pi f_{LO}t, \beta = 2\pi f_c t$ and write

$$-\sin 2\pi f_{LO}t \cdot \cos 2\pi f_c t = -0.5\sin 2\pi(f_c + f_{LO})t + 0.5\sin 2\pi(f_c - f_{LO})t$$

The signal at the difference frequency is written

$$Q(t) = 0.5\sin 2\pi(f_c - f_{LO})t = 0.5\sin 2\pi f_b t$$

We can write these two signals $I(t)$ and $Q(t)$ as one complex signal $\tilde{s}(t) = I(t) + jQ(t) = a(t)e^{j\phi(t)}$ that has time varying amplitude and phase $a(t), \phi(t)$

<u>Mathematics of complex downconversion in complex notation</u>

The IQ receiver function can also be described using complex signals as follows.

The complex RF signal $\hat{r}(t) = a(t)e^{j\phi(t)}e^{j2\pi f_c t}$ is multiplied by the complex local oscillator $e^{-j2\pi f_{LO}t|} = \cos 2\pi f_{LO}t - j\sin 2\pi f_{LO}t$
to yield
$$\hat{r}(t)e^{-j2\pi f_{LO}t} = [a(t)e^{j\phi(t)}e^{j2\pi f_c t}]e^{-j2\pi f_{LO}t} = a(t)e^{j\phi(t)}e^{j2\pi f_b t} = I(t) + jQ(t)$$

with $f_b = f_c = -f_{LO}$

The received complex baseband signal is

$\tilde{r}(t) = I(t) + jQ(t)$

The diagram below using complex signals performs the same function as the previous diagram above using real signals. Note that the complex signal diagram does not use the low pass filters.

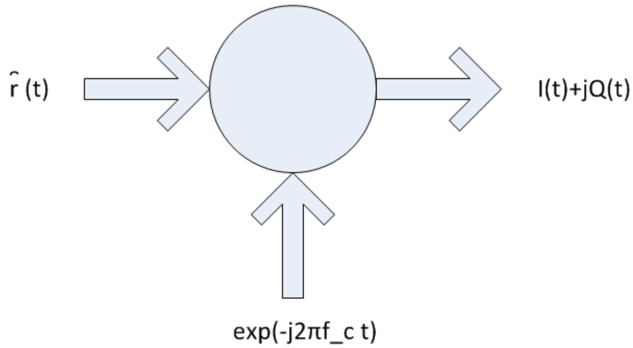


Figure caption: IQ receiver in complex notation, in this diagram f_c = f_{LO}

For the case $a(t) = 1, \phi(t) = 0$ where the RF input signal is a simple carrier wave
$\hat{r}(t) = e^{j2\pi f_c t}$, and
$I(t) = \cos 2\pi f_b t$,
$Q(t) = \sin 2\pi f_b t$,
we can write
$\tilde{r}(t) = I(t) + jQ(t) = \cos 2\pi f_b t + j\sin 2\pi f_b t = e^{j2\pi f_b t}$
the same result as above, apart from a factor 0.5 arising from the complex notation.

$I(t)$ and $Q(t)$ as displayed on a *x-y* scope will show a sinusoidal wave at the difference frequency , a circle is displayed. If $f_b < 5$ Hz or so, then the dot on the scope can be seen tracing out the circle.

Mathematics of complex downconversion in complex notation with a real carrier wave

Assume signal input, $s(t) = a(t)\cos[2\pi f_c t + \phi(t)] = \mathrm{Re}\{a(t)e^{j\phi(t)}e^{j2\pi f_c t}\}$ with
$a(t) = 1, \phi(t) = 0$
Multiply $s(t) = \cos[2\pi f_c t]$ by $e^{-j2\pi f_{LO} t|} = \cos 2\pi f_{LO} t - j\sin 2\pi f_{LO} t$ to obtain complex downconverter outputs $I(t) = \cos 2\pi f_b t$ and $Q(t) = \sin 2\pi f_b t$

Exercise for the student