

Université d'Ottawa  
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University of Ottawa  
Faculty of Engineering

Department of  
Civil Engineering

## CVG 3106 SOIL MECHANICS - II MIDTERM EXAM

**Length of Examination: 1hr 15 min**  
**Professor: Won Taek Oh**

**18<sup>th</sup> Feb, 2011 (11:30 to 12:45 PM)**  
**Page 1 of 13**

Family Name: \_\_\_\_\_

Other Names: \_\_\_\_\_

Student Number: \_\_\_\_\_

Signature \_\_\_\_\_

- (i) This is a closed book exam. No textbooks are allowed
- (ii) Formula sheet is available on last pages of this question paper
- (iii) If you do not understand a question, clearly state an assumption and proceed.**
- (iv) Non programmable calculators are permitted
- (v) Questions have the values shown next to the question.
- (vi) Marks will be taken out for missing units and labels.**
- (vii) Answers should be succinct.

At the end of the exam, when time is up:

- Stop working and turn your exam upside down.
- Please remain silent.
- Do not move or speak until ALL exams have been picked up, and a TA or the Professor gives the go-ahead to leave.

<u>Question</u>	<u>Max Marks</u>	<u>Marks Awarded</u>
1	30	
2	20	
3	50	
<b>Total</b>	<b>100%</b>	

## Question 1 (30 Marks)

A soil fill ( $\gamma = 1.6 \text{ Mg/m}^3$ ) is deposited on an overconsolidated clay ground ( $\text{OCR} = 1.5$ ). Find the shear strength of the soil at the base of the fill (i.e. point A in Figure 1) just after the height of fill has been raised from 3 m to 6 m (i.e. from Layer 1 to Layer 2 in Figure 1). The consolidated-drained test results on two identical specimens collected from the clay ground are shown in Table 1.

Assumption and information:

- The pore-water pressure after the construction of the Layer 1 has been completely dissipated.
- Dissipation of pore pressure during the construction of Layer 2 is negligible.
- The lateral pressure at any point is one half of the vertical pressure.
- $G_s = 2.72$ ,  $e = 0.72$ , and  $w = 26\%$  for the clay ground just before the construction of Layer 2.
- Use Figure 2(a) and (b) (see page 3) to estimate the pore pressure parameters  $A$  and  $B$ , respectively.

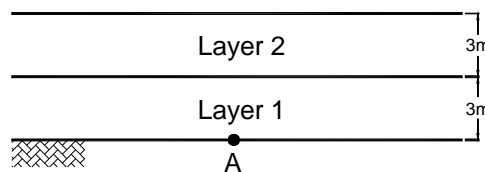
f) 
$$\gamma = \frac{G_s + Se}{1 + e} \gamma_w$$

$$Se = G_s w$$

$$\tau = c' + \sigma' \tan \phi'$$

$$\Delta u = B \left[ \Delta \sigma_3 + A (\Delta \sigma_1 - \Delta \sigma_3) \right]$$

$$\sigma'_1 = \sigma'_3 \tan^2 \left( 45 + \frac{\phi'}{2} \right) + 2c' \tan \left( 45 + \frac{\phi'}{2} \right)$$

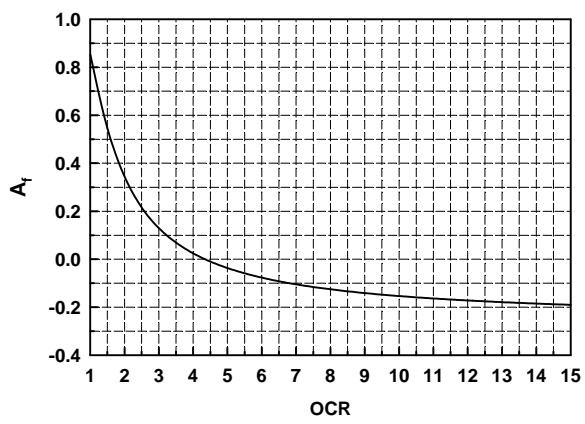


Overconsolidated clay  
( $G_s = 2.72$ ,  $e = 0.72$ ,  $w = 26\%$ )

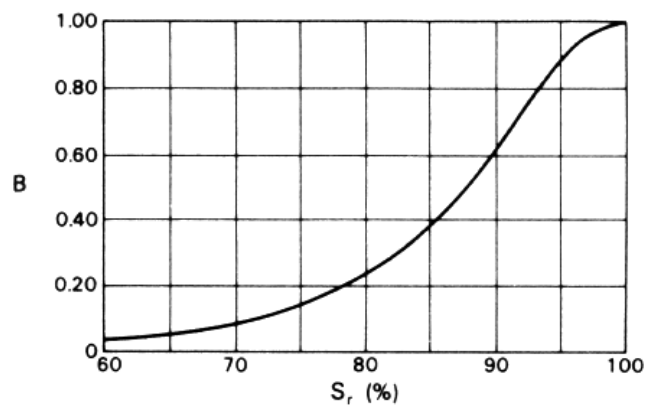
Figure 1

Table 1. Major and minor principal stresses at failure

	$\sigma_3$ (kPa)	$\sigma_1$ (kPa)
Test 1	82.8	329.2
Test 2	165.6	558.6



(a)



(b)

Figure 2

$$\sigma'_1 = \sigma'_3 \tan^2 \left( 45 + \frac{\phi'}{2} \right) + 2c' \tan \left( 45 + \frac{\phi'}{2} \right)$$

$$(a) \quad 329.2 = 82.8 \tan^2 \left( 45 + \frac{\phi'}{2} \right) + 2c' \tan \left( 45 + \frac{\phi'}{2} \right)$$

$$(b) \quad 558.6 = 165.6 \tan^2 \left( 45 + \frac{\phi'}{2} \right) + 2c' \tan \left( 45 + \frac{\phi'}{2} \right)$$

Subtracting Equation (a) from Equation (b);

$$229.4 = 82.8 \tan^2 \left( 45 + \frac{\phi'}{2} \right)$$

$$\phi' = 28^\circ$$

From Equation (a);

$$c' = \frac{329.2 - 82.8 \tan^2 (45 + 14)}{2 \tan (45 + 14)}$$

$$c' = 30 \text{ kN} / \text{m}^2$$

Shear strength =  $\tau = c' + \sigma' \tan \phi'$

$$\sigma' = \sigma_{(layer1)} + (\Delta\sigma)_{layer2} - (\Delta u)_{layer2}$$

$$\sigma_{(layer1)} = 1.6 \times 9.81 \times 3 = 47 \text{ kPa}$$

$$(\Delta\sigma)_{layer2} = 1.6 \times 9.81 \times 3 = 47 \text{ kPa}$$

$$(\Delta u)_{layer2} = B \left[ \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3) \right] \text{ and } \Delta\sigma_3 = \frac{\Delta\sigma_1}{2} = \frac{47}{2} = 23.5 \text{ kPa}$$

$$Se = G_s w$$

$$S = \frac{G_s w}{e} = \frac{2.72 \times 0.26}{0.72} = 0.98$$

From Figure 2,  $A_f = 0.55$  (OCR = 1.5) and  $B = 0.98$

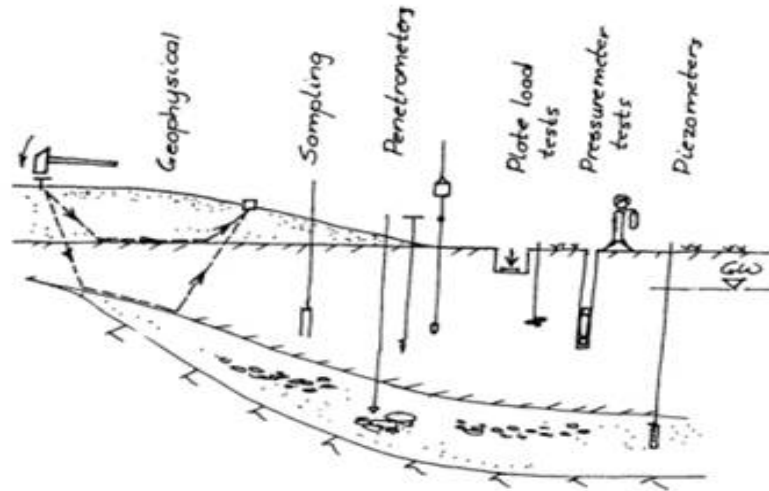
$$\Delta u = B \left[ \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3) \right]$$

$$= 0.98 \times [23.5 + 0.55 \times 23.5] = 35.7 \text{ kPa}$$

$$\tau = c' + \sigma' \tan \phi' = 30 + (47 + 47 - 35.7) \times \tan 28^\circ = 61 \text{ kPa}$$

## **Question 2 (20 Marks)**

What is the key information that can be derived from different tests listed in Figure 3.



**Figure 3**

Test	Key information
Geophysical	
Sampling	
Penetrometer	
Plate load	
Pressuremeter	
Piezometer	

### Question 3 (50 Marks)

For the continuous (i.e. strip) foundation ( $B = 1.5\text{m}$ ) given in Figure 4

- Calculate allowable bearing capacity ( $FS = 3$ ) using **Terzaghi Bearing Capacity Equation**. The ground water table is at a depth of 1m below the ground surface. (15 Marks)
- Repeat Q3.(a) for a square foundation (i.e.  $B \times L = 1.5\text{m} \times 1.5\text{m}$ ) using **Terzaghi Bearing Capacity Equation**. The level of ground water table has dropped to 2m below the ground surface in connection with the active use of the groundwater. (assume that  $e$  and  $w$  above ground water table are the same as Q3.(a)) (15 Marks)
- Repeat Q3.(b) for a load inclined at an angle of  $15^\circ$  to the vertical using **General Bearing Capacity Equation**. (20 Marks)

(♣ sees pages 6 - 9 for i) Terzaghi bearing capacity equation, ii) General bearing capacity equation, and iii) Bearing capacity factors)

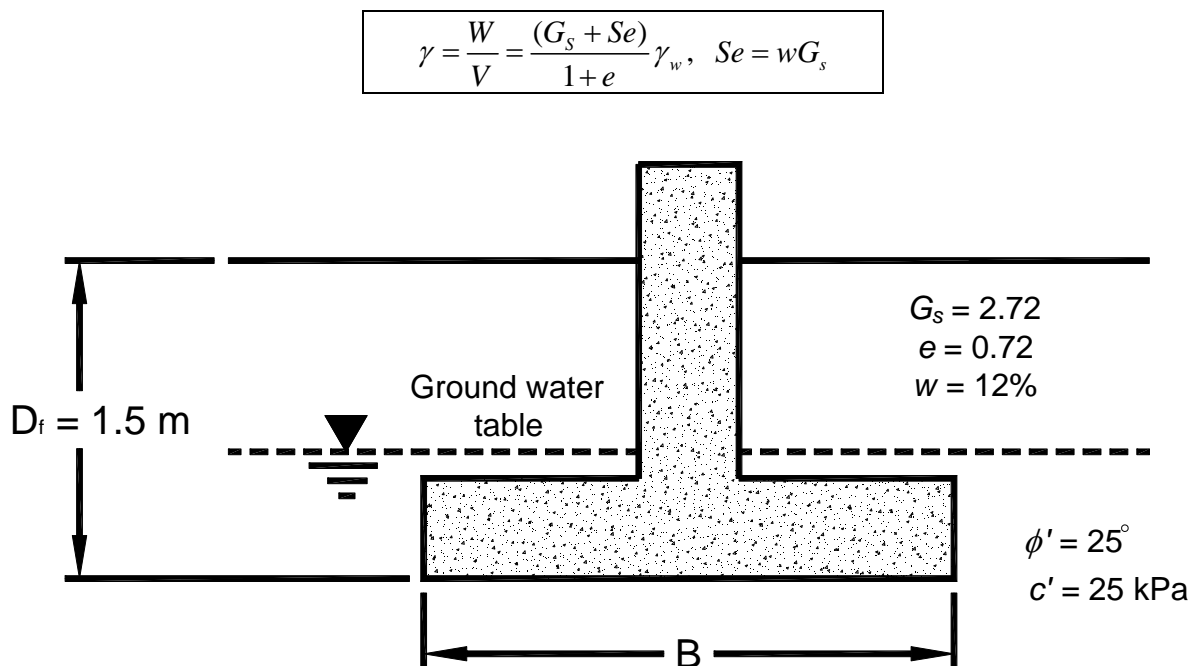


Figure 4

### **Q3(a)**

Local shear failure ( $\phi' = 25^\circ$ )

$$\bar{\phi}' = \tan^{-1}(\frac{2}{3} \tan 25^\circ) = 17.27^\circ$$

$$N'_c = 14.8, N'_q = 5.6, N'_\gamma = 2.25 \text{ for } \phi' = 17.27^\circ$$

unit weight of soil above ground water table

$$\gamma = \frac{G_s + Se}{1 + e} \gamma_w = \frac{2.72 + 2.72 \times 0.12}{1 + 0.72} \times 9.81 = 17.4 \text{ kN} / \text{m}^3$$

$$\gamma_{sat} = \frac{G_s + Se}{1 + e} \gamma_w = \frac{2.72 + 1 \times 0.72}{1 + 0.72} \times 9.81 = 19.6 \text{ kN} / \text{m}^3$$

$$\begin{aligned} q_{ult} &= \frac{2}{3} c' N'_c + q N'_q + \frac{1}{2} B \gamma N'_\gamma \\ &= \frac{2}{3} \times 25 \times 14.8 + [17.4 \times 1 + (19.6 - 9.81) \times 0.5] \times 5.6 + \frac{1}{2} \times 1.5 \times (19.6 - 9.81) \times 2.25 \\ &= 388 \text{ (kPa)} \end{aligned}$$

$$\therefore q_{all} = q_{ult} / 3 = 388 / 3 = 129 \text{ kPa}$$

**Q3(b)**

$$\begin{aligned}\gamma_b &= \frac{1}{B}[\gamma d + \gamma'(B - d)] \quad \text{for } d \leq B \\ &= \frac{1}{1.5}[17.4 \times 0.5 + (19.6 - 9.81) \times (1.5 - 0.5)] \\ &= 12.3(kN / m^3)\end{aligned}$$

$$\begin{aligned}q_{ult} &= 0.867c'_c + qN'_q + 0.4B\gamma N'_\gamma \\ &= 0.867 \times 25 \times 14.8 + 17.4 \times 1.5 \times 5.6 + 0.4 \times 1.5 \times 12.3 \times 2.25 \\ &= 483.6(kPa)\end{aligned}$$

$$\therefore q_{all} = q_{ult} / 3 = 483.6 / 3 = 161kPa$$



### **Q3(c)**

$$N_c = 20.72, N_q = 10.66, N_\gamma = 10.88 \text{ for } \phi' = 25^\circ$$

$$q_{ult} = c'N_c F_{cs} F_{cd} F_{ci} + qN_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

$$F_{cs} = 1 + \left( \frac{B}{L} \right) \left( \frac{N_q}{N_c} \right) = 1 + \frac{10.66}{10.88} = 1.98$$

$$F_{qs} = 1 + \left( \frac{B}{L} \right) \tan \phi' = 1 + \tan 25^\circ = 1.57$$

$$F_{\gamma s} = 1 - 0.4 \left( \frac{B}{L} \right) = 0.6$$

$$F_{cd} = 1 + 0.4 \left( \frac{D_f}{B} \right) = 1.4$$

$$F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \left( \frac{D_f}{B} \right) = 1 + 2 (\tan 25^\circ) (1 - \sin 25^\circ)^2 = 1.3$$

$$F_{\gamma d} = 1$$

$$F_{ci} = F_{qi} = \left( 1 - \frac{\beta^\circ}{90^\circ} \right)^2 = \left( 1 - \frac{15^\circ}{90^\circ} \right)^2 = 0.694$$

$$F_{\gamma i} = \left( 1 - \frac{\beta}{\phi'} \right)^2 = \left( 1 - \frac{15}{25} \right)^2 = 0.16$$

$$\begin{aligned} q_{ult} &= c'N_c F_{cs} F_{cd} F_{ci} + qN_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i} \\ &= 25 \times 20.72 \times 1.98 \times 1.4 \times 0.694 + (17.4 \times 1.5) \times 10.66 \times 1.57 \times 1.3 \times 0.694 \\ &\quad + 0.5 \times 12.3 \times 1.5 \times 10.88 \times 0.6 \times 1 \times 0.16 \\ &= 1400(kPa) \end{aligned}$$

$$\therefore q_{all} = q_{ult} / 3 = 1400 / 3 = 467(kPa)$$

# Terzaghi's ultimate bearing capacity theory

## □ General shear failure

### ✓ Continuous or strip footing

$$q_u = c'N_c + qN_q + \frac{1}{2}\gamma B N_\gamma$$

### ✓ Square footing

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma B N_\gamma$$

### ✓ Circular footing

$$q_u = 1.3c'N_c + qN_q + 0.3\gamma B N_\gamma$$

$$N_c, N_q, N_\gamma = f(\phi')$$

## □ Local shear failure

### ✓ Continuous or strip footing

$$q_u = \frac{2}{3}c'N'_c + qN'_q + \frac{1}{2}\gamma B N'_\gamma$$

### ✓ Square footing

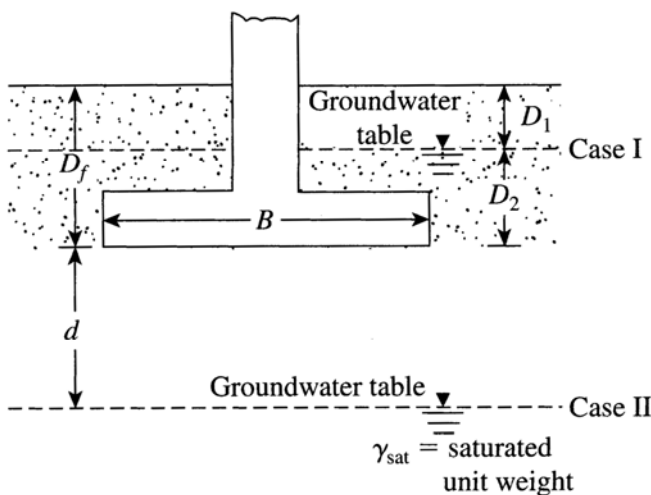
$$q_u = 0.867c'N'_c + qN'_q + 0.4\gamma B N'_\gamma$$

### ✓ Circular footing

$$q_u = 0.867c'N'_c + qN'_q + 0.3\gamma B N'_\gamma$$

$$N'_c, N'_q, N'_\gamma = f(\bar{\phi}'), \bar{\phi}' = \tan^{-1}(\frac{2}{3}\tan\phi')$$

## Effect of groundwater table on the bearing capacity



## □ Case I ( $0 \leq D_1 \leq D_f$ )

## □ Case II ( $0 \leq d \leq B$ )

$$\gamma_b = \gamma' + \frac{d}{B}(\gamma - \gamma')$$

## □ Case III ( $B \leq d$ ): no effect

**Table 2. Bearing capacity factors for Terzaghi bearing capacity equation.**

$\phi'$	$N_c$	$N_q$	$N_\gamma^a$	$\phi'$	$N_c$	$N_q$	$N_\gamma^a$
0	5.70	1.00	0.00	26	27.09	14.21	9.84
1	6.00	1.10	0.01	27	29.24	15.90	11.60
2	6.30	1.22	0.04	28	31.61	17.81	13.70
3	6.62	1.35	0.06	29	34.24	19.98	16.18
4	6.97	1.49	0.10	30	37.16	22.46	19.13
5	7.34	1.64	0.14	31	40.41	25.28	22.65
6	7.73	1.81	0.20	32	44.04	28.52	26.87
7	8.15	2.00	0.27	33	48.09	32.23	31.94
8	8.60	2.21	0.35	34	52.64	36.50	38.04
9	9.09	2.44	0.44	35	57.75	41.44	45.41
10	9.61	2.69	0.56	36	63.53	47.16	54.36
11	10.16	2.98	0.69	37	70.01	53.80	65.27
12	10.76	3.29	0.85	38	77.50	61.55	78.61
13	11.41	3.63	1.04	39	85.97	70.61	95.03
14	12.11	4.02	1.26	40	95.66	81.27	115.31
15	12.86	4.45	1.52	41	106.81	93.85	140.51
16	13.68	4.92	1.82	42	119.67	108.75	171.99
17	14.60	5.45	2.18	43	134.58	126.50	211.56
18	15.12	6.04	2.59	44	151.95	147.74	261.60
19	16.56	6.70	3.07	45	172.28	173.28	325.34
20	17.69	7.44	3.64	46	196.22	204.19	407.11
21	18.92	8.26	4.31	47	224.55	241.80	512.84
22	20.27	9.19	5.09	48	258.28	287.85	650.67
23	21.75	10.23	6.00	49	298.71	344.63	831.99
24	23.36	11.40	7.08	50	347.50	415.14	1072.80
25	25.13	12.72	8.34				

## General bearing capacity equation

□ Shape factor (De Beer, 1970)    □ Depth factor (Hansen, 1970)

$$F_{cs} = 1 + \left( \frac{B}{L} \right) \left( \frac{N_q}{N_c} \right)$$

$$F_{qs} = 1 + \left( \frac{B}{L} \right) \tan \phi'$$

$$F_{\gamma s} = 1 - 0.4 \left( \frac{B}{L} \right)$$

$$\text{for } D_f / B \leq 1$$

$$F_{cd} = 1 + 0.4 \left( \frac{D_f}{B} \right)$$

$$F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \left( \frac{D_f}{B} \right)$$

$$F_{\gamma d} = 1$$

□ Inclination factor (Meyerhof, 1963; Hanna and Meyerhof, 1981)

$$F_{ci} = F_{qi} = \left( 1 - \frac{\beta^\circ}{90^\circ} \right)^2$$

$$F_{\gamma i} = \left( 1 - \frac{\beta^\circ}{\phi'} \right)^2$$

$$\text{for } D_f / B > 1$$

$$F_{cd} = 1 + (0.4) \tan^{-1} \left( \frac{D_f}{B} \right)$$

$$F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \tan^{-1} \left( \frac{D_f}{B} \right)$$

$$F_{\gamma d} = 1$$

**Table 3. Bearing capacity factors for General Bearing Capacity Equation**

$\phi'$	$N_c$	$N_q$	$N_\gamma$	$\phi'$	$N_c$	$N_q$	$N_\gamma$
0	5.14	1.00	0.00	26	22.25	11.85	12.54
1	5.38	1.09	0.07	27	23.94	13.20	14.47
2	5.63	1.20	0.15	28	25.80	14.72	16.72
3	5.90	1.31	0.24	29	27.86	16.44	19.34
4	6.19	1.43	0.34	30	30.14	18.40	22.40
5	6.49	1.57	0.45	31	32.67	20.63	25.99
6	6.81	1.72	0.57	32	35.49	23.18	30.22
7	7.16	1.88	0.71	33	38.64	26.09	35.19
8	7.53	2.06	0.86	34	42.16	29.44	41.06
9	7.92	2.25	1.03	35	46.12	33.30	48.03
10	8.35	2.47	1.22	36	50.59	37.75	56.31
11	8.80	2.71	1.44	37	55.63	42.92	66.19
12	9.28	2.97	1.69	38	61.35	48.93	78.03
13	9.81	3.26	1.97	39	67.87	55.96	92.25
14	10.37	3.59	2.29	40	75.31	64.20	109.41
15	10.98	3.94	2.65	41	83.86	73.90	130.22
16	11.63	4.34	3.06	42	93.71	85.38	155.55
17	12.34	4.77	3.53	43	105.11	99.02	186.54
18	13.10	5.26	4.07	44	118.37	115.31	224.64
19	13.93	5.80	4.68	45	133.88	134.88	271.76
20	14.83	6.40	5.39	46	152.10	158.51	330.35
21	15.82	7.07	6.20	47	173.64	187.21	403.67
22	16.88	7.82	7.13	48	199.26	222.31	496.01
23	18.05	8.66	8.20	49	229.93	265.51	613.16
24	19.32	9.60	9.44	50	266.89	319.07	762.89
25	20.72	10.66	10.88				