Department of Computer Science

Degree Title: BEng

Using Formula One data captured during practice and qualifying sessions to create race pit strategies using Machine Learning

Vivek Mistry

Formative Two: Literature Review

Supervisor: William Smith

To all students everywhere

Acknowledgements

I would like to thank my goldfish for all the help it gave me writing this document.

As usual, my boss was an inspiring source of sagacious advice.

Contents

List of Figures

List of Tables

1 Introduction

To understand the project, it is important to understand the domain of Formula One. This will allow for the successful evaluation of prior work and the creation of an appropriate model. This introduction aims to introduce the role of a strategy engineer, some key terminology and explain the race format.

1.1 What is F1 Strategy?

Formula One strategy can be described as a “continuous game of multi-dimensional chess” [1]. It is about plotting the best possible path to a Grand Prix finish by working with tire engineers, race engineers and drivers. This can be done by focusing on car setup and finding the optimal pit window during the race. [2] For this project, we will concentrate on finding the optimal pit window.

Throughout the weekend, the teams have access to 8 sets of soft compounds, 3 sets of medium compounds and 2 sets of hard compound tires. After each free practice, you must return two sets of tires. If you get to the final qualifying session, you must return the tires used during this section. In the event of rain, the teams will have access to four sets of intermediate tires and three sets of wet tires. [3]

1.1.1 Weekend Format

Most Formula One weekends start with three free practice sessions which provide live information on tire performance. [4] This allows teams to start building a model of the car on the specific track. The types of activity they will do include:

- Run for a longer amount of time to simulate race pace and understand more about tire degradation.

- Run for one lap with the softer tire compound to simulate the qualifying pace.[4]

The strategy model evolves weekend by weekend and the team often build models of their strategy which get updated at the end of every weekend. [1] Attaining the methods used within the teams will be inaccessible for this project as they do not want their competitors to see it.

During qualifying, teams will try to set the fastest lap to have the best starting position possible.

1.1.2 Race Day

Before race day, engineers would check the weather forecast and make decisions about what strategies could be used based on different events during the race. [5] A lot of these models will not be used, and these pit strategies are dynamic as they need to be reevaluated on every lap. [[6]] Often a more aggressive strategy can result in a high reward but carries a higher risk. [2] An example of this is Russia 2015 where Sergio Perez qualified 7th. The race engineer team decided to pit him when the **safety car** came out, meaning that he had to complete more than 40 laps on one tire. This bold strategy, led to him finishing in 3rd place. [6] It is important to note, that during the race there is a mandatory compound change.

Here are a few factors that may lead to a pit stop being made:

- **Undercut:** The driver pits before the car in front to try and gain a position. Fresher tires will yield more pace and create a net gain over the next few laps meaning that they will be in the lead once the competitor pits. The risk is that it requires conceding track positions to cars behind and having to get past traffic after the pit stop.

- **Overcut:** The driver remains on track longer than a competitor to try to gain a place. The issue with this is that your tires will be older than your competitors once they pit.

- **DRS Train:** DRS stands for Drag Reduction System; this makes the cars faster to assist with overtaking and can be used when you are 1 second within a car and can only be enabled in certain parts of the track. [7] A DRS Train is where everyone has DRS so there is no benefit, so you get stuck behind each other. This often leads to drivers trying an undercut.

- **Virtual Safety Car (VSC):** In the event of a safety incident, the **FIA (Fédération Internationale de l'Automobile)** will tell the drivers how fast they can drive at certain points of the track. This results in a quicker pit stop as everyone is under a reduced speed.

- **Safety Car (SC):** In the event of a safety incident, the SC will lead the pack of cars at a reduced speed. No overtaking can take place. These factors allow for less time lost during a pit stop compared to regular conditions.

- **Red Flag:** In the event of a safety incident, the FIA will tell drivers to return to the pits. Teams can change tires and fix damage, which allows you can switch compounds without a loss. [5]

When evaluating existing methods simulating and modeling pit stops, we can analyse how effectively these factors are shown in the data model.

1.1.3 Sprint Weekend

Some Formula One weekends are sprint weekends. A sprint weekend consists of Free Practice 1 and 2. Sprint Qualifying (known as “Sprint Shootout”) and the Sprint race which is held over one-third race distance. Then, Qualifying and Race as usual. [8]

The format used during 2022 and 2023 involved having Free Practice 1, 2 and 3. Qualifying which set the start positions for the Sprint race. The result of the Sprint race determined the start positions for the Race. [9] Deciding in what capacity to include Sprint Weekends will be a crucial step in defining my data model.

1.2 Movitation

Formula One is a Data-Driven sport. I have discovered FastF1 API [10] which can be used to gather lap data, telemetry and weather data and has not been used in any research yet. My motivation here is to start by creating a basic model before enhancing it using different architectures and adding more data inputs. The sources I have seen have used other techniques such as Game Theory or Optimization problems. The sources that have been used to explore Machine Learning in this domain are sparse and exclude data such as wet races and do not look at goals such as enhancing pre-existing pit stop decisions.

1.3 Objectives

- Investigate what factors and strategies engineers take into consideration over a formula one weekend.
- Critically evaluate existing methods of simulating pit strategies
- Investigate what data inputs increase the effectiveness of the model from the FastF1 API
- Investigate what iterative methods there are to developing neural networks.
- Effectively use an experiment management tool when creating different models.
- Investigate what neural network (NN) architectures would be most effective for time series data I have.
- Create a suitable method to evaluate NN decisions which involve checking whether the NN makes a better decision to what happened in real life.

2 Literature Review

There is limited literature on Machine Learning in this domain, but game theory and simulation have also been used to model strategy. Closed source models will be built by the team to assist them during the race weekend and by Formula One themselves who use it to display statistics on the broadcast feed.

2.1 Game Theory

Aguad and Thraves look at modelling a race through a zero-sum feedback Stackelberg game using dynamic programming, as they believe most other literature does not focus on competition. This is modelled by having drivers A and B compete in N laps of a race. For each lap $n \in \{1, \ldots, N\}$, you choose whether to stop and change tires or continue.

There are three tire choices, denoted as T, where $T = \{1, 2, 3\}$ and this set maps to {soft, medium, hard}. Each tire $t \in T$ has a lifespan of u_t laps.

During each lap, you decide on a tire choice T or opt not to pit, represented by 0.

There are variables to keep track of the current tire compound, the amount of laps the tire has done so far and whether the mandatory pit stop has taken place yet. Then, as n increases, these variables are modified to ensure that the state stays up to date. The constraints placed upon this model are that there cannot be a stop in the first and last lap as the start-finish line is parallel to the pits so you must start and finish there.

At the beginning of the race, they find an equilibrium using the Lemme-Howson algorithm. One issue here is that if we view race as a selfish game, there is not a place where equilibrium can be achieved. Lap time is formulated as a function of three elements: a ghost lap (represents a perfect lap), an interaction function (which models racing conditions) and pit stop time. An exponential function is used to ensure a substantial decrease in the interaction effect as the gap between drivers widens. Fuel loss is modeled as linear despite having no access to the sensor data to validate this.

Within the model, Aguad and Thraves make a successful modeling decision as they model that one driver will start in front of the other as qualifying will set the grid for race day. During a SC, there is an equation which models that the driver in first will be leading the race by 0.5 seconds. The only way of overtaking here would be through a pit stop.

Stochastic modeling is used for any yellow flags here. For any VSC, the time decrease is fixed to 40%. The time left under VSC or SC is modeled by tracking this under a variable. This enables the model to have 3 different lap time functions, either under VSC, SC or with regular conditions. The stochastic modeling here is suitable as it allows us to consider randomness in lap times as the result of yellow flag events.

The objective given is to maximise the time gap with the other driver by the end of the race. The cost function represents the driver finishing the race adequately without the laps the tires have used being greater than ut. The paper also explores an alternative objective function where you maximise the odds of winning. In both cases, we see winning strategies from both objective functions. [11]

The dynamic programming algorithm decides the tire compound to start the race. For every lap, it will look through the set of states, compute the objective function, make the pit stop decision and then recalculate values for each state.

Some disadvantages of this method the first driver will make the first pit stop and then the second place reacts to this. This would expose the second driver to an undercut. Some factors that are ignored here include weather, rather than rely on another driver's decision, it would make more sense to just look at weather data in the event of rain.

The issue with modeling this as a 2 person game is that it could be viewed as being too fine grained. At Silverstone 2022, Hamilton managed a double overtake into the final corner while Perez and Leclerc were battling. At this point, I would view that at a minimum it should be viewed as a 3 person game. [12]

The formulaic approach makes it harder to reuse. DRS is modelled with a fixed equation to say it will be activated after the second lap. Since this year, it has been enabled after the first lap. [13]

We have to calculate every possible combination at every state, making this computationally expensive. This is one reason why I shall be implementing via machine learning as we can use batching to overcome memory and processing limitations.

2.2 Simulation

Heilmeier et al created a model to load into a software tool to simulate race outcomes based on user inputs. They successfully split the race strategy into 3 discrete categories: pit stops, driver style and response to event races (further classification can be seen at A.1). This is the only source that addressed energy consumption as a factor in lap times.

Unlike game theory, which needs to search quite a breadth of nodes at the end of each lap n , the simulation in this paper was intended to provide robust results with a fast calculation time. The simulation is performed on a lap-by-lap basis and the user is required to pick and provide the pit stop information.

The starting point for the modeling algorithm here is to model the race without any competitor interaction using the formulas shown below:

$$t_{\text{race,currentlap}} = \sum_{\text{lap}=1}^{\text{current lap}} t_{\text{lap}}(\text{lap}) \tag{2.1}$$

$$\begin{aligned} t_{\text{lap}}(\text{lap}) = t_{\text{base}} + t_{\text{tire}}(a_{\text{tire}}, c_{\text{tire}}) + t_{\text{fuel}}(\text{lap}) + t_{\text{car}} \\ + t_{\text{driver}} + t_{\text{grid}}(\text{lap}) + t_{\text{pit,inlap/outlap}}(\text{lap}) \end{aligned} \tag{2.2}$$

$$t_{\text{base}} = t_Q + t_{\text{gap,racepace}} \tag{2.3}$$

t_{base} allows us to change times for all drivers at once. For example, if there is rain, it will slow everyone down. A good assumption here comes from Equation (2.3), where t_Q represents the fastest qualifying lap. This is one of the most recent sets of data we would have before the race. To improve this model during sprint weekends, we could investigate the effect of taking the fastest lap from a sprint race instead.

To model the tires, they acknowledge that there is a period when drivers will set slower times as they put heat into their tires. They model this with a linear function in the event of a low amount of data or a log function. Another assumption made is that the lap time lost to fuel mass changing as shown below:

$$t_{\text{tire,log}}(a_{\text{tire}}, c_{\text{tire}}) = \log(a_{\text{tire}} \cdot k_{1,\log}(c_{\text{tire}}) + 1) \cdot k_{2,\log}(c_{\text{tire}}) + k_3(c_{\text{tire}}) \tag{2.4}$$

$$t_{\text{tire,lin}}(a_{\text{tire}}, c_{\text{tire}}) = a_{\text{tire}} \cdot k_{2,\text{lin}}(c_{\text{tire}}) + k_3(c_{\text{tire}}) \quad (2.5)$$

Mass sensitivity is gathered using an engine map. Despite this being a suitable model, questions could be raised about the availability of this data. Another successful modeling technique is the handicap which is given to each driver based upon the starting position. As the driver qualifies further down, the racing line will be further away from the driver.

At this point, we can introduce competitor interaction. At first, the algorithms calculate the lap time for the current lap. If DRS is active and reduces the lap time based upon that. It also has a minimum gap for overtakes can take place to model the change in position. However, it does include slipstream effects.

In the event of an overtake, each driver will receive a time malus as the time will be less than optimal. This model successfully looks at the effect on both drivers and not only one. [14]

The results are accurate and show that simulated time is quite close to what happened in real life. The outcome of this source is that I could use it to assist me with modeling a race if the NN takes an outcome that didn't occur in real life. However, it does not automatically make decisions and includes yellow flags events, which is where Aguad and Thraves excel.

2.3 Optimization

Garcia employs a data-driven approach where he starts by looking at regression analysis on tire wear and then uses his findings to formulate a Mixed-Quadrant Programming (MIQP) optimisation problem. The dataset he uses can be seen in /A.2. Everything apart from the time differences between the first car behind and the first car in front well as the time difference to the front car lapped in front is taken from the pre-made database on Kaggle. [15] The time difference data is added by Garcia because these are places where interaction could affect lap time.

Initially, when estimating time performance using linear regression, Garcia takes Hungary 2022 as his dataset. However, there is a lack of justification regarding why he has exclusively chosen this event. Another weakness is that a high number of variables are placed into the regression equation (these can be seen in A.3). This becomes quite complex and raises the question about the effectiveness of using this form of analysis.

During the statistical analysis, Garcia successfully identified the relevant variables Lap, Inpits, Outpits, Hard, and some teams by analysing p-values. Outlier analysis takes place by comparing using the interquartile range and confidence region method. The confidence region method works better due to a smaller deviation of values.

Initially, if a race started yet, Bayesian statistics are used, more specifically priors. As the race goes on, the relevance of the priors is reduced as we have more recent data points. The prior data used here is Hungary 2020 rather than Hungary 2021 as it rained during this race. This means that we aren't using the most recent data point in exchange for having identical weather conditions.

The way the optimisation works is by fixing the lap we are on. Running the previous regression models to get co-efficiencies. Then, we estimate the race time for a profile strategy. The profile strategy is just an estimate of the full race time given that we make specific strategy calls. A MIQP is then formed, where the objective function is defined as the remaining race time relative to the hypothetical race future laps with new soft tires in each lap time with no stops. However, this is physically impossible to do in real life which raises whether there is a more appropriate objective function.

Garcia then evaluates the model every 5 laps. This means the lap we fix before running the model will increase by 5 every time. The evaluation showed that these strategies reflected mostly real life. However, some drivers had good strategies but skill levels and the car had an impact on the race as well but was not modeled. [16]

2.4 Neural Networks

The applications for NN within sports are betting shops, fans, commentators and the teams themselves. The application of ML has been applied across football, NFL, etc. However, this will not be covered as they have fewer variables and can viewed as being a simpler multi-class classification problem (win, lose and draw).

2.4.1 NASCAR

Choo looks at using NNs for choosing NASCAR pit stops; NASCAR is an American Racing Series. The key rules differences are that:

- You can change only 2 tires or all 4.

- Only 6 pit crew are allowed to work on the car which increases pit stop time compared to Formula One.

In this model, they model the NASCAR race as a series of outputs and caution periods (similar to yellow flags). They measure pit crew performance by looking at previous stop-time data.

Some initial data analysis is carried out, where they create various plots related to pit stop time and position. I suggest that this can be used in this project as the only inputs are time and position which is universal. The conclusions provided are:

- Drivers in the front tended to have more consistent and lower stop times, however, a more holistic view of the race would be suitable
- Drivers with high stop times tended to lose track position; such losses could be mitigated by making strategic tire change decisions later in the race

The second point here brings another motivation for teams as implementing this well could still lead to an increase in race positions.

The model takes the variable “average previous rate of change in rank” which is updated throughout the race. They are assuming that the performance of a driver can be estimated in the early part of each race, drivers that gain track position usually continue to do so.

When applying this to the domain of Formula 1, take a driver in the x^{th} fastest car. If he qualifies for a position, that is much lower than they expected. They could make a lot of overtakes during the start, then when he gets to the $x\text{-}1^{th}$ fastest car, they may not be able to get past. However, in the paper, they use a small subset of data to test the initial hypothesis so it is justified within the NASCAR domain.

Another difference between Formula One and NASCAR is that they could go to the same track multiple times a year which makes the NASCAR data source richer when looking at the effect of tire on a specific track.

This paper looks at tire strategy choices uniquely by studying historical records of tire change decisions used for previous races on the same track. From a training perspective, the concern here would be about overfitting to data in specific tracks (more specifically ones visited multiple times a year), however this has not been discussed,

Once all the parameters are decided, Choo proceeds to make 5 different datasets and use the root-mean-square deviation to evaluate his model. Overall, the lowest loss came when he inputted all races. This suggested that the effects of aggregation across a larger sample of races could outstrip differences in track characteristics for predictions. [16]

2.4.2 Formula 1

Heilmeier et al proceeded to write a paper on using an ANN (Artificial Neural Network) for strategy decisions, two years after writing the simulation piece.

A key idea presented is that a simple model where we look at making just one-stop on track with just one car on the track is unrealistic as battling for the position is an essential part of real-world racing.

The key factors are tire degradation, burned fuel mass and interaction between drivers and use equations we have seen in the simulation section. Applications for the model are that it can make a plan before the race and during the race. One issue with game theory related to making decisions based upon the first driver is identified here as it acknowledges that it is not possible to optimize each driver's strategy simultaneously from a selfish perspective.

Heilmeier et al built two neural networks here, one which decides whether to put and one which decides what tire compound to move onto. A point for improvement would be to merge both models.

One of the inputs added to the whether to pit model was a category in the set 1,2,3 which represents the absolute level of tire stress for the respective race track. However, this is not the only way you can classify tracks as you could classify whether it is a street circuit or group them by continent.

For model one, one of the stages of preprocessing are drivers making a final pit stop after 90% of race progress is removed. This happened in the Azerbaijan Grand Prix 2023, where Ocon pitted in the penultimate lap of the race. Hence, you may think it is a valid set of data but Ocon lost 3 places [[17]]. The second model is a multi-class specification problem. This takes a smaller data set than the first model because there are only 4087 relevant laps.

An MIQP is used to calculate the set's optimum stint length by calculating the race duration for each possible combination of stint length. This takes extra computations so raises the question of whether it is worth letting our model just evaluate lap times on a lap-by-lap basis. As previously mentioned, Ocon completed n-1 laps of the Azerbaijan Grand Prix 2023 on one tire.

When creating the network to choose whether to pit, a feed-forward neural network was used. This approach doesn't do a good job of gradually increasing the probability of higher output as a race progresses under normal conditions. However, the system should still be able to react quickly when there's a sudden, unexpected event, like a yellow flag. Hence, a recurrent NN was used which means that units of several consecutive laps are processed at once allowing them to learn relationships between each lap. Although this gave a better training loss, it gave some incomprehensible outputs. Therefore, they made a hybrid of both models, that worked better. [18] In this text, there is no mention of any pre-trained or pre-existing models, using them could be viewed as a better starting point for this project.

2.4.3 Formula E

* INSERT TEXT HERE *

2.5 Assumptions to Question

Various assumptions are raised throughout the papers used in this section, to start my data analysis I would like to explore these questions:

- When looking at race data, can we see linear tire degradation?
- What tracks are low and high degradation based upon our dataset, do they match up with Categories in "Virtual Strategy Engineer" paper?
- Using the data available, how can we model pit stops? Will we need to use any of the equations provided in the simulation section?
- Is there any way the following statement can be validated - “winning odds increase by more than 15% compared to when both drivers race strategically”() using our dataset?

2.6 Sources related to AI development Management

* INSERT TEXT HERE *

A Appendix

A.1 Literature Review

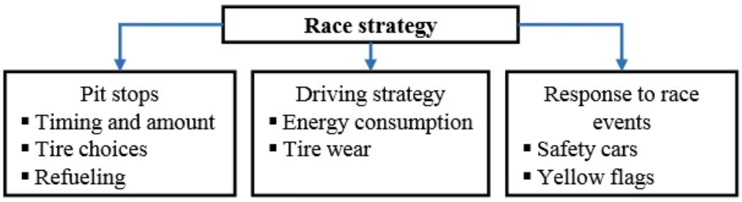

Figure A.1: Heilmeier et al classification of race strategy []

Column name	Description
Lap	Lap of the race
Position	Position of the driver
Lap Time	Lap time
Accumulated Lap Time	Accumulated lap time
Driver	Driver's code. Each driver has a unique code that represents them during the race. It has three letters, and they usually are their first three last name letters.
Team	Driver's team
Tire Compound	Tire compound
Lapwear	Lapwear of the current tire they are using
Inpits	Binary variable that will be 1 if the driver is pitting on the lap being evaluated, and 0 if not
Outpits	Binary variable that will be 1 if the driver is leaving pits on the lap being evaluated, and 0 if not
DRS	Binary variable that will be 1 if the driver has DRS, and 0 if not
Δ_1	Time difference to the first car not lapped behind
Δ_2	Time difference to the first car not lapped in front
Δ_3	Time difference to the first car lapped in front

Figure A.2: Kaggle Formula One World Championship Source [15] database Columns

Variable	Values	Description
X_{lap}	$\{1, \ldots, 70\}$	Race lap.
X_{inpits}	$\{0, 1\}$	Binary variable that will have a value of 1 if the driver is pitting on the lap being evaluated, and 0 if not.
X_{inpits}	$\{0, 1\}$	Binary variable that will have a value of 1 if the driver is pitting on the lap being evaluated, and 0 if not.
$X_{constructor}$	{Red Bull, Mercedes, Ferrari, Aston Martin, Alpine, Alpha Tauri, Haas, McLaren, Alfa Romeo, Williams }	Qualitative variable that returns the team of the pilot.
$X_{\Delta 1}$	$[0, 1.5]$	Time difference to the first car not lapped behind.
$X_{\Delta 2}$	$[0, 1.5]$	Time difference to the first car not lapped in front.
$X_{\Delta 3}$	$[0, 1.5]$	Time difference with the first car lapped in front.
X_{w}	$\{1, \ldots, 70\}$	Number of laps a type of tire is used during a stint.
X_{s0}	$\{0, 1\}$	Binary variable that will have a value of 1 if the car is going to use a soft tire, and 0 otherwise.
X_{m0}	$\{0, 1\}$	Binary variable that will have a value of 1 if the car is going to use a medium tire, and 0 otherwise.
X_{h0}	$\{0, 1\}$	Binary variable that will have a value of 1 if the car is going to use a hard tire, and 0 otherwise.

Figure A.3: Garcia's regression dependent variable descriptions []

Bibliography

[1] 'What goes into f1 strategy?' Mercedes-AMG PETRONAS F1 Team. (), [Online]. Available: https://www.mercedesamgf1.com/news/what-goes-into-f1-strategy (visited on 07/10/2024).

[2] 'So, you want to be an f1 strategy engineer?' (), [Online]. Available: https://www.astonmartinf1.com/en-GB/news/feature/so-you-want-to-be-an-f1-strategy-engineer (visited on 07/10/2024).

[3] GPfans.com. 'Everything you need to know about f1 tyres in 2024,' GPfans. (21st Jul. 2024), [Online]. Available: https://www.gpfans.com/en/f1-news/100347/f1-tyres-changes-compounds-rules-colours-blankets/ (visited on 16/11/2024).

[4] 'Insider's guide: How f1 race strategy works.' (31st Jan. 2022), [Online]. Available: https://www.motorsport.com/f1/news/how-f1-race-strategy-works/6791893/ (visited on 07/10/2024).

[5] 'F1 strategy explained: What's an undercut, overcut, a DRS train and more.' (29th Jun. 2023), [Online]. Available: https://www.autosport.com/f1/news/f1-strategy-explained-whats-an-undercut-overcut-a-drs-train-and-more/10488979/ (visited on 07/10/2024).

[6] FORMULA 1, *Bernie collins: Strategy risks, racing rewards + pitwall pressure | f1 beyond the grid podcast*, 29th Nov. 2023. [Online]. Available: https://www.youtube.com/watch?v=H6gFB3UUt_o (visited on 07/10/2024).

[7] 'What is DRS and why does the device raise criticism? - the athletic.' (), [Online]. Available: https://www.nytimes.com/athletic/4266856/2024/02/23/formula-one-f1-drs-explained/ (visited on 10/10/2024).

[8] 'Everything you need to know about the 2024 f1 sprint format,' Formula 1® - The Official F1® Website. (), [Online]. Available: https://www.formula1.com/en/latest/article/2024-f1-sprint-rules-format-explained.5pQvaAY52nnX9vZYAYgf3O (visited on 10/10/2024).

[9] 'How does the 2022 f1 sprint format work? | formula 1®,' Formula 1® - The Official F1® Website. (), [Online]. Available: https://www.formula1.com/en/latest/article/how-does-the-f1-sprint-work-the-format-explained-ahead-of-imola.4ep6uU8VYFMgftvTYxkZdI (visited on 10/10/2024).

[10] 'FastF1 3.4.3.' (), [Online]. Available: https://docs.fastf1.dev/ (visited on 17/11/2024).

[11] F. Aguad and C. Thraves, 'Optimizing pit stop strategies in formula 1 with dynamic programming and game theory,' *European Journal of Operational Research*, vol. 319, no. 3, pp. 908–919, 16th Dec. 2024, ISSN: 0377-2217. DOI: 10.1016/j.ejor.2024.07.011. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0377221724005484 (visited on 10/10/2024).

[12] FORMULA 1, *THROUGH GOES HAMILTON*, 3rd Jul. 2024. [Online]. Available: https://www.youtube.com/watch?v=FM7wyRivEJY (visited on 16/11/2024).

[13] 'Five f1 regulation changes to watch out for in 2024,' Mercedes-AMG PETRONAS F1 Team. (), [Online]. Available: https://www.mercedesamgf1.com/news/five-f1-regulation-changes-to-watch-out-for-in-2024 (visited on 16/11/2024).

[14] A. Heilmeier, M. Graf and M. Lienkamp, 'A race simulation for strategy decisions in circuit motorsports,' in *2018 21st International Conference on Intelligent Transportation Systems (ITSC)*, ISSN: 2153-0017, Nov. 2018, pp. 2986–2993. DOI: 10.1109/ITSC.2018.8570012. [Online]. Available: https://ieeexplore.ieee.org/abstract/document/8570012 (visited on 11/10/2024).

[15] 'Formula 1 world championship (1950 - 2024).' (), [Online]. Available: https://www.kaggle.com/datasets/rohanrao/formula-1-world-championship-1950-2020 (visited on 16/11/2024).

[16] C. L. W. Choo, 'Real-time decision making in motorsports : Analytics for improving professional car race strategy,' Accepted: 2015-12-16T15:54:53Z Journal Abbreviation: Analytics for improving professional car race strategy, Thesis, Massachusetts Institute of Technology, 2015. [Online]. Available: https://dspace.mit.edu/handle/1721.1/100310 (visited on 18/10/2024).

[17] 'Esteban ocon gives his honest opinion on the pit lane incident at the azerbaijan grand prix - YouTube.' (), [Online]. Available: https://www.youtube.com/watch?v=W-K60Q342Lw (visited on 16/11/2024).

[18] A. Heilmeier, A. Thomaser, M. Graf and J. Betz, 'Virtual strategy engineer: Using artificial neural networks for making race strategy decisions in circuit motorsport,' *Applied Sciences*, vol. 10, no. 21, p. 7805, Jan. 2020, Number: 21 Publisher: Multidisciplinary Digital Publishing Institute, ISSN: 2076-3417. DOI: 10.3390/app10217805. [Online]. Available: https://www.mdpi.com/2076-3417/10/21/7805 (visited on 14/10/2024).