

- Now the problem is you have to compute $nCr \% P$, where $n, r \leq 10^{18}$ but $P \leq 10^6$.
- We need to solve this problem using Lucas theorem. Because in Lucas theorem problem will be reduced to sub problems. In this theorem the n and r are converted to P base number and then we compute the same digit-location wise binomial coefficients. Lucas theorem is given below :

$$\binom{n}{r} \equiv \prod_{i=0}^k \binom{n_i}{r_i} \pmod{P}$$

Where $n = (n_k \dots n_2 n_1 n_0)_P$, $r = (r_k \dots r_2 r_1 r_0)_P$

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long long in[1000001], fact[1000001], ifact[1000001];

void generate(long long MX, long long P)
{
    fact[0]=1;
    for(int i=1;i<=MX;i++) fact[i]=(1LL * fact[i-1] * i)%P;
    in[0]=0,in[1]=1;
    for(int i=2;i<=MX;i++) in[i]= (1LL * ( (P-1)*(P/i) )%P * in[P%i] )%P;
    ifact[0]=1;
    for(int i=1;i<=MX;i++) ifact[i]=(1LL * ifact[i-1] * in[i] )%P;
}

long long small_nCr(long long n, long long r, long long P)
{
    if(r>n)return 0;

    long long ans;
    ans=(1LL * fact[n] * ifact[n-r])%P;
    ans=( ans * ifact[r] )%P;

    return ans;
}

long long nCr(long long n, long long r, long long P)
{
    if(r==0)return 1;

    long long ni=(n%P), ri=(r%P);

    return ( nCr(n/P, r/P, P)* small_nCr(ni, ri, P) ) %P;
}

long long Lucas(long long n, long long r, long long P)
{
    generate(P, P);
    return nCr(n, r, P);
}
```

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int main()
{
    long long prime[]={13,29,67,113,157,223};
    for(int i=0;i<6;i++)
    {
        cout<<( 126%prime[i] )<<' '<<Lucas(9, 5, prime[i] )<<endl;
    }
}

```

- Now the problem is, if we want to find $nCr \% P$, where **P is not a prime** number.
Solution: We can split P into its prime divisors and count binomial coefficients for each divisor and marge them using Chinese remainder theorem.

Chinese Remainder Theorem:

P can be written as, $P = P_1 P_2 \dots P_{n-1} P_n$ (prime divisors)

Now we can separately calculate,

$$n_{Cr} = X$$

$$X \equiv r_1 \pmod{P_1}$$

$$X \equiv r_2 \pmod{P_2}$$

... ..

$$X \equiv r_n \pmod{P_n}$$

Now we can marge the above equations using Chinese remainder theorem. By using this theorem we can find the minimum value of X for which all the equation are true along with the below one.

$$X \equiv r \pmod{P}$$

X can found by using followed equation,

$$X = \sum_{i=1}^n (r_i * pd_i * inv(pd_i, P_i))$$

$$pd_i = \frac{P}{P_i} \quad \& \quad r = X \% P$$

```

long long n, prime[20], rim[20];

long long bigMod(long long b, long long p, long long M)
{
    if(p==0)return 1;

    long long tmp= bigMod (b, p/2,M);

    tmp = (tmp * tmp)% M;

    return (p%2==0)? tmp : ( b * tmp) % M;
}

long long INV(long long num, long long M)
{
    return bigMod(num,M-2,M);
}

long long ChineseRemainder( )
{
    long long product=1, x=0, pd;
    for(int i=0;i<n;i++)
        product*=prime[i];

    for(int i=0;i<n;i++)
    {
        pd=product/prime[i];

        x+=( rim[i] * pd * INV( pd, prime[i]) );

        x%=product;
    }

    return x;
}

```

“Just don’t laugh at my English”- Kamol :D