

Lean4

lecture 1

Quick Tour of Lean4

Metaprogramming

- Programming: Writing code to manipulate data
- Metaprogramming: Writing code to manipulate *code*
- Reflection: Programs examining their own code

Why Metaprogramming

- Extend Lean's syntax
- Create new tactics
- Interface with Lean's compiler
- Execute tactics based on data
- Inspect the content of compiled Lean

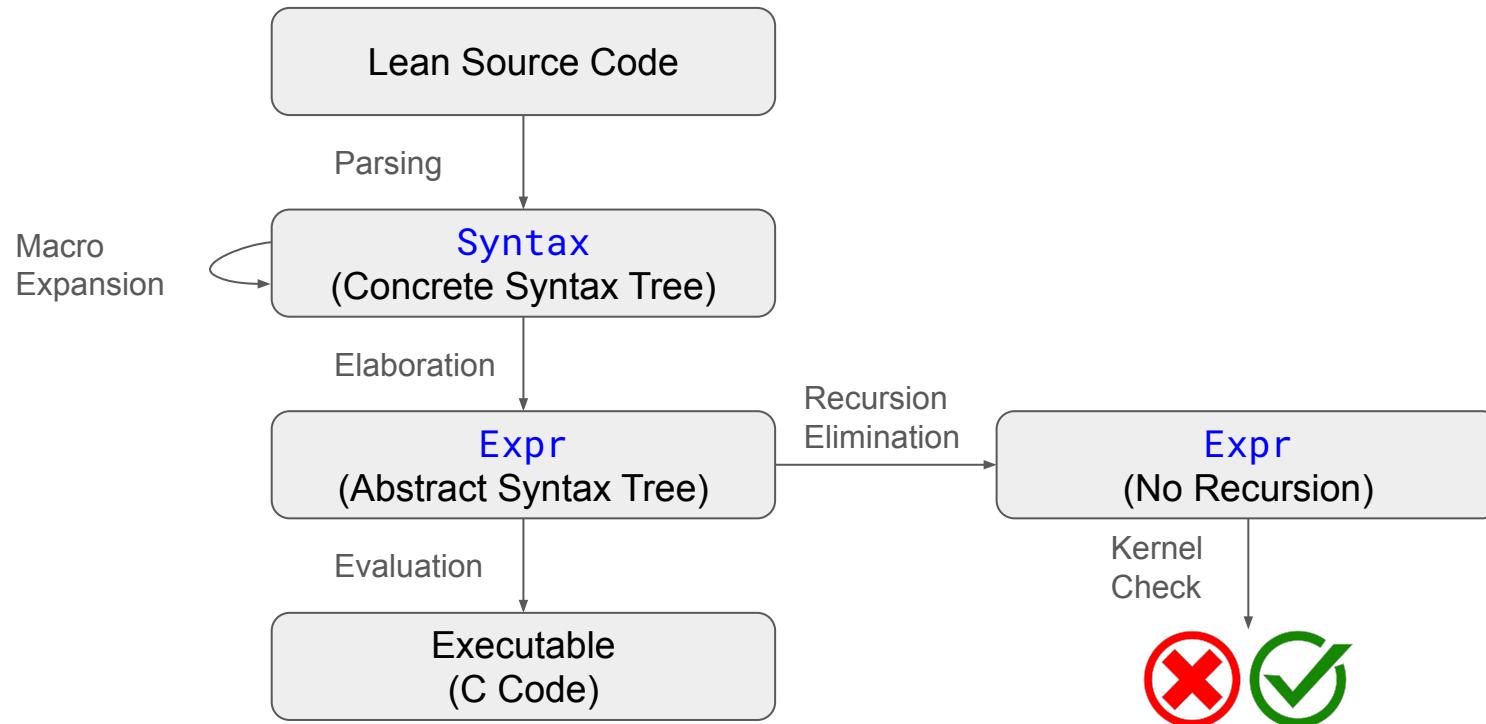
What's Unique about Lean?

Language	Meta-Level Language
C	C Macros (e.g. #define)
Rocq	Ltac, OCaml
Isabelle	SML, Scala
Agda	Haskell
Lean4	?

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Lean4	Lean4!

Lean Compiler Overview



Compiler Intermediates

```
/--  
Lean syntax trees.  
-/  
inductive Syntax where  
...  
...
```

```
/--  
Lean expressions.  
-/  
inductive Expr where  
...  
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De Bruijn Index

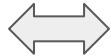
- Nameless variable representation: Replaces variable names with natural numbers indicating the number of binders in scope between def and use
- Easier to do capture-avoiding substitution
- Easier to check for alpha-equivalence
- Need additional context for free variables

$$\lambda z. (\lambda y. y (\lambda x. x)) (\lambda x. z x) \quad \leftrightarrow \quad \lambda (\lambda 0 (\lambda 0)) (\lambda 1 0)$$

$$\lambda x. (\lambda y. x z) \quad \leftrightarrow \quad \lambda (\lambda 1 3) \\ [z \rightarrow 3]$$

Locally Nameless

- Bound variables: De Bruijn Index
- Free variables: User-defined names

$$\lambda x. (\lambda y. x z)$$

$$\lambda (\lambda 1 z)$$

Lean's Expression Type

```
inductive Expr where
| bvar      : Nat → Expr
| fvar      : FVarId → Expr
| mvar      : MVarId → Expr
| sort      : Level → Expr
| const     : Name → List Level → Expr
| app       : Expr → Expr → Expr
| lam       : Name → Expr → Expr → BinderInfo → Expr
| forallE   : Name → Expr → Expr → BinderInfo → Expr
| letE      : Name → Expr → Expr → Expr → Bool → Expr
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```

Bound variables: occurrences of variables where there's a binder above it.

The second `x` in `fun x => x + y.`

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```

Free variables: occurrences of variables without an explicit binder. Can loop up it's type in [LocalContext](#).

The `y` in `fun x => x + y.`

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```

Meta variables: existential variables in Rocq. Placeholder values to be filled in during elaboration.

The _ in `fun (x : _) => x + 1.`

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```

Polymorphic type universe levels

Lean's Universe Structure

sort	Prop (Sort 0)	Type (Sort 1)	Type 1 (Sort 2)	Type 2 (Sort 3)
type	True	Bool	Nat -> Type	Type -> Type 1
term	True.intro	true	fun n => Fin n	fun (_ : Type) => Type

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```

Constants: things defined earlier
in some Lean document
(functions, theorems, etc.)

They may be polymorphic over a
list of universe levels

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Function applications

`f x y` becomes `app (app f x) y`

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Function abstraction

`fun (x : t) => e` becomes
`lam x t e`

`BinderInfo` specifies if the binder is implicit or a typeclass argument

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Dependent arrow expression

$(x : t) \rightarrow e$ becomes
`forallE x t e`

Non-dependent arrows like $x \rightarrow e$ is
also encoded this way

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Let expressions

`let (x : t) := e; body`
becomes
`letE x t e body`

Last boolean argument tells the
elaborator whether the let is
dependent or not

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Numeric and string literals

Exists mostly for performance reasons so we don't represent `(1000 : Nat)` as `Nat.succ (Nat.succ (Nat.succ ...))`

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Stores meta-data along with an expression

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Projection: used to access fields in a structure

Redundant constructor, purely for performance

Examples in Lean Code