Exercises for OpenMP

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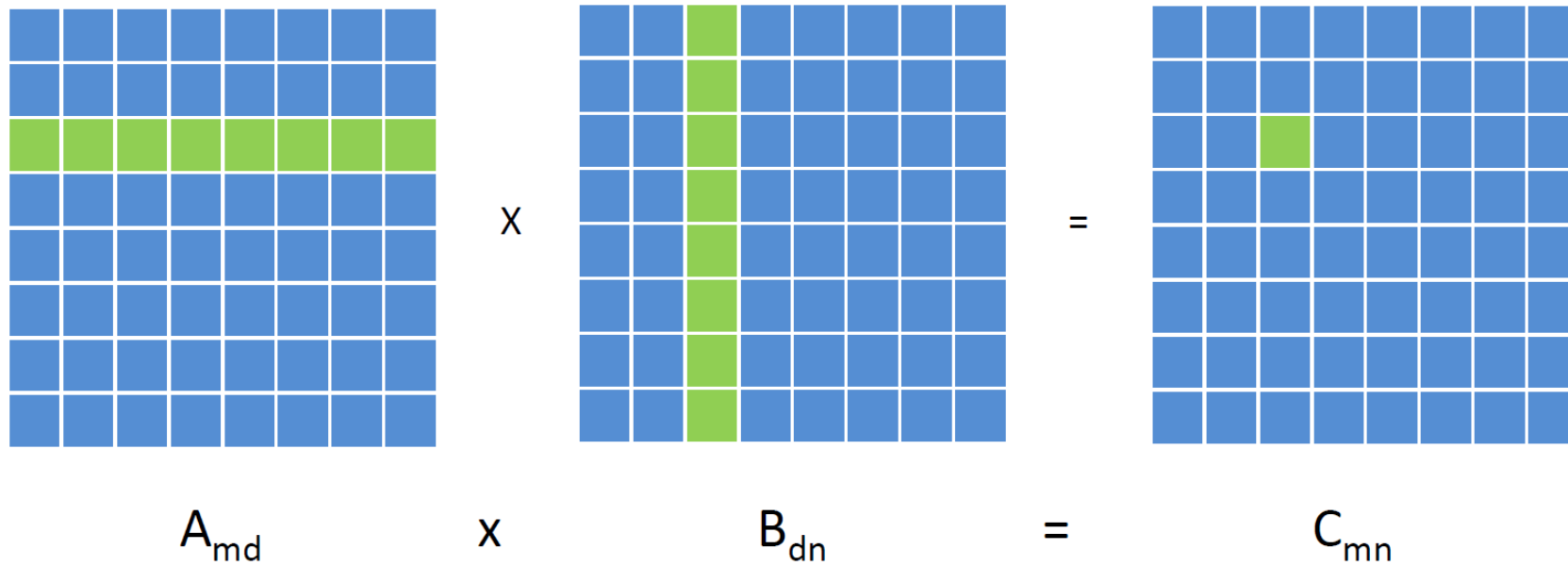
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Exercise 1: SAXPY

- SAXPY: s = a*x + y

  adds a scalar multiple of a real vector to another real vector.

- Use OpenMP to parallelize the SAXPY codes.

```
int i;
#pragma omp parallel for private(i)
  for (i = 0; i < n; i++){
    y[i] = a*x[i] + y[i];
  }
```

Exercise 2: Matrix Multiplicatoin

$$A_{md} \quad \times \quad B_{dn} \quad = \quad C_{mn}$$

- Matrix element

$$c_{i,j} = \sum_{k=1}^{d} a_{i,k} \cdot b_{k,j}$$

- Use OpenMP to parallelize the matrix-multiplication codes.

Notes: 1. The three matrices are shared data, meaning that all threads can read and write them.

2. Distribute the works of the most outer loop to minimize overheads.
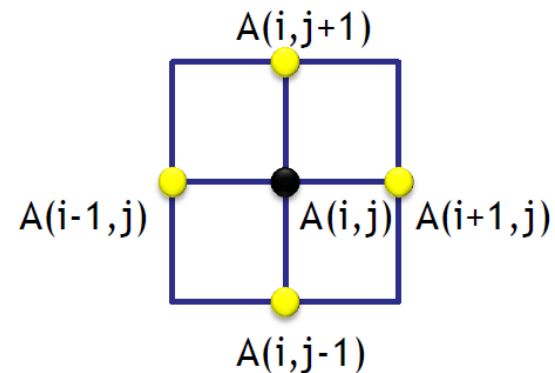
```
#pragma omp parallel for shared(nra,ncb,nca) private(sum,i,j,k)
for (i = 0; i < nra; i++){
    for (j = 0; j < ncb; j++){
        sum = 0.0;
        for (k = 0; k < nca; k++){
            sum = sum + a[i][k] * b[k][j];
        }
        c[i][j] = sum;
    }
}
```

Exercise 3: Laplacian solver

- Two-dimensional Laplace equation: $\nabla^2 f(x, y) = 0$

- Discretize the laplacian with first-order differential method and express the solution as

$$A_{k+1}(i, j) = \frac{A_k(i - 1, j) + A_k(i + 1, j) + A_k(i, j - 1) + A_k(i, j + 1)}{4}$$

- The solution on one point only depends on the four neighbor points:

- Jacobi iterative algorithm:

1. Give a trial solution A depending on a provided initial condition.

2. Calculate the new value for every element of the solution, that is A_new(i,j), based on the old values of the four neighbor points.

3. Update the solution, i.e. A=A_new,

4. Iterate steps 2 and 3 until converged, i.e. max(| A_new(i,j)-A(i,j)|)<tolerance.

5. Finally the converged solution is stored at A.

- Use Jacobi iterative algorithm to solve Laplace equation.

- Use OpenMP to parallelize the program for Laplacian solver.

```
int iter = 0;
while ( error > tol && iter < iter_max ) {        // iterate until converged

    error = 0.0;

    #pragma omp parallel for shared(m, n, Anew, A) private(i,j)

    for( j = 1; j < n-1; j++)    for( i = 1; i < m-1; i++ ) {

            Anew[j][i] = 0.25 * ( A[j][i+1] + A[j][i-1] + A[j-1][i] + A[j+1][i]);   //calculate new value from neighbors

            error = fmax( error, fabs(Anew[j][i] - A[j][i]));   // calculate the maximum error

    }

    #pragma omp parallel for shared(m, n, Anew, A) private(i,j)

    for( j = 1; j < n-1; j++)   for( i = 1; i < m-1; i++ ) {

            A[j][i] = Anew[j][i];     // Update the solution

    }

    iter++;

}
```
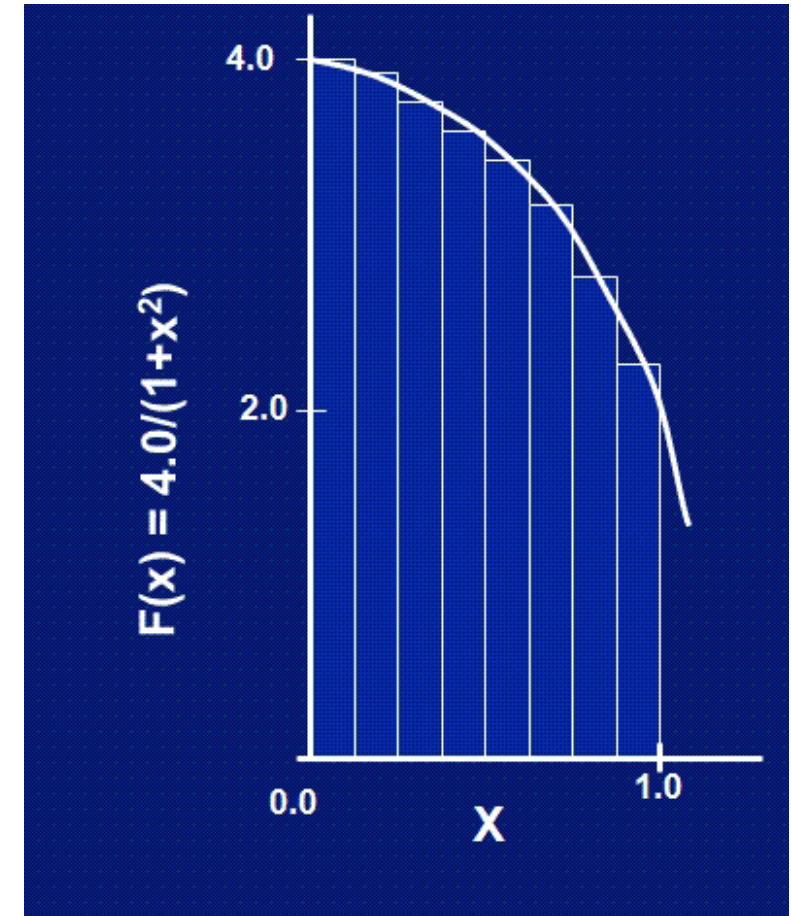
Exercise 4: Compute the value of Pi

- The value of pi can be computed by the following integral formula

$$\int_0^1 \frac{4.0}{(1+x^2)} \, dx = \pi$$

  Numerically, we can approximate the value of pi as the sum of a number of rectangles.

$$\sum_{i=0}^{N} F(x_i)\Delta x \approx \pi$$

1. Provided the serial code for computing the value of pi, parallelize it using OpenMP directives.

2. Compare the performance between seral and OpenMP codes.

- Use OpenMP to parallelize the program for calculating pi.

Notes: Use reduction clause to avoid a data racing condition.

```
#pragma omp parallel for default(shared) private(i,x) reduction(+:sum)
{
    for (i = 0; i < n; i++) {
        x = (i+0.5)*step;
        sum +=  4.0/(1.0+x*x);
    }
}
```

Exercise 5: Matrix-vector multiplication (in Fortran)

- Calculate a=B*c, where a is an m dimension vector, c is an n dimension vector and B is an m*n dimension matrix.

- Serial Fortran code

```fortran
do i = 1, m
     a(i) = b(i,1)*c(1)
     do j = 2, n
      a(i) = a(i) + b(i,j)*c(j)
     end do
end do
```

- Use OpenMP to parallelize the code for matrix-vector multiplication (version 1)

```fortran
!$OMP PARALLEL DO DEFAULT(shared) PRIVATE(i,j)
   do i = 1, m
      a(i) = b(i,1)*c(1)
      do j = 2, n
         a(i) = a(i) + b(i,j)*c(j)
      end do
   end do
!$OMP END PARALLEL DO
```

- Use OpenMP to parallelize the code for matrix-vector multiplication (version 2, only for Fortran)

```fortran
!$OMP PARALLEL DEFAULT(shared) PRIVATE(i,j)
    !$OMP WORKSHARE
    a(1:m) = b(1:m,1)*c(1)
    !$OMP END WORKSHARE
    !$OMP DO REDUCTION(+:a)
    do j = 2, n
      do i = 1, m
        a(i) = a(i) + b(i,j)*c(j)
      end do
    end do
    !$OMP END DO
!$OMP END PARALLEL
```