Parallel Laplace Solver with MPI

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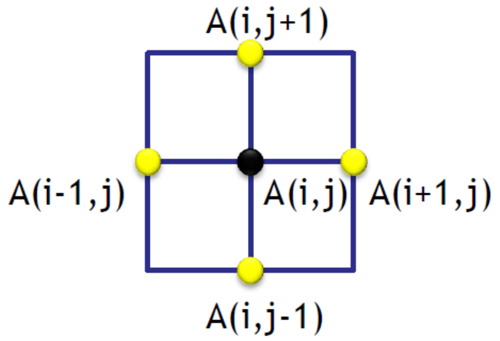
Laplace solver (1)

- Two-dimensional Laplace equation: $\nabla^2 f(x, y) = 0$

- Discretize the Laplacian using first-order differential method and obtain the solution as following,

$$A_{k+1}(i,j) = \frac{A_k(i-1,j) + A_k(i+1,j) + A_k(i,j-1) + A_k(i,j+1)}{4}$$

- The solution on one grid only depends on the four neighbor grids:



Laplace solver (2)

- Use iterative algorithm to obtain a converged solution.

- Jacobi iterative algorithm:

1. Give a trial solution A based on a provided initial condition.
2. Calculate a new solution, that is A_new(i,j), using the old values of the four neighbor points that are stored in A.
3. Update the solution: A=A_new.
4. Iterate steps 2 and 3 until converged, i.e. max(|A_new(i,j)-A(i,j)|)<tolerance.
5. Finally the converged solution is stored in A.

Serial code in C (kernel)

```
while ( dA > tolerance && iteration <= max_iterations ) {      // do until error is minimal or until max steps
  for(i = 1; i <= ROWS; i++)          // main calculation: average my four neighbors
     for(j = 1; j <= COLUMNS; j++) {
        A_new[i][j] = 0.25 * (A[i+1][j] + A[i-1][j] +A[i][j+1] + A[i][j-1]);
  }
  dA = 0.0;       // reset largest change
  for(i = 1; i <= ROWS; i++)
      for(j = 1; j <= COLUMNS; j++){
        dA = fmax( fabs(A_new[i][j]-A[i][j]), dA);      // find the latest change
        A[i][j] = A_new[i][j];          // copy grid to old grid for next iteration
   }
   iteration++;
}
```

Serial code in Fortran (kernel)

```
do while ( dA > tolerance .and. iteration <= max_iterations)      ! do until error is minimal or until max steps
     do j=1,columns            ! main calculation: average my four neighbors
         do i=1,rows
             A_new(i,j)=0.25*(A(i+1,j)+A(i-1,j)+A(i,j+1)+A(i,j-1) )
         enddo
     enddo
     dA=0.0                    ! reset largest change
     do j=1,columns
         do i=1,rows
             dA = max( abs(A_new(i,j) - A(i,j)), dA )      ! find the latest change
             A(i,j) = A_new(i,j)          ! copy grid to old grid for next iteration
         enddo
    enddo
    iteration = iteration+1
enddo
```

Parallelize with MPI

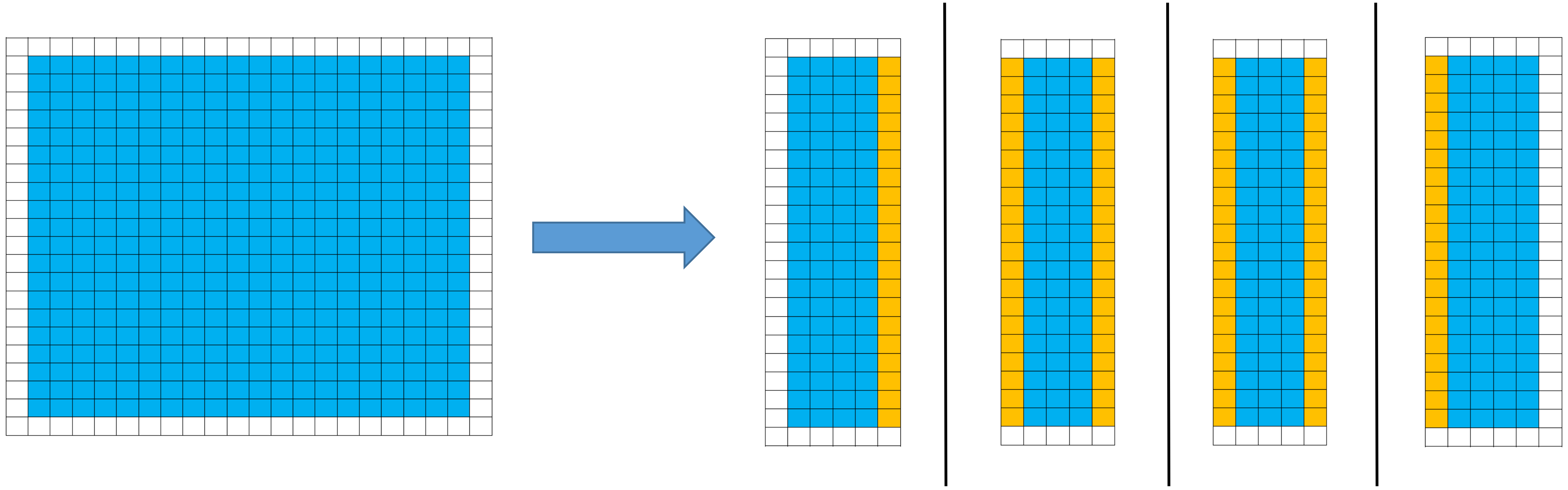
Analysis for parallelism:

1. Find the “hot spots”, the most time-consuming parts of the code.
2. Decompose the grids into sub-grids. Each process owns one sub-grid.
3. Pass necessary data between processes. (e.g. use MPI_Send and MPI_Recv). Be careful to avoid dead locks.
4. Pass “shared” data between the root process and all other process (e.g. use MPI_Bcast and MPI_Reduce).

Domain Decomposition (1)

- 1D Decomposition:

  Divide columns in Fortran or divide rows in C.



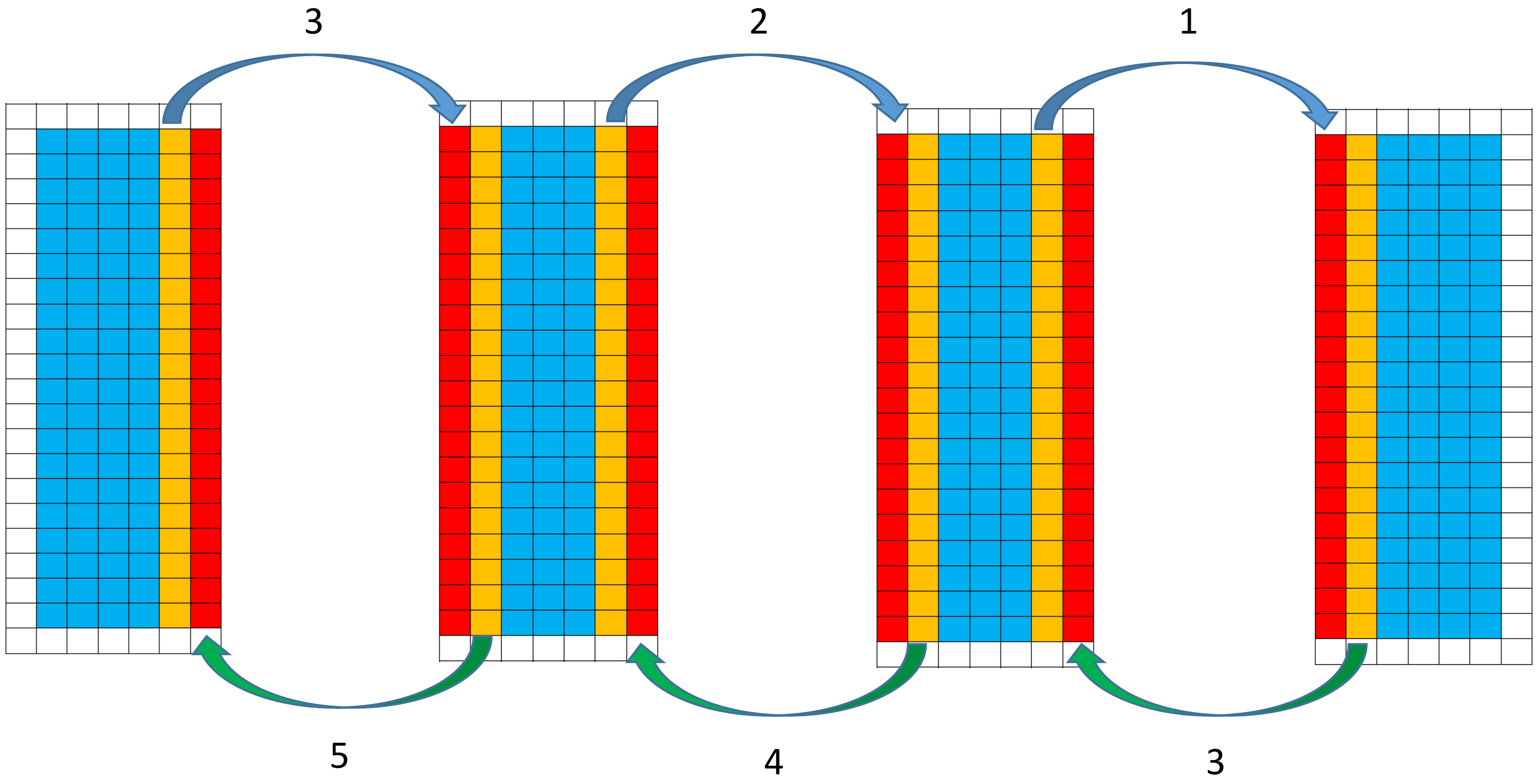
✓ Blue zone: The real grids.

✓ White zone: An additional layer for boundary condition.

✓ Yellow zone: The data on these grids need to be sent to neighbor process(es).

Ghost Zone (1)

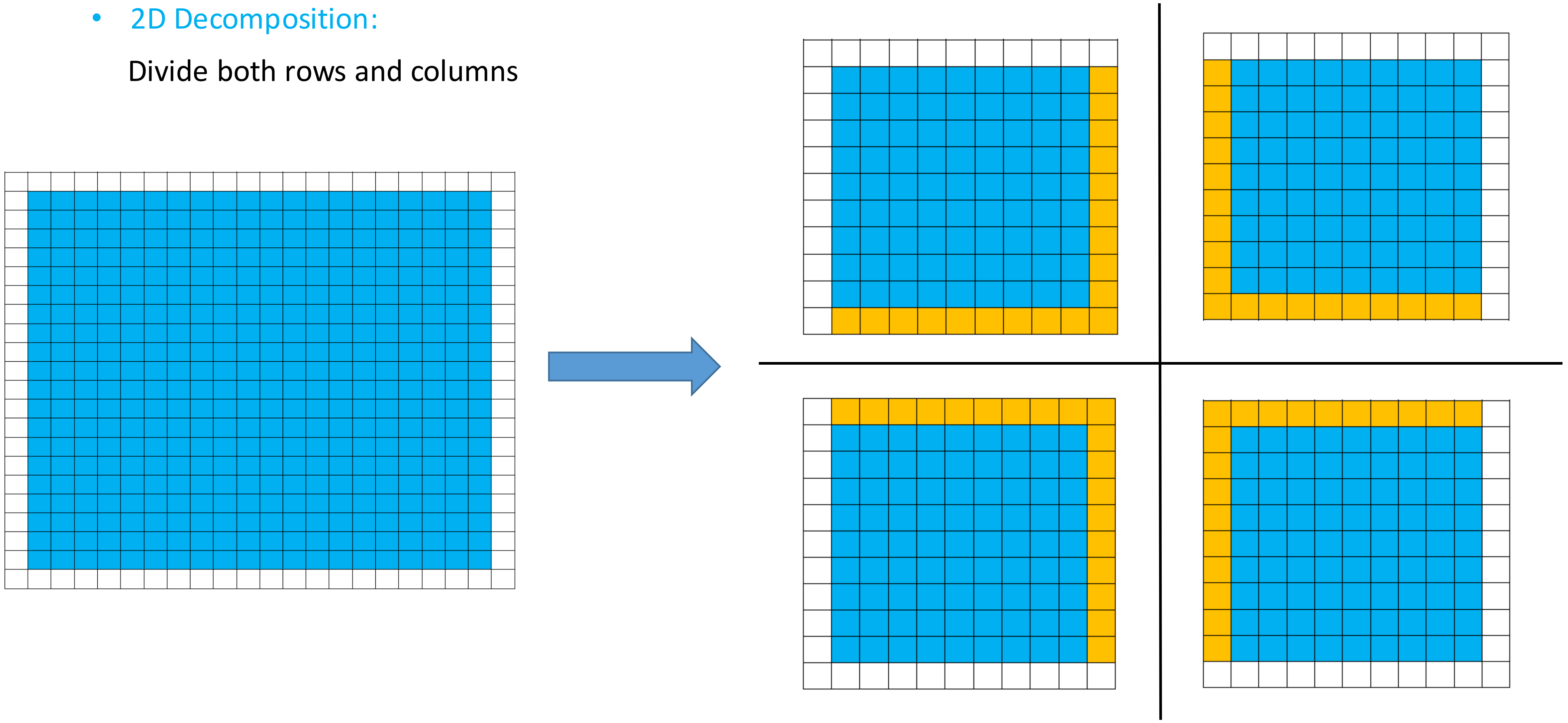
- ✓ Red zone: An additional layer to receive the data from neighbor process(es). Also called “ghost zone”.



Domain Decomposition (2)

- 2D Decomposition:

  Divide both rows and columns



Ghost Zone (2)

- ✓ Send and receive row-type and column-type data.