Lab 5 - Localization

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Part A

Question 1

i.

$$T\left(\boldsymbol{x}_i = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}\right) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & x \\ \sin(\theta) & \cos(\theta) & y \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_\Delta = T(\boldsymbol{x}_{k-1})^{-1} T(\boldsymbol{x}_k) = \begin{pmatrix} 0.999 & -0.048 & 0.223 \\ 0.048 & 0.999 & -0.013 \\ 0 & 0 & 1 \end{pmatrix}$$

We can retrieve $\Delta \boldsymbol{x}$ by doing the opposite operation of T (i.e. x, y are right-most column and $\theta = \cos^{-1}(0.997)$)...

$$\Delta \boldsymbol{x} = (0.223 \;\; -0.013 \;\; 0.0480)^T$$

ii.

$$\boldsymbol{x}_k = T(\boldsymbol{x}_{k-1}) T(\Delta \boldsymbol{x}) = T(\boldsymbol{x}_{k-1}) T_\Delta = \begin{pmatrix} 0.457 & -0.887 & 3.123 \\ 0.887 & 0.457 & 4.186 \\ 0 & 0 & 1 \end{pmatrix}$$

Again, retrieve $\boldsymbol{x}_k$ from the transformation matrix $T(\boldsymbol{x}_k)$...

$$\boldsymbol{x}_k = (3.123 \;\; 4.186 \;\; 1.096)^T$$

Question 2

For $\alpha_{\text{hit}} = 0.74$, $\alpha_{\text{short}} = 0.07$, $\alpha_{\text{max}} = 0.07$, $\alpha_{\text{rand}} = 0.12$, $\sigma = 0.5m$, $z_{\text{max}} = 10m$ and $d = 7m$, we can model the probability $p\left(z_k^{(i)} | x_k, m\right)$ with the following python script:

```python
from math import sqrt, exp, pi

a_hit = 0.74
a_short = 0.07
a_max = 0.07
a_rand = 0.12
sigma = 0.5
z_max = 10
d = 7

def p_hit(z):
    nu = 1
    if 0 <= z <= z_max:
        return nu * (1/sqrt(2*pi*sigma**2)) * exp(-((z-d)**2)/(2*sigma**2))
    return 0

def p_short(z):
    return (2/d) * ((1 - z/d) if 0 <= z < d and d != 0 else 0)

def p_max(z):
    epsilon = 0.1
    return 1/epsilon if z_max - epsilon <= z <= z_max else 0
```

```python
def p_rand(z):
    return 1/z_max if 0 <= z <= z_max else 0

def p(z):
    return a_hit*p_hit(z) + a_short*p_short(z) + a_max*p_max(z) + a_rand*p_rand(z)

for i, z in enumerate([0, 3, 5, 8, 10]):
    print(f"[{i}] z = {z} -> p(...) = {p(z)}")
```

This gives us the following results:

i. $z_k^{(i)} = 0m \implies p\left(z_k^{(i)} | x_k, m\right) = 0.032$

ii. $z_k^{(i)} = 3m \implies p\left(z_k^{(i)} | x_k, m\right) = 0.023428571428578904$

iii. $z_k^{(i)} = 5m \implies p\left(z_k^{(i)} | x_k, m\right) = 0.017912354448417746$

iv. $z_k^{(i)} = 8m \implies p\left(z_k^{(i)} | x_k, m\right) = 0.09190663043951833$

v. $z_k^{(i)} = 10m \implies p\left(z_k^{(i)} | x_k, m\right) = 0.7120000089923066$