Chapter 5: Two Dimensional Kinematics

Peter Dourmashkin

September 6, 2023

Contents

5 Two Dimensional Kinematics **1**

5.1 Introduction to the Vector Description of Motion in Two Dimensions . . . 1

5.2 Projectile Motion 2

5.2.1 Example: Time of flight and maximum height of a projectile . . . 7

5.2.2 Orbit equation 9

5.2.3 Example: Apple target practice 11

Chapter 5

Two Dimensional Kinematics

Where was the chap I saw in the picture somewhere? Ah yes, in the dead sea floating on his back, reading a book with a parasol open. Couldn't sink if you tried: so thick with salt. Because the weight of the water, no, the weight of the body in the water is equal to the weight of the what? Or is it the volume equal to the weight? It's a law something like that. Vance in High school cracking his fingerjoints, teaching. The college curriculum. Cracking curriculum. What is weight really when you say weight? Thirtytwo feet per second per second. Law of falling bodies: per second per second. They all fall to the ground. The earth. It's the force of gravity of the earth is the weight. [1]

James Joyce

5.1 Introduction to the Vector Description of Motion in Two Dimensions

We have introduced the concepts of position, velocity and acceleration to describe motion in one dimension; however we live in a multidimensional universe. In order to explore and describe motion in more than one dimension, we shall study the motion of a projectile in two-dimension moving under the action of uniform gravitation.

We extend our definitions of position, velocity, and acceleration for an object that moves in two dimensions (in a plane) by treating each direction independently, which we can do with vector quantities by resolving each of these quantities into components. For example, our definition of velocity as the derivative of position holds for each component separately. In Cartesian coordinates, the position vector $\vec{\mathbf{r}}(t)$ with respect to a choice of origin for the object at time t is given by

$$\vec{\mathbf{r}}(t) = x(t)\,\hat{\mathbf{i}} + y(t)\,\hat{\mathbf{j}} \tag{5.1}$$

[1] James Joyce, *Ulysses*, The Corrected Text edited by Hans Walter Gabler with Wolfhard Steppe and Claus Melchior, Random House, New York.

Choose a coordinate system with the positive y-axis in the upward vertical direction and the positive x-axis in the horizontal direction in the direction that the object is moving horizontally. Choose the origin at the ground immediately below the point the object is released. Figure 5.1 shows the coordinate system with the position of the object $\vec{\mathbf{r}}(t)$ at time t, and the coordinate functions $x(t)$ and $y(t)$. The velocity vector

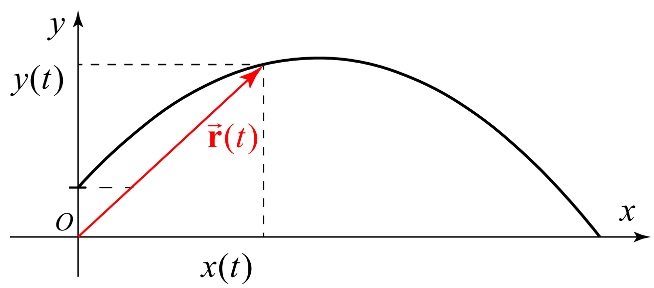

Figure 5.1: Coordinate system and position function for object undergoing twe-dimensional motion.

$\vec{\mathbf{v}}(t)$ at time t is the derivative of the position vector,

$$\vec{\mathbf{v}}(t) = \frac{dx(t)}{dt}\,\hat{\mathbf{i}} + \frac{dy(t)}{dt}\,\hat{\mathbf{j}} \equiv v_x(t)\,\hat{\mathbf{i}} + v_y(t)\,\hat{\mathbf{j}}. \tag{5.2}$$

where $v_x(t) = dx/dt$ and $v_y(t) = dy/dt$ denote the x- and y-components of the velocity respectively.

The acceleration vector $\vec{\mathbf{a}}(t)$ is defined in a similar fashion as the derivative of the velocity vector,

$$\vec{\mathbf{a}}(t) = \frac{dv_x(t)}{dt}\,\hat{\mathbf{i}} + \frac{dv_y(t)}{dt}\,\hat{\mathbf{j}} \equiv a_x(t)\,\hat{\mathbf{i}} + a_y(t)\,\hat{\mathbf{j}}. \tag{5.3}$$

where $a_x(t) = dv_x/dt$ and $a_y(t) = dv_y/dt$ denote the x- and y-components of the acceleration respectively.

5.2 Projectile Motion

Consider the motion of a body that is released at time $t = 0$ with an initial velocity $\vec{\mathbf{v}}_0$. Two paths are shown in Figure 5.2. The dotted path represents a parabolic trajectory and the solid path represents the actual trajectory. The difference between the two paths is due to air resistance acting on the object, $\vec{\mathbf{F}}^{air}$. For a rapidly moving object, $\vec{\mathbf{F}}^{air} = -bv^2\hat{\mathbf{v}}$ where $\hat{\mathbf{v}}$ is a unit vector in the direction of the velocity $\vec{\mathbf{v}}$. (For the orbits shown in Figure 5.2. $b = 0.01\ \mathrm{N}\cdot\mathrm{s}^2\cdot\mathrm{m}^{-2}$, $\left|\vec{\mathbf{v}}_0\right| = 30.0\ \mathrm{m}\cdot\mathrm{s}$, the initial launch angle

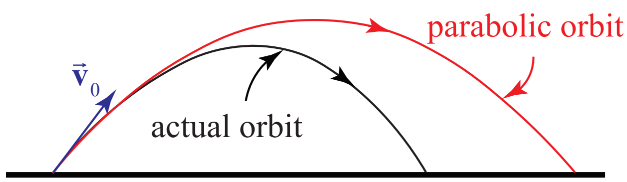

Figure 5.2: Actual orbit accounting for air resistance and parabolic orbit of a projectile.

with respect to the horizontal $\theta_0 = 21°$, and the actual horizontal distance traveled is 71.7% of the projectile orbit.).

There are other factors that can influence the path of motion; for example density of air, height above the surface of the earth, or wind. Many other factors can also come in play: the body may be rotating, a special shape can alter the flow of air around the body, which may induce a curved motion or lift like the flight of a baseball or golf ball, to cite a few. We shall begin our analysis by neglecting all interactions except the gravitational interaction.

Initial conditions Decompose the initial velocity vector into its components:

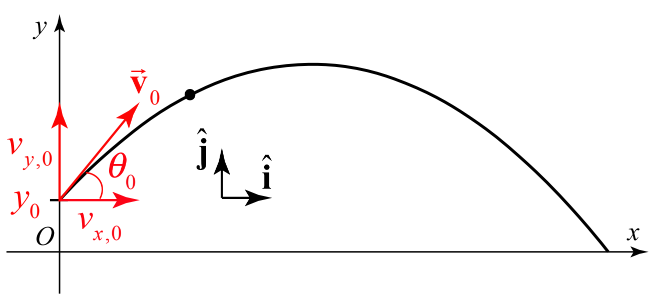

Figure 5.3: Initial conditions for projectile motion

$$\vec{\mathbf{v}}_0 = v_{x,0}\,\hat{\mathbf{i}} + v_{y,0}\,\hat{\mathbf{j}}. \tag{5.4}$$

The vector decomposition for the initial velocity is shown in Figure 5.3. Often the description of the flight of a projectile includes the statement, "a body is projected with an initial speed v_0 at an angle θ_0 with respect to the horizontal." The components of the initial velocity can be expressed in terms of the initial speed and angle according to

$$\begin{aligned} v_{x,0} &= v_0 \cos\theta_0, \\ v_{y,0} &= v_0 \sin\theta_0. \end{aligned} \tag{5.5}$$

Because the initial speed is the magnitude of the initial velocity, therefore

$$v_0 = (v_{x,0}^2 + v_{y,0}^2)^{1/2}. \tag{5.6}$$

The angle θ_0 is related to the components of the initial velocity by

$$\theta_0 = \tan^{-1}(v_{y,0}/v_{x,0}). \tag{5.7}$$

Equation 5.7 will give two values for the angle θ_0, so care must be taken to choose the correct physical value.

The initial position vector generally is given by

$$\vec{\mathbf{r}}_0 = x_0\,\hat{\mathbf{i}} + y_0\,\hat{\mathbf{j}}. \tag{5.8}$$

Note that the trajectory in Figure 5.3 starts at the point $(x_0 = 0, y_0)$, but this will not always be the case.

Experimental measurement establish that the vertical component of the acceleration for motion near the surface of Earth neglected all resistive force is

$$a_y = -g, \tag{5.9}$$

The acceleration is a constant and is independent of the mass of the object. Notice that for our choice of coordinate system, $a_y < 0$. This is because we chose our positive $\hat{\mathbf{j}}$-direction to point upwards. The sign of the y-component of acceleration is determined by how we choose our coordinate system. This result known as *Galileo's*

Figure 5.4: Model of device to measure gravitational acceleration.

Law of Free Falling Bodies was initially measured by Galilei Galileo with an ingenious device. He rolled a ball down a wooden ramp with negligible friction (possibly made form lignum wood)). A mounted bell on a small arch would ring when the ball passed through. Galileo would adjust the spacing between the arches so that the bells would ring in even intervals. (His father was a musician and it is most likely that Galileo could accurately hear even intervals of ringing more accurately than any clock could measure.)

When care is taken to minimize all other interactions, the horizontal acceleration vector is essentially zero:

$$\vec{\mathbf{a}}_x = \vec{\mathbf{0}} \tag{5.10}$$

Application of Newton's Second Law

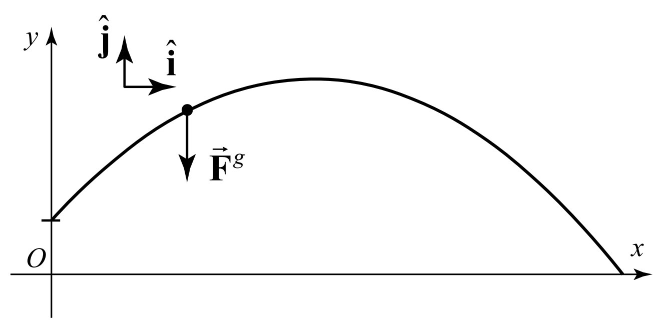

Figure 5.5: Free-body force diagram on the object in projectile motion.

As a prelude to Chapter 8, we will briefly discuss how to apply Newton's Second Law to justify the above experimental results for the components of acceleration for two dimensional motion near the surface of Earth. We begin by neglecting all forces other than the gravitational interaction between the object and Earth. This force acts downward with magnitude mg, where m is the mass of the object and $g = 9.8\ \mathrm{m} \cdot \mathrm{s}^{-2}$. Figure 5.5 shows a free-body force diagram on the object. The vector decomposition of the force is

$$\vec{\mathbf{F}}^g = -mg\hat{\mathbf{j}}. \tag{5.11}$$

The gravitational force is acting in the negative $\hat{\mathbf{j}}$-direction. Newton's Second Law (see Chapter 8 for a detailed discussion of Newton's Laws of Motion) states that the vector sum of the forces acting on the object is equal to the product of the mass m and the acceleration vector $\vec{\mathbf{a}}$.

$$\sum_i \mathbf{F}_i = m\,\mathbf{a} \tag{5.12}$$

For projectile motion, neglecting air resistance, the only force acting on the object is the gravitational force, $\vec{\mathbf{F}}^g$. Newton's Second Law is a vector equation, therefore the components are equated separately. For the vertical direction $-mg = m\,a_y$ and for the horizontal direction $0 = m\,a_x$. Therefore $a_y = -g$ and $a_x = 0$ verifying Galileo's experimental observations.

The acceleration in the vertical direction is constant for all bodies near the surface of the Earth, independent of the mass of the object, thus confirming Galileo's Law of Free Falling Bodies. The application of Newton's Second Law generalizes the experimental observation that objects fall with constant acceleration. Our statement about the acceleration of objects near the surface of Earth depends on our model force law, Equation 5.11. If subsequent observations show that the acceleration is not constant then we either must include additional forces (for example, air resistance), or modify the force law (for objects that are no longer near the surface of Earth, or consider that Earth is a non-symmetric non-uniform body), or take into account the rotational motion of the Earth.
In order to determine the velocity components as functions of time, first integrate Equations 5.9 and 5.10. The change in the y-component of velocity is

$$v_y(t) - v_{y,0} = \int_{t'=0}^{t'=t} a_y(t')\,dt' = -\int_{t'=0}^{t'=t} g\,dt' = -gt. \tag{5.13}$$

Therefore

$$v_y(t) = v_{y,0} - gt. \tag{5.14}$$

The change in the x-component of velocity is zero,

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t')\,dt' = 0. \tag{5.15}$$

Therefore $v_x(t)$ remains constant throughout the flight of the object.

$$v_x(t) = v_{x,0} \tag{5.16}$$

The change in the y-component of the position function can now be determined by integrating Equation 5.14

$$y(t) - y_0 = \int_{t'=0}^{t'=t} v_y(t')\,dt' = \int_{t'=0}^{t'=t} (v_{y,0} - gt)dt' = v_{y,0}t - (1/2)gt^2. \tag{5.17}$$

Therefore the coordinate function $y(t)$ is

$$y(t) = y_0 + v_{y,0}t - (1/2)gt^2. \tag{5.18}$$

The change in the x-component of the position function can now be determined by integrating Equation 5.16

$$x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t')\,dt' = \int_{t'=0}^{t'=t} v_{x,0}\,dt' = v_{x,0}t \tag{5.19}$$

Therefore the coordinate function $x(t)$ is

$$x(t) = x_0 + v_{x,0}t. \tag{5.20}$$

The complete set of vector equations for position, velocity and acceleration for *free fall* are given by

$$\vec{\mathbf{r}}(t) = x(t)\,\hat{\mathbf{i}} + y(t)\,\hat{\mathbf{j}} = (x_0 + v_{x,0}t)\,\hat{\mathbf{i}} + (y_0 + v_{y,0}\,t - (1/2)g\,t^2)\,\hat{\mathbf{j}}, \tag{5.21}$$

$$\vec{\mathbf{v}}(t) = v_x(t)\,\hat{\mathbf{i}} + v_y(t)\,\hat{\mathbf{j}} = v_{x,0}\,\hat{\mathbf{i}} + (v_{y,0} - g\,t)\,\hat{\mathbf{j}}, \tag{5.22}$$

$$\vec{\mathbf{a}}(t) = a_x(t)\,\hat{\mathbf{i}} + a_y(t)\,\hat{\mathbf{j}} = -g\,\hat{\mathbf{j}}. \tag{5.23}$$

5.2.1 Example: Time of flight and maximum height of a projectile

A person throws a stone at an initial angle $\theta_0 = 45^\circ$ from the horizontal with an initial speed of $v_0 = 20.0\ \mathrm{m \cdot s}$. The point of release of the stone is at a height $h = 2\mathrm{m}$ above the ground. Neglect air resistance.

a) How long does it take the stone to reach the highest point of its trajectory?

b) What was the maximum vertical displacement of the stone?

Answer

Choose the origin on the ground directly underneath the point where the stone is released. We choose the positive y-axis in the upward vertical direction and the positive x-axis in the direction that the object is moving horizontally. Set $t = 0$ the instant the stone is released. At $t = 0$ the initial conditions for the position are $x_0 = 0$ and $y_0 = h$ The initial x- and y-components of the velocity are given by Equations 5.5. At time t the stone has coordinates $(x(t), y(t))$. These coordinate functions are shown in Figure 5.6.

The slope of this graph at any time$t$ yields the instantaneous y-component of the velocity $v_y(t)$. Figure 5.6 is a plot of $y(t)$ vs.$x(t)$ and Figure 5.7 is a plot of $y(t)$ vs. t. There are several important things to notice about Figures 5.6 and 5.7. The first point is that the abscissa axes are different in both figures. The second thing to notice is that at $t = 0$, the slope of the graph in Figure 5.6 is equal to $\tan(\theta_0)$:

$$\left.\frac{dy}{dx}\right|_{t=0} = \left.\left(\frac{dy/dt}{dx/dt}\right)\right|_{t=0} = \frac{v_{y,0}}{v_{x,0}} = \tan\theta_0. \tag{5.24}$$

In Figure 5.7, the slope fo the graph at $t = 0$ is $v_{y,0}$:

$$\left.\frac{dy}{dt}\right|_{t=0} = v_{y,0}. \tag{5.25}$$

The slope of this graph in Figure 5.7 at time t yields the instantaneous y-component of the velocity $v_y(t)$. Let $t = t_1$ correspond to the instant the stone is at its maximal

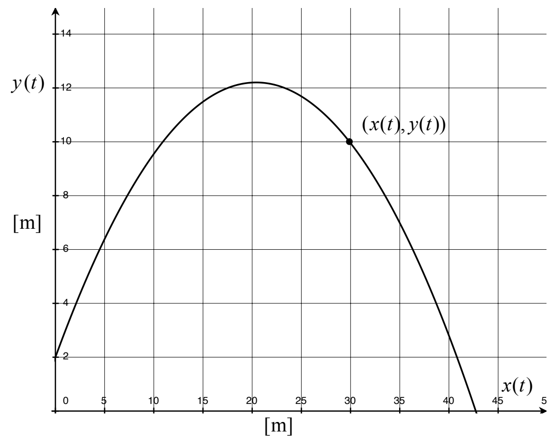

Figure 5.6: Coordinate functions for stone shown as a curve in plot of $y(t)$ vs. $x(t)$.

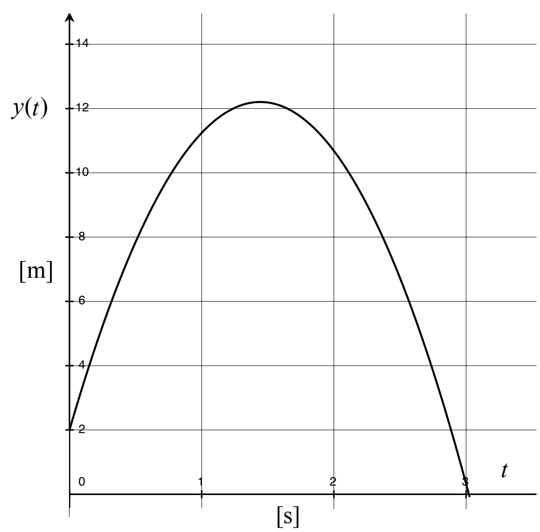

Figure 5.7: Plot of $y(t)$ vs. t.

vertical position, the highest point in the flight. The final thing to notice about Figure 5.7 is that at $t = t_1$ the slope is zero, hence $v_y(t_1) = 0$. Therefore

$$v_y(t_1) = v_0 \sin\theta_0 - g t_1 = 0, \tag{5.26}$$

which we can solve for the time t_1:

$$t_1 = \frac{v_0 \sin\theta_0}{g} = \frac{(20\ \mathrm{m \cdot s^{-1}})\sin(45°)}{9.8\ \ \mathrm{m \cdot s^{-2}}} = 1.44\ \mathrm{s}. \tag{5.27}$$

The graph in Figure 5.8 shows a plot of $v_y(t)$. At $t = 0$ the intercept is positive

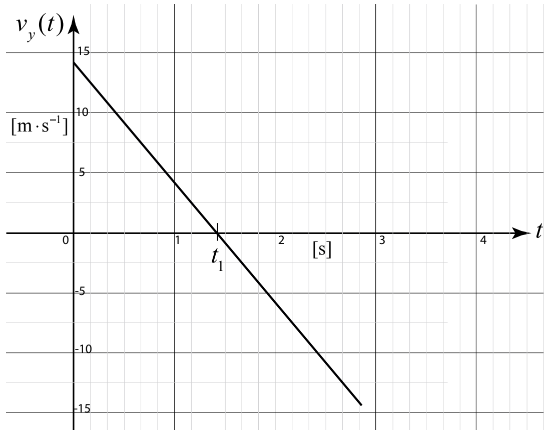

Figure 5.8: Plot of $v_y(t)$ vs. t.

indicting that $v_{y,0} > 0$is positive which means that the stone was thrown upwards. The y-component of the velocity changes sign at $t = t_1$ indicating that the stone is reversing its direction and starting to move downwards.
Substitute the expression for t_1 (Equation 5.27) into the y-component of the position function in Equation 5.18 to find the maximal height of the stone above the ground

$$\begin{aligned} y(t = t_1) &= h + (v_0 \sin\theta_0)\left(\frac{v_0 \sin\theta_0}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\theta_0}{g}\right)^2 \\ &= h + \frac{{v_0}^2 \sin^2\theta_0}{2g} = 2\ \mathrm{m} + \frac{(20\ \mathrm{m}\cdot\mathrm{s}^{-1})^2 \sin^2(45^\circ)}{2(9.8\ \ \mathrm{m}\cdot\mathrm{s}^{-2})} = 12.2\ \mathrm{m}. \end{aligned} \tag{5.28}$$

5.2.2 Orbit equation

Our description of parabolic motion has emphasized the independence of the spatial dimensions, treating all of the kinematic quantities as functions of time. We shall now eliminate time from our equation and find the orbit equation of the body undergoing projectile motion. We begin with the x-component of the position in Equation 5.20 and solve for the time variable t, where we are now writing $x \equiv x(t)$:

$$t = \frac{x - x_0}{v_{x,0}}. \tag{5.29}$$

Substitute the expression for t into the y-component of the position in Equation 5.18 yields $y(x)$:

$$y(x) = y_0 + v_{y,0}\left(\frac{x - x_0}{v_{x,0}}\right) - \frac{1}{2}g\left(\frac{x - x_0}{v_{x,0}}\right)^2. \tag{5.30}$$

A little algebraic manipulation yields the equation for a parabola:

$$y(x) = -\frac{1}{2}\frac{g}{v_{x,0}^2}x^2 + \left(\frac{g\,x_0}{v_{x,0}^2} + \frac{v_{y,0}}{v_{x,0}}\right)x - \frac{v_{y,0}}{v_{x,0}}x_0 - \frac{1}{2}\frac{g}{v_{x,0}^2}x_0^2 + y_0. \tag{5.31}$$

The graph of $y(x)$ vs. x is shown in Figure 5.9. The velocity vector is given by

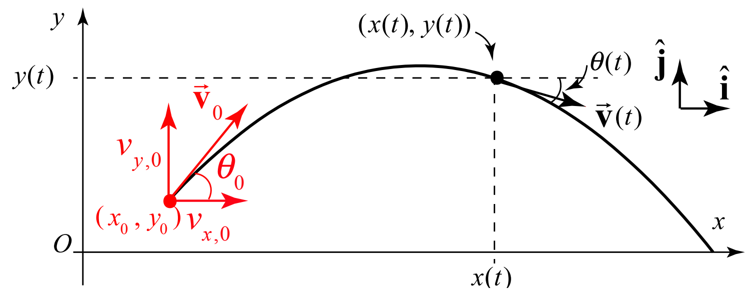

Figure 5.9: Plot of $y(x)$ vs. x illustrating a parabolic orbit.

$$\mathbf{v}(t) = \frac{dx(t)}{dt}\ \hat{\mathbf{i}} + \frac{dy(t)}{dt}\ \hat{\mathbf{j}} = v_x(t)\ \hat{\mathbf{i}} + v_y(t)\ \hat{\mathbf{j}}. \tag{5.32}$$

The direction of the velocity vector at a point $x(t), y(t))$ can be determined from the components. Let $\theta(t)$ be the angle that the velocity vector forms with respect to the positive x-axis, which is given by

$$\theta(t) = \tan^{-1}\left(\frac{v_y(t)}{v_x(t)}\right) = \tan^{-1}\left(\frac{dy/dt}{dx/dt}\right) = \tan^{-1}\left(\frac{dy}{dx}\right). \tag{5.33}$$

The function $y(x)$ is given by Equation 5.31. Differentiating $y(x)$ with respect to x yields

$$\frac{dy}{dx} = -\frac{g}{v_{x,0}^2}x + \left(\frac{g\,x_0}{v_{x,0}^2} + \frac{v_{y,0}}{v_{x,0}}\right). \tag{5.34}$$

The direction of the velocity vector $\vec{\mathbf{v}}(t)$ at a point $(x(t), y(t))$ is therefore

$$\theta(x) = \tan^{-1}\left(-\frac{g}{v_{x,0}^2}x + \left(\frac{g\,x_0}{v_{x,0}^2} + \frac{v_{y,0}}{v_{x,0}}\right)\right). \tag{5.35}$$

Although we can determine the angle of the velocity, we cannot determine how fast the body moves along the parabolic orbit from our graph of $y(x)$; the magnitude of the velocity cannot be determined from information about the tangent line.

If we choose our origin at the initial position of the body at $t = 0$, then $x_0 = 0$ and $y_0 = 0$. The orbit equation, Equation 5.31, can now be simplified to

$$y(x) = -\frac{1}{2}\frac{g}{v_{x,0}^2}x^2 + \frac{v_{y,0}}{v_{x,0}}x. \tag{5.36}$$

5.2.3 Example: Apple target practice

An apple is suspended at a height h above the ground. A physics demo instructor has set up a projectile gun a horizontal distance d away from the apple. When the the projectile is launched it is a height s above the ground. The demo instructor fires the projectile with an initial velocity of magnitude v_0 just as the apple is released. Find the angle at which the projectile gun must be aimed in order for the projectile to strike the apple. Ignore air resistance.

Answer

Understand the problem There are two objects involved in this problem. Each object is in free fall. The apple is undergoing one-dimensional motion while the projectile is undergoing two-dimensional motion. A sketch of the motions is shown in Figure 5.10(a).

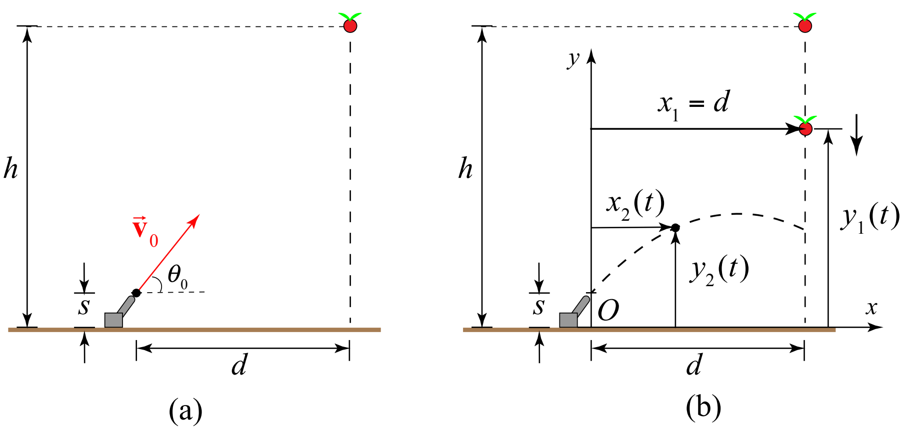

Figure 5.10: (a) Sketch of motions, (b) Coordinate system.

Set up the mathematical frmaework The quantities h, d and s are unspecified, so our answers will be functions of the symbolic expressions for those quantities. Because the acceleration is unidirectional and constant, we will choose Cartesian coordinates, with one axis along the direction of acceleration. Choose the origin on the ground directly underneath the point where the projectile is released. Choose upwards for the positive y-direction and horizontally tot eh right (towards the apple) for the positive x-direction. Choose position functions for the apple at time t as follows. The horizontal coordinate is constant and given by $x_1 = d$. The vertical coordinate represents the height above the ground and is denoted by $y_1(t)$. The projectile has coordinates $(x_2(t), y_2(t))$. We show these coordinates in Figure 5.10(b).

Devise a plan Because we are ignoring air resistance, the apple undergoes constant

acceleration downwards with the y-component $a_{y,1} = -g$,.The projectile undergoes uniform motion in the horizontal direction (the x-component of the velocity is constant) and acceleration components $a_{x,2} = 0$ and $a_{y,2} = -g$.
We can write down the equations for motion for each object. The key point is that at the instant when the collision occurs the projectile and apple are located at the same position. This extra condition will provide us with enough information to find the angle θ_0 at which the projectile gun must be aimed in order for the projectile to strike the apple.

Carry out the plan The initial conditions in the y-direction for the apple are $y_{1,0} = h$, and $v_{y,1,0} = 0$. Because the apple moves vertically, the apple always satisfies the constraint condition $x_1 = d$ and $v_{x,1} = 0$. The equations for the y-components of the position and velocity of the apple are then

$$\begin{aligned} y_1(t) &= h - \tfrac{1}{2}gt^2, \\ v_{y,1}(t) &= -gt. \end{aligned} \tag{5.37}$$

The initial position for the projectile is given by $x_{2,0} = 0$ and $y_{2,0} = s$. The components of the initial velocity are given by $v_{x,2,0} = \cos\theta_0$ and $v_{y,2,0} = \sin\theta_0$, where v_0 is the magnitude of the initial velocity and θ_0 is the initial angle with respect to the horizontal. The equations for the x-components of the position and velocity of the projectile are

$$\begin{aligned} x_2(t) &= v_0\cos(\theta_0)t, \\ v_{x,2}(t) &= v_0\cos(\theta_0). \end{aligned} \tag{5.38}$$

The equations for the y-components of the position and velocity of the projectile are

$$\begin{aligned} y_2(t) &= s + v_0\sin(\theta_0)t - \tfrac{1}{2}gt^2, \\ v_{y,2}(t) &= v_0\sin(\theta_0) - gt. \end{aligned} \tag{5.39}$$

At the time $t = t_c$, when the collision occurs, the projectile and apple are located at the same position. Therefore the constraint conditions are

$$\begin{aligned} y_1(t_c) &= y_2(t_c), \\ x_1(t_c) &= x_2(t_c) = d. \end{aligned} \tag{5.40}$$

Set $t = t_c$ in the equations of motion for the components of the position of the projectile and apple and apply the constraint conditions (Equations 5.40), first to the y-components:

$$h - \frac{1}{2}g\,{t_c}^2 = s + v_0\sin(\theta_0)\,t_c - \frac{1}{2}g\,{t_c}^2, \tag{5.41}$$

which simplffies to

$$v_0\sin(\theta_0)\,t_c = h - s. \tag{5.42}$$

The constraint condition for the x-components is

$$v_0\cos(\theta_0)\,t_c = d. \tag{5.43}$$

Dividing Equation 5.42 by 5.43 yields

$$\frac{v_0\sin(\theta_0)\,t_c}{v_0\cos(\theta_0)\,t_c} = \tan(\theta_0) = \frac{h-s}{d}. \tag{5.44}$$

From Figure 5.11 we can see that $\tan\theta_0 = (h - s)/d$ therefore the demo instructor aims the projectile at the initial position of the apple. For the short times involved in the demo, the air resistance is not a significant factor.

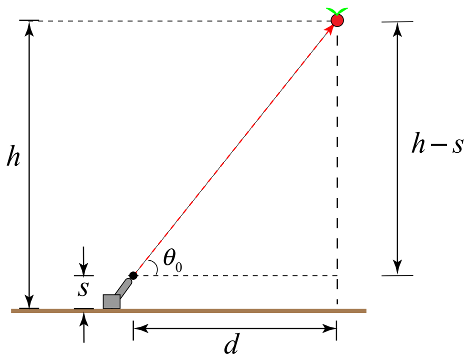

Figure 5.11: $\tan\theta_0 = (h - s)/d$.

Review At first the result may seem surprising, however both the projectile and the apple have the same acceleration so the change in the y-component of the position for either object due to the acceleration is the same $-(1/2)gt_c^2$. Suppose you are freely falling with the apple. From your perspective the apple is at rest. If you choose your origin at the ground at $t = 0$, then the y-component of the position of the apple is $y_1' = h$. The projectile is also not accelerating according to you but the y-component has changed due to the initial y-component of velocity. So at time t_c, according to you, the y-component of the position of the projectile is $y_2' = s + v_0 \sin(\theta_0)\, t_c$. At time $t = t_c$, you observe that the two objects collide so $h = s + v_0 \sin(\theta_0)\, t_c$.

We have assumed that the quantities h, d, and s are related in such a way that the projectile travels the distance d horizontally before striking the ground. Finding the relation is not part of this problem; we will just trust the demo instructor to have rehearsed prior to class. For the short times involved in the demo, the air resistance is not a significant factor.