5 Algebra Self-Tests

5.1 Algebra Diagnostic Exam #1

Problem 35: Evaluate and express in scientific notation with the correct number of significant figures:

$$\frac{1}{8 \cdot 10^{12}} \quad \text{and} \quad \frac{160}{32 \cdot 10^{-4}}.$$

Problem 36: Jack and Janet are buying an assortment of fruit. Jack wants the number of apples to be three less than twice the number of oranges. Janet wants to buy two more apples than oranges. How many oranges and apples should they get?

Problem 37: Expand the following polynomial and determine its degree:

$$x^4 - 3x^3 + 3x^2 + (2 - x^2)(x^2 + 2).$$

Problem 38: A man throws a rock off the ledge of a 100 foot high cliff at time $t = 0$. The height of the rock above ground level is then given in terms of the time t (seconds) by $40t - 16t^2 + 100$. After how many seconds does the rock hit the ground?

Problem 39: Factor $x^3 - x^2 - 16x + 16$.

Problem 40: Simplify $\dfrac{(x+h)^3 - x^3}{h}$.

5.2 Algebra Diagnostic Exam #1 Solutions

Solution 35: Evaluate and express in scientific notation with the correct number of significant figures:

$$\frac{1}{8 \cdot 10^{12}} \text{ and } \frac{160}{32 \cdot 10^{-4}}.$$

$$\frac{1}{8 \cdot 10^{12}} = \frac{1}{8} \times 10^{-12} = .125 \times 10^{-12} = \boxed{1.3 \times 10^{-13}}$$

$$\frac{160}{.32 \times 10^{-4}} = \frac{16 \times 10^{1}}{32 \times 10^{-6}} = .5 \times 10^{7} = \boxed{5 \times 10^{6}}$$

Solution 36: Jack and Janet are buying an assortment of fruit. Jack wants the number of apples to be three less than twice the number of oranges. Janet wants to buy two more apples than oranges. How many oranges and apples should they get?

Let x be the number of apples and y be the number of oranges. Then we have the system of equations:

$$x = 2y - 3$$
$$x = y + 2$$

Solve simultaneously. Subtracting, $0 = y - 5$ so $y = 5$ and $x = 7$. Answer: $\boxed{5 \text{ oranges}, 7 \text{ apples}}$.

Solution 37: Expand the following polynomial and determine its degree:

$$x^4 - 3x^3 + 3x^2 + (2 - x^2)(x^2 + 2)$$

$$= x^4 - 3x^3 + 3x^2 + (2x^2 + 4 - x^4 - 2x^2)$$
$$= -3x^3 + 3x^2 + 4 \qquad \boxed{\text{degree } 3}$$

Solution 38: A man throws a rock of the ledge of a 100 foot high cliff at time $t = 0$. The height of the rock above ground level is then given

in terms of the time t (seconds) by $40t - 16t^2 + 100$. After how many seconds does the rock hit the ground?

The problem states the height at time t is $40t - 16t^2 + 100$, and asks what is t when the height is 0? Thus, we solve:

$$\begin{aligned} -16t^2 + 40t + 100 &= 0 \\ 4t^2 - 10t - 25 &= 0. \end{aligned}$$

This gives:

$$t = \frac{10 \pm \sqrt{100 - 4 \cdot 4 \cdot (-25)}}{2 \cdot 4} = \frac{10 \pm \sqrt{500}}{8}$$

and we reject the negative root to get $t = \frac{10}{8}\left(1 + \sqrt{5}\right) = \boxed{\frac{5}{4}\left(1 + \sqrt{5}\right).}$

Solution 39: Factor $x^3 - x^2 - 16x + 16$.

$f(x) = x^3 - x^2 - 16x + 16$ By inspection, $f(1) = 0$, so $x - 1$ is a factor.
$f(x) = (x-1)(x^2 - 16)$ (by division or inspection)
$f(x) = \boxed{(x-1)(x-4)(x+4).}$

Solution 40: Simplify $\frac{(x+h)^3 - x^3}{h}$.

$$\begin{aligned} \frac{(x+h)^3 - x^3}{h} &= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\ &= \boxed{3x^2 + 3xh + h^2} \end{aligned}$$

5.3 Algebra Diagnostic Exam #2

Problem 41: 6.25×10^{24} molecules of water fill a .2 liter glass. Approximately how much of this volume (in liters) does one molecule account for?

Problem 42: Solve for x and y in terms of a and b, simplifying your answer as much as possible:

$$\begin{aligned} x + 2y &= a \\ x - y &= b \end{aligned}$$

Problem 43: Write $(x+1)^3$ as a polynomial in x.

Problem 44: Suppose a and b are the distinct roots of $x^2 - 4x - 2$, with $a < b$. Which of the following correctly compares a and b to 1?

i. $a < b < 1$

ii. $a < 1 < b$

iii. $a < b < 1$

Problem 45: Does $x - 1$ divide evenly into $5x^8 - 3x^5 + x^4 - 2x - 1$? (Hint: do not attempt to divide.)

Problem 46: First solve for x^2, then for x:

$$\frac{1 + x^2}{1 - x^2} = y$$

5.4 Algebra Diagnostic Exam #2 Solutions

Solution 41: 6.25×10^{24} molecules of water fill a .2 liter glass. Approximately how much of this volume (in liters) does one molecule account for?

$$\frac{.2}{6.25 \times 10^{24}} = \frac{20 \times 10^{-2}}{6.25 \times 10^{24}} = 3 \times 10^{-26} \text{ liters}$$

(to one significant figure.)

Solution 42: Solve for x and y in terms of a and b, simplifying your answer as much as possible:

$$\begin{aligned} x + 2y &= a \\ x - y &= b \end{aligned}$$

Subtracting: $3y = a - b$, so $\boxed{y = \frac{a-b}{3}}$.

Substituting: $x = y + b$, so $\boxed{x = \frac{a+2b}{3}}$.

Solution 43: Write $(x+1)^3$ as a polynomial in x.

$(x+1)^3 = x^3 + 3x^2 + 3x + 1$ by the binomial theorem.

Solution 44: Suppose a and b are the distinct roots of $x^2 - 4x - 2$, with $a < b$. Which of the following correctly compares a and b to 1?

i. $a < b < 1$

ii. $a < 1 < b$

iii. $a < b < 1$

The solutions to (roots of) $x^2 - 4x + 2 = 0$ are:

$$x = \frac{4 \pm \sqrt{16 - 4 \cdot 2}}{2} = \frac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2}.$$

Thus $a = 2 - \sqrt{2} \approx .6$, and $b = 2 + \sqrt{2} \approx 3.4$, so the correct comparison is $\boxed{a < 1 < b}$.

Solution 45: Does $x-1$ divide evenly into $5x^8 - 3x^5 + x^4 - 2x - 1$? (Hint: do not attempt to divide.)

$x-1$ is a factor of $f(x)$ if and only if $f(1) = 0$. But evaluating $f(1)$ gives us $5-3+1-2-1=0$, so $x-1$ is a factor.

Solution 46: First solve for x^2, then for x:

$$\frac{1+x^2}{1-x^2} = y$$

Solving:

$$\begin{aligned} 1+x^2 &= y - x^2y \\ x^2(y+1) &= y-1 \\ x^2 &= \frac{y-1}{y+1} \\ x &= \pm\sqrt{\frac{y-1}{y+1}} \end{aligned}$$

6 Algebra Self-Evaluation

You may want to informally evaluate your understanding of the various topic areas you have worked through in the *Self-Paced Review*. If you meet with tutors, you can show this evaluation to them and discuss whether you were accurate in your self-assessment.

For each topic which you have covered, grade yourself on a one to ten scale. One means you completely understand the topic and are able to solve all the problems without any hesitation. Ten means you could not solve any problems easily without review.

1.	Scientific Notation	____
2.	Significant Figures	____
3.1	Linear Equations in One Variable	____
3.2	Simultaneous Linear Equations in Two Variables	____
4.1	Polynomials	____
4.2	Binomial Theorem	____
5.	Quadratic Equations	____
6.1	Factoring	____
6.2	Long Division	____
7.1	Algebraic Manipulations: Eliminating Radicals	____
7.2	Algebraic Manipulations: Combining Fractions	____
8.1	Geometric Series	____
8.2	Geometric Progressions	____