1.144: Applied Category Theory for Engineering Design (ACT4ED)

Fall 2025

Instructor:
Prof. Gioele Zardini, gzardini@mit.edu, zardini.mit.edu
Room: 32-716

Teaching Assistant:
Yujun Huang, yujun233@mit.edu
Office hour: TBD

Units: 12 units, 4-0-8, G
Lectures: Mondays and Wednesdays 11:00-12:30 pm (1-150)
Recitations: Fridays 1:00-2:00 pm (1-246)
Class website: CANVAS (all class materials, submissions, and announcements) and Piazza for discussions

---

Course Summary When designing complex, multi-component systems, we need to consider multiple trade-offs at various abstraction levels and scales, and choices of single components need to be studied jointly. For instance, the design of future mobility solutions (e.g., autonomous vehicles, micromobility) and the design of the mobility systems they enable are closely coupled. Indeed, knowledge about the intended service of novel mobility solutions would impact their design and deployment process, while insights about their technological development could significantly affect transportation management policies. Approaching the co-design of these systems is a complex task for at least two reasons. On one hand, the co-design of interconnected systems (e.g., large networks of cyber-physical systems) involves the simultaneous choice of components arising from heterogeneous natures (e.g., hardware vs. software parts) and fields, while satisfying systemic constraints and accounting for multiple objectives. On the other hand, components are connected via collaborative and conflicting interactions between different stakeholders (e.g., within an intermodal mobility system). In short, we lack tools that can enable the clear formulation and efficient solution of such complex, compositional design problems.

In various engineering and applied science domains, recognizing the significance of abstraction and compositionality can markedly enhance both problem understanding and solution development. Applied Category Theory is a branch of mathematics that offers valuable insights into these very aspects. A problem, however, is that this type of mathematics is not traditionally taught – to date, there exists no easy path for engineers to learn category theory that is approachable and emphasizes engineering applications, such as the ones presented above. This course aims at filling this gap. It is designed not merely to teach category theory, but to foster the compositional engineering way of thinking. Category theory will just be the means towards this end.

The class covers topics from foundational principles to advanced applications, emphasizing the role of compositional thinking in engineering. It showcases successful applications in areas such as dynamical systems and automated system design optimization, with a focus on autonomous robotics and mobility. Additionally, the course offers students the opportunity to work on their own application through a dedicated project in the second half of the semester.

Pre-requisites Calculus, linear algebra, and dynamical systems at undergraduate level; or permission of instructor.

Objectives
- Learn concepts from algebra and category theory, as well as methods to leverage such concepts in engineering applications;
- Become familiar with case studies of applied category theory, involving dynamical systems, databases, and complex systems design (e.g., autonomous robots, mobility systems);
- Be able to recognize compositional structures in concrete situations, at different levels of abstraction;
- Understand the "compositional engineering way of thinking" as an approach to systems analysis, co-design, and computation.
- Develop communication skills;

Literature The course textbook is the own work-in-progress (currently in dicussions with publishers) book "Applied Category Theory for Engineering", by Censi, Lorand, and Zardini. Occasionally, we will refer to the book "Seven Sketches in Compositionality", by Fong and Spivak.

Homework Problem sets will be released on a weekly basis, and are due approximately 9 days later (refer to CANVAS for deadlines). Problem sets are weighted equally.
The policy for late submissions is the following. Submissions during the 24h period after the deadline are graded at 80% of the total score. Submissions during the 24 to 48h are graded at 50%. Submissions later than 48h after the deadline *will not be accepted*.
Please ask questions during the office hours, as well as on the public forum on Piazza. Students are expected to turn in all completed problem sets on time.

Final Project The project will leverage the knowledge developed during the class to model a co-design problem in a domain of choice (ideally, in the students' discipline of interest). The project will include the formalization of the problem, its solution, a public presentation to the class, and a report. For the project, students will be able to work in groups. Allowed group sizes will be determined based on enrollment. A detailed document presenting the project rules and tasks will be made available during the first half of the class.

Midterm The course includes a midterm, testing the understanding of the concepts presented in the first half of the class. During the midterm, no collaboration and no aids are allowed. Details about the midterm will be communicated in due time.

Collaboration Policy Students are encouraged to form study groups. However, each student must write up their own, original solutions for problem sets. Whenever you use books or online resources to solve a problem, please report the sources, and never report material verbatim. Furthermore, please include the name of your collaborators in the headers of the problem set. Finally, the use of Generative AI tools is not permitted in any form.

Course Requirements The course includes a midterm, as well as a (group) project for the second-half of the term. The grading is decided as follows: The final grade will be determined by combining scores from the problem sets (40%), the midterm (30%), and the final project (30%). Additionally, students are given the chance to be beta testers of a work-in-progress book. Their feedback can contribute to up to 10%.

Lecture Schedule (subject to change)

When	Day	Session	Topic	Homework
09/03	W	L1	*Introduction: Logistics, Motivation, Compositionality*	
09/05	F		NO RECITATION – SKIP	
09/08	M	L2	*Sets, functions, relations*	PS 1 out
09/10	W	L3	*Algebra*	
09/12	F	L4-1	*Trade-offs, Monotonicity*	
09/15	M	R1		PS 2 out
09/17	W	L4-2	*Trade-offs, Monotonicity, and Recap*	
09/19	F		HOLIDAY – NO RECITATION	
09/22	M	L5	*Categories*	PS 1 in
09/24	W	L6	*Actions*	
09/26	F	R2		
09/29	M	L7	*Functors*	PS 3 out
10/1	W	L8	*Co-design*	
10/3	F	R3		PS 2 in
10/6	M	L9	*Brainstorming for projects*	
10/8	W	L10	*Natural transformations*	
10/10	F	R4		PS3 in, PS 4 out
10/13	M		HOLIDAY – NO CLASS	
10/15	W	L11	*Parallel composition (monoidal categories)*	
10/17	F	R5		
10/20	M		**Midterm**	PS5 out
10/22	W	L12	*Modeling co-design problems, projects kick-off*	
10/24	F	R6		
10/27	M	L13	*Feedback (traced monoidal categories)*	PS 4 in, PS 6 out
10/29	W	L14	*Functorial computation*	
10/31	F	R7		
11/3	M	L15	*Monads*	PS5 in
11/5	W	L16	*Midterm presentations for projects*	
11/7	F	R8		
11/10	M		HOLIDAY – NO CLASS	
11/12	W	L17	*Project time, feedback*	
11/14	F	R9		PS6 in
11/17	M	L18	*Enriched and Negative category theory*	
11/19	W	L19	*Project time, feedback*	
11/21	F	R10		
11/24	M	L20	Invited Talk	
11/26	W	L21	Invited Talk	
11/28	F		HOLIDAY – NO RECITATION	
12/1	M	L22	Final Presentations	
12/3	W	L23	*Invited Talk: Prof. Markus Bühler (MIT)*	
12/5	F	R11	*Final Presentations*	

Extended Syllabus

L1 *Introduction: Logistics, Motivation, Compositionality*: Introducing the class and its logistics; Introducing compositionality, with motivation stemming from en gineering design, dynamical systems, robotics, and transportation.

L2 *Sets, functions, relations*: Introduction to basic notation needed for the class, sets, functions, and relations, as their use in modeling engineered systems.

L3 *Algebra*: Semigroups, monoids, and groups, as well as the concept of homomorphisms. Illustration of various applications in robotics, dynamical systems, and signal processing will be discussed.

L4 *Trade-offs, Monotonicity*: Introduction to the multi-objective nature of complex systems design. Formalization of different forms of ordered sets, and their manipulation. Monotonicity, and relation to compositional properties.

L5 *Categories*: Presentation of the central concept of the class: categories, or, everything is the same! Showing how the various structures presented so far generalize, and how they relate to applications.

L6 *Actions*: From passive to active: can categories do something for us? Compositionality of dynamical systems, and applications to multi-objective design, and motion planning.

L7 *Functors*: Generalization of the concept of "action" and extended examples.

L8 *Co-design*: Compositional, collaborative, computational, continuous design framework, to design multi-compo nent, multi-objective complex engineering systems. Applications to transportation and robotics.

L10 *Natural transformations*: Up the ladder of abstraction: introducing natural transformations and diagrams, as well as their employment into engineering applications

L11 *Parallel composition (monoidal categories)*: Monoidal categories and various forms of parallel composition, with applications to systems, control, and design.

L12 *Feedback (traced monoidal categories)*: Fundamental notion of feedback via traced monoidal categories, motivated by applications in engineering design and control.

L13 *Modeling co-design problems, projects kick-off*: Contextualize learned concepts in the context of a framework for engineering co-design. Methodology to model society-critical engineering problems as co-design problems, and presentation of the project challenges for the second half of the semester.

L14 *Functorial computation*: How to solve co-design optimization problems? Explaining the functorial approach to efficient computation, and introducing the algorithms behind this new class of optimization problems.

L15 *Monads*: Introducing monads, and their employment in engineering (e.g., uncertainty quantification in design and decision processes).

L16 *Universal constructions*: Bringing together all the introduced notions and contextualizing into a single formalism.

L17 *Midterm presentations for projects*: Time to present to the rest of the class the first steps of the project, and receive feedback to be implemented for the rest of the term.

L18 *Enriched category theory*: Customizing your categories via enrichment, and application to routing problems and co-design;

- L19 *Negative category theory*: Extending notions of categories via negative information, and fundamental implications for co-design problems with negative results, as well as other problems in engineering (e.g., motion planning).
- L20-23 *Special topics in applied category theory*: Special lectures in applied category theory, featuring invited speakers. The topics include categorical machine learning (e.g., deep learning), cybernetics, hybrid systems, hierarchies for decision making, polynomial functors, categorical control systems design, and compositional computing.
- L24-25 *Final presentations*: Final classes dedicated to final project presentations and feedback from colleagues.

General remarks Grades at MIT are not awarded according to a predetermined distribution of letter grades. The grade for each student will be determined independent of the performance of other students in the class and will be related to the student's mastery of the material based on the grade descriptions as specified by MIT rules and regulations.

Please contact the course staff if there is any confusion about the following points: (i) the number and kinds of required assignments; (ii) schedule of tests and major due dates; (iii) grading criteria and procedures; (iv) relative contribution of each assignment and course component to the final grade; and (v) expectations regarding collaboration and academic conduct.

Student Holidays are reported in the detailed schedule.

According to MIT's policy on academics and student religious observances: "Any student who is unable to attend classes or participate in any examination, study, or work requirement on a particular day because of his or her religious beliefs is excused from any such activity. The student will be given the opportunity to make up the work that was missed, provided that the makeup work does not create an unreasonable burden upon MIT... The Institute will not level fees or charges of any kind when allowing the student to make up missed work. In addition, students will suffer no adverse or prejudicial effects because they have made use of these provisions." If you find that an assignment due date, midterm, or class falls on the same day as an observance in your faith, please contact the course staff to discuss options and make a plan for your absence.

Please, do make an effort to communicate with the instructors if you find yourself in difficulty, such as family emergencies, health problems, etc. Under such notice from the student and on agreement between the student and the course staff, the assignment with the lowest grade will be discarded from evaluation. It is possible to obtain extensions of deadlines, but your issues should rise to the level of a certain level of difficulty, not just inconvenience. At this time, we do not plan to offer alternate midterms.

If your personal circumstances do not allow you to take the midterm on the specified date, please discuss this with the course staff so that we can try to accommodate your schedule.

Civil & Environmental Engineering Communication Lab (CEE Comm Lab) The CEE Comm Lab provides discipline-specific peer coaching for undergrads, grads, and postdocs in our community. Are you working on a scientific paper, presentation, or application? Our team of trained

CEE PhD and Postdoc Comm Fellows can provide actionable feedback at any stage in your process, from initial brainstorming to final polishing. If you're affiliated with CEE, which includes taking a Course 1 class or conducting research in a CEE lab, you can schedule free in-person or virtual appointments throughout the year. Since 2020, the Comm Fellows have hosted more than 600 coaching sessions, developed scientific communication workshops, and published online resources. To learn more and to schedule a one-on-one session please visit our website: https://mitcommlab.mit.edu/cee/