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Introduction to Robotics

February 23-24, 2023

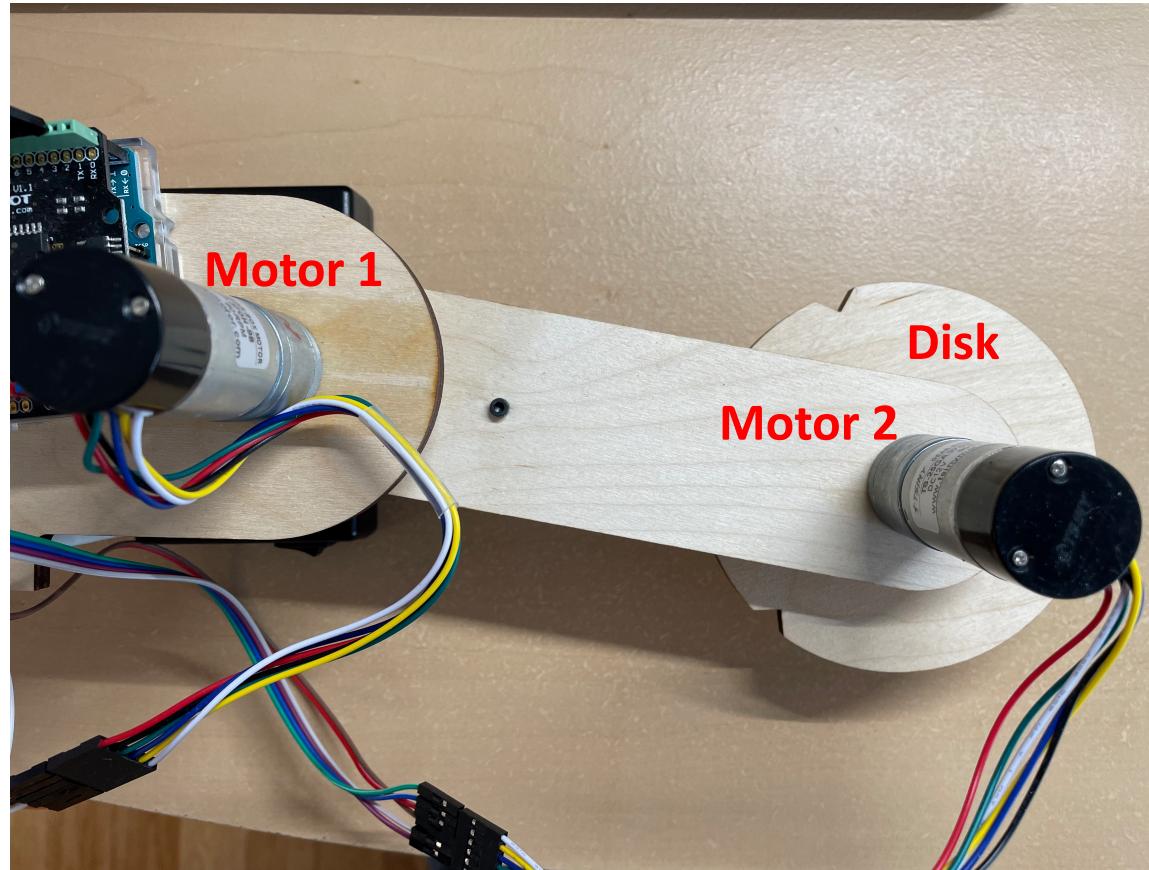
Lab 3: DC Motor Control

Objectives

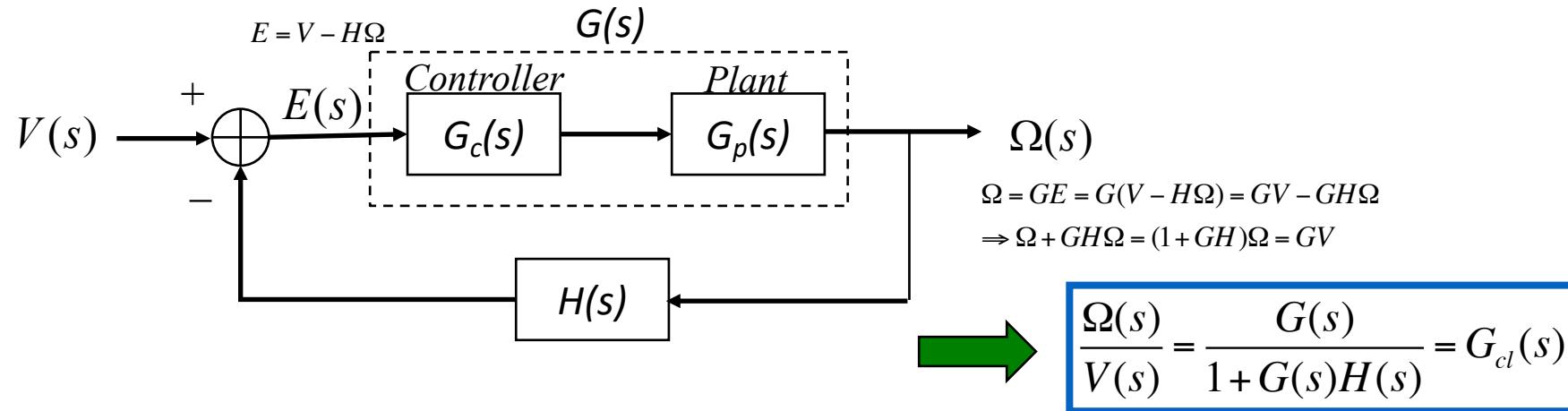
- Understand the Proportional, Integral, and Derivative (PID) control actions.
- Derive closed-loop transfer functions.
- Design and fine-tune a PI controller for velocity control.
- Design and fine-tune a PD controller for position control.
- Term Project: Task strategy.

System Setup

- Detach the second link and attach the disc to Motor 2.



Standard Closed-Loop PID Control



PID controller transfer function

$$\begin{aligned}
 G_c(s) &= K_p + \frac{K_i}{s} + K_d s \\
 &= \frac{K_d s^2 + K_p s + K_i}{s} \\
 &= K_d \left(\frac{s^2 + \left(\frac{K_p}{K_d} \right) s + \left(\frac{K_i}{K_d} \right)}{s} \right)
 \end{aligned}$$

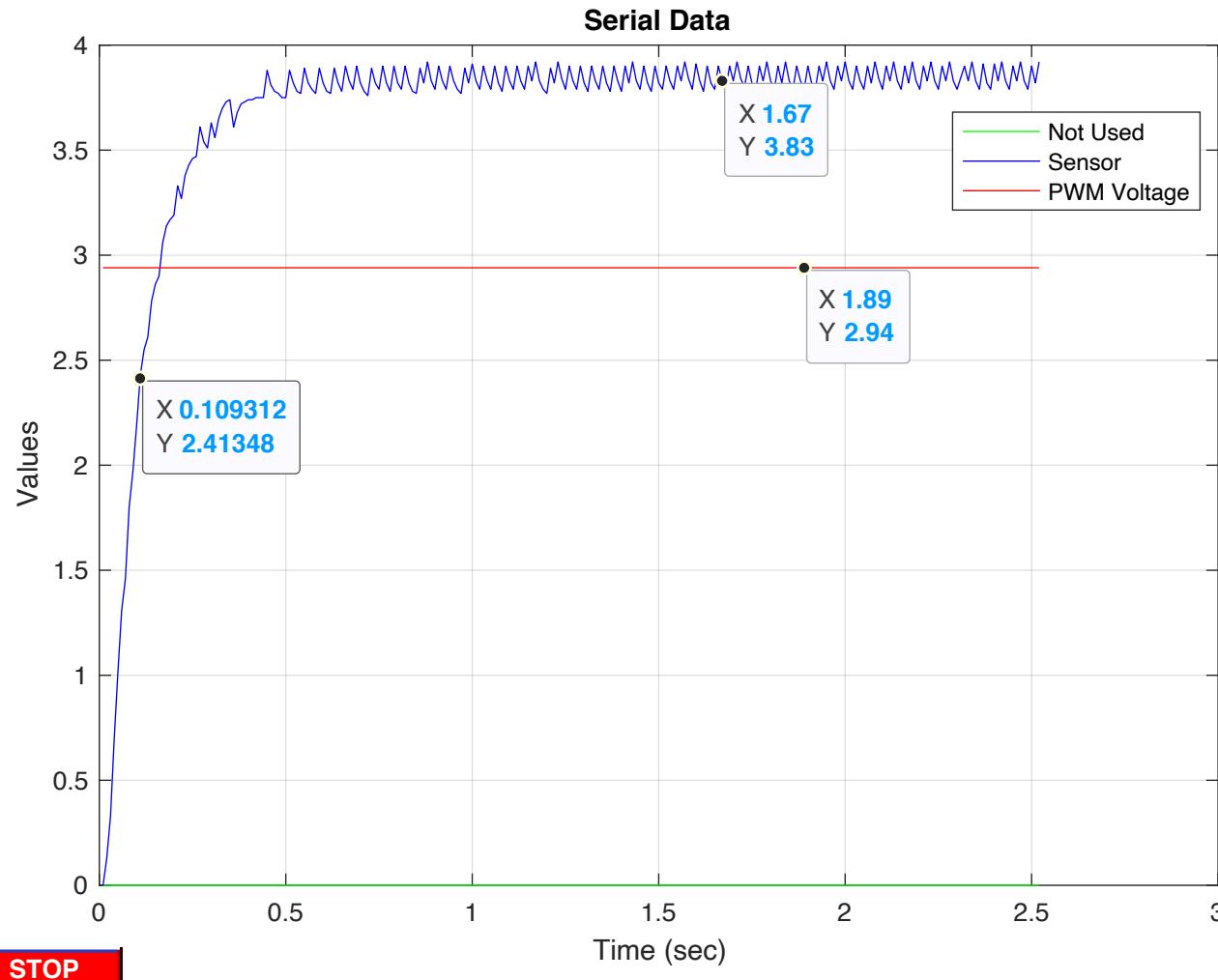
PID controller pseudo code

$$PID_output = K_p e(t) + K_i \int e(t) dt + K_d \frac{d}{dt} e(t)$$

$$e(t) = \text{set_point} - \text{sensor_output}$$

$$\int e(t) dt \approx \sum_t e(t) \Delta t$$

$$\frac{d}{dt} e(t) \approx \frac{e(t) - e(t-1)}{\Delta t}$$



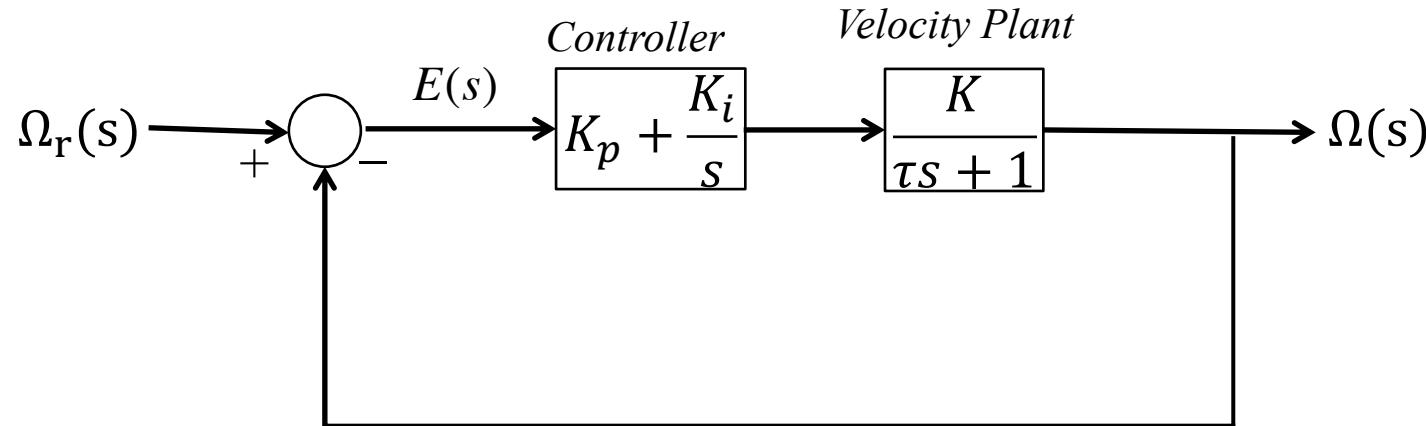
Plant transfer function:

$$G_p(s) = \frac{\Omega(s)}{V_{pwm}(s)} = \frac{K_{dc}}{\tau s + 1} = \frac{1.3}{0.11s + 1}$$

PWM = 255 corresponds to 5V from PWM pin (1V = 51 PWM) so we can further represent the transfer function as:

$$G_p(s) = \frac{\Omega(s)}{PWM(s)} = \frac{\frac{K_{dc}}{51}}{\tau s + 1} = \frac{0.0255}{0.11s + 1}$$

$$K = \frac{K_{dc}}{51}$$



Closed-Loop Transfer Function: $G_{cl}(s) = \frac{K(K_p s + K_i)}{\tau s^2 + (1 + K_p K)s + K_i K}$

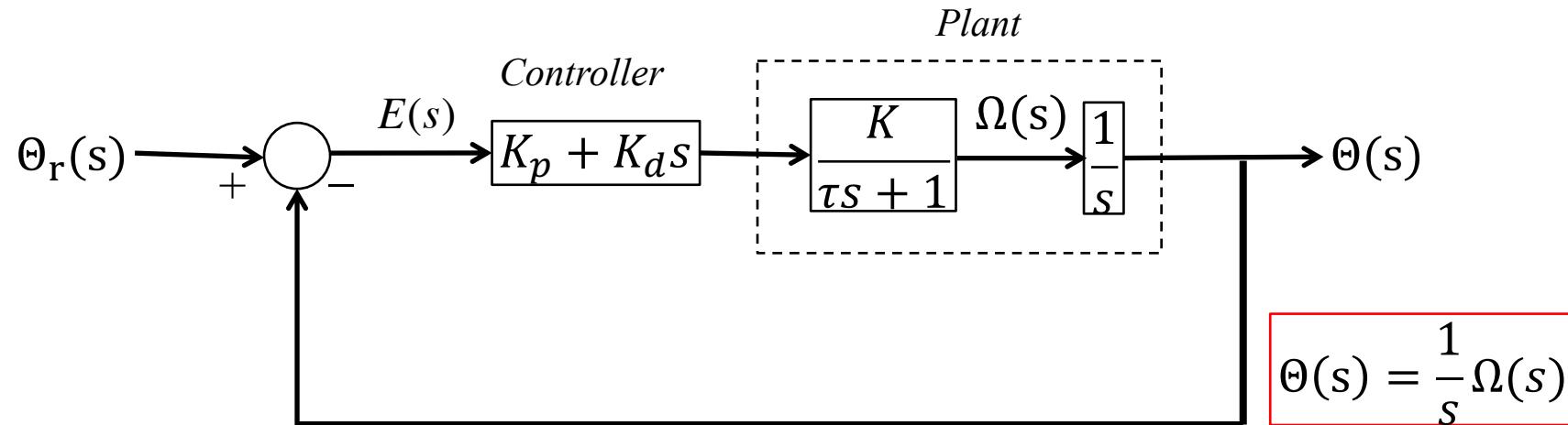
One zero
Two poles

Steady-state

$$\Omega_{ss} = \lim_{t \rightarrow \infty} \Omega(t) = \lim_{s \rightarrow 0} s \left(\frac{1}{s} \right) G_{cl}(s) = \left(\frac{K_i K}{K_i K} \right) = 1$$

No steady-state error with a step input

Position Control with P&D Control Actions



Closed-Loop Transfer Function: $G_{cl}(s) = \frac{K(K_p + K_d s)}{\tau s^2 + (1 + K_d K)s + K_p K}$

One zero Two poles

↑
A “free” integrator

No steady-state error with a step input

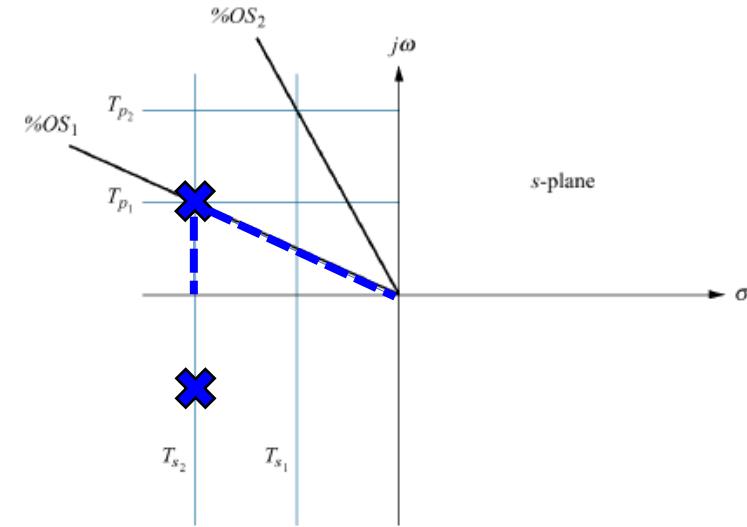
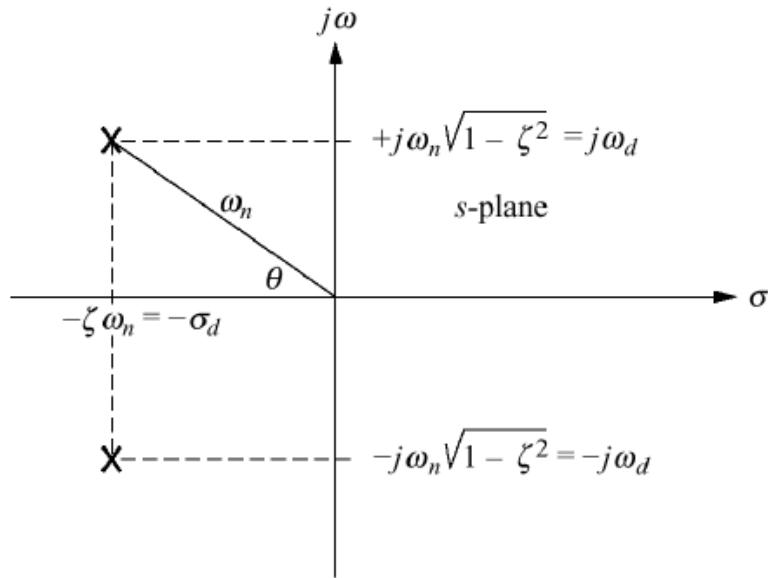
Comparison of Closed-Loop Transfer Functions

PI Control of Velocity: $G_{cl}(s) = \frac{K(K_p s + K_i)}{\tau s^2 + (1 + K_p K)s + K_i K}$

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

PD Control of Position: $G_{cl}(s) = \frac{K(K_p + K_d s)}{\tau s^2 + (1 + K_d K)s + K_p K}$

2nd Order System Poles



$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

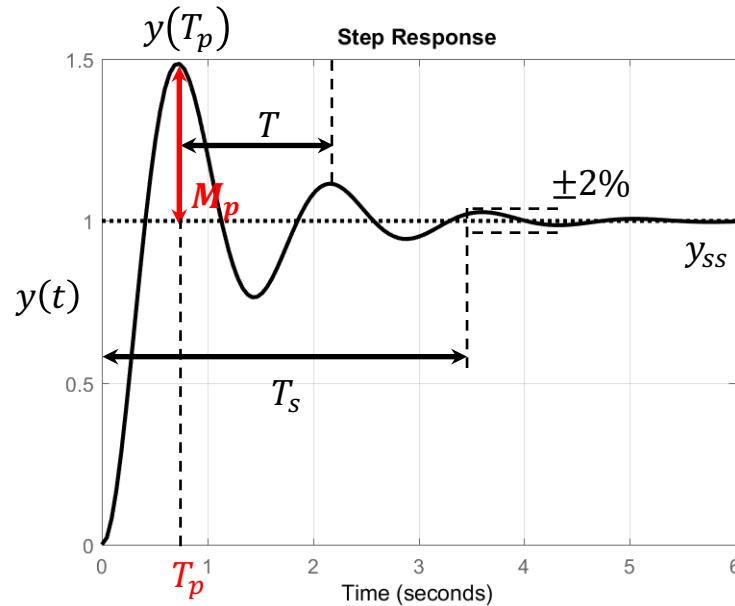
$$s_{1,2} = -\sigma_d \pm j\omega_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$T_s = \frac{4}{\zeta\omega_n} \quad (\text{within } 2\%)$$

$$\%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100$$

Dynamic Response Characteristics



Example: Figure 1

$$y_{ss} = 1 \quad y(T_p) = 1.5$$

$$M_p = 0.5 \quad \%OS = 50\%$$

$$\zeta = -\frac{\ln(0.5)}{\sqrt{\pi^2 + \ln^2(0.5)}} = 0.215$$

$$T_s \approx \frac{4}{\sigma} = \frac{4}{\zeta \omega_n} \quad 2\% \text{ Settling Time}$$

$$T_s \approx \frac{4.6}{\sigma} = \frac{4.6}{\zeta \omega_n} \quad 1\% \text{ Settling Time}$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = \frac{2\pi}{T} \quad \text{Damped Natural Frequency}$$

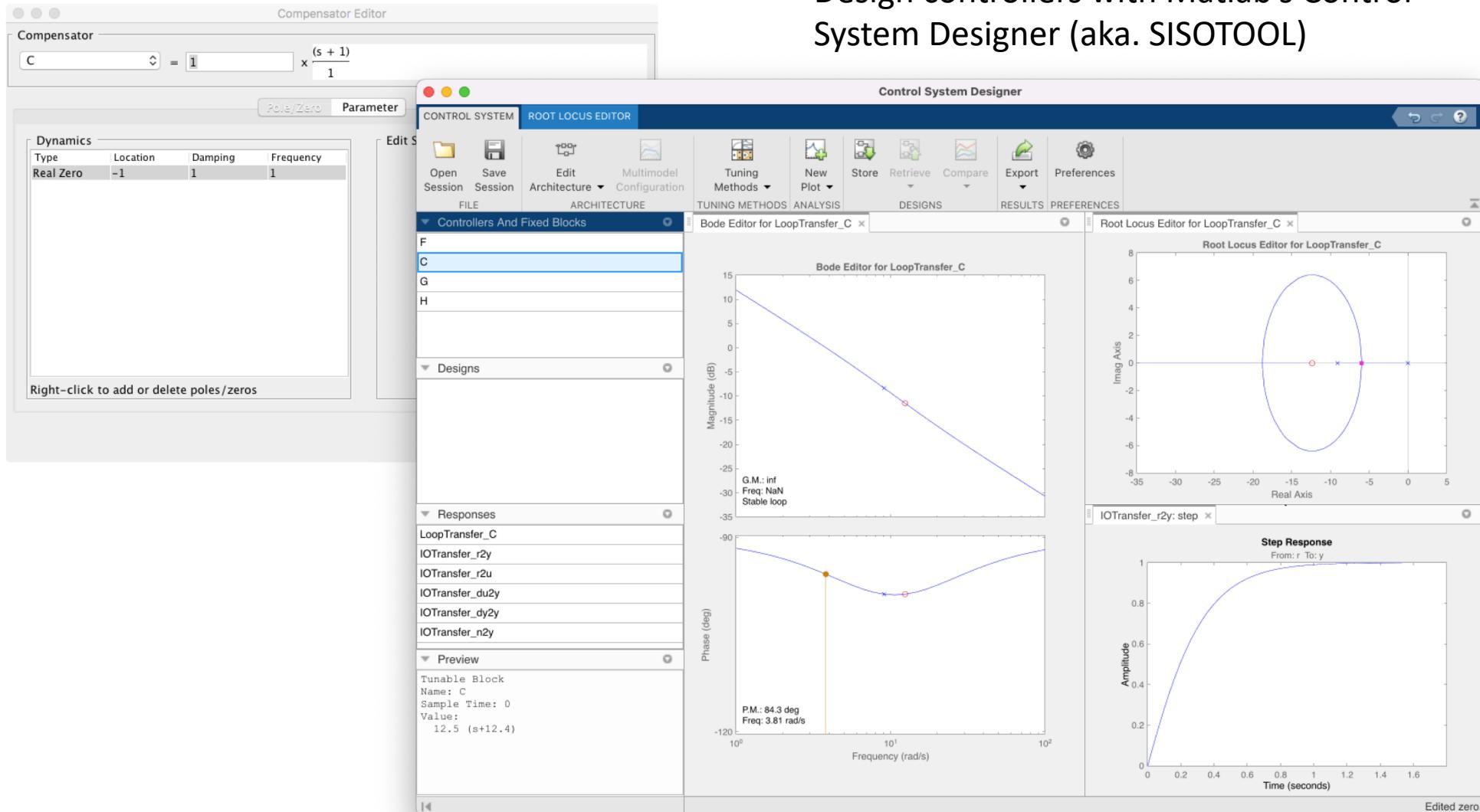
$$T_p = \frac{\pi}{\omega_d} \quad \text{Peak Time}$$

$$M_p \triangleq y(T_p) - y_{ss} \quad \text{Overshoot (Magnitude)}$$

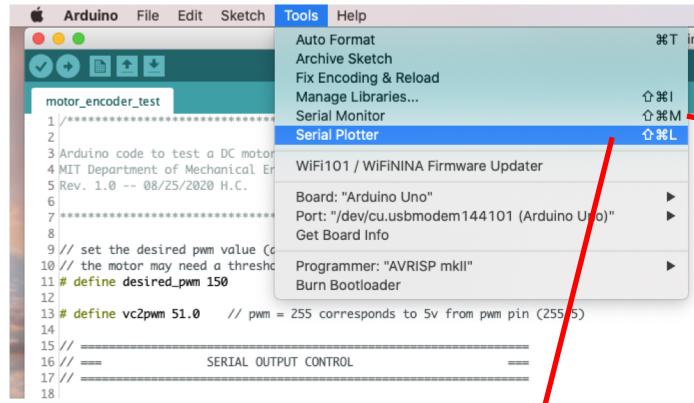
$$\%OS \triangleq \frac{y(T_p) - y_{ss}}{y_{ss}} \times 100 \quad \text{Overshoot (Percent)}$$

$$\frac{M_p}{y_{ss}} = \frac{\%OS}{100} = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \quad , 0 \leq \zeta < 1$$

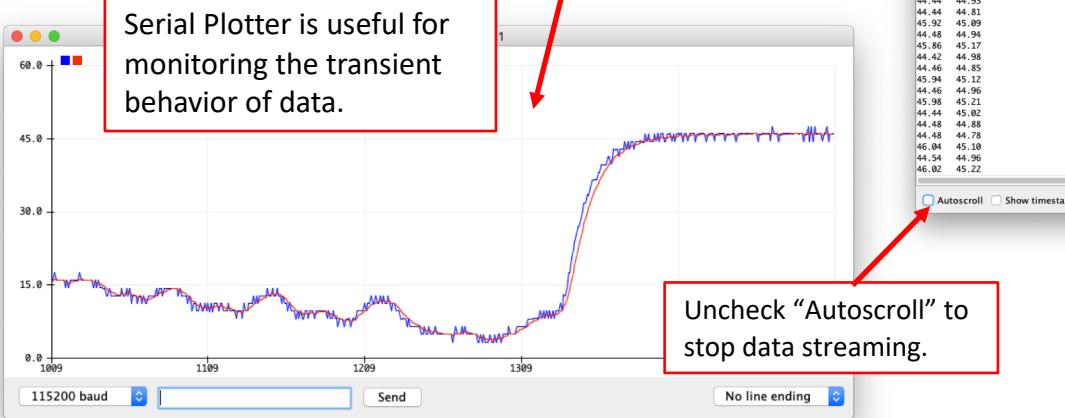
- Control action comparison:
 - *Proportional* – improves speed but with steady-state error in some cases
 - *Integral* – improves steady state error but with less stability; may create overshoot, longer transient, or integrator windup
 - *Derivative* – improves stability but sensitive to noise; may create large output when the input is not a continuous signal
- Reduce overall gain can increase stability but with slower response
- Avoid saturations
- Set integrator limits to prevent windups



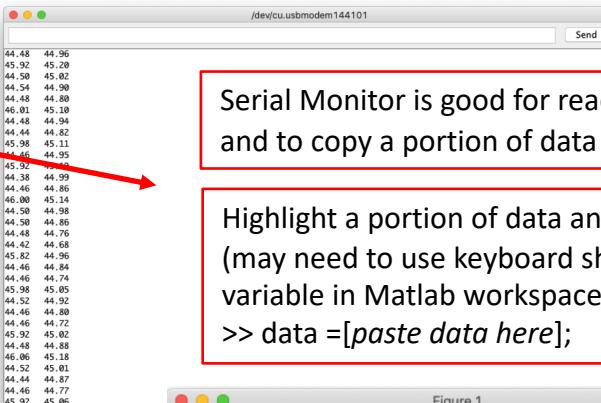
Monitoring/Capturing Serial Data



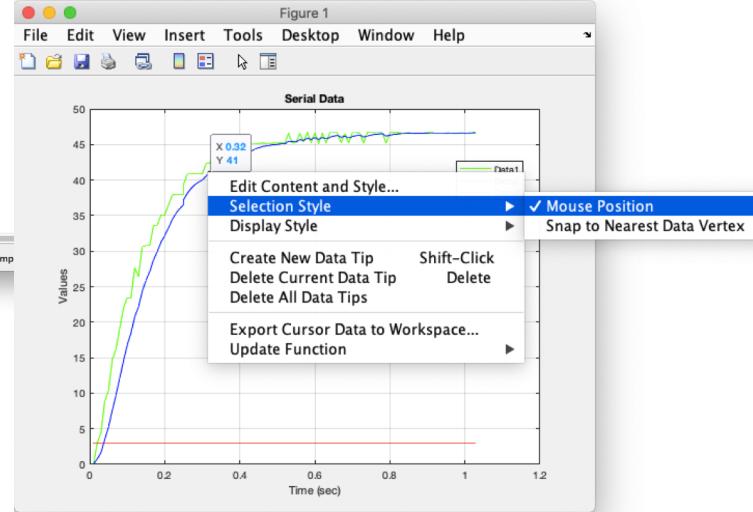
Serial Plotter is useful for monitoring the transient behavior of data.



Uncheck "Autoscroll" to stop data streaming.



Serial Monitor is good for reading data values and to copy a portion of data record to clipboard.



Highlight a portion of data and copy and paste (may need to use keyboard shortcuts) to a variable in Matlab workspace, for example:
`>> data =[paste data here];`

Neither "Serial Monitor" nor "Serial Plotter" is capable of saving data to a file.

The Matlab script "SerialRead.m" can be used to stream serial data directly into Matlab workspace for plotting and for post processing.

- Required code in GitHub: https://github.com/mit212/lab3_2023
- Demonstrate to the lab staff your closed-loop responses.

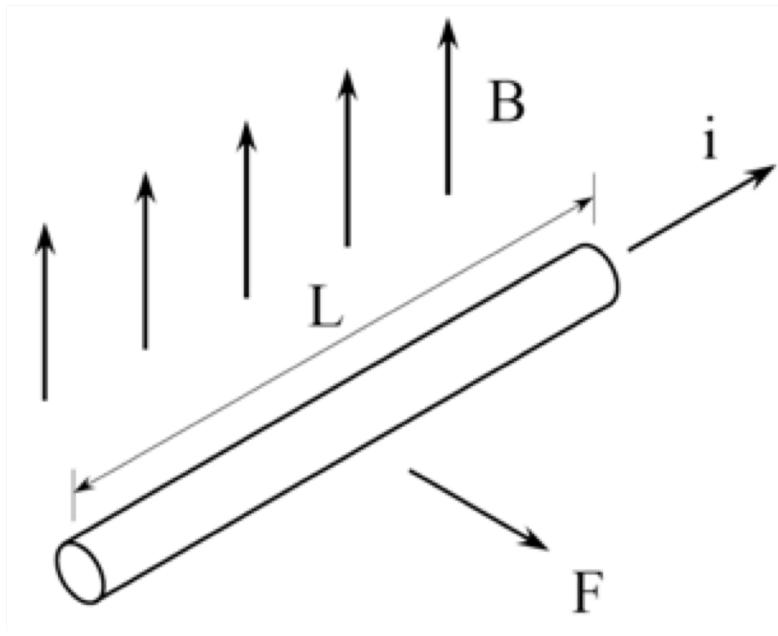
- Manipulator:
 - Design and fabricate an end-effector to perform CPR (Cardiopulmonary Resuscitation).
 - Detect and guide the end-effector to the chest of the patient.
 - May need computer vision and force feedback control.
- Mobile Robot:
 - Design and fabricate a mechanism to get an AED (Automated External Defibrillator) bag.
 - Deliver the AED bag to the destination.
 - May need obstacle avoidance, path planning, and autonomous navigation.
- Avatar:
 - Interact with the patient and the environment with VR headset and controllers.
 - Tele-operation of the manipulator.

DC Motor

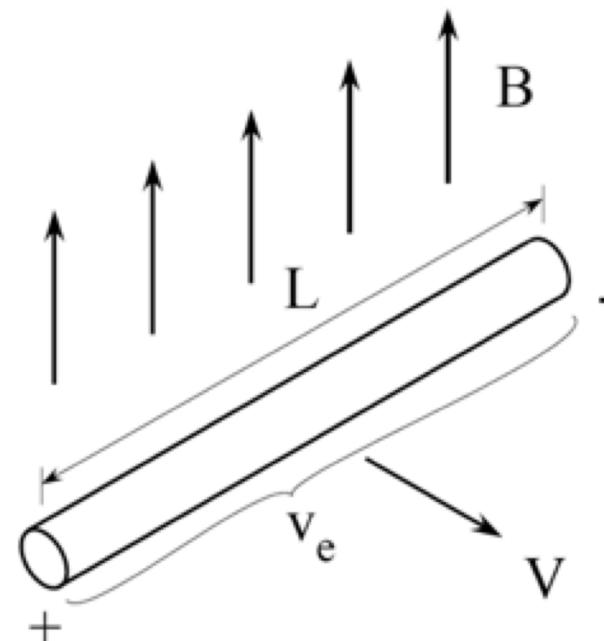
DC Motor Principle

Lorentz law:

magnetic field applies force to a current
(Lorentz force)

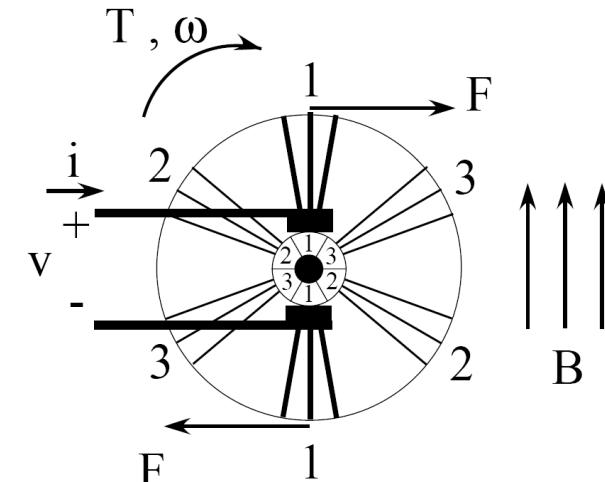
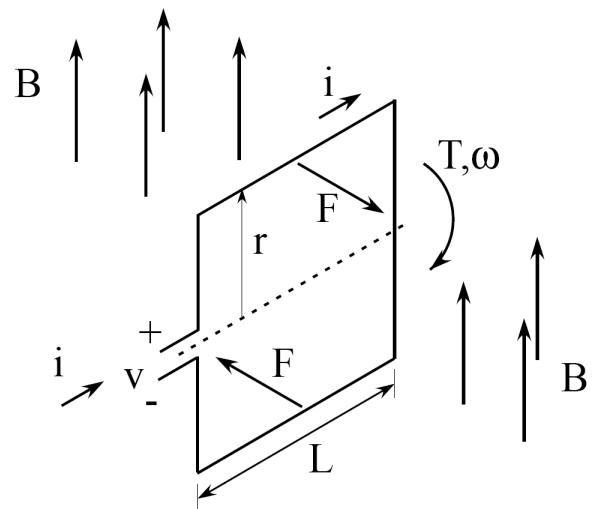
**Faraday law:**

moving in a magnetic field results
in potential (back EMF)



$$F = (\mathbf{i} \times \mathbf{B}) \cdot l = iBl \quad (\mathbf{i} \perp \mathbf{B}) \quad v_e = \mathbf{V} \times \mathbf{B} \cdot l = VBl \quad (\mathbf{V} \perp \mathbf{B})$$

DC Motor Principle (Cont.)



multiple windings N :
continuity of torque

$$T = 2Fr = 2(iBNl)r \quad (\text{Lorentz law})$$

$$v_e = 2VBNl = 2(\omega r)BNl \quad (\text{Faraday law})$$

or

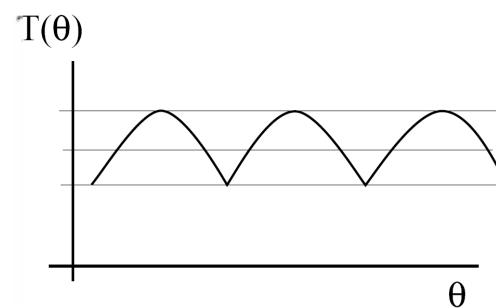
where

$$T = K_m i$$

- $K_m \equiv 2BNlr$ torque constant

$$v_e = K_v \omega$$

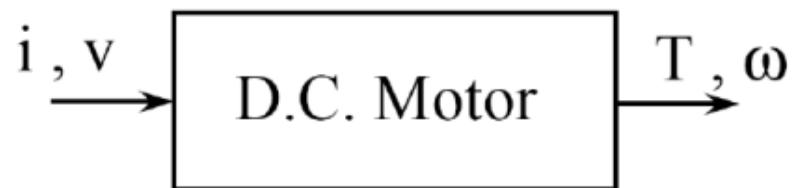
- $K_v \equiv 2BNlr$ back-emf constant



DC Motor as A Transducer

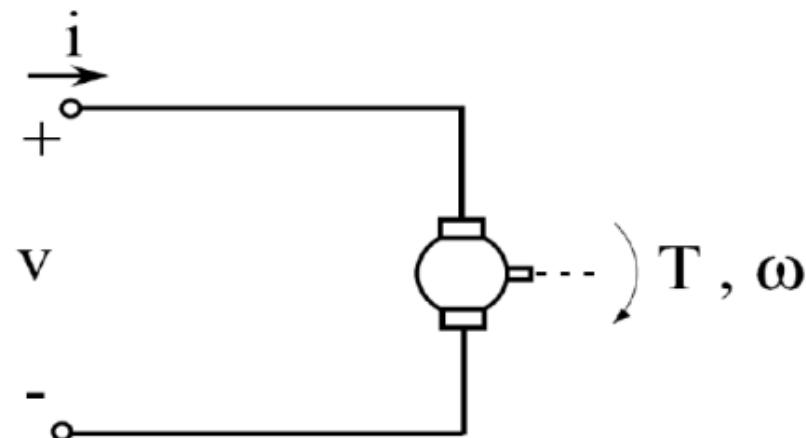
DC motor is an electromechanical system that may have a time constant close to the plant.

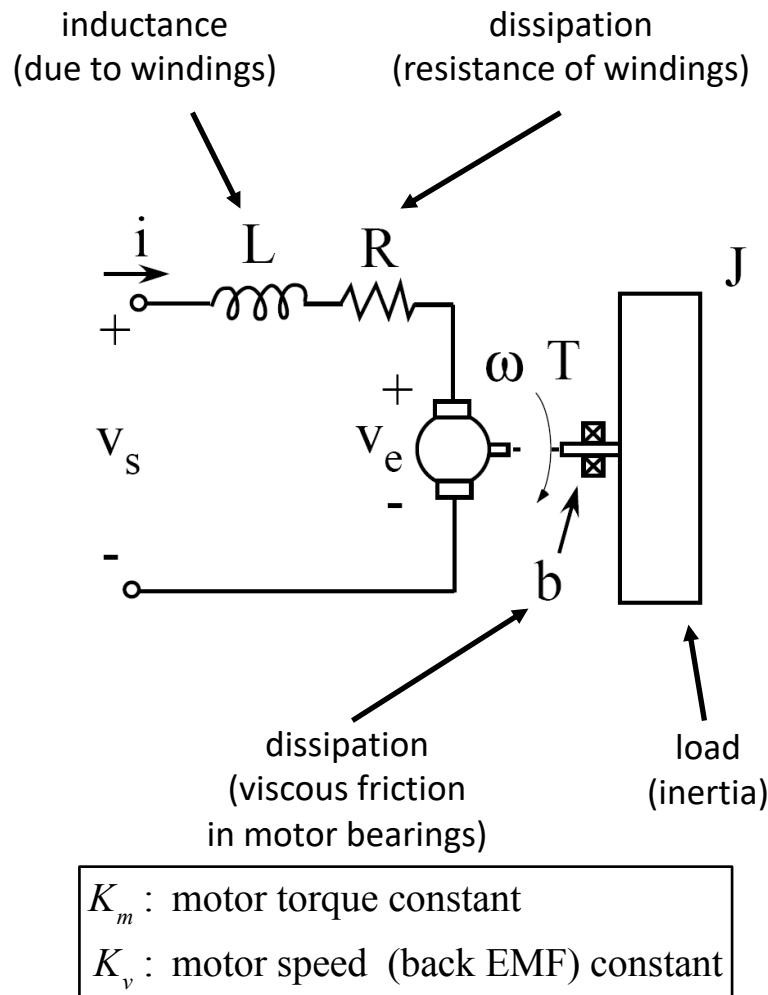
A transducer converts energy from one domain (e.g., electrical) to another (e.g., mechanical), or vice versa.



$$P_{in} = P_{out}$$

$$i(t) * v(t) = T(t) * \omega(t)$$





Equation of motion – Electrical

$$\text{KVL: } v_s - v_L - v_R - v_e = 0$$

$$\Rightarrow v_s - L \frac{di}{dt} - Ri - K_v \omega = 0$$

Equation of motion – Mechanical

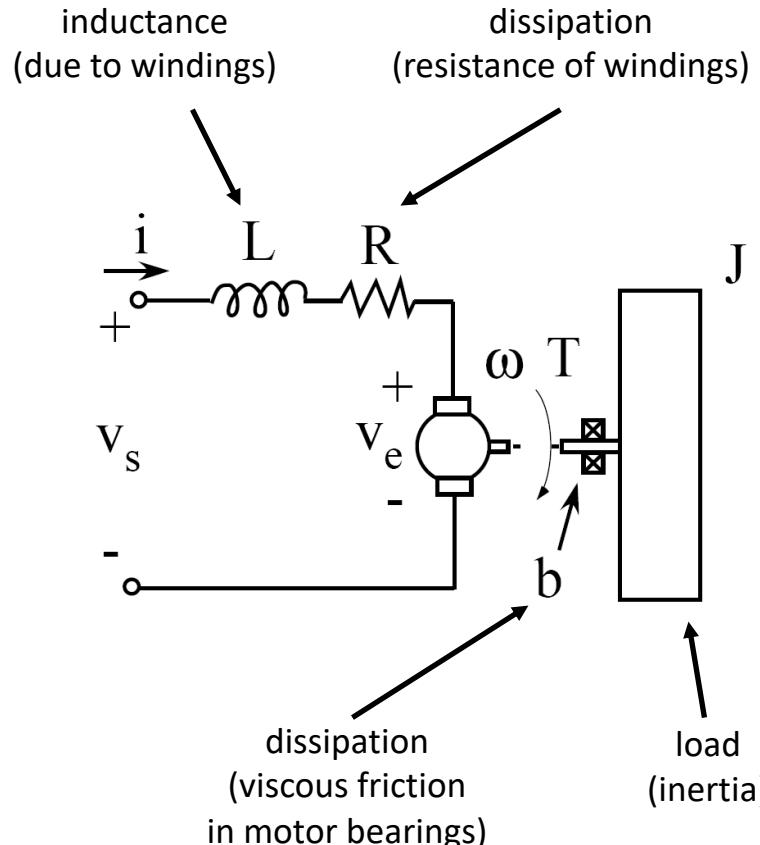
$$\text{Torque Balance: } T = T_b + T_J$$

$$\Rightarrow K_m i - b\omega = J \frac{d\omega}{dt}$$

Combined equations of motion

$$L \frac{di}{dt} + Ri + K_v \omega = v_s$$

$$J \frac{d\omega}{dt} + b\omega = K_m i$$



Equation of motion – Electrical

$$\text{KVL: } V_s(s) - V_L(s) - V_R(s) - V_e(s) = 0$$

$$V_s(s) - LsI(s) - RI(s) - K_v\Omega(s) = 0$$

Equation of motion – Mechanical

$$\text{Torque Balance: } T(s) = T_b(s) + T_J(s)$$

$$K_m I(s) - b\Omega(s) = Js\Omega(s)$$

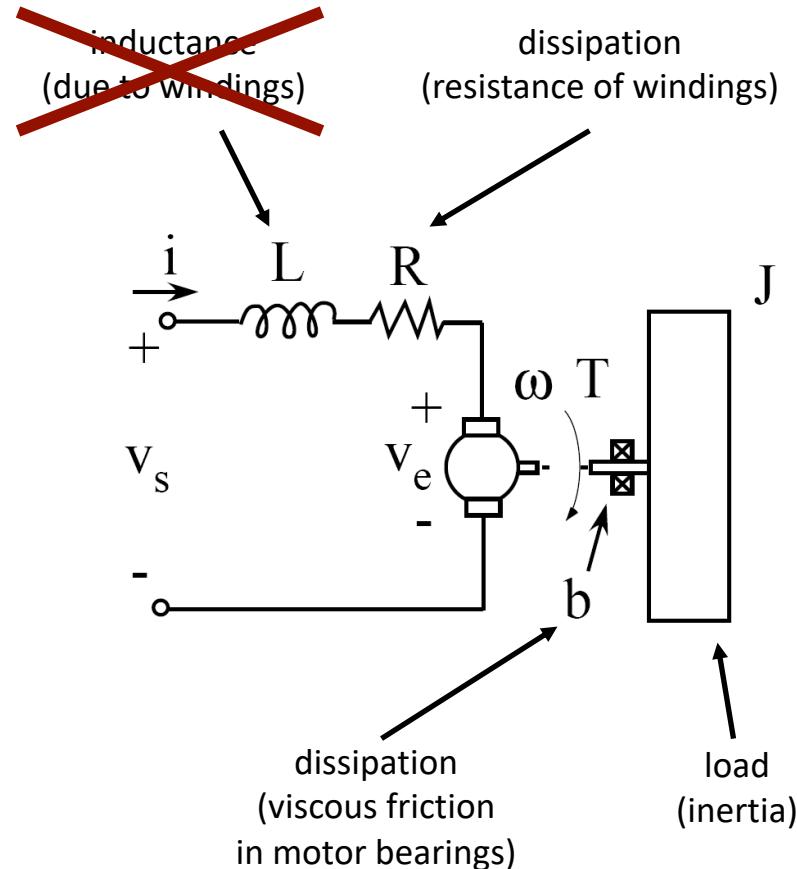
Combined equations of motion

$$LsI(s) + RI(s) + K_v\Omega(s) = V_s(s)$$

$$Js\Omega(s) + b\Omega(s) = K_m I(s)$$

$$\Rightarrow \left[(Ls + R) \left(\frac{Js + b}{K_m} \right) + K_v \right] \Omega(s) = V_s(s)$$

$$\Rightarrow \boxed{\left[\frac{LJ}{R}s^2 + \left(\frac{Lb}{R} + J \right)s + \left(b + \frac{K_m K_v}{R} \right) \right] \Omega(s) = \frac{K_m}{R} V_s(s)}$$



Neglecting the inductance (why?)

$$L \approx 0$$

$$\Rightarrow \left[Js + \left(b + \frac{K_m K_v}{R} \right) \right] \Omega(s) = \frac{K_m}{R} V_s(s)$$

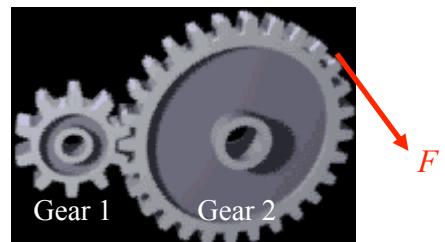
This is our familiar 1st-order system!

If we are given step input $v_s(t) = V_0 u(t)$
 \Rightarrow we already know the step response

$$\omega(t) = \frac{K_m}{R} V_0 \left(1 - e^{-t/\tau} \right) u(t),$$

where now the time constant is

$$\boxed{\tau = \frac{J}{\left(b + \frac{K_m K_v}{R} \right)}}.$$



$$\text{Gear Ratio } N: \frac{n_1}{n_2} = \frac{r_1}{r_2} = \frac{\Omega_2}{\Omega_1} = \frac{T_1}{T_2} = \frac{F_1^T r_1}{F_2^T r_2}$$

$$\frac{n_1}{n_2} = \frac{44}{180}$$

Unit Conversion:

$$rpm = \frac{2\pi}{60} (rad / s)$$

$$N = \frac{kg \times m}{s^2}$$

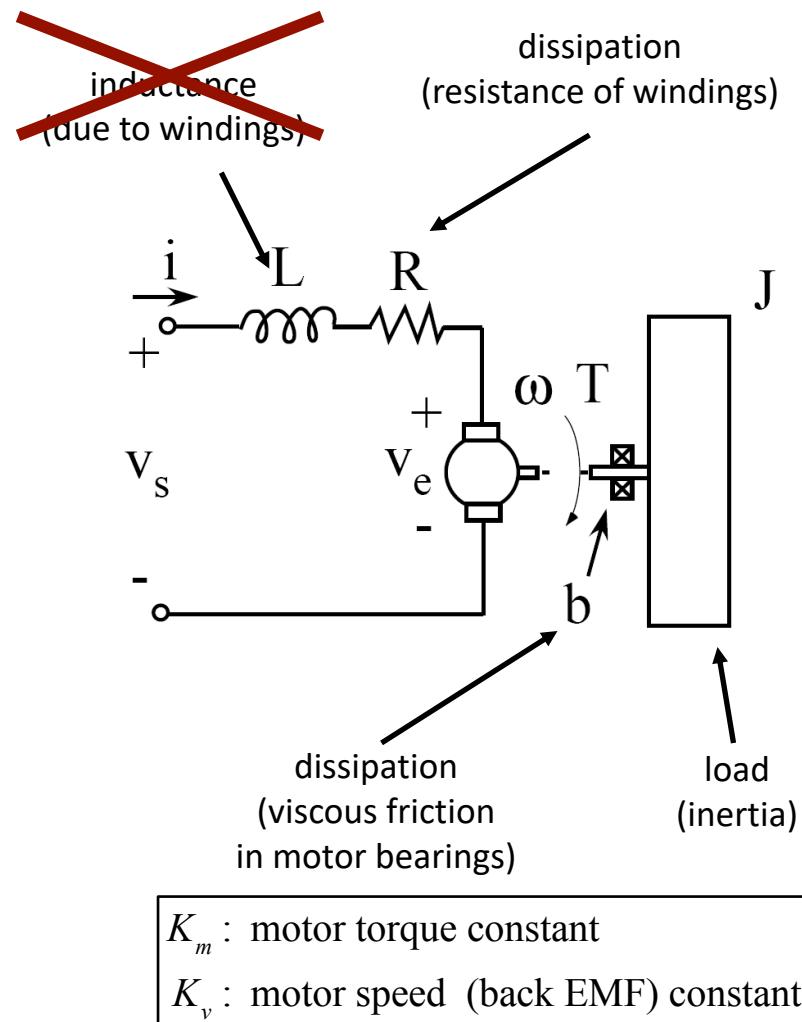
$$V(voltage) = \frac{kg \times m^2}{s^3 \times A}$$

Power Conservation:

$$P_{mechanical} = P_{electrical}$$

$$\begin{aligned} P_{mechanical}(t) &= T(t) \times \Omega(t) \\ &= K_m \times i(t) \times \Omega(t) \end{aligned}$$

$$\begin{aligned} P_{electrical}(t) &= v_b(t) \cdot i(t) \\ &= K_v \cdot \Omega(t) \cdot i(t) \end{aligned}$$



Neglecting the inductance...

$$L \approx 0$$

$$\Rightarrow \left[Js + \left(b + \frac{K_m K_v}{R} \right) \right] \Omega(s) = \frac{K_m}{R} V_s(s)$$

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