RGB Microscopy Image

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(a) MRF Prior Model

Prior beliefs on uncorrupted images-

1. Images are spatially piecewise smooth (along all three channels).
2. Discontinuities/large changes occur only at object boundaries, and these occur at the same location across the different channels.
3. Number of objects<<number of pixels.

Points 1 and 3 are similar to greyscale images' MRF prior models. However, due to point 2, our model should assign a higher probability to a large deviation in the clique of one channel if there exists such a deviation in the other channels.

Topological space

S is a set of points on a $2D$ Cartesian grid. Each point is a vector having three values.

Random Field

X is the collection of all the random variables (points in the image) belonging to S. A single image x is a realisation of this field.

Neighbours

Each site has 4 neighbours, and the sites on the border are wrapped around around to have an equal number of neighbours for each point.

Penalty function

Let $\mathbf{y_i}$ represent the neighbours of a single pixel of $\mathbf{x}$

$$\mathbf{y_i} = \mathbf{x_i} + \mathbf{r_i}$$

For a single pixel, we want to penalise deviations, especially if only across a single channel. One such formulation is-

$$\mathbf{u_i} = \left[\frac{r_i[1]}{|r_i[2]| + |r_i[3]|}, \frac{r_i[2]}{|r_i[1]| + |r_i[3]|}, \frac{r_i[3]}{|r_i[2]| + |r_i[1]|}\right]$$

and an energy function $E(\mathbf{x}) = \sum_i g(|u_i|)$

where $\frac{\partial g}{\partial |u|} = 2\mathbf{u}h(|u|)$ where $h(u) = \frac{\partial g}{\partial |u|^2}$ There may be cases where the change only occurs across one channel (for some particular colour shades). Thus, $|u| \to \infty$ should not result in the amount of force $|\frac{\partial g}{\partial |u|}| \to \infty$

Conditions

1. Continuous
2. Real valued
3. Non-negative
4. $\lim_{u\to\infty} h(u) = 0$ and $\lim_{u\to\infty} 2|u|h(u) \le C$ where C is a positive constant.

$g(u) = -\gamma\ exp(-u^2/\gamma)$ satisfies all these conditions. Thus, by calculating the gradient of $E(\mathbf{x})$, we can perform gradient descent on each pixel $\mathbf{x}$. Updates for step size τ-

$$\mathbf{x_u} = \mathbf{x} - \tau \sum_i 2(\mathbf{x} - \mathbf{y_i})h(|u_i|)$$

(b) Noise Model: Poisson

A Poisson distribution of noise occurs in light microscopy because of the discrete nature of photons. We assume that the variance of X_i depends on the true signal value of only that channel. Therefore, the likelihood for one pixel is given by-

$$P(y_i|x_i) = \prod_j x_{ij}^{y_{ij}}\, exp(-x_{ij})/(y_{ij}!)$$

Here, j is for indexing the three colour channels. Since they are considered as independent in the noise model, their joint likelihood is just the product of the channels' individual likelihoods.

(c) Bayesian Formulation

We aim to maximize the Bayesian posterior probability-

$$\begin{aligned}
\max_{x_i} P(x|y) &= \max_{x_i} P(y_i|x_i)P(x_i|x_{Ni}, \gamma) \\
&= \max_{x_i}(\log P(y_i|x_i) + \log P(x_i|x_{Ni}, \gamma)) \\
&= \max_{x_i} \left(\sum_j (y_{ij}\log(x_{ij}) - x_{ij} - \log(y_{ij}!)) + \sum_{a\in A_i} -V(x_a) \right) \\
&= \min_{x_i} \left(\sum_j (x_{ij} - y_{ij}\log(x_{ij}) + \sum_{a\in A_i} V(x_a) \right)
\end{aligned}$$

Here, the expression of $\sum\limits_{a\in A_i} V(x_a)$ is given by the energy function $E(\mathbf{x_i})$, since there are 4 cliques, each having 2 members, in a 4-neighbour system and they make up the energy function.
The index j again refers to the three colour channels.
Optimizing this expression for all the pixels of the image will give us the solution to this Bayesian denoising formulation.